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Impact ionization in semiconductors: Effects of high electric fields and high scattering rates

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We present a theory of impact ionization in semiconductors that expands an earlier theory of Kane and includes the effects of high electric fields and high scattering rates on the electron-electron collision process. We show that their combined effect, i.e., the intracollisional field effect and collision broadening, leads to a softening of the threshold energy for impact ionization and a marked increase in the anisotropy of the ionization rate with respect to the direction of the electric field.

I. INTRODUCTION

The multiplication of carriers by impact ionization is of central importance in the theory of semiconductor devices both as limiting mechanism and as a basis of device functionality. Impact ionization results from a two-electron effect corresponding to the exact inverse of the Auger process: a highly energetic conduction-band electron collides with a valence-band electron which is ionized over the band gap, leaving two conduction electrons and a hole. The ionizing electron gains its energy typically in a high electric field. The subject of this gain of energy in the presence of phonon scattering was originally developed by Wolff[1] (electron temperature model) and Shockley.[2] The ionization rate itself has been treated by Keldysh[3] and Kane.[4] In the Keldysh and Kane theories the electron energy is assumed, and the influence of the high electric field and phonon scattering are disregarded. More recent theories of carrier multiplication by impact ionization have employed the Boltzmann equation[5] also including energy band structure[6] and have used the Keldysh formula in addition to phonon scattering in the collision terms of their equations. Subsequently, it has become clear that the Keldysh formula represents a coarse approximation, and that the Kane approach is more representative.[7] We have, therefore, developed a numerical algorithm to compute the ionization rate similar to Kane's. However, as a major addition, we have also included in our calculation the effects of the high electric field and collision broadening on the inverse Auger process itself by using a method developed by Iafrate and Krieger.[8] In addition we have used a more advanced calculation of band structure and dielectric function than was available to Kane in his original work. In our results we note that the influence of the electric field and collision broadening results in a dramatic softening of the ionization threshold and an increase in the anisotropy with respect to the field direction of the ionization rate at a given energy of the primary electron.

The organization of this paper is the following. First, we introduce a quantum transport approach which offers a more consistent method to calculate the impact ionization scattering rate in the high field limit. Next we present the analytical basis for the calculation of this rate, and then discuss our numerical approach to this more complete solution of the problem. Finally, we discuss our results and conclusions for the impact ionization rate in Si.

II. INFLUENCE OF THE ELECTRON-PHONON INTERACTION AND HIGH FIELDS ON IMPACT IONIZATION

The conventional treatment of impact ionization in semiconductors assumes that it is a two-particle scattering event which enters the Boltzmann equation in the collision term. This process exhibits a strong threshold behavior as a function of incident electron energy as a consequence of the requirement of energy and momentum conservation. However, the threshold energy for impact ionization in most important semiconductors is of the order of electron volts and the operating electric fields are extremely high (10^5–10^6 V/cm). Under these conditions, the assumption of universal energy conservation is certainly suspect, as is the applicability of the Boltzmann equation.

Many authors have attacked the problem of finding quantum transport equations which improve upon the Boltzmann equation in the high field limit.[9] Here we start with a form of an equation derived by Levinson[10] and Barker and Ferry[9] which is applicable for impact ionization. We first consider only electron-phonon scattering. One then obtains the following equation for the distribution function f:

$$f(n,k;t)=\frac{1}{\hbar^2}\sum_{q,n'}\sum_{\eta=-1,1}[N_q+\tfrac{1}{2}(1+\eta)]|M(q)|^2 \times[f(n',k+q;t)S_\eta(n,k;n',k+q;t)-f(n,k;t)S_\eta(n',k+q;n,k+q;t)] ,$$

$$S_\eta(n_1,k_1;n_2,k_2;t)\equiv 2\,\mathrm{Re}\int_0^t dt' \exp\left[-\frac{i}{\hbar}\left[\int_{t'}^t E_{n_1}(k_1-E_f(t-\tau))d\tau - \int_{t'}^t [E_{n_2}(k_2-E_f(t-\tau))-\eta\hbar\omega_q]d\tau\right]\right] \times \exp\left[-\frac{1}{\hbar}\left|\int_{t'}^t \Gamma_{n_1}(k_1-E_f(t-\tau))+\Gamma_{n_2}(k_2-E_f(t-\tau))d\tau\right|\right] . \quad (1)$$

