Brief Reports

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Twisted boundary conditions and effective mass close to a Mott transition

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We use some recent results, relating the optical mass to the sensitivity of a system of interacting lattice fermions to boundary conditions, to obtain the scaling properties of the effective mass close to a metal-insulator transition due to correlations.

In a recent Letter, Shastry and Sutherland[1] (SS) made the remark that the coefficient governing the finite-size scaling corrections to the ground-state energy density of an interacting system is the same as what appears in the free-acceleration term of the conductivity. That is, they obtained the relation $\Delta E/L^d \propto D_c/L^2$, where $\Delta E/L^d$ is the difference in the ground-state energy density between periodic and twisted boundary conditions imposed on a d-dimensional hypercubic system of finite size L. Then they showed that the charge stiffness D_c appearing in this relation is associated with the free-acceleration term of the real part of the frequency-dependent conductivity, i.e., $\mathrm{Re}\sigma_{xx}^{f}(\omega)=(2\pi e^2/\hbar)D_c\delta(\hbar\omega)$. Shastry and Sutherland discuss the possibility of a metal-insulator transition, due to correlations (Mott transition), occurring in an interacting lattice fermion system associated with the vanishing of D_c. In fact, the scaling behavior of D_c close to the *zero-temperature* metal-insulator transition can be obtained from finite-size-scaling theory[2] under the assumption that this transition, as described by the Hubbard model, is continuous. This is supported by renormalization group[3] and other approaches[4] to this model. We find $D_c \propto |g|^{2-\alpha-2\nu}$, where the dimensionless quantity g measures the distance in parameter space to the transition ($g=0$ defines the critical point). In this expression α and ν are standard critical exponents associated with the unstable zero-temperature fixed point controlling the transition.[4] Specializing to the fixed density transition of the half-filled Hubbard band model[5] ($d>1$) and using the modified hyperscaling relation[4,6] $2-\alpha=\nu(d+z)$, we have $D_c \propto |U-U_c|^{\nu(d+z-2)}$, where U_c is the critical value of the Coulomb repulsion U and z is the dynamical critical exponent, which plays a central role in the theory of quantum critical phenomena.[4,6]

The critical behavior of the charge stiffness, when expressed as $D_c \propto \xi^{-(d+z-2)}$, implies that the frequency-dependent conductivity of the interacting system behaves as $\sigma_{xx} \propto \xi^{2-d} f(\omega\tau_\xi)$, where $\tau_\xi \propto \xi^z$, as expected from purely dimensional arguments.[7] We may then identify the diverging length $\xi \propto |g|^{-\nu}$ with the conductivity correlation length in the metallic phase.[7] As emphasized by SS, the quantity D_c is the inverse of the conductivity effective mass $m^* \propto 1/D_c$, which diverges at the Mott transition due to the vanishing of D_c for $d+z-2>0$. Alternatively the scaling properties of m^* can be obtained directly from the expression for the frequency-dependent conductivity if, following Kohn,[8] we define the effective mass m^* through the relation $(-ne^2/m^*) = \lim_{\omega\to 0}\omega\,\mathrm{Im}\sigma_{xx}(\omega)$ and take $\mathrm{Im}f(\omega\tau_\xi) \propto (1/\omega\tau_\xi)$ in the limit $\omega\tau_\xi \to 0$ as is the case for a perfect conductor.[8] It is interesting that the optical mass m^* considered above scales differently from the thermal mass[4] m_T, defined through the linear term of the specific heat, $m_T \propto C/T$, close to the metal-insulator transition.

The scaling arguments presented above turn out to be sufficiently general to apply for lattice boson systems at the continuous superfluid-to-insulator transition.[6] In this case the charge stiffness is replaced by the generalized superfluid density ρ_s which scales as D_c and vanishes at the transition.[6] For one-dimensional quantum systems and in the Lorentz-invariant case, i.e., when the dynamic exponent $z=1$, we get $d+z-2=0$ and D_c or ρ_s becomes a constant independent of the interactions. This result is expected from conformal invariance since the coefficient of the L^{-2} term of the finite-size-scaling corrections to the ground-state energy at criticality is in this case a universal constant.[9] It does not apply, however, for the half-filled Hubbard band at $d=1$, the lower critical dimension for the fixed density ($n=1$) transition.[5] In this case $U_c=0$ and the charge stiffness[10] $D_c=0$, i.e., the system is an insulator for all $U>0$.

The authors in Ref. 1 have shown the existence of a metal-insulator transition in the one-dimensional Hubbard model as the electron density n approaches the critical value $n_c = 1$ for a fixed $U > 0$. For small U the charge stiffness D_c vanishes linearly with the hole concentration,[11] i.e., $D_c \propto (n - n_c)$. With the critical behavior of the relevant physical quantities expressed in terms of $(\mu - \mu_c)$, where μ is the chemical potential and μ_c is its value at the phase boundary, this exact result, together with the fact that $\nu z = 1$ for this density-driven transition,[4,6] allows us to determine unambiguously the exponents $\nu = \frac{1}{2}$, $z = 2$, and $\alpha = \frac{1}{2}$. The same exponents have been obtained for an interacting one-dimensional Bose system at the density-driven Mott insulator to superfluid transition.[6,12]

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