Homogeneous and inhomogeneous states of a two-dimensional electron liquid in a strong magnetic field

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A theory of linear screening by a two-dimensional electron liquid in a strong magnetic field is formulated quantitatively. It takes into account electron-electron interaction and it is valid in the limit of small q. The results of the theory, including the negative sign of the screening radius, are shown to be confirmed by magnetocapacitance experiments. Near the fractional quantum Hall states the screening length tends to infinity, and the long-range potential of the remote donors makes the electron liquid very inhomogeneous. Different manifestations of this effect are discussed.

The theory of zero-temperature linear screening (LS) by the two-dimensional electron gas without magnetic field is based upon the Thomas-Fermi approximation, which relates the reciprocal screening length q_s to the thermodynamic density of states (TDS):[1]

$$q_s = 2\pi(e^2/\kappa)G, \quad (1)$$

where κ is a dielectric constant and G is the TDS. The reciprocal TDS is the derivative of the chemical potential E_F with respect to the electron density n, or the second derivative of the energy density H :

$$G^{-1} = dE_F/dn = d^2H/dn^2. \quad (2)$$

Equation (1) is valid in the limit of small q.

The LS in a magnetic field is a more difficult problem. At zero temperature TDS in the free-electron approximation is zero if the Landau level is completely occupied and it is infinite if it is partly occupied. Thus Eqs. (1) and (2) become meaningless. Das Sarma and co-workers[2] and Murayama and Ando[3] have proposed a self-consistent procedure that takes into account a broadening of the Landau level due to disorder. I think this approach is good if the resulting width of the Landau level, as obtained in this approximation, is larger than the electron-electron interaction energy. Otherwise the "compressibility" that is necessary for screening is provided by the interaction. This is the case for structures with high mobility.

A different approach to the LS theory has been put forward recently,[4,5] which takes into account the electron-electron interaction. The most interesting result of the theory is that the TDS $G(B)$ in a magnetic field B is negative in a wide range of the filling factor ν of the Landau level. In the first part of this paper a quantitative formulation of this theory is given and its experimental verification[6–8] is discussed. The theory does not work in the vicinity of the filling factors ν corresponding to integer or fractional gaps because the two-dimensional electron liquid (TDEL) becomes incompressible. I show at the end of the paper that this state may be very inhomogeneous.

At a given level of a disorder the smallest fractional gaps are smeared out. So the energy density H_N of the interacting electrons on the positive background with the given number N of the Landau level is a smooth function of ν within some intervals of ν between the survived fractional singularities. It can be shown[4] that the Thomas-Fermi scheme of a LS is applicable within such intervals, and Eqs. (1) and (2) are exact for the TDEL with an arbitrarily strong interaction, but only in the limit of small q ($q \ll q_s$), so that the theory can describe, for example, the LS by the TDEL of a charge, located far from the plane with the TDEL, but it does not work if the external charge is in the plane. In the case of a strong magnetic field, H_N can be substituted in Eq. (2) instead of H. [Another derivation of Eqs. (1) and (2) for the case of a strong interaction can be done by the Green's-function method using relation (19.4) of Ref. 9.]

Of course, the exact expression for $H_N(\nu)$ does not exist. At $N = 0$ one can use the result by Fano and Ortolani,[10] which gives the Wigner crystal energy if ν is close to integer and also takes into account the results of some numerical calculations and electron-hole symmetry:

$$H_0(\nu) = \sqrt{2\pi}(e^2/\kappa){n_0}^{3/2}g(\nu), \quad (3)$$

$$g(\nu) = -0.6267\nu^2 - 0.7821x^{3/2} + 0.55x^2 - 0.463x^{5/2}, \quad (4)$$

where $x = \nu(1-\nu)$, $\nu = n/n_0$, and $n_0 = eB/2\pi\hbar c$. The screening length, as obtained in this approximation, is shown in Fig. 1. In fact, two different approximations are proposed in Ref. 10. One can get an idea about their accuracy by comparing the results for q_s. The most sensitive point is $\nu = \frac{1}{2}$, where the difference is about 20%.