An equivalent form of this equation has been discussed in detail in Ref. 11. Here, Γ is the imaginary part of the electronic self-energy. The real part of the self-energy has been neglected, $M(q)$ is the electron-phonon matrix element, and $\hbar\omega_q$ is the phonon energy. $\mathbf{E}_f$ is related to the field $\mathbf{F}$ through $\mathbf{E}_f=(e/\hbar)\mathbf{F}$. In the Fock approximation, Γ can be expressed by the scattering rate through

$$\hbar^{-1}\Gamma(E)\approx R_{\mathrm{ph}}(E)/2 , \quad (2)$$

where $R_{\mathrm{ph}}(E)$ is the total phonon scattering rate. Note that the distribution function has been taken out of the time integration. The justification and strategy for this is given in Ref. 11 and concerns the rate of change of the distribution function as the wave vector k is accelerated during the collision. Reference 11 also gives a detailed overview of the physics involved in Eq. (1). The resulting scattering rate must, of course, retain a dependence on the field, although the distribution function $f(n,k;t)$ has been taken out of the time integrals. This is particularly important in cases where the zero-field scattering rate changes rapidly with energy.

Furthermore, once $f(n,k;t)$ has been removed from the time integrals, a collision time is easily identifiable. The time integration in (1) is of the form

$$\int_0^t dt' \exp\left[i\int_{t'}^t d\tau\,\phi(t-\tau)\right]e^{-\eta(t,t')} , \quad (3)$$

where ϕ is a function of the interacting states and t and t'. It involves the differences (in energy) of the accelerating states and is real. $\eta(t,t')$ expresses the decay of the interacting states and is also real and positive. Either the integration over the phase factor or the decaying exponential will limit the values to t' for which there are contributions to the integral. We shall call this lower limit $t-t_c$, where t_c measures the duration of the collision. It is difficult to determine t_c in a complicated band structure, and therefore we take a pragmatic approach. Since the phonon scattering rate is not a strong function of energy,

$$\eta(t,t')\approx[\Gamma_{n_1}(k_1)+\Gamma_{n_2}(k_2)]|t-t'\equiv|t-t'|/t_0 .$$

Certainly, t_c is less than some multiple of t_0. Consequently, a very good approximation of (3) is given by

$$\int_{t-2t_0}^t dt' \exp\left[i\int_t^{t'}\phi(t,t')\right]e^{-|t-t'|/t_0} = \int_0^{2t_0} dt' \exp\left[i\int_0^{t'}\phi(\tau)d\tau\right]e^{-t'/t_0} . \quad (4)$$

The last equality results from a change of variables in both integrals. From (4), we see that the scattering kernel $S_\eta(n_1,k_1;n_2,k_2;t)$ is independent of t for $t>t_c$. These same arguments can be used to describe the scattering kernel for additional scattering mechanisms such as impact ionization which will be discussed in detail below. However, we can always choose a t_0 based on the total scattering rate, which leads to the result $1/t_0=\sum_i\Gamma_i$, where Γ_i is the self-energy of the ith interacting particle from all scattering processes.

The use of Eq. (1) with the above prescriptions permits the inclusion of two effects which go beyond the semiclassical treatment based on the golden rule. These effects are collisional broadening and the well-known intracollisional field effect.[9,11] In the zero-field limit the scattering event $k\rightarrow k'$ is described by a scattering rate $R(k\rightarrow k')$ given by

$$R(k\rightarrow k')=\frac{2}{\hbar^2}\,\mathrm{Re}\sum_{k'}|V_{k,k'}|^2 A(E(k),E(k')) , \quad (5)$$

where $A(E(k),E(k'))$ is the joint spectral density function (see Ref. 9) which is just a Lorentzian in the energy arguments and has the typical collision-broadened form

$$A(E,E')=\frac{\hbar}{\pi}\frac{\Gamma(E)+\Gamma(E')}{[E+\Delta(E)-E'-\Delta(E')]^2+\hbar^2[\Gamma(E)+\Gamma(E')]^2} . \quad (6)$$

In the limit of small Γ, one recovers of course, the familiar golden rule expression $A(E,E')\rightarrow\delta(E-E')$.

