

## Shubnikov–de Haas effect in a metallic impurity band

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Magnetotransport measurements are reported near the magnetic-field-induced metal-insulator transition in Si-doped GaAs. Optical measurements reported earlier gave evidence for a metallic impurity band for donor densities up to approximately five times the zero-field Mott density. Shubnikov–de Haas–like oscillations are observed at all metallic donor densities and magnetic fields. Enhanced carrier effective masses are found in the metallic-impurity-band range of densities.

Semiconductors doped with shallow donors undergo an insulator to metal transition as the dopant density  $n$  exceeds a critical density  $n_c$ . For lower  $n$  the hydrogenic shallow donor model successfully accounts for the optical properties and thermally induced hopping between the donors in the impurity band accounts for the low-temperature transport properties.<sup>1</sup> As  $n$  is increased overlap of the donor wave functions broadens and shifts the energy of the impurity band until it merges with the conduction band at some density  $n_i$ . At sufficiently high  $n$ , screening by the metallic carriers is effective and a nearly-free-electron metal results. Near the metal-insulator transition (MIT) both disorder and electron-electron interactions are important ingredients in any description of the system.<sup>2,3</sup> The critical density  $n_c$  for the metal-to-insulator transition is approximately the Mott density defined by  $n_c^{1/3}a^* \cong 0.25$  where  $a^*$  is the effective donor Bohr radius.<sup>2</sup> It is interesting to consider the relationship between  $n_c$  and  $n_i$ . The question is the following: does the metal-insulator transition take place in the impurity band or in the conduction band? The MIT literature contains conflicting opinions on this question.<sup>2,3</sup> We have recently reported results of far-infrared magneto-optical measurements on  $n$ -type GaAs which gives new insight to this issue.<sup>4,5</sup> The measurements were made in both Voigt and Faraday geometries using polarized light to check the optical selection rules. At 4.2 K absorption bands were found that were consistent with donor  $1s \rightarrow 2p$  transitions from their resonance frequencies and selection rules. They were observed at both insulating and metallic conditions near the metal-insulator transition. In samples with donor densities below about  $4 \times 10^{16} \text{ cm}^{-3}$  the magnetic-field-induced MIT was within our range of magnetic fields ( $\lesssim 11 \text{ T}$ ) and the  $1s \rightarrow 2p$  transitions could be observed in both the metallic and insulating states within the same sample. The oscillator strength of the absorption in the  $1s \rightarrow 2p$  resonance was consistent with the conclusion that nearly all of the carriers are in the impurity band. As the sample tempera-

ture was raised these absorption lines reduced in oscillator strength and were replaced by cyclotron resonance absorption in the Faraday geometry and a zero frequency Drude response in the Voigt geometry. At low temperatures these impurity band optical transitions were observed in all samples with densities  $n \lesssim 5n_c$ . For higher density samples only the free-carrier response was observed.

These optical results are strong evidence for the existence of a metallic impurity band in this system, i.e.,  $n_c < n_i$ . They indicate that the metallic impurity band persists in the  $n_c \leq n \lesssim 5n_c$  density range and for  $n \gtrsim 5n_c$  the impurity band merges with the conduction band. These observations raise the question, which we address in this paper, of the nature and properties of the electrical transport in a metallic impurity band. We present, here, the results of magnetotransport measurements on  $n$ -type GaAs doped near  $n_c$ . In the metallic impurity band range of densities we observe weak localization as expected and a feature that is found to evolve into the Shubnikov–de Haas (SdH) effect at higher donor densities. This feature is the focus of this paper.

The samples used in this study were 2–4- $\mu\text{m}$ -thick  $n$ -type GaAs (Si doped) epitaxial films grown on semi-insulating GaAs substrates by molecular-beam epitaxy. They are similar to, and in some cases the same as, the samples used in the optical studies.<sup>3,4</sup> The transport measurements were made on specimens with a 0.5 mm  $\times$  3 mm lithographically defined Hall bar pattern contacted with annealed indium. Carrier densities and compensations were deduced from room-temperature and 77-K low-field magnetotransport. The densities of several samples were also checked by capacitance measurements and good agreement with the transport results was found. Compensations ( $n_d/n_a$ ) were found to vary from 0.1 to 0.3 and carrier density inhomogeneities in all samples but one were less than 2%. Both the longitudinal ( $R_{zz}$ ) and transverse ( $R_{xx}$ ) resistances were measured for these samples by ac techniques for temperatures  $T \geq 0.36 \text{ K}$ ,

and magnetic fields  $B \leq 9.2$  T. The currents were sufficiently low that all of the measurements reported here were in the linear transport regime.