In this approximation both $G(B)$ and q_s are negative everywhere within the interval $0 < \nu < 1$. It does not mean any instability. The negative compressibility relates to the compression of the electrons together with the background. This comes from the negative sign of the correlation energy. The work for a compression of electrons at a given background density is positive due to the attraction to the background. The negative q_s

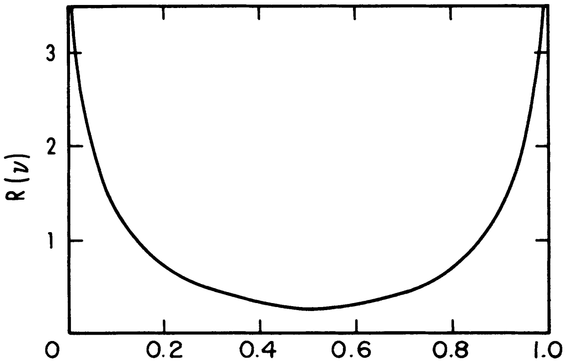

FIG. 1. Dimensionless screening length $R(\nu) = -2\sqrt{n_0}/q_s$.

means some overscreening: the sign of a resulting potential is opposite the sign of a bare one. This means also attraction of a probe charge to the screened charge of the same sign. One can show, however, that the interaction of two real charges of the same sign is repulsive.

The negative sign of $G(B)$ in the wide range of ν has been recently directly confirmed in an acute experiment by Eisenstein, Pfeiffer, and West.[8] It can be obtained also from the capacitance measurements. It is well known that the capacitance C between metallic electrode and the TDEL is mainly the plane condenser capacitance C_0, but it has a small correction C^*, depending on B:

$$C^* = -C_0^2/e^2G(B)A, \quad (5)$$

where A is the area of the condenser. The sign of $G(B)$ effects the sign of C^*. The experimental data give the magnetocapacitance, i.e., the difference $C(B) - C(0)$, where $C(0)$ stands for zero-field capacitance. One gets

$$C(B) - C(0) = C_0^2[G^{-1}(0) - G^{-1}(B)]/e^2A. \quad (6)$$

The experimental data on magnetocapacitance are usually interpreted in the framework of a one-electron picture of the Landau levels smeared by a disorder.[6] However, in the LS regime the electron-electron interaction plays a more important role than a disorder. In this range the magnetocapacitance data can be described in terms of the TDS obtained from Eqs. (2)–(4) without taking into account any disorder. It is important, however, that zero-field TDS $G(0)$ is not equal to the free-electron TDS but it is also renormalized by the electron-electron interaction. In the electron density range ($\sim 10^{11}$ cm^{-2}) a good approximation for the chemical potential is[11]

$$E_F = n\pi\hbar^2/m - 2(e^2/\kappa)\sqrt{2n/\pi}. \quad (7)$$

The effect of interaction, described by the second terms in Eq. (12), is very important for the TDS at $B = 0$.

Figure 2 shows the magnetocapacitance data obtained by Smith III, Wang, and Stiles[6] on the modulation-doped GaAs/$Ga_{1-x}Al_xAs$ heterostructures and my calculations, which use Eqs. (2)–(7). The theory does not contain any fitting parameters. Thus, in the region $\frac{1}{3} < \nu < \frac{2}{3}$ it is in very good agreement with the data. To describe the drops near the integer filling factors one should take into account a disorder, because the LS regime is not applicable there. A negative correction to the capacitance has been observed recently[7] in the Si-MIS (metal-insulator structure) also, but it has been smaller than the theoretical estimate.

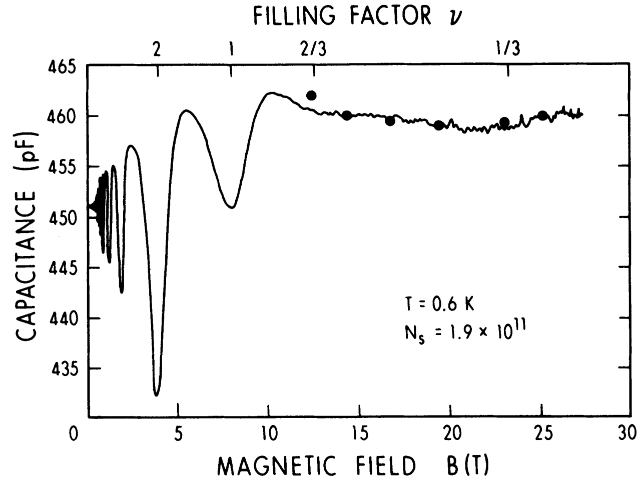

FIG. 2. Experimental data on magnetocapacitance (Ref. 6) (full line) and the results of my calculations (full circles) obtained without any fitting parameters.