In the presence of high electric fields, the intracollisional field effect can become important. This effect is similar to the Franz-Kledysh effect in which energy is exchanged with the field during the time of the collision, further broadening the effective spectral density. This broadening is also manifested as a spatial broadening due to the tunneling nature of the field-assisted processes (see Ref. 9). The two broadening mechanisms, collision

broadening and the intracollisional field effect, have little effect on the phonon scattering rates which do not involve a threshold at high energies.[11] However, impact ionization is a threshold process which varies exponentially over a large range of energies. In fact, Kane has found that in silicon, the impact ionization rate for electrons varies from 10^{10} s^{-1} for energies below 2 eV to 10^{15} s^{-1} for very high energy electrons.[4] Therefore we expect that both collisional broadening and the intracollisional field effect will have a much larger effect on the impact ionization rate than on phonon scattering. (For example, see Ref. 11.)

Therefore, we generalize Eq. (1) to include impact ionization by adding the electron-electron interaction to the Hamiltonian which then can be written as

$$H_{\rm tot}=H_0+H_B+H_{AB}+V_{AA} \ . \tag{7}$$

Here, H_0 is the single-particle electron Hamiltonian in the presence of an electric field F, and H_B is the uncoupled phonon bath Hamiltonian. H_{AB} couples the electrons and phonons and V_{AA} couples the electronic states to each other via the screened Coulomb interaction. We assume *a priori* that the phonon scattering rate $R_{\rm ph}(k)$ is larger than the electron-electron scattering rate for impact ionization $R_{ii}(k)$. Then, we can treat impact ionization as a simple first-order scattering event, but we must keep the higher-order terms in the electron-phonon interaction. In other words, we ignore the effect of the electron-electron interaction on the electron propagator, but dress it with the phonons. This is valid for all but extremely high energies as will be shown later.

The effect of the electric field on the electron-electron interaction introduces several new terms to the equation of motion (1) including Zener tunneling, and field-assisted impact ionization and has been considered already by Keldysh,[12] Kane,[13] Krieger and Iafrate,[14] and Bude, Hess, and Iafrate.[15] Here, we will concentrate solely on impact ionization. We have generalized this treatment by including contributions of the electron-phonon interaction through collision broadening. For a detailed discussion of the physics of these effects, we refer the reader to the review of Capasso,[16] who originally suggested that collision broadening be included in Monte Carlo simulations.

The addition of the electron-electron interaction in the Hamiltonian of Eq. (7) changes Eq. (1) in the following ways. First, the collision kernel takes on a two-body form as a result of the two-body nature of the electron-electron interaction. As justified above, only the influence of the electron-phonon interaction is included in the calculation of the electronic self-energy. Therefore we can replace $S_\eta(n_1,k_1;n_2,k_2)$ with $S(12;34)$ given by

$$S(12,34)=\int_0^{2t_0}dt'\left\{\exp\frac{1}{\hbar}\left[i\int_0^{t'}d\tau\left[\sum_{i=1}^{2}E_{n_i}(k_i-E_f\tau)-\sum_{i=3}^{4}E_{n_i}(k_i-E_f\tau)\right]+\sum_{i=1}^{4}-\Gamma(n_i,k_i)t'\right]\right\}, \tag{8}$$

where the indices $n_1k_1,n_2k_2,\ldots$ are designated by the numbers $1,2,\ldots$ for convenience. For electron initiated impact ionization, n_1, n_3, and n_4 are conduction bands and n_2 is a valence band. As stated above, t_0 is the inverse of the sum of the imaginary self-energy of the interacting particles in the presence of phonon scattering.

The second change of Eq. (1) concerns the matrix elements. The impact ionization process involves the interaction of a conduction band and a valence band electron via the Coulomb potential $V(r-r')$,

$$V(r-r')=\frac{e^2}{4\pi\epsilon_0}\frac{1}{|r-r'|} \tag{9}$$

screened by the wave vector and frequency-dependent dielectric function $\epsilon(q,\omega)$, where q and $\hbar\omega$ are interpreted in the context of electron-electron scattering as the momentum and energy exchanged during the collision.

