Exchange modes in ferromagnetic superlattices

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The spin-wave spectrum of infinite, semi-infinite, and finite ferromagnetic superlattices with arbitrary elementary units is analyzed theoretically in the exchange-dominated region. The general dispersion equations for spin-wave modes are derived in the framework of the transfer-matrix formalism. Infinite structures with layered defects are also analyzed. Some numerical results are presented for structures with $N=1$, 2, and 6 atomic planes in the elementary unit. The considerations are restricted to simple cubic structures with the exchange coupling between nearest neighbors.

I. INTRODUCTION

Recent advances in epitaxial growth techniques have made possible the preparation of a large variety of magnetic layered structures as, for example, alternating magnetic-nonmagnetic and magnetic-magnetic superlattices with the magnetic materials being either ferromagnetic or antiferromagnetic. The quality of magnetic superlattices is presently approaching the quality of layered structures formed from nonmagnetic semiconducting materials, which are known to have almost perfect interfaces. Macroscopic properties of magnetic layered structures are subject to design and control on atomic scale by varying the thickness and composition of the constituent films. In addition, the existence of collective excitations in magnetic superlattices, which have properties distinctly different from those of their bulk counterparts, has opened up the possibility of fabricating new materials with controllable magnetic and electromagnetic properties.

The simplest structures consist of magnetic (ferromagnetic or antiferromagnetic) films alternating with nonmagnetic ones. For the nonmagnetic layers thick enough, the magnetic films are exchange decoupled and interact only via long-range dipolar stray fields. The corresponding spin-wave spectrum in the exchange-dominated region is then composed of spin waves confined to a particular magnetic film, contrary to the dipolar-dominated region, where the normal modes are collective excitations of the whole system and form a characteristic subband structure. The dipolar-dominated region was analyzed theoretically in the magnetostatic limit[1-8] as well as with retardation effects included.[9-14] If the intralayer exchange and dipolar contributions to the spin-wave energy are comparable, one obtains collective exchange-dipolar modes.[15]

If the thickness of the nonmagnetic films is small enough or if the magnetic films are in direct contact, a ferromagnetic or antiferromagnetic exchange coupling between the magnetic films can occur. In that case the single-film modes interact also in the exchange-dominated region.[6,16-22] The mixed exchange-dipolar modes of exchange-coupled multilayers have also been the subject of intensive theoretical studies.[23-26]

Theoretical analyses of spin-wave excitations in magnetic superlattices are usually restricted to the low-temperature limit, although some temperature dependences have also been discussed.[27,28] To find the appropriate spectrum, one usually proceeds by considering the nature of the solution for the relevant wave field in each layer of the elementary unit. The general solution inside each film is written as a linear combination of a few terms (usually two, four, or six terms) which describe waves with opposite wave-vector component (real, imaginary, or complex) along the superlattice direction. The allowed spin-wave energies are then determined by the appropriate boundary conditions at the interfaces. This method is commonly used for the continuum models of the superlattices. It is also applicable to discrete systems.[6,16] In the latter case, however, one may also use the approach based on the transfer-matrix formalism in the site representation. This method is applied in this paper to spin-wave modes in the exchange-dominated region of ferromagnetic superlattices with an arbitrary elementary unit. A similar method has been recently applied to the spin-wave spectrum in sandwich structures.[29] Applying the transfer-matrix formalism, we derive the general dispersion equations for bulk and surface waves in semi-infinite layered structures. Apart from this, we analyze the modes in structures with finite thickness. Two different cases are here distinguished: (i) the case of complete units and (ii) the case with one incomplete unit at a surface. The modes in infinite structures with some layered defects are also discussed. Such defects break the translational symmetry and, in the general case, can localize some of spin-wave modes at the defect. The considerations are restricted to the case of the exchange coupling between nearest neighbors.

The model and transfer-matrix method are described in Sec. II. The eigenvalue problem for the transfer matrix is discussed in Sec. III. In Secs. IV and V the general dispersion equations, respectively, for bulk and surface modes are derived. The modes localized at layered defects in infinite structures are analyzed in Sec. VI. In Sec. VII the spin-wave spectrum of finite structures is considered. In subsequent sections we apply the derived

dispersion equations to particular layered systems. The case of one atomic plane in the elementary unit (uniform system) is briefly discussed in Sec. VIII. Some numerical and analytical results for structures with $N=2$ atomic planes in the elementary unit are given in Sec. IX, whereas some results for structures with higher N are presented in Sec. X. Finally, concluding remarks are given in Sec. XI.