Typical traces of  $R_{xx}$  as a function of magnetic field for several films are shown in Fig. 1. The results for  $R_{zz}$  ( $\mathbf{B} \parallel \hat{z}$ ) are qualitatively similar but the features are somewhat smaller in amplitude. At low temperatures and magnetic fields all of the samples doped above  $n_c$  exhibit negative magnetoresistance which is understood in terms of weak localization effects.<sup>5</sup> At higher fields one or more Shubnikov-de Haas-like features are seen and then a strongly temperature-dependent increase of resistance, which corresponds to the magnetic-field-induced MIT. No SdH oscillations are observed for samples with  $n \leq n_c$ . Although several SdH oscillations are found for the high-density ( $n \gtrsim 5n_c$ ) samples, only the last oscillation is observed for samples with carrier densities  $n_c < n \lesssim 5n_c$ . The magnetic-field position for the last SdH oscillation is denoted  $B_s$ . The critical magnetic field  $B_c$  for the magnetic-field-induced MIT is determined by extrapolating the conductance  $\sigma_{xx}$  to zero temperature on a  $T^r$  plot.<sup>3,6</sup> A  $r = \frac{1}{2}$  dependence of the conductance has been widely observed on the metallic side of the MIT and it has been attributed to electron-electron interactions in disordered metals.<sup>5,6</sup> In GaAs doped close to  $n_c$ , however,  $r = \frac{1}{3}$  has been reported.<sup>6</sup> The differences in the deduced values for  $B_c$  for the two cases are small ( $\approx 5\%$ ) and this uncertainty is not significant for the purposes of this paper. The values that we obtain for  $B_c$  and  $B_s$  are nearly identical for both  $\sigma_{xx}$  and  $\sigma_{zz}$ . The results for  $B_c$  for these samples are plotted against  $n$  in Fig. 2.  $B_c$  is seen to increase very rapidly as  $n$  is increased above  $n_c$ . Therefore,  $n_c(B)$  depends only weakly on  $B$ .

A plot of the magnetic field position  $B_s$  of the last SdH oscillation versus donor density  $n$  is also shown in Fig. 2. In the figure we have plotted both the field position of the peak in  $R_{xx}$  and the position of its minimum derivative. For nearly free electrons it is generally argued that the minimum derivative of  $R_{xx}$  gives the correct SdH posi-

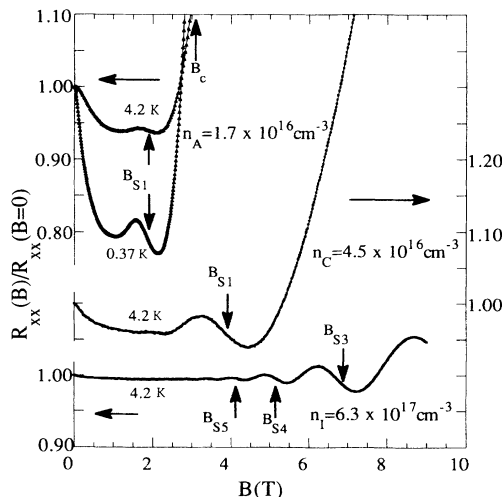


FIG. 1.  $R_{xx}$  vs  $B$  for three samples at 4.2 K and sample  $A$  at 0.37 K.

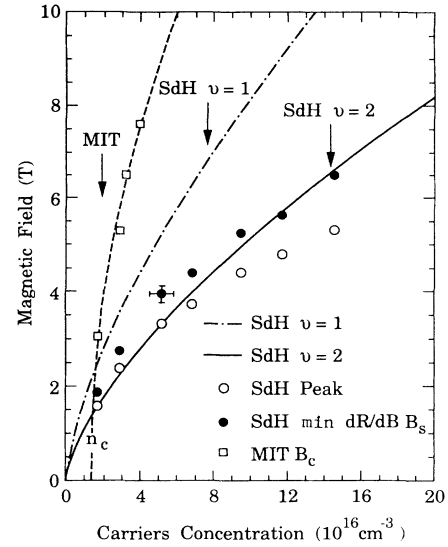


FIG. 2.  $B_c$  and  $B_s$  vs donor density. The curves are the free-carrier predictions for  $B_s$  with and without spin degeneracy. Except for one sample, the errors are within the size of the dots.