Because the interaction is between two-particle states, the matrix elements contain direct and exchange terms. When summed over all spins except for the initial state spin, the matrix elements for impact ionization are then

$$M_{\rm tot}^2=[2|M_1|^2+2|M_2|^2-(M_1^*M_2+M_1M_2^*)] \ , \tag{10}$$

with

$$\begin{aligned} M_1&\equiv\langle n_1k_1;n_2k_2|V|n_3k_3;n_4k_4\rangle \ , \\ M_2&\equiv\langle n_1k_1;n_2k_2|V|n_4k_4;n_3k_3\rangle \ , \end{aligned} \tag{11}$$

where n and k are band indices and Bloch wave vectors. Here the subscripts 1 and 2 refer to the initial conduction and valence electrons and 3 and 4 refer to the final conduction electrons.

When the Bloch wave functions are expanded in Fourier series using reciprocal lattice vectors G, $u_{nk}(r)=e^{i\mathbf{k}\cdot\mathbf{r}}\sum_G z_G(n,k)e^{i\mathbf{G}\cdot\mathbf{r}}$, we have for the quantities in (11)

$$M_i=\sum_{G_1,G_3,G_4}\frac{z_1(G)z_2(G_3+G_4-G_1-G_0)z_3^*(G_3)z_4^*(G_4)}{\mathcal{V}q_i^2\epsilon(q,\omega_i)} \ ,$$

$$q_1=k_1-k_3+G_1-G_3 \ , \quad \omega_1=[E_{n_1}(k_1)-E_{n_3}(k_3)]/\hbar \ ,$$

$$q_2=k_1-k_4+G_1-G_4 \ , \quad \omega_2=[E_{n_1}(k_1)-E_{n_3}(k_3)]/\hbar \ ,$$

$$G_0=k_1+k_2-k_3-k_4 \ ,$$

as derived by Kane.[4] $\mathcal{V}$ represents the volume.

To obtain the total scattering rate we need then to sum over the two independent k vectors and the secondary particle band indices in $S(12;34)$. Given n_1 and k_1, the total impact ionization scattering rate R_{ii} from that state is simply

$$R_{ii}(n_i,k_1)=\sum_{n_1,n_2,n_3}\sum_{k_3,k_4}S(12;34) \ . \tag{12}$$

The wave vector sums in Eq. (12) run over the first Bril-

louin zone (BZ). G_0 is the umklapp reciprocal lattice vector necessary to ensure that k_2 lies in the first BZ for a given k_1. Note that the matrix elements M_1 and M_2 conserve crystal momentum.

III. NUMERICAL APPROACH

In order to calculate the scattering rate from Eq. (12), a local pseudopotential band-structure calculation was performed for the excitation spectrum and Bloch wave functions for Si following the local-pseudopotential approach. We have used 825 points in the irreducible wedge defined by $k_x+k_y+k_z \leq \frac{3}{2}(2\pi/a)$ and $k_y \geq k_x \leq k_z \geq 0$. The mesh points in this wedge were spaced by $2\Delta=\frac{1}{20}(2\pi/a)$.

We write the total scattering rate from state (n_1,k_1) as a sum of integrals over small regions of hyper-k-space, centered at the mesh points k_3^0,k_4^0. The total rate $R_{\rm tot}(n_1,k_1)$ is then given by

$$R_{\rm tot}(n_1,k_1)=\frac{2}{\hbar^2}\sum_{n_2,n_3,n_4}\left[\frac{V}{8\pi^3}\right]^2 \times \sum_{k_3^0,k_4^0} R_\Delta(k_1,k_3^0,k_4^0,k_2,n)\ , \tag{13}$$

where

$$R_\Delta(k_1,k_3^0,k_4^0,k_2,n)\equiv M_{\rm tot}^2\int_{-\Delta}^{\Delta} d^3k_1 d^3k_2 S(12;34)\ . \tag{14}$$

