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Self-consistent addition spectrum of a Coulomb island in the quantum Hall regime

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Coulomb interactions are shown to influence the addition spectrum of a small electron gas in the quantum Hall regime in ways that cannot be described by a classical charging energy. The interaction energy between electrons is observed to depend upon Landau-level index, and the evolution of the addition spectrum with magnetic field is found to depend strongly on Coulomb interactions. A self-consistent model of the island is introduced that can account for these results.

The energy-level spectrum of a two-dimensional island of electrons in a high magnetic field is a subject of considerable recent interest.[1–11] For noninteracting electrons residing in a circularly symmetric external confinement potential $V_{\rm ext}(r)$, the behavior is well known. In the limit where $V_{\rm ext}(r)$ varies slowly on the scale of the magnetic length $l_B=(\hbar/eB)^{1/2}$, the kinetic and spin energies of the electrons are quantized into Landau levels (LL's), and the single-particle energies are given by

$$E(n,m,S_z)\approx[(n+\tfrac{1}{2})\hbar\omega_c+g\mu_B BS_z]+V_{\rm ext}(r_m)\,, \qquad (1)$$

where $\omega_c=eB/m^*$ is the cyclotron frequency, $n=0$, 1, 2, ... is the orbital Landau-level index, $S_z=\pm\frac{1}{2}$ (denoted ↑ and ↓) is the spin LL index, and $r_m=(2m\hbar/eB)^{1/2}$ is the radius of the drifting cyclotron orbit that encloses m flux quanta. The behavior of these single-particle energies, calculated for $V_{\rm ext}(r)=(\frac{1}{2})m^*\omega_0^2r^2$, is shown in Fig. 1(a). A rich spectrum of level crossings is observed with increasing B as the LL degeneracy (given by $1/2\pi l_B^2$) increases, and the number of occupied LL's (given by $\nu=2\pi l_B^2 n_s$, where n_s is the sheet electron density) decreases.

Coulomb interactions are expected to strongly alter this picture, however, as studies of both microscopic[3–7] and phenomenological[1,2] models have shown. In the microscopic models, Coulomb repulsion causes electrons to spread out and partially occupy higher r_m orbitals,[4] and correlations lead to the formation of fractional quantum Hall states.[6] These microscopic models have been solved, however, only for small numbers of electrons[3–6] or for oversimplified forms for the interaction,[6,7] making comparison to experiment difficult. Because of this, a phenomenological model, which we call the constant-interaction (CI) model,[1,2] has typically been employed to interpret experiments.[1,8,9] In this CI model, the electrochemical potential, i.e., the energy required to add the Nth electron to the island, is given by (Ref. 1)

$$\mu_e(N)=(N-\tfrac{1}{2})U+E_N\,, \qquad (2)$$

where U is the Coulomb interaction energy between electrons on the island and E_N is the Nth quantized single-particle energy state of the island. U is assumed to be a *constant*, independent of both magnetic field and particle number, and E_N is calculated for a *noninteracting* system [cf. Fig. 1(a)].

Despite its simplicity, the CI model has been quite suc-

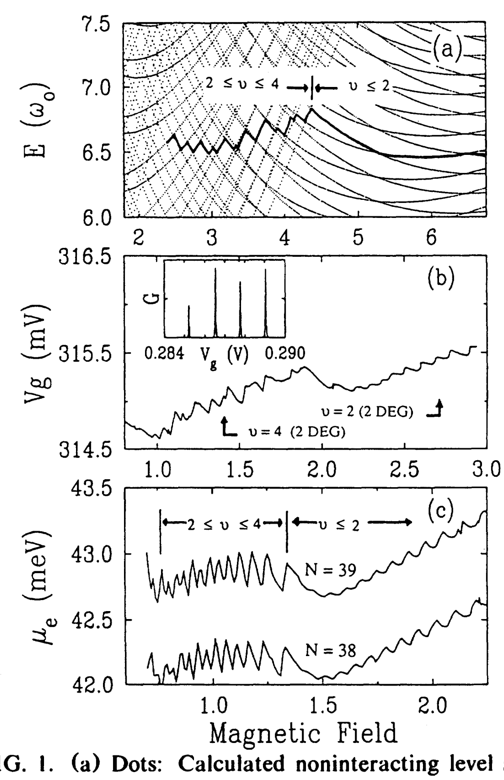

FIG. 1. (a) Dots: Calculated noninteracting level spectrum of an island vs B (in units of ω_c/ω_0). Thick solid line: Energy of the 39th electron. The filling factor ν of the island is as indicated. (b) Position in back-gate voltage V_g of a conductance peak as a function of B (in T) at $T\approx30$ mK. The measured filling factors ν of the 2DEG are also shown. Inset: Conductance vs V_g at $B=2.5$ T. (c) Electrochemical potential of the 38th and 39th electron vs B (in T) calculated using the self-consistent (SC) model described in the text.

cessful in describing a variety of transport experiments by our group[8] and others.[1] In this Rapid Communication, however, we present new experimental results that are inconsistent with the CI model. These experiments lead us to reinterpret our previous results[8] and to develop a different *self-consistent* (SC) model of the island. This SC model, whose predictions are in good agreement with experiment, gives an interesting picture of a small electron gas in the quantum Hall regime.

