Scaling behavior of fluctuation conductivity of high-temperature superconductors in a magnetic field

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Fluctuation conductivities of $YBa_2Cu_3O_x$ and $Tl_2Ba_2CaCu_2O_x$ films have been analyzed beyond the Gaussian approximation by using the two- (2D) and three-dimensional (3D) scaling functions in the critical region. Near the mean-field transition temperature $T_c(H)$, the fluctuation conductivity of $YBa_2Cu_3O_x$ shows a 3D scaling behavior similar to that seen in the single crystal, suggesting that fluctuations are coherent over the adjacent Cu-O layers in this region. The fluctuation conductivity of $Tl_2Ba_2CaCu_2O_x$ shows good agreement with 2D scaling behavior for temperatures higher than a few degrees above $T_c(H)$, but a deviation occurs very near and below $T_c(H)$. The poor scaling in that region may be caused by double transitions, which are systematically seen in the highly anisotropic high-T_c superconductors like $Tl_2Ba_2CaCu_2O_x$, preventing the determination of the validity of a 2D scaling formula near and below $T_c(H)$.

One of the most interesting features of the high-temperature superconductors (HTS) is a large fluctuation extended well above the mean-field transition temperature, T_c. This is due to the very short coherence lengths which determine the unit volume of the fluctuation and high operating temperatures because each fluctuation mode is associated with the energy of $\sim k_B T$. In addition, the layered structure of the conducting Cu-O planes in HTS may further reduce the effective dimensionality of the fluctuation, which enhances the fluctuation compared to the three-dimensional (3D) case. Among all the thermodynamic properties subject to the fluctuations near T_c, we discuss the in-plane field-dependent fluctuation conductivity, σ', of highly c-axis-oriented $Tl_2Ba_2CaCu_2O_x$ and $YBa_2Cu_3O_x$ films. Previous studies on polycrystalline $Tl_2Ba_2CaCu_2O_x$ films[1,2] showed two-dimensional behavior of the fluctuations in the mean-field region. In a strong magnetic field, it was shown[2] that σ' agrees well with 2D prediction (including the Lawrence-Doniach[3] term) based on the Gaussian fluctuation theory for $T>T_c(0)+10$ K, but a deviation from the theory was observed below that temperature. Such a deviation could result from entering the critical region, where correction terms of higher order in the superconducting order parameter, ψ, are not negligible in the Ginzburg-Landau (GL) free energy. In the Gaussian approximation, individual fluctuations are considered to be independent and only the $|\psi|^2$ terms are included. Thus, the Gaussian fluctuation theory predicts diverging σ' and other physical properties at T_c, but it works very well for $T \gg T_c$, where fluctuations are small in magnitude. Therefore, it is quite natural to observe a deviation of the experimental σ' from the Gaussian theory as T approaches T_c. However, a divergence in σ' is cut off in the critical region by the quartic term in ψ. In this work, we extend the previous analysis of the fluctuation conductivity to the critical region using the scaling functions obtained by Ullah and Dorsey.[4] We first show that σ' of an epitaxial $YBa_2Cu_3O_x$ film in the critical region follows a 3D behavior agreeing with the results of the single crystal by Welp *et al.*[5] Fluctuation conductivities of $Tl_2Ba_2CaCu_2O_x$ films (epitaxial and polycrystalline films are the same) show 2D scaling behavior for temperatures higher than a few degrees above $T_c(H)$, but a deviation occurs very near and below $T_c(H)$.

The critical region is where the free energy of a typical fluctuation is roughly less than the thermal energy and the effect of the quartic term in ψ is no longer negligible; for $T>T_c$, that is, $(\alpha^2/\beta)V \lesssim k_B T_c$, where $\alpha=\alpha'(T/T_c-1)$ and β are the coefficients of the $|\psi|^2$ and $|\psi|^4$ term in the GL free energy, and V is a volume of such a fluctuation.[6] In strong magnetic fields, electrons are confined to the Landau levels due to the orbital motion around an axis along the field direction. This limits the spatial correlation transverse to the field direction to the magnetic length $l_H=\sqrt{\Phi_0/2\pi H}$ in strong fields.[4,6,7] Thus, the effective dimension is reduced because the only free motion available is along the field direction and the effect of the fluctuations becomes more important. In the case of a field parallel to the c axis ($\mathbf{H}\|\mathbf{c}$), a volume of a typical fluctuation in a field is given by $l_H^2\xi_c$ for 3D and $l_H^2 d$ for 2D, where ξ_c is the coherence length along the c axis and d is the film thickness. Since $(\alpha'^2/\beta)=T_c\Delta C$, the critical region, in terms of a reduced temperature $\Delta t(H)=[T-T_c(H)]/T_c(0)$, is given by[6]