The LS theory assumes that the redistribution of electron density, which suppresses the external potential, is small as compared with the density. This is an important point where the physical picture of the LS theory differs very strongly from the one-electron approximation. In the one-electron approximation the density of electrons in an external potential (Fig. 3) is very inhomogeneous and it is quantized in units of n_0 at any point $\mathbf{r}$, because the Landau level is entirely occupied if it is lower than the Fermi level, and it is empty otherwise. This is true at any small external potential $F(\mathbf{r})$ and at low enough temperature. The above theory of the LS obviously contradicts this statement. It claims that in the ground state of the system a long-range external potential is screened almost perfectly by a *small* redistribution of the electron density and the density is not necessarily quantized. The physical reason for this important difference is that electron-electron interaction tends to keep the TDEL homogeneous.

Now we discuss the applicability of the LS theory in a magnetic field to the most important case when the external potential is created by the fluctuations of the density of the remote charged donors.

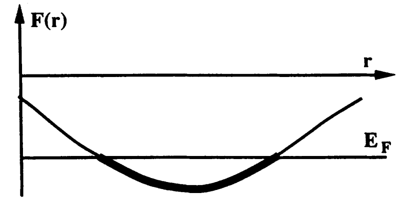

FIG. 3. The Landau level, which is bent by a random potential. In a one-electron approximation it is either completely occupied (thick line) or empty (thin line).

The electrostatic model of a heterojunction with a wide spacer is a plane condenser with a TDEL as one plate and with a thin layer of the randomly distributed donors as the other plate. The distance between two plates is the spacer width s. The average density of the charged donors c is close to the density n of the TDEL. The fluctuations of the density of charged donors create a random potential $F(\mathbf{r})$ in the plane of the TDEL. It has been shown[12,13] that for a Gaussian distribution of charged donors the mean-square potential diverges logarithmically due to the long-range fluctuations. It has a form

$$\sqrt{\langle F^2(\mathbf{r})\rangle} = W\sqrt{\ln\frac{S}{2s}}, \tag{8}$$

where

$$W = \sqrt{2\pi}\frac{e^2\sqrt{c}}{\kappa}. \tag{9}$$

Here S is a size of a sample.We suppose below that the spacer width s is much larger than all other microscopic lengths in the problem. The characteristic length of the random potential acting on the TDEL is of the order of s, but the characteristic energy W is independent of s. For $c = 5\times10^{10}$ cm^{-2} and $\kappa = 12.5$ one obtains $W = 6.5$ meV. This is a very large value, but usually the long-range random potential is essentially suppressed by electron screening. The lack of screening leads to the metal-nonmetal transition driven by the long-range potential.[13]

In the Thomas-Fermi scheme of the LS the Fourier transform of the bare potential F_q becomes $F_q(q/q_s)$ after the screening[1] if $q \ll |q_s|$. The LS removes the divergency of the mean-square potential, and one gets[13]

$$\sqrt{\langle F^2(\mathbf{r})\rangle} = \frac{W}{2|q_s|s} \tag{10}$$

instead of Eq. (1), and this is valid at $2|q_s|s \gg 1$.

In the LS theory the redistribution of the electron density n' is connected with the resulting potential $F(\mathbf{r})$ after the screening as follows:

$$n' = Fq_s\kappa/2\pi e^2. \tag{11}$$

Making use of Eqs. (9)–(11), one gets the criterion of the applicability of the LS:

$$\frac{n'}{n} = 0.063\left(\frac{100\text{ nm}}{s}\right)\sqrt{\frac{10^{11}\text{ cm}^{-2}}{n}}\sqrt{\frac{c}{n}} \ll 1. \tag{12}$$

This criterion is valid both with and without magnetic field, but in the case of a strong magnetic field n is the electron density *of the partly filled Landau level* rather than the total density.