We begin with some experimental observations. The device, which has been discussed in detail previously,[8,10] consists of a two-dimensional electron gas (2DEG) in an inverted GaAs/Al_xGa_{1-x}As heterostructure with electrostatic gates above and below it. A negative bias applied to the upper depletion gate defines an island of 2DEG (lithographic dimensions, 450×900 nm^2) containing fewer than 100 electrons. The island is probed by weakly coupling it to two large 2DEG regions and measuring the conductance as a function of the voltage V_g applied to the lower (n^+-type GaAs) gate. The positions in V_g of the observed conductance peaks [Fig. 1(b) inset] are a direct measurement of the energies for adding additional electrons to the island:[1] $V_g(N)=\mu_e(N)/e\alpha$, where $\alpha[\approx 0.4$ (Ref. 8)] is a constant. Figure 1(b) shows the position of a particular conductance peak as a function of magnetic field. Also shown are the filling factors ν of the 2DEG adjacent to the island, as determined by Shubnikov–de Haas measurements.

In Ref. 8, we presented similar results from a different device. Based on the predictions of the CI model, the structure in $V_g(N)$ in the region above 1.9 T was attributed to level crossings between states in the lowest two *orbital* Landau levels. In that interpretation, $B=1.9$ T in Fig. 1(b) would correspond to a filling factor of $\nu=4$ in the island. Comparison with the filling factors in the 2DEG, however, show that this interpretation is unreasonable. It would require the filling factor (and density) of the island to be *larger* that that of the 2DEG, while other experiments[1,9] (and common sense) indicate that it should be *smaller. We are thus led to a different interpretation than* the one given in Ref. 8; namely, that 1.9 T corresponds to $\nu=2$, and the region above 1.9 T to $\nu\leq 2$. The structure in the peak position above 1.9 T thus represents crossings between states in the two lowest *spin-split* LL's. This interpretation presents a dilemma, however. The experimental data does *not* resemble the predictions of the constant-interaction model. The CI model predicts very *infrequent* level crossings in the $\nu\leq 2$ regime [Fig. 1(a)] since, in GaAs, the spin-splitting $g\mu_B B$ is small. In the experiment, however, *frequent* level crossings are observed [Fig. 1(b)].

Motivated by this discrepancy, we have developed an alternative self-consistent (SC) model of the island, which we now discuss in detail. In this model, the total energy of the island, $E_{\rm tot}(N)$, is given by

$$E_{\rm tot}(N)=\left\{\sum_n\sum_s[(n+\tfrac{1}{2})\hbar\omega_c+g\mu_B BS_z]\int d^2r\,\rho_{ns}(r)\right\}+\int d^2r\,\rho(r)\left[V_{\rm ext}(r)+\tfrac{1}{2}\int d^2r'\rho(r')V_{ee}(r,r')\right], \quad (3)$$

where $\rho_{ns}(r)$ is the electron density in the n, S_z spin-polarized Landau level, $\rho(r)$ is the total electron density, $V_{\rm ext}(r)$ is the bare (unscreened) confining potential created by the upper gates, and $V_{ee}(r,r')$ is the electron-electron interaction. The electron density in each LL is limited by the Landau-level degeneracy:

$$\rho_{ns}(r)\leq 1/2\pi l_B^2\,. \quad (4a)$$

We further assume that the *charge in each Landau level is quantized,* i.e.,

$$\int d^2r\,\rho_{ns}(r)=N_{ns},\quad \sum_n\sum_s N_{ns}=N\,, \quad (4b)$$

and N_{ns} is an integer. This is expected to be a valid approximation if the coupling between states in different Landau levels is small. According to simulations of our device by Kumar,[11] $V_{\rm ext}$ is roughly parabolic with oscillator frequencies given by $\hbar\omega_y=3.5$ meV, and $\hbar\omega_x=0.8$ meV. For computational simplicity, we will assume radial symmetry: $V_{\rm ext}(r)=(\frac{1}{2})m^*\omega_0^2r^2$ and use a single parameter, $\hbar\omega_0=[(\hbar\omega_x)(\hbar\omega_y)]^{1/2}=1.6$ meV, to characterize the potential. Electrons added to this bare confinement potential interact with each other and screen the potential. This interaction $V_{ee}(r,r')$ is cut off at short distances by the finite z-extent δz of the 2DEG wave function (~ 10 nm) and is screened at long distances by the image charges associated with the nearby metallic n^+ region. We use the following form for the interaction to account for these effects:

$$V_{ee}(r,r')=e^2/\epsilon(|r-r'|^2+\delta z^2)^{1/2} - e^2/\epsilon(|r-r'|^2+4d^2)^{1/2}\,, \quad (5)$$

where d is the distance from the 2DEG to the gate (100 nm) and $\epsilon=13.6$ is the dielectric constant of GaAs. Finally, we will use the bare g factor of GaAs $g=-0.44$ to describe the spin splitting, although the exchange enhancement of the g factor[12] can be included in a straightforward manner. As we will see, the size of the spin splitting does not qualitatively affect the predictions.

We now discuss the results of the SC model, obtained by numerically minimizing (3) subject to the constraints (4), and by using the definition of the electrochemical potential:

$$\mu_e(N)\equiv E_{\rm tot}(N)-E_{\rm tot}(N-1)\,. \quad (6)$$

Figure 2(c) shows the calculated $\mu_e(N)$ for $N=38$ and 39 as a function of magnetic field. The overall shape, as well as the scale, of the structure in μ_e is quite similar to that in the experimental data of Fig. 1(b). In addition, the separation between successive μ_e curves in the SC model [~ 0.6 meV between $\mu_e(N=38)$ and $\mu_e(N=39)$] is in reasonable agreement with the experimentally observed peak spacing ($\alpha\Delta V_g\sim 0.48$ mV). The density of the island in the model is $\sim 30\%$ less than in the experiment, but uncertainties in the number of electrons and in the pa-

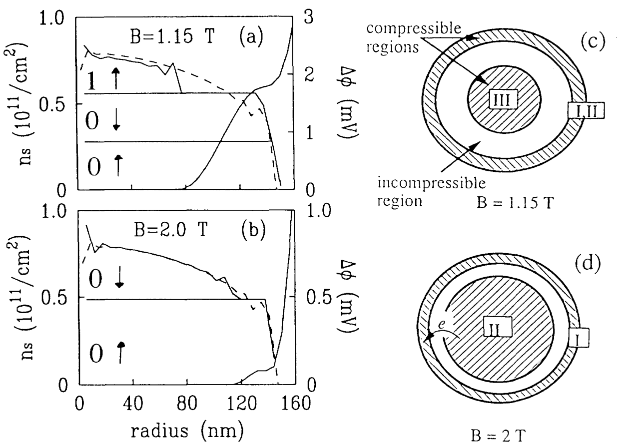

FIG. 2. Charge density and electrostatic potential in the self-consistent model for 39 electrons and (a) $B=1.15$ T and (b) 2.0 T. The dashed line is the classical electrostatic solution. Partially filled (compressible) LL's screen the confining potential, while full (incompressible) LL's do not. (c),(d) Schematic top views of the island showing the compressible regions of the 0↑ LL (region I), the 0↓ LL (region II), and the 1↑ LL (region III).

rameter ω_0 can easily account for this difference. Most importantly, there are frequent level crossings in the $\nu \leq 2$ regime; the SC model can thus account for the level crossings observed in the experiment [Fig. 1(b)].

To understand the physics underlying this behavior, we first note that in the SC model *the shape of the charge distribution closely approximates the classical electrostatic solution.*[13] This can be seen in Figs. 2(a) and 2(b), where the SC charge density $\rho(r)$ (solid lines) is given at two very different magnetic fields and compared to the classical result (dashed line). While electrons can lower their kinetic and spin energies by shifting to lower LL's, these quantum effects are only a small perturbation. We can now immediately understand the origin of frequent level crossings in the $\nu \leq 2$ regime. An electron transfers from the upper to the lower LL roughly whenever a flux quantum h/e is added to the area of the island and an additional electron can be accommodated in the lower LL. The frequency of these level crossings is governed by classical electrostatics and LL degeneracies, and *not* by the spin-splitting energy between LL's.

