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Nonlinear fluctuation conductivity of a layered superconductor: Crossover in strong electric fields

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The fluctuation conductivity of a clean layered superconductor in an arbitrary electric field is studied. It is shown that in the vicinity of T_c an unusual crossover from three-dimensional to two-dimensional behavior in the non-Ohmic region of the electric field excess current dependence takes place. In spite of the large size of the fluctuation Cooper pairs, the strong electric field gives rise to additional decay and to an effective freezing of the number of degrees of freedom.

Recent advances in the technology of monocrystalline film growth has permitted the creation of perfect samples of high-T_c superconductors with a sharp transition in the superconductive state. In the vicinity of T_c and in sufficiently strong electric fields, a nonlinear fluctuation conductivity was observed.[1,2] It is interesting that in the immediate vicinity of the critical temperature and in sufficiently strong electric fields the critical exponents deviate from the three-dimensional (3D) case.[3,4] This fact is nontrivial because, as it is well known, in the immediate vicinity of T_c [where $\xi(T) \gg a$] the behavior of the fluctuations has to be 3D; it is really so[2] in the same temperature range but in a weak field where the nonlinear effects are not important.

The problem of non-Ohmic behavior of the fluctuation conductivity in sufficiently strong fields was discussed many years ago.[3,5] It was shown that the fluctuation conductivity may be calculated in the linear-response approximation only for sufficiently weak fields, when they do not perturb the fluctuation spectrum. Beginning from some characteristic field strength,[3] the acceleration of electrons is so large that, on the path of the order of ξ, they change their energy by an order of $T-T_c$, which characterizes the binding energy of the fluctuating Cooper pairs. As a result, the fluctuating Cooper pairs decay additionally and the deviation from the Ohm's law takes place. The characteristic electric field E_c, at which the nonlinear effects are exhibited, decreases as $\epsilon^{3/2}$ (here $\epsilon = T - T_c/T_c$ is the reduced temperature) as the temperature approaches T_c.

In this Rapid Communication we shall study the fluctuation conductivity of a layered superconductor in an arbitrary electric field and show that the crossover from 3D to 2D behavior of the conductivity takes place not only upon the change of the temperature but also upon the change of the electric field. This means that in very strong electric fields even at temperatures close to T_c the pair motion becomes 2D again due to the pair-breaking effect of the electric field.

Let us begin with the expression for the fluctuation correction δJ_{fl} which was calculated in the case of dirty isotropic superconductors by Gorkov for the electric current:[5]

$$\delta J_{\mathrm{fl}} = cE \int_0^\infty v\,dv \int \frac{d^3q}{(2\pi)^3} e^{-2Dq^2v} \exp\{-\tfrac{2}{3} v[\pi^2 \tau_s (T^2 - T_c^2) + e^2 D E^2 v^2]\} , \tag{1}$$

where D is the diffusion coefficient, τ_s is the relaxation time of the inelastic scattering [$\tau_s(T-T_c) \ll 1$], and c is a constant dependent on the geometry of the spectrum ($\hbar = k_B = 1$).

We consider now the case of a clean superconductor ($l \gg \xi_0$), which corresponds to the experiments under discussion.[6] Hence, the anomalous Maki-Thompson contribution, which is stipulated by the coherent electron scattering on impurities, does not appear at all and we need not introduce the hypothesis of the presence of paramagnetic impurities in the superconductor (as was done in Ref. 5). So, in the vicinity of the critical temperature, we can restrict ourselves by considering the paraconductive contribution only. However, the term originating from the pair breaking in (1) has to be omitted and some necessary modifications of (1), required by the specifics of our model, have to be made.

Let us begin with the electron spectrum. In order to take into account the layered structure of high-T_c superconductors, we adopt its Fermi surface in the form of a modulated cylinder which describes the 2D movement of electrons in layers and, simultaneously, the possibility of their hopping between layers:

$$\xi(\mathbf{p}) = \epsilon(\mathbf{p}) - \epsilon_F = v_F(|\mathbf{p}_\parallel| - p_F) + w\cos(p_\perp a) , \tag{2}$$

where $\xi(\mathbf{p})$ is the electron energy measured from the Fermi level, v_F and p_F are the Fermi velocity and momentum in the plane of the layers, and w is the electron hopping integral.