$$\Delta t(H)_{2\mathrm{D}} \lesssim \left[\frac{k_B}{\Delta C}\frac{H}{\Phi_0 d}\right]^{1/2} \quad \text{for 2D} , \tag{1}$$

$$\Delta t(H)_{3\mathrm{D}} \lesssim \left[\frac{k_B}{\Delta C}\frac{H}{\Phi_0 \xi_c(0)}\right]^{2/3} \quad \text{for 3D} , \tag{2}$$

where ΔC is the specific-heat jump at $T_c(0)$, and $T_c(H)$ is the mean-field transition temperature in a magnetic field. The critical region grows with applied field due to a decrease of l_H and it is easily accessible in HTS.

Recently Ikeda, Ohmi, and Tsuneto[6] calculated the fluctuation conductivity based on the model of Lawrence and Doniach[3] but including the $|\psi|^4$ term with the renormalization effect due to interactions between fluctuations. They could fit the resistive transitions of $YBa_2Cu_3O_x$ in a magnetic field with suitable choice of parameters. Ullah and Dorsey[4,8] included the quartic term within the Hartree approximation and showed that the experimentally obtained Ettingshausen coefficient of $YBa_2Cu_3O_x$ was in quantitative agreement with their calculation.[8] They later[4] obtained other transport properties including electrical conductivity due to the fluctuations. In either two or three dimensions, they found a scaling form of σ' for **H**||**c** in high field where only the lowest landau level is occupied by electrons. These scaling forms are given, in terms of unspecified scaling functions F_{2D} and F_{3D}, by

$$\sigma'(H)_{2D} = \left[\frac{T}{H}\right]^{1/2} F_{2D}\left[A\frac{T-T_c(H)}{(TH)^{1/2}}\right] \quad \text{for 2D}, \qquad (3)$$

$$\sigma'(H)_{3D} = \left[\frac{T^2}{H}\right]^{1/3} F_{3D}\left[B\frac{T-T_c(H)}{(TH)^{2/3}}\right] \quad \text{for 3D}, \qquad (4)$$

where A and B are appropriate constants characterizing the materials. The arguments of the scaling functions are another representation of the critical region in a strong magnetic field [cf. Eqs. (1) and (2)]. Any physical properties subject to the fluctuations will follow the same scaling behavior. Note that only the effect of the Aslamazov-Larkin term[9] has been considered in the above formulation, but near T_c it dominates additional terms such as the Maki-Thompson term[10] and the Zeeman effect[11], due to the large phase relaxation rate[2,11,12] and small Zeeman energy compared to the thermal energy,[11] respectively. Although the lowest-Landau-level expression [Eqs. (3) and (4)] is not strictly valid for HTS in the fluctuation regime within the available magnetic fields, inclusion of the higher Landau will still retain their functional forms as noted by Ullah and Dorsey.[4] Ikeda *et al.*[6] found that, for $\mu_0 H \geq 2$ T, the lowest-Landau-level approximation works pretty well for $YBa_2Cu_3O_x$.

Recently, a 3D scaling behavior in fluctuation conductivity and other thermodynamic properties has been observed in $YBa_2Cu_3O_x$ single crystals near $T_c(H)$.[5] Thus, it is interesting to perform the same analysis of σ' on the epitaxially grown films of $YBa_2Cu_3O_x$ and those with much greater anisotropy, e.g., $Tl_2Ba_2CaCu_2O_x$. In the following, we analyze the resistive transitions of $YBa_2Cu_3O_x$ and $Tl_2Ba_2CaCu_2O_x$ films with the above scaling formula. A film of $YBa_2Cu_3O_x$, grown *in situ* by dc magnetron sputtering from a stoichiometric target onto rotating (100) $SrTiO_3$ substrates, was shown by transmission electron microscopy (TEM) to be epitaxial. Two kinds of $Tl_2Ba_2CaCu_2O_x$ films, epitaxial and polycrystalline, were investigated in this study. The epitaxial film was prepared by laser ablation onto (100) $SrTiO_3$ substrates followed by a post anneal and confirmed by electron channeling to be epitaxial. The polycrystalline film was prepared on (100) MgO by dc magnetron sputtering from a three-gun source followed by post annealing. All films were found to be highly c-axis oriented by x-ray diffraction. The fluctuation conductivity was first determined by subtracting normal-stat conductivity from the total conductivity. The linear background conductivity is determined by either extrapolation of the high-temperature resistivity or fitting zero-field conductivity to the Lawrence-Doniach model. They both resulted in the same scaling behavior near $T_c(H)$.