We seek an analytic expression for the k-space integrals and the inner time integral. This is achieved by expanding the band structure in a Fourier series in terms of the reciprocal lattice vectors which lie in the field direction. In our example, we have chosen the field to be in the $\langle 100\rangle$ crystallographic direction. Then,

$$E(k_x^0,\ k_y-E_f t,\ k_z^0)=\sum_m \varepsilon_m^n(k_x^0,k_z^0)\cos[m\pi(k_y-E_f t)] \tag{15}$$

for all t and k_y. We use eleven expansion coefficients $(m=0,1,\ldots,10)$ for each unique pair (k_x^0,k_z^0). Also, since Δ is small, we use a linear interpolation algorithm in the x and z directions for the energies of points near (k_x^0,k_z^0). Our result can be written as

$$R_\Delta=M_{\rm tot}^2\int_0^{2t_0}dt'\frac{2^5}{\hbar^2}\int_{-\Delta}^{\Delta}dk_{1y}dk_{2y}\prod_{i=1}^{4}\frac{\sin[\Delta(A_i k_{1y}+B_i k_{2y}+C_i)]}{A_i k_{1y}+B_i k_{2y}+C_i}\cos(X_a k_{1y}+X_b k_{2y}+X_c) \times \exp\left[-\sum_{i=1}^{4}\Gamma(n_i,k_i)t'\right]\ , \tag{16}$$

where A_i, B_i, C_i, X_a, X_b, and X_c are time-dependent coefficients, and $\Gamma_{\rm tot}$ is the sum of the imaginary parts of the electronic self-energies for the four states involved. Here we have approximated $\Gamma_{\rm tot}$ as $R_{\rm ph}/2$ where $R_{\rm ph}$ is the total energy-dependent phonon scattering rate for these states. By using a proper change of coordinates, we remove one of the k-space integrals and create a two-parameter lookup table for the remaining k-space integral. Thus only the last time integral remains to be performed numerically in Eq. (16) during run time.

The matrix elements in $M_{\rm tot}$ are evaluated using the Fourier-series expansions for the pseudopotential wave functions and the frequency- and wave-vector-dependent dielectric function $\epsilon(q,\omega)$, which has been calculated from the band structure, using a random-phase approximation. We include 15 bands to calculate the dielectric function, and 8 bands to calculate the rate.

Because it is impractical to actually evaluate all of the terms in the summation of Eq. (13), we employ a Monte Carlo algorithm to evaluate this summation. This leads to an interpretation of the R_Δ terms as local averages, in which the function being integrated over k_3 and k_4 is piecewise continuous. Then the Monte Carlo evaluation of the sum in Eq. (13) is a Monte Carlo integration of this approximate, piecewise continuous function.

IV. RESULTS AND DISCUSSION

Figure 1 shows impact ionization scattering rates as a function of initial electron energy for Si. [Note that the rate at a given energy E is an average over all wave vectors and bands with $E_n(\mathbf{k})=E$.] The lower, dashed curve is Kane's result[4] for the impact ionization rate in Si using the energy-conserving δ-function expression, and a much larger mesh $2\Delta=\frac{1}{8}(2\pi/a)$. Our result using the same method is shown as the solid line just above Kane's. Although Kane has used a rather coarse grid and a much less accurate expression for the δ function (he allows energy differences of up to 0.2 eV between initial and final states), his result agrees surprisingly well with ours.

The dotted curve represents our zero field using Eq. (13). In the zero-field limit, there is still a finite collision broadening due to the phonons which lowers the threshold for impact ionization from the golden rule's result. This may seem somewhat surprising, for if collision broadening had any effect on the impact ionization rate near threshold, one would expect that this effect would be more pronounced in Kane's work than in ours because Kane has considered energy conservation only up to 0.2 eV (as a numerical approximation), which is greater than the phonon-induced collision broadening for electrons