First of all let us mention that, in the case of anisotropic spectrum (2), even for the diffusive character of the electron motion, the product Dq^2 has to be treated not as the first term of the expansion, which is valid for small $\mathbf{q}$ only, but for any $\mathbf{q}$ from the Brillouin zone. So, for the spec-

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trum (2), one can find[6]

$$Dq^2=\langle\tau(\mathbf{v}\cdot\mathbf{q})^2\rangle=\langle\tau[\epsilon(\mathbf{p}+\mathbf{q})-\epsilon(\mathbf{p})]^2\rangle_{\mathbf{q}\to 0}\Longrightarrow\frac{12T\eta^{\text{dirty}}}{\pi}[q_\parallel^2+(4w^2/v_F^2)\sin^2(q_\perp a/2)]\,, \tag{3}$$

where $\langle\cdots\rangle$ means the averaging over the Fermi surface and $\eta^{\text{dirty}}=\pi\tau v_F^2/24T_c^2$ is the value of the Ginzburg-Landau parameter[7]

$$\eta=-\frac{v_F^2\tau^2}{3}[\psi(\tfrac{1}{2}+1/4\pi T\tau)-\psi(\tfrac{1}{2})-(1/4\pi T\tau)\psi'(\tfrac{1}{2})] \tag{4}$$

in the limit of a dirty metal [$\psi(x)$ is the digamma function]. Hence, the second modification of (1) for the case of clean superconductors is evident: we have to use below not η^{dirty} but $\eta^{\text{clean}}=\eta_0=\lim_{T\tau\to\infty}\eta=7\zeta(3)v_F^2/48\pi^2T_c^2$ [$\zeta(3)=1.202$ is the Riemann zeta function of argument 3].

The last that we have to do with (1) is to take into account the "mass" of the fluctuating Cooper pairs, that was omitted in Ref. 5 because of the presence of strong pair breaking. It is easy to do if one remembers that it appears in the denominator of the fluctuation propagator $L^R(\mathbf{q},w)$, side by side with Dq^2:[7]

$$L^R(\mathbf{q},\omega)=-\frac{8T}{\pi\rho}\frac{1}{(8/\pi)(T-T_c)-i\omega+Dq^2}\,. \tag{5}$$

Hence, in (1) it is sufficient to substitute

$$Dq^2\Longrightarrow\frac{8T}{\pi}\left[\epsilon+\tfrac{3}{2}\eta_0q_\parallel^2+6\frac{w^2}{v_F^2}\eta_0\sin^2\frac{q_\perp a}{2}\right]$$

to obtain the generalization of the Gorkov result for the case of a clean layered superconductor:

$$\delta J_\parallel=\frac{14\zeta(3)v_F^2e^2}{\pi^3}\int_0^\infty v\,dv\int_{-\pi/a}^{\pi/a}\frac{dq_\perp}{2\pi}\int_0^\infty\frac{q_\parallel dq_\parallel}{2\pi}\exp\left[-\frac{16T_cv}{\pi}\left[\epsilon+\tfrac{3}{2}\eta_0q_\parallel^2+6\frac{w^2\eta_0}{v_F^2}\sin^2\frac{q_\perp a}{2}\right]\right]\times\exp[-(8T/\pi)\eta_0e^2E^2v^3] \tag{6}$$

(we adopted the electric field $\mathbf{E}$ parallel to the planes of the layers).

The integrations over $q_\parallel$ and $q_\perp$ are trivial and after some calculations one can find

$$\delta J_\parallel(E,\epsilon)=\frac{e^2E}{16a\epsilon}\int_0^\infty dxe^{-x\{1+[\xi_\perp(\epsilon)/a]^2\}}\times I_0\{[\xi_\perp(\epsilon)/a]^2x\}e^{-(E/E_c)^2x^3}\,, \tag{7}$$

where $\xi_\perp(\epsilon)=a(3w^2\eta_0/v_F^2\epsilon)^{1/2}$ is the coherent length in the direction perpendicular to the layers, $E_c=E_0\epsilon^{3/2}$ ($E_0=2^{9/2}T_c/\pi e\eta_0^{1/2}=54T_c^2/ev_F$) is some characteristic value of the electric field and $I_0(z)$ is the Bessel function of the imaginary argument. From this general result it is easy to reproduce the static paraconductive contribution in the fluctuation conductivity of a layered superconductor above T_c.[8] It is sufficient to set $E\to 0$ and the integral in (7) may be carried out exactly:

$$\sigma_\parallel(0,\epsilon)=\lim_{E\to 0}\frac{\delta J_\parallel(E,\epsilon)}{E}=\frac{e^2}{16a\epsilon}\frac{1}{\{1+2[\xi(\epsilon)/a]^2\}^{1/2}}=\frac{e^2}{16a}\begin{cases}\dfrac{1}{\epsilon}\ \text{for}\ \xi_\perp(\epsilon)\ll a\\[2ex]\dfrac{4\pi}{\sqrt{14\zeta(3)}}\left[\dfrac{T_c}{w}\right]\dfrac{1}{\sqrt{\epsilon}}\ \text{for}\ \xi_\perp(\epsilon)\gg a\,.\end{cases} \tag{8}$$

The opposite limit $E>E_c$ of the nonlinear fluctuation conductivity is more interesting to us now. To obtain the asymptotic expressions we shall adopt $E\gg E_c$. In this case the integral in (7) converges at $x\sim(E_c/E)^{2/3}\ll 1$. However, the result of the integration strongly depends on the relation between $\xi_\perp(\epsilon)$ and a.

If the temperature is sufficiently far away from the critical value [$\xi(\epsilon)<a$], then, in the region of the convergency of the integral in (7), the product of the first two functions may be adopted as 1 and

$$\delta J_\parallel^{(2D)}(E\gg E_c)=\frac{e^2\Gamma(\tfrac{1}{3})E_0}{48a}\left[\frac{E}{E_0}\right]^{1/3}. \tag{9}$$

The opposite case $\xi_\perp(\epsilon)\gg a$ is more complex for the analysis. Indeed, there are two parameters competing with each other.

(i) If the electric field is so strong that

$$\frac{E}{E_c}\gg[\xi_\perp(\epsilon)/a]^3\,,$$

then the region of the integral convergence turns out to be so small that, in spite of the large factor $(\xi_\perp/a)^2\gg 1$ in the argument of Bessel function, the product

$$f(x)=e^{-x[1+(\xi_\perp/a)^2]}I_0[(\xi_\perp/a)^2x] \tag{10}$$

may be adopted at 1 as above. Hence, in a very strong field $E\gg E_c(\xi_\perp/a)^3$, the same 2D behavior of the fluctuation conductivity (9) takes place even in the vicinity of T_c, when $\xi_\perp(\epsilon)\gg a$. The physical reasons of such an effective reduction of dimensionality will be discussed later.

(ii) In the case of intermediate fields

$$E_c\ll E\ll E_c(\xi_\perp/a)^3\,,$$

in the domain of the integral convergence in (8), the function $f(x)$ varies considerably. Due to the condition $[\xi_\perp(\epsilon)/a]^3\gg E/E_c$ one can use the asymptotic expression

$$I_0(z\gg 1)=(1/2\pi z)^{1/2}e^z \tag{11}$$

and the integral in (7) can be rewritten as

$$\delta J_{\parallel}^{(3D)} = \frac{e^2 E}{16 a \epsilon}\left[\frac{E_c}{E}\right]^{1/3}\left[\frac{a}{\xi_\perp}\right]\left[\frac{1}{2\pi}\right]^{1/2} \times \int_0^\infty \frac{1}{\sqrt{y}} e^{-(E_c/E)^{2/3}y - y^3} dy . \quad (12)$$

Taking into account the fact that $E_c \ll E$, one can find

$$\delta J_{\parallel}^{(3D)}(E \gg E_c) = \frac{\Gamma(\frac{1}{6}) e^2 \sqrt{\pi} E_0}{12 a \sqrt{14\zeta(3)}}\left[\frac{T_c}{w}\right]\left[\frac{E}{E_0}\right]^{2/3} . \quad (13)$$

The results of the numerical calculations for the typical cases $\xi_\perp(\epsilon_1)/a = 10.0$ and $\xi_\perp(\epsilon_2)/a = 0.3$ are presented in a ln-ln scale in Fig. 1 (w was adopted equal to T_c in these calculations). The asymptotes (8), (9), and (13) are shown by the tangent lines.