Figure 1 shows the scaled resistive transitions of epitaxial $YBa_2Cu_3O_x$ films for the field parallel to the c axis ($H\|c$) with the 3D scaling formula [Eq. (4)]. The y axis is $\sigma'(H/T^2)^{1/3}$ and the x axis is $[T-T_c(H)]/(TH)^{2/3}$, the argument of the 3D scaling function. The data near $T_c(H)$ at fields higher than 2 T show a good agreement with Eq. (4) consistent with the single-crystal results. Furthermore, higher field data (>4 T) exhibit very impressive 3D scaling behavior down to the vanishing resistance region shown in the inset of Fig. 1 in a semilogarithmic plot. However, this result could be fortuitous since the scaling formula applied here does not explicitly contain any dissipation associated with flux motion which supposedly plays a dominant role in the tail region. An attempt to fit with the 2D formula was unsuccessful resulting in unphysical parameters. At $T=T_c(H)$, the fluctuation conductivity does not diverge in contrast to the Gaussian theory. The lower-field data, not shown here, (≤ 1 T) showed further deviation from high-field ones without any sharp crossover. The same trend was also observed in $Tl_2Ba_2CaCu_2O_x$ samples. The two free parameters used in Fig. 1 are the upper critical field slope, $dH_{c2}/dT = -1.7$ T/K and zero-field mean-field transition temperature, $T_c(0) = 88.6$ K. [A scaling of similar quality could be obtained with slightly different parameter sets, for example, $dH_{c2}/dT = -1.9$ T/K and $T_c(0) = 88.7$ K.] The value of $T_c(0)$ is higher than 88.1 K of the zero-resistance temperature at 0 T as expected and the value of dH_{c2}/dT agrees very well with the magnetization data of the single crystal[13] and the results of Ref. 5. A similar scaling behavior, $\Delta t(H)_{3D} \sim H^{2/3}$, has been used by Tinkham[14] to explain the resistive broadening of $YBa_2Cu_3O_x$ single crystals.

A previous study[2] on polycrystalline $Tl_2Ba_2CaCu_2O_x$

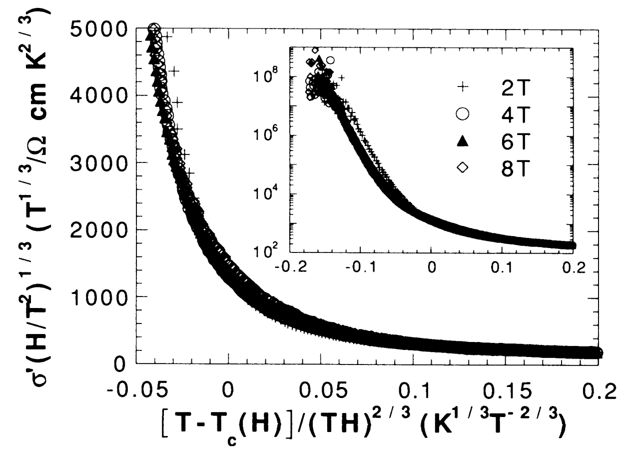

FIG. 1. 3D scaling of the fluctuation conductivity of an epitaxial $YBa_2Cu_3O_x$ film is plotted. A semilogarithmic plot is shown in the inset with the same coordinates.

showed a 2D behavior of the fluctuations for $T > T_c(0) + 10$ K, but a deviation from the 2D Gaussian fluctuations was observed below that temperature. Since such a deviation could result from entering the critical region, which is larger than that of $YBa_2Cu_3O_x$ due to the higher anisotropy, it is interesting to check the performance of the 2D scaling functions in the critical region over the Gaussian theory. We studied epitaxial as well as polycrystalline films to further see if there is any morphology dependence of the fluctuation conductivity. Scaled resistive transitions of epitaxial $Tl_2Ba_2CaCu_2O_x$ films for $H \| c$ with the 2D scaling formula [Eq. (3)] are shown in Fig. 2(a), with the parameters of $T_c(0) = 101$ K and $dH_{c2}/dT = -2$ T/K. Overall agreement with 2D scaling is fairly good. Scaling with the 3D formula resulted in a poorer collapse of the data. For the scaled temperature range of $t_s \equiv T - T_c(H)/\sqrt{TH} > 0.3$, 2D scaling works very well as expected from the previous studies.[2] In this region, the fluctuations might be confined to the individual Cu-O layers. As T passes through $T_c(H)$, the data begin to spread out. Essentially the same behavior is observed in the case of polycrystalline $Tl_2Ba_2CaCu_2O_x$ as shown in Fig. 2(b), indicating that the 2D nature of the fluctuation is not affected by the different morphology such as the existence of the high-angle grain boundaries. The parameters used in Fig. 2(b) are $T_c(0) = 102.5$ K and $dH_{c2}/dT = -2$ T/K. We fixed the parameter $dH_{2C}/dT = -2$ T/K during the scaling process, but a similar quality of the scaling could be obtained with different sets of parameters ranging, for example, from -1.5 to -4 T/K with appropriate choices of $T_c(0)$. Higher $T_c(0)$ was needed to match higher $(-dH_{c2}/dT)$. Magnetization measurements, which give the thermodynamic $H_{c2}(T)$ and, hence, dH_{c2}/dT, have not been reported for $Tl_2Ba_2CaCu_2O_x$.