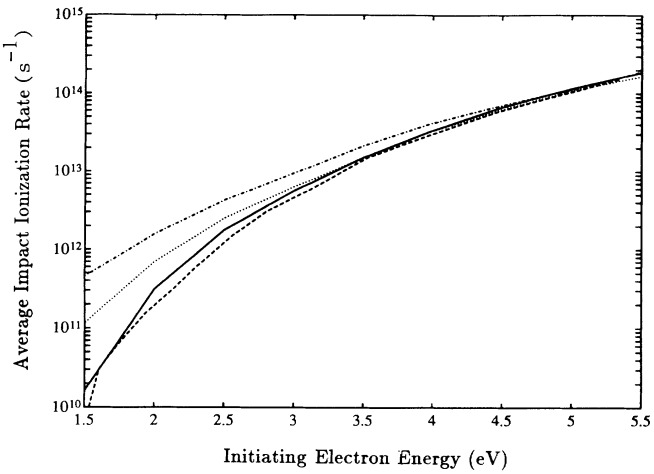

FIG. 1. Impact ionization rates for Si averaged over all initial electron states with a given energy measured from the bottom of the conduction band. Dashed curve, Kane's result; solid curve, zero-field, no collision broadening; dotted curve, collision broadening; dash-dotted curve, $F=5\times10^5$ V/cm and collision broadening.

near threshold, ≈ 30 meV. However, the collision broadening replaces the δ function with a Lorentzian, see Eq. (6), whose full width at half maximum is $\Gamma_0=\Gamma_{\text{tot}}$, which is approximately 30 meV, but the tail extends much farther than the 0.2 eV which Kane has used, decaying slowly to zero. Because the density of final states for impact ionization increases rapidly as a function of initial electron energy for electrons near threshold, otherwise rare processes with $|E_i-E_f| \gg \Gamma_0$ can actually be favored. It is just these processes, which are far from energy conserving, that inflate the impact ionization rate near threshold. The collision broadening effect becomes small for higher initial electron energies since the rate is much less sensitive to changes in energy in this region. A similar and additional effect is seen in the high field case as discussed below.

A note of caution has to be added for the long time limit. As steady state is approached, the electron system cannot "borrow" more energy from the phonon bath than it returns. We believe that a more exact analysis than that given by the Levinson equation is needed in order to correct any energy imbalances which may result from the inclusion of collision broadening into the impact ionization processes, but that for short times, the above broadening and threshold shifting represents a good approximation. If this is not the case, the problem is considerably more complicated, and its solution is beyond the capabilities of modern computational resources. The importance of these higher-order terms is currently under investigation. It may also be necessary to examine the effect of the averaging on the heat bath in a more microscopic way.

The dash-dotted curve at the top is our result for $F=5\times10^5$ V/cm. Here it is seen that the field also substantially softens the threshold energy while for higher electron energies, the field has little effect on the average scattering rate. We interpret these results in the following way: according to Eqs. (8) and (13) the scattering rate associated with an electron in state k depends on $E(\mathbf{k})$ and energies $E(\mathbf{k}+\mathbf{E}_f t)$ with $t<t_c$. This means that electrons experience energy gains and losses as they are accelerated through the band structure during the time of the collision (limited by the phonon scattering rate which is much larger than the impact ionization rate for most energies). Therefore the scattering rate associated with these electrons is "averaged" over these energies. Electrons whose energy is near the zero-field threshold may gain sufficient energy during the collision to cross over the threshold. As mentioned before, the field exhibits its strongest effect on electrons near threshold. For higher energies, the effect of the field, commonly known as the intracollisional field effect, is drastically reduced since the functional energy dependences are weak.

Figures 2(a) and 2(b) show the secondary electron and hole distributions as a function of initial electron energy for $|\mathbf{F}|=0$ V/cm and $|\mathbf{F}|=5\times10^5$ V/cm, respectively. The distributions for $|\mathbf{F}|=5\times10^5$ V/cm are slightly broader than those for zero field for the same reason as discussed above. Even in the zero-field case, lower energy electrons are subject to considerable energy broadening. For instance, classically, the secondary electrons and holes produced by an initiating electron of energy E_i can have energies E_{sec} (measured from band edges) of up to E_i-E_{gap} disregarding conservation of crystal momentum. (E_{gap} is the energy band gap.) However, the secondary distribution functions for $E_i=1.5$ eV show E_{sec} of up to 1.5 eV. This agrees with the conclusions reached earlier regarding collision broadening. Again, our results represent an upper limit on the effect of collision broadening, since in the long time limit, energy conservation of the total system must be maintained. Secondaries produced by high-energy impact ionizing electrons show little energy broadening. We would like to emphasize the fact that for the extreme energy tails of the broadening, our calculation may become increasingly inaccurate. As stated, we believe that this problem is not amenable to easy fixes. We have checked that the non-Markovian corrections to the electron propagator giving rise to the self-energy which are usually cited to correct optical spectra, do not reduce the broadening significantly.