Let us now discuss the obtained results in more detail.

Far from T_c, when $\xi_\perp(\epsilon) < a$, there is no problem in the treatment of (9). Cooper pairs "rotate" in every plane separately and we reproduce the result of Ref. 3 only:

$$\delta J_{\parallel}^{(2D)}(E,\epsilon) = \frac{e^2}{16a}\begin{cases} E/\epsilon & \text{for } E \ll E_c \\ \dfrac{\Gamma(\frac{1}{3}) E_0}{3}\left[\dfrac{E}{E_0}\right]^{1/3} & \text{for } E \gg E_c . \end{cases} \quad (14)$$

In the immediate vicinity of T_c (but still out of the critical region), when $\xi_\perp(\epsilon) \gg a$, in small fields the movement of the fluctuating Cooper pairs has a 3D character. In accordance with Ref. 3 this situation remains at fields even higher then E_c. The nontrivial fact, which follows from (7), is the crossover in a sufficiently strong electric field $E_{cr} \sim E_c[\xi_\perp(\epsilon)/a]^3$ from 3D behavior ($\delta J_{\parallel}^{(3D)} \sim E^{2/3}$) to 2D behavior ($\delta J_{\parallel}^{(2D)} \sim E^{1/3}$), in spite of the large size of pairs $\xi_\perp(\epsilon) \gg a$ at these temperatures:

$$\delta J_{\parallel}^{(3D\to 2D)}(E,\epsilon) = \frac{e^2}{4a}\begin{cases} \dfrac{\pi E}{\sqrt{14\zeta(3)}}\left[\dfrac{T_c}{w}\right]\dfrac{1}{\epsilon} & \text{for } E \ll E_c , \\ \dfrac{\Gamma(\frac{1}{6})\sqrt{\pi} E_0}{3\sqrt{14\zeta(3)}}\left[\dfrac{T_c}{w}\right]\left[\dfrac{E}{E_0}\right]^{2/3} & \text{for } E_c \ll E \ll E_c\left[\dfrac{\xi_\perp(\epsilon)}{a}\right]^3 , \\ \dfrac{\Gamma(\frac{1}{3}) E_0}{12}\left[\dfrac{E}{E_0}\right]^{1/3} & \text{for } E \gg E_c\left[\dfrac{\xi_\perp(\epsilon)}{a}\right]^3 . \end{cases} \quad (15)$$

The nontrivial fact may be understood from the following qualitative consideration. In the region $E \ll E_c$, all types of 3D rotations of the fluctuating Cooper pairs are possible. The picture is 3D. When $E \sim E_c$, the rotations of the AB type (Fig. 2) (the electron pair rotates in the ZX plane so that one of the electrons goes from point A to point B) lead to displacements of particles a distance on the order of $\xi_\perp$. From the definition of E_c, $E_c \xi_\perp \sim T - T_c$ (it is the characteristic energy of fluctuating Cooper pairs) and at $E > E_c$, such rotations begin to decay. The next step of the freezing of the degrees of freedom is the decay of rotations from the polar point A to the nearest