II. MODEL AND TRANSFER-MATRIX METHOD

Consider a layered structure in which an elementary unit is repeated periodically along the stacking direction which is parallel to the x axis of the coordinate system. Let the elementary unit consist of N (100) atomic planes of the simple cubic lattice with the lattice constant a. In a general case all N atomic planes of the elementary unit can be occupied by different magnetic atoms. However, the translational symmetry in the atomic planes is assumed. We restrict considerations to the ferromagnetic-type exchange interaction between all nearest neighbors (NN's). Apart from this we assume that an external static magnetic field H_0 is applied along one of the in-plane crystallographic axes (parallel to the axis z), which are also easy directions for the static magnetization. We will neglect any out-of-plane magnetic anisotropy which may occur if two adjacent atomic planes are occupied by different magnetic atoms. Consequently, we assume that the magnetic anisotropy is described approximately by a uniaxial-type term with the easy direction along the axis z. The system can be then described by the Hamiltonian

$$\begin{aligned} H=&-\tfrac{1}{2}\sum_l\sum_\rho\sum_{\delta_\parallel} J_l \mathbf{S}_{l,\rho}\cdot\mathbf{S}_{l,\rho+\delta_\parallel} \\ &-\sum_l\sum_\rho J_{l,l+1}\mathbf{S}_{l,\rho}\cdot\mathbf{S}_{l+1,\rho} \\ &-\sum_l\sum_\rho D_l(S^z_{l,\rho})^2+\mu_0\mu_B H_0\sum_l\sum_\rho g_l S^z_{l,\rho} , \end{aligned} \tag{1}$$

where l is the atomic plane index, $\boldsymbol{\rho}$ determines the position of a spin in the atomic plane, $\boldsymbol{\delta}_\parallel$ is the position vector to the in-plane nearest neighbors, J_l is the exchange coupling constant for both NN's lying in the lth atomic plane, and $J_{l,l+1}$ is the exchange integral for NN's lying in the adjacent, lth, and $(l+1)$st atomic planes. The parameters D_l and g_l are, respectively, the anisotropy constant (assumed positive, $D_l>0$) and the Landé factor for magnetic ions lying in the lth atomic plane. Finally, μ_0 and μ_B are the magnetic permeability of the vacuum and Bohr magneton, respectively. The index l runs over all atomic planes. The N atomic planes of the elementary unit will be indexed by n ($n=1,2,\ldots,N$), whereas the elementary units will be indexed in the following by m.

Applying the equation of motion for the spin operators and performing the two-dimensional in-plane Fourier transformation, one obtains the following linearized equation for the spin-wave amplitudes u_l, which is valid in the low-temperature limit:[6]

$$(E-E_l)u_l+S_lJ_{l,l+1}u_{l+1}+S_lJ_{l,l-1}u_{l-1}=0 , \tag{2}$$

where S_l is the spin number for magnetic atoms in the lth atomic plane and E_l is defined as

$$\begin{aligned} E_l=&g_l\mu_0\mu_B H_l^{\rm eff}+4S_lJ_l(1-\gamma_\parallel)+S_{l+1}J_{l,l+1} \\ &+S_{l-1}J_{l,l-1} , \end{aligned} \tag{3}$$

with $\gamma_\parallel$ given by the expression

$$\gamma_\parallel=\tfrac{1}{2}[\cos(q_y a)+\cos(q_z a)] , \tag{4}$$

and $\mathbf{q}$ being the two-dimensional in-plane wave vector. The effective field $H_l^{\rm eff}$ which occurs in Eq. (3) consists of the external field H_0 and anisotropy contribution $H_l^{\rm an}$:

$$H_l^{\rm eff}=H_0+H_l^{\rm an}=H_0+\frac{2D_l}{g_l\mu_0\mu_B}\left[S_l-\frac{1}{2}\right] . \tag{5}$$

To obtain Eq. (2) we used the random-phase approximation for all terms arising from the exchange and Zeeman parts of the Hamiltonian (1). In the expressions which follow from the single-ion anisotropy term, we used the approximation

$$S^+S^z+S^zS^+\rightarrow 2(\langle S^z\rangle+\tfrac{1}{2})S^+ , \tag{6}$$

which is consistent with the Lines approximation in the zero-temperature limit. Restricting considerations to low T, we finally replaced the thermal average $\langle S^z\rangle$ by its zero-temperature value $\langle S^z\rangle=-S$.

It is worth noting that the model discussed here applies also to ferromagnetic multilayers with the easy axis and external magnetic field along the stacking direction. The macroscopic demagnetization field, if necessary, can be simply taken into account by replacing Eq. (5) by a similar one with an additional term $H_l^{\rm dem}$: $H^{\rm eff}=H_0+H_l^{\rm an}+H_l^{\rm dem}$.

Equation (2) is a recurrence equation for the spin-wave amplitudes u_l. For further purposes we write this equation in the matrix form

$$\begin{bmatrix} u_{l+1} \\ u_l \end{bmatrix}=\underline{M}_l\begin{bmatrix} u_l \\ u_{l-1} \end{bmatrix} , \tag{7}$$

where

$$\underline{M}_l=\begin{bmatrix} (E_l-E)/S_lJ_{l,l+1} & -J_{l,l-1}/J_{l,l+1} \\ 1 & 0 \end{bmatrix} . \tag{8}$$

The transfer matrix $\underline{M}_l$ obeys the condition

$$\det M_l=J_{l,l-1}/J_{l,l+1} . \tag{9}$$

Because of translational symmetry along the stacking direction, the material parameters fulfill the conditions $D_{mN+n}=D_n$, $S_{mN+n}=S_n$, $g_{mN+n}=g_n$, and $J_{mN+n}=J_n$ for $n=1,2,\ldots,N$. Apart from this one has $J_{mN+n,mN+n+1}=J_{n,n+1}$ for $n=1,2,\ldots,N-1$. The exchange coupling across the interface between two adjacent elementary units will be denoted as $J_