The effect of the field on the wave-vector-dependent scattering rate is also dramatic. The rates in Fig. 1 only demonstrate the effect of the field averaged over all $\mathbf{k}$ with the same energy. Because Monte Carlo transport simulations show that in silicon most impact ionization events occur for 2.5 eV $\leq E \leq$ 3.5 eV (Ref. 17) and the effect of the field is reduced, it is instructive to investigate the anisotropy of the impact ionization rate for energies in this range.

Figure 3(a) shows the effect of the field as a function of wave vector for states with $E(\mathbf{k})=2.5$ and 3.0 eV in the second conduction band, where $k_x=\cos\theta$, $k_y=\sin\theta$, $k_z=0$, where $-\pi/2\leq\theta\leq\pi/2$ with the field applied along the y direction in real space. The zero-field result is shown by the dotted and dashed lines for $E(\mathbf{k})=2.5$ and 3.0 eV, respectively. For these energies, the impact ionization rate is almost isotropic. (For energies near the

zero-field threshold, the anisotropy becomes more pronounced.) However, for high fields, the impact ionization rate becomes quite anisotropic as demonstrated by the dashed-dotted and solid lines in Fig. 3(a) which show the **k**-dependent rates for $E(\mathbf{k})=2.5$ and 3.0 eV for $|\mathbf{F}|=5\times10^5\hat{y}$ V/cm applied along the y direction in real space. This behavior can be understood by viewing the constant-energy contours in the $k_z=0$ plane for the second conduction band in Fig. 3(b). The electronic states are accelerated toward negative k_y by the field where they gain energy for $\theta\leq\pi/2$ and lose energy for $\theta\geq\pi/2$.