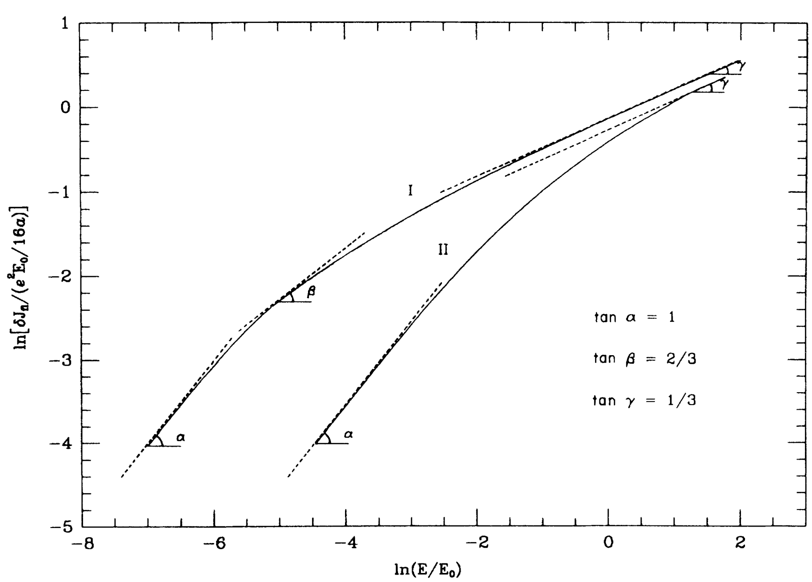

FIG. 1. The dependence of the normalized excess current $\delta J_{\parallel}/(e^2 E_0/16a)$ on the dimensionless electric field E/E_0 in the ln-ln scale for the typical cases $\epsilon_1 = 5\times 10^{-4}$, for which $\xi_\perp(\epsilon_1)/a = 10.0$ (curve I) and for $\epsilon_2 = 0.15$, for which $\xi_\perp(\epsilon_2)/a = 0.3$ (curve II). For simplicity the case $w = T_c$ is adopted in these calculations. The tangent lines show the asymptotes (8), (9), and (13).

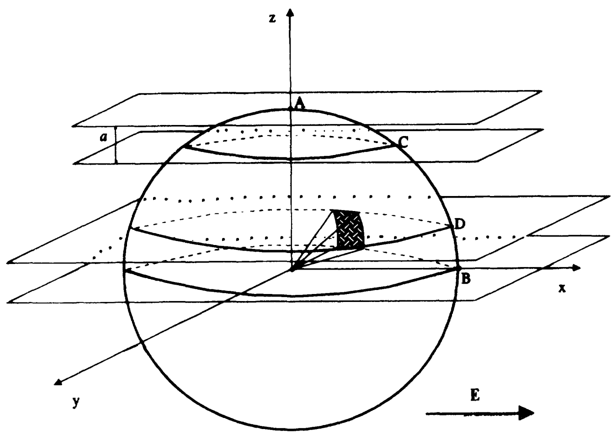

FIG. 2. The layered structure of the superconductor and the sphere of all possible rotations of the Cooper pairs in the presence of the in-plane electric field (see the text for a detailed explanation).

layer (point C), in the plane ZX. In this case the vertical displacement is a, and the corresponding displacement in the field direction X is of the order of $(a\xi_\perp)^{1/2}$. Hence, such rotations decay at $E_c^{(1)}$, which is determined from the condition

$$E_c^{(1)}(a\xi_\perp)^{1/2} \sim T - T_c \sim E_c\xi_\perp ,$$

$$E_c^{(1)} \sim E_c(\xi_\perp/a)^{1/2} .$$

The rotations of the pairs from the layer XY to the nearest one (one of the electrons of the Cooper pair hops from point B to point D) are more stable with increasing electric field. This leads to a displacement in the X direction $\Delta x \sim a^2/\xi_\perp$ and the characteristic decay field $E_c^{(2)}$ of such rotations is $E_c^{(2)} \sim E_c(\xi_\perp/a)^2$. But even for fields $E > E_c^{(2)}$, some 3D rotations are still possible. They are the rotations in the plane YZ. The minimum of them has the same hopping of an electron from the layer (XY) to the nearest one, during which the displacement in the Y direction is of the order of $a^2/\xi_\perp$. But this movement has to be slightly 3D, hence the angular displacement in the XY plane has to be at least of the same order of that in the YZ plane: $\Delta\phi \sim a/\xi_\perp$. It will give $\Delta x \sim a^3/\xi_\perp^2$ and a corresponding field $E_c^{(3)} \sim E_{\mathrm{cr}} \sim E_c(\xi_\perp/a)^3$, in accordance with the result of the integration in (8). In higher fields all the possibilities of 3D behavior of the Cooper pairs are exhausted and the 2D regime of $\delta J_{\mathrm{fl}}(E,\epsilon)$ takes place in spite of the condition $\xi_\perp(\epsilon) \gg a$.

It is important to emphasize that this is not a 2D motion within layers but rather within the plane perpendicular to the external field (which was adopted to be parallel to the layers) to avoid its strong pair-breaking effect.

In connection with experiment[1,2] we have to mention that in high-T_c superconductors the critical field E_c appears to be rather high. For example, for $T_c = 80$ K, $v_F = 3\times 10^7$ cm/sec, $\epsilon = 0.01$, and $E_c \sim 10^2$ V/cm. This means that a voltage on the order of 100 mV for a sample of the size 10 μ has to be applied.

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