To understand the oscillations in greater detail, we first note that the readjustment of charge to minimize kinetic and spin energy leaves the island in a configuration, as shown in Figs. 2(c) and 2(d), where compressible regions (partially filled LL's) are separated by incompressible regions (full LL's). The compressible regions can be thought of as metallic regions that screen the external potential,[14] leaving the self-consistent electrostatic potential flat [see Figs. 2(a) and 2(b)]. The incompressible regions, on the other hand, can be thought of as insulators that do not screen the external potential. Now consider adding an additional electron to the island, e.g., in the $\nu \leq 2$ regime shown in Fig. 2(d). The electron may be added to either compressible region I or II, and the behavior of $\mu_e(N)$ with B depends upon where it is added. To see this, note that when the magnetic field is increased slightly, the degeneracy of the LL's increases. The charge density in the lower LL then increases in the center of the island and thus decreases near the edges, thereby increasing the electrostatic potential in region II and decreasing it in region I. The rising (falling) portions of the $\mu_e(N)$ vs B curves in Fig. 1(c) thus correspond to the Nth electron being added to region II (region I). As the magnetic field is further increased the electrostatic potential disparity between regions II and I becomes large and discrete charge e will move from the upper to the lower LL, as shown in Fig. 2(d). These electron transfers show up as crossovers between the rising and falling parts of the $\mu_e(N)$ traces. The oscillations of μ_e with B are thus due to the periodic buildup and release of electrostatic frustration in the island that occurs because the charge must distribute itself among the LL's in multiples of e.

Another important result from the SC model is that the magnitude of the interaction between electrons depends upon the shape of the charge distributions and upon the width of the incompressible region separating them. This can be seen by constructing "level spectra"[8,15] in different magnetic-field regimes, as is done in Fig. 3. These spectra are made by subtracting a constant between successive μ_e curves in the model, or between peak position curves in the experiment. Physically, this constant represents the interaction energy U between an electron in one metallic region and an electron in another. In particular, it is the U between electrons whose electrochemical potentials are *crossing* with increasing B. Figures 3(a) and 3(b) are level spectra in the $3 \geq \nu \geq 2$ regime. The amount subtracted ($U_{0,1}=0.45$ meV in the model, $\Delta V_{g0,1}=1.175$ mV in the data) is the Coulomb interaction between an electron in the $n=0$ LL and an electron in the $n=1$ LL. A

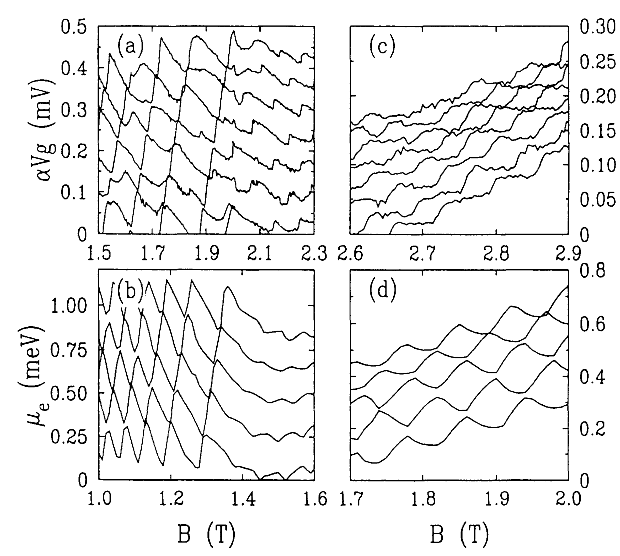

FIG. 3. (a) Experimental spectrum and (b) SC model spectrum in the $3 \geq \nu \geq 2$ regime. These spectra are constructed by subtracting a constant between successive peak position traces. (c) Experimental spectrum and (d) SC model spectrum in the $\nu \leq 2$ regime, constructed by subtracting slightly larger constants than in (a) and (b).

in the $n=1$ LL. A *different, larger* constant must be subtracted to construct a level spectrum in the $\nu \leq 2$ regime. This is done in Figs. 3(c) and 3(d), where the amounts subtracted are $U_{0\uparrow,0\downarrow}=0.55$ meV in the model, and $\Delta V_{g0\uparrow,0\downarrow}=1.35$ mV in the experiment. The Coulomb interaction U thus depends upon LL index. In addition, U depends on the magnetic field, since the level spectra in general line up only over a limited range of B.[7] These variations in U again illustrate the limitations of the constant-interaction model and the necessity of a self-consistent approach.

In conclusion, we have studied the addition spectrum of a small electron gas in the quantum Hall regime. We find that the magnitude of the Coulomb interaction between electrons is a function of the Landau-level index and magnetic field, and that Coulomb interactions strongly influence the evolution of the addition spectrum with B. The experimental results are in good agreement with a self-consistent model of the island.

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