In the opposite case, when $n' \geq n$, the electron density is inhomogeneous, the potential created by electrons is smaller than the bare potential, and in a length scale of the order of s the picture is similar to that of the one-electron approximation (Fig. 3). This case can be called the "dirty" regime. Its important feature is a complete localization of electron states at filling factor $\nu < \frac{1}{2}$. Indeed, if the electron repulsion can be neglected, the local electron density is either zero or close to n_0. Thus, at $\nu < \frac{1}{2}$ the part of the plane occupied by the electrons is less than $\frac{1}{2}$, and there is no percolation through occupied regions. The experimental manifestation of this picture looks like a "freeze-out" when ρ_{xx} tends to *infinity* as T tends to zero. It has been observed by Paalanen, Tsui, and Lin[14] in a heterostructure with a thin spacer layer exactly at $\nu = \frac{1}{2}$.

In the "clean" regime, when the LS prevents electrons from separation and the density fluctuations are small, the fractional quantum Hall effect can be observed at $\nu < \frac{1}{2}$.

The condition of the validity of the LS theory is also the absence of the fractional gaps within the interval of the order of n' near n. Now I consider the electrostatic potential and the distribution of density in the case when ν is close to a fractional value ν_f, and the fractional gap is big enough to survive at a given disorder. Let Δ be the discontinuity of the chemical potential at $\nu = \nu_f$. (Δ is larger[15] than the fractional gap as obtained from the activation energy.) I argue below that if $\Delta \ll W$, the random potential makes the state with $\nu = \nu_f$ very inhomogeneous (Fig. 4). In this case most of the plane is occupied by the TDEL with filling factors either larger or less than ν_f. The statistical properties of these two areas are the same. Their characteristic size is of the order of the correlation length s of the random potential. These areas are separated by the thin strips of the incompressible liquid with $\nu = \nu_f$. The following arguments give an order-of-magnitude estimate of the width R of these strips. The electron density within the strips is constant and it does not follow the change of the random potential. The unscreened charge creates an electric field E, and the width R can be found from the condition $eER = \Delta$. To estimate E notice that the variation of the electron density, which is necessary for perfect compensation of the random potential, is of the order of $\sqrt{c}/s$ in a length scale of the order of s. Thus, the density gradient is $\sqrt{c}/s^2$, and the variation of the density at a distance R is $\sqrt{c}R/s^2$. This is an estimate for the unscreened density, and the electric field is of the order of $e\sqrt{c}R/s^2\kappa$. Finally, one obtains $R \sim s\sqrt{\Delta/W}$. The data obtained by both activated transport[16] and photoluminescence experiments[17] show that Δ is a few times smaller than W even for the largest fractional gaps at $B < 25$ T. Therefore the case $R \ll s$ seems to be reasonable at least for small gaps. In

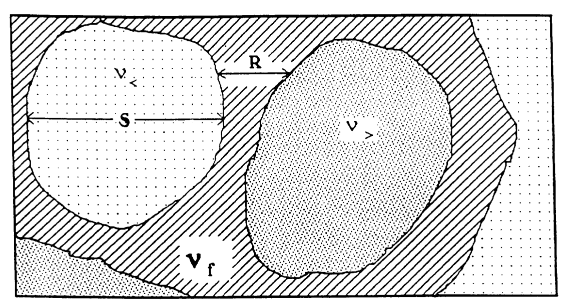

FIG. 4. The map of the compressible ($\nu_<$ and $\nu_>$) and incompressible (ν_f) states.

this case the strips of the incompressible liquid are small.

The width of the incompressible strip R decreases with Δ, and the incompressible state obviously disappears at $R \approx 1/q_s$. Thus, it is possible to estimate the minimum fractional gap $\Delta_{\rm min}$ surviving at a given long-range disorder.

If the incompressible liquid occupies a small part of the area, the thermodynamic functions, such as magnetocapacitance, are much less sensitive to the fractional singularities than the transport properties, which are determined by the connectivity of the different regions.

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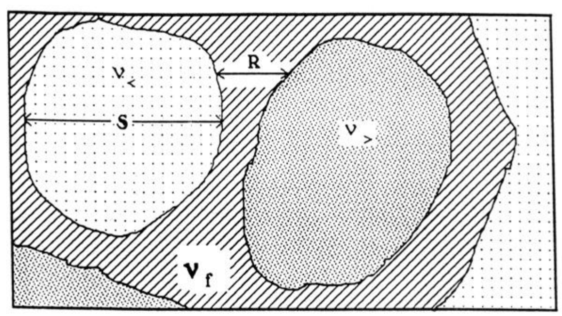

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