The temperature of $t_s = 0.3$, at which noticeable spread out begins, is about 3–5 K above $T_c(0)$ for field from 1 to 10 T. This is at least an improvement toward the critical region over the result of Ref. 2 based on the Gaussian theory. A poor scaling behavior for $t_s < 0.3$ could be due to the effect of the apparent double superconducting transitions. In the semilogarithmic plot (insets of Fig. 2), the double transitions can be clearly seen with the onsets of $t_s \approx 0$. This is common to the highly anisotropic (2D-like) HTS.[15] Different dissipation mechanisms, such as Josephson phase fluctuations[16] or due to the change in the vortex state configuration, could be responsible for double transitions. Since these mechanisms are not considered in the present scaling formula which includes only the amplitude fluctuations of ψ, the possibility that double transitions are responsible for poor scaling below $T_c(H)$ prevents us from determining the validity of the proposed 2D scaling. Perhaps this will be an unavoidable problem for the highly anisotropic HTS.

In summary, the fluctuation conductivities of $YBa_2Cu_3O_x$ and $Tl_2Ba_2CaCu_2O_x$ films has been analyzed using the 2D and 3D scaling functions. Near $T_c(H)$, a 3D scaling behavior has been observed in $YBa_2Cu_3O_x$ that is similar to the single-crystal case, meanwhile, $Tl_2Ba_2CaCu_2O_x$ shows better agreement with the 2D scaling function for temperatures higher than a few degrees above $T_c(H)$, consistent with the previous observation, but a deviation occurs very near and below $T_c(H)$. The double transitions could be responsible for the poor scaling below $T_c(H)$, thus preventing the confirmation of the 2D scaling formula in this region.

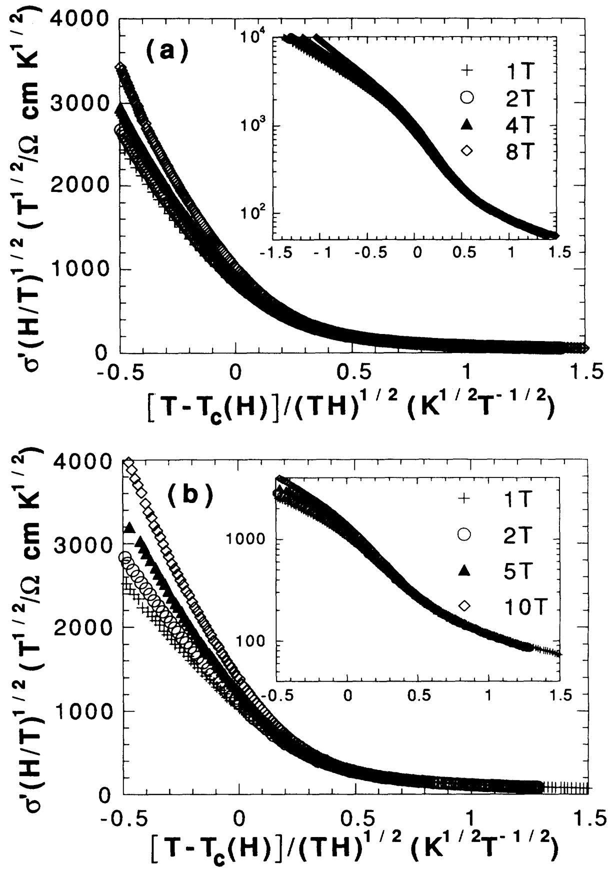

FIG. 2. (a) 2D scaling of the fluctuation conductivity of an epitaxial $Tl_2Ba_2CaCu_2O_x$ film is plotted. Double transitions are evident in the semilogarithmic plot. 2(b). 2D scaling of the fluctuation conductivity of a polycrystalline $Tl_2Ba_2CaCu_2O_x$ film shows the same behavior as the epitaxial one.

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