tion. The SdH positions predicted from the free-electron model under the assumption of spin degeneracy ( $\nu=2$ ) and spin polarization ( $\nu=1$ ),

$$B_s = (2\pi n^2 / \nu^2)^{1/3} (hc / 2e), \quad (1)$$

are also plotted in Fig. 2. In GaAs the spin splitting due to the band  $g^*$  factor is much smaller than the Landau-level spacing so that spin degeneracy is expected for these broad features at these low fields. For  $n \gtrsim 5n_c$ ,  $B_s$  is seen to approach the  $\nu=2$  free-carrier result as expected since the free-carrier model should apply in the high-density limit. The results of the optical studies discussed earlier imply that the SdH oscillations observed for  $n \lesssim 5n_c$  are to be associated with a metallic impurity band. In this impurity band range the experimental  $B_s$  appears to deviate toward the  $\nu=1$  spin-polarized curve. Within the context of the SdH effect in an impurity band this behavior may be understood in terms of an exchange-enhanced  $g^*$  factor. The electron wave functions in the impurity band will have a large amplitude on the donor sites so that double occupancy of a donor leads to a large exchange energy. This effect would lift the spin degeneracy as observed.

From Fig. 1 it is seen that the SdH feature is temperature dependent. We have analyzed its temperature dependence within the framework of the conventional theory of the SdH effect. For the nearly-free-carrier model the temperature dependence of the SdH amplitude is given by<sup>7</sup>

$$\frac{A(T)}{A(T_0)} = \frac{T_0 \sinh(2\pi kT / \hbar\omega_c)}{T \sinh(2\pi kT_0 / \hbar\omega_c)}. \quad (2)$$

The conditions for the validity of Eq. (2) are (a)  $\omega_c \tau \gg 1$ , where  $\omega_c$  is the cyclotron frequency and  $\tau$  the carrier relaxation time, and (b)  $E_F > \hbar\omega_c$ , i.e., more than one Landau level is occupied. Both of these conditions fail for

our low-density GaAs magnetoresistance measurements. Only the first SdH oscillation is observed. Also, for our lowest density metallic sample (sample *A*)  $\omega_c\tau \approx 0.2$  at  $B_s$  and  $\omega_c\tau \approx 1$  is achieved only for  $n \gtrsim 7 \times 10^{16} \text{ cm}^{-3}$  (the values for  $\tau$  are deduced from 4.2-K mobilities). Therefore, in order to proceed we have tested the usefulness of Eq. (2) by examining the temperature dependence of the first SdH oscillation for several other materials. We have looked at the published data on bulk InAs and InP which are materials with small values of  $g^*m^*$  so that the spin splitting is small compared with the Landau-level spacing as is the case for GaAs.<sup>7,8</sup> In the literature in general only the  $n > 2$  oscillations have been analyzed for the effective mass  $m^*$ . We have compared the  $m^*$  obtained from an analysis of the amplitude data for the first SdH oscillation with that for the  $n > 2$  oscillations. For the SdH amplitude we take two cases: (1) the high-field amplitude, which is the difference between the peak and the valley at higher fields, and (2) the low-field amplitude, which is the difference between the peak and the lower-field valley. The analysis of the high-field amplitude based on Eq. (2) does not give a good result for  $m^*$ . However, the results for the low-field amplitude are in good agreement with  $m^*$  obtained from the  $n > 2$  data.

We have also made transport measurements on bulk *n*-type InSb for which the spin splitting is comparable to the Landau-level spacing. From low-field magnetotransport at 77 K the sample was found to have a carrier density  $n = 5.2 \times 10^{15} \text{ cm}^{-3}$  and a 77-K mobility  $\mu = 230\,000 \text{ cm}^2/\text{Vs}$ . At this density there is no metal-insulator transition in InSb up to 9 T and at 2 K we observe five well-defined SdH oscillations. The  $n = 1$  feature is well separated from the other oscillations but for  $n > 1$  the features are not well resolved because of the spin splitting. Nevertheless, the  $m^*$  given by fitting the low-field amplitude for the  $n = 1$  feature is in good agreement with the known conduction-band effective mass in InSb. Therefore, we conclude that analysis of the low-field amplitude data for the first SdH feature gives a reliable measure of the carrier effective mass in these materials.