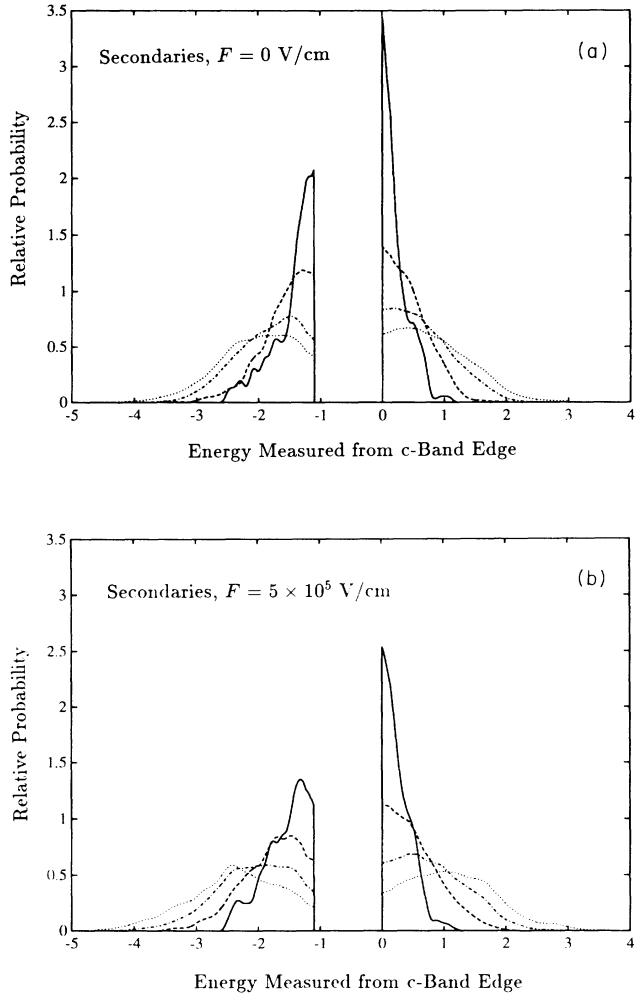

FIG. 2. (a) Secondaries produced by impact ionizing electrons in the conduction band for no field. Solid curve, initial electron of 1.5 eV; dashed curve, 2.5 eV; dash-dotted curve, 3.5 eV; dotted curve, 4.5 eV. The two sets of curves correspond to holes for the $E<-E_G$, and electrons, for $E>0$. Energies are referenced from the bottom of the conduction band. (b) Secondaries produced by impact ionizing electrons in the conduction band $F=5\times10^5$ V/cm. Solid curve, initial electron of 1.5 eV; dashed curve, 2.5 eV; dash-dotted curve, 3.5 eV; dotted curve, 4.5 eV. The two sets of curves correspond to holes for the $E<-E_G$, and electrons, for $E>0$. Energies are referenced from the bottom of the conduction band.

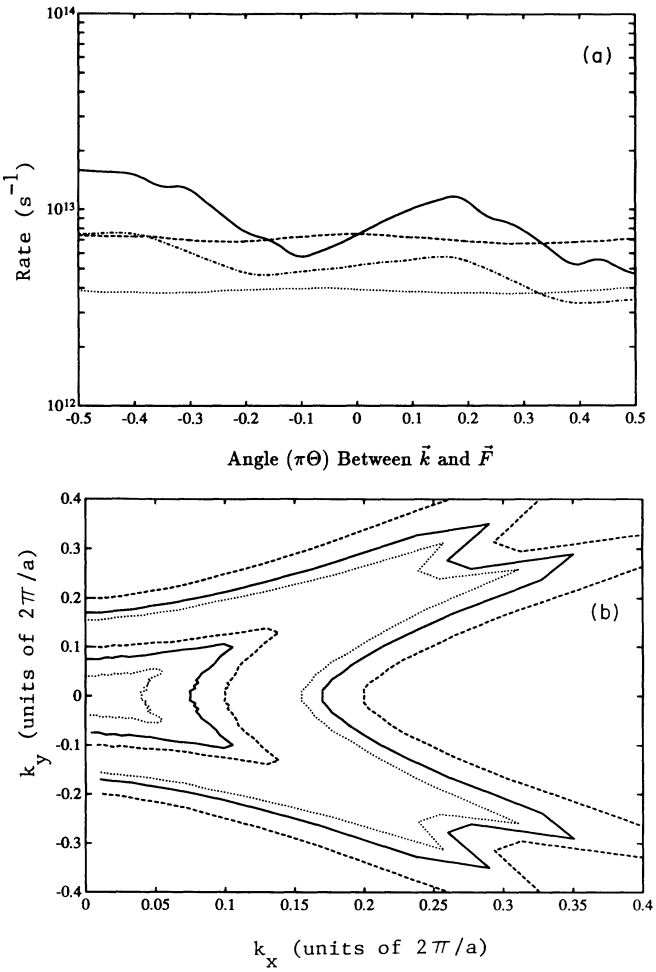

FIG. 3. (a) Wave-vector dependence of impact ionization scattering rate. The four plots are the scattering rates on constant energy contours in the k_x,k_y plane in the second conduction band plotted as against the angle from the k_y axis in units of π. The field is applied in the $\hat{y}$ direction in real space. Solid curve, $F=5\times10^5$, $E=3.0$ eV; dashed curve, no field, $E=3.0$ eV; dash-dotted curve, $F=5\times10^5$, $E=2.5$ eV; dotted curve, no field $E=2.5$ eV. (b) Constant energy contours for the second conduction band of Si. The outer solid curve is the $E=3.0$ eV contour, and the inner solid curve is the $E=2.5$ eV contour, referred to in (a).

V. CONCLUSIONS

We have presented a theory of field-assisted impact ionization expanding Kane's theory. Our investigation shows that the already soft threshold of the Kane theory is considerably broadened by collision broadening and the intracollisional field effect. Therefore a threshold energy cannot be precisely defined. The onset of a steep increase of the ionization rate typically appears at conduction-band energies close to the value of the band gap, hence threshold energy calculations, that conserve both energy and momentum, arrive at much larger values, and appear to be invalid. Furthermore, the electric field causes a dramatic anisotropy of the impact ionization scattering rate.

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