We have performed a similar analysis of the SdH-like feature observed in GaAs in order to get a measure of the carrier effective mass in the impurity band. We take the low-field amplitude of the SdH feature in the  $R_{xx}$  data. A plot of  $A(T)/A(T_0)$  vs  $T$  for the  $1.7 \times 10^{17} \text{ cm}^{-3}$  sample is shown in Fig. 3 together with the best fit to Eq. (2). We have examined the possible effects of magnetic-field-dependent background resistance coming from weak localization and the metal-insulator transition features on the SdH oscillation. The differences in the values of  $m^*$  obtained by using different reasonable background subtractions do not vary by more than 10%. From this analysis we conclude that the background effects are sufficiently weak to be ignored for these data.

The results for  $m^*/m_0$  from this analysis of several samples are shown in Fig. 4. At high densities ( $n \gtrsim 5n_c$ ) the effective mass approaches the GaAs band-edge value ( $m^* = 0.067m_0$ ). As the density is lowered toward  $n_c$  the effective mass increases. This result is consistent with a metallic impurity band. As the donor density decreases the impurity band is expected to narrow (this effect is ob-

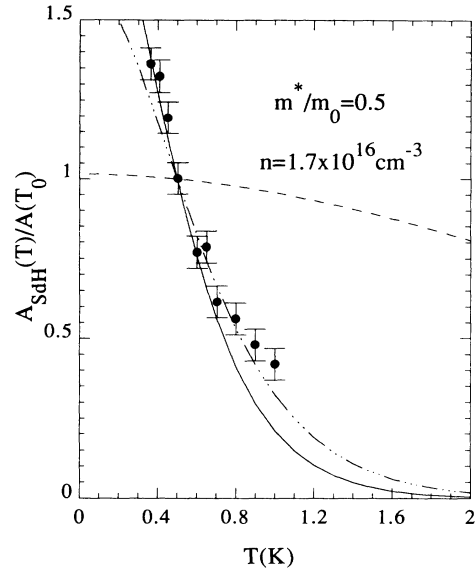


FIG. 3. Shubnikov-de Haas amplitudes plotted against  $T$ . The solid curve is the best fit based on Eq. (2) with  $m^*/m_0 = 0.5$ . The dashed curves are calculations based on Eq. (2) with  $m^*/m_0 = 0.067$  and  $0.4$ .

served in the optical data<sup>3,4</sup>). Within a tight-binding model for the impurity band the effective mass is inversely proportional to its band width.<sup>2</sup>

It is interesting to consider these SdH features from another point of view. Within a tight-binding model for a metallic impurity band electrical transport takes place by the tunneling of the electrons between the disordered donor sites. Interference between different paths that the electrons can take affects the conductivity. Since there are many different paths in the metallic state, and because the donors are disordered, a calculation of the con-

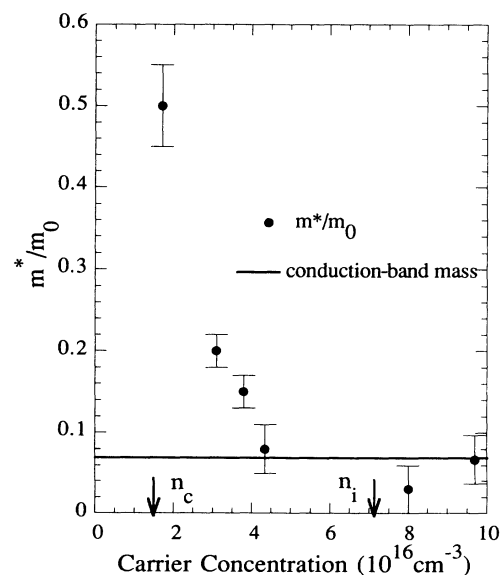


FIG. 4.  $m^*/m_0$  vs donor density for several samples as determined from the temperature dependence of the SdH amplitudes.

ductivity for this case is very difficult.<sup>9</sup> If, however, we hypothesize that the most probable minimal loops, corresponding to a triangle with sides  $\cong r = (3/4\pi n)^{1/3}$ , give the characteristic interference effect then we can derive a condition for the interference maximum. Assuming that the interference peak corresponds to one quantum of flux in the average minimal loop  $B_s$  is given by  $B\pi r^2 \simeq 2\pi\phi_0$  (where  $\phi_0$  is the quantum of flux). This condition gives the characteristic field  $B_s \propto (hc/2e)n^{2/3}$ . This result is seen to be consistent with the free-electron SdH pre-

diction to within a numerical factor of order 1. Therefore, within the tight-binding picture, the peaks in  $R_{xx}$  for a metallic impurity band are to be thought of as the Aharonov-Bohm effect in a random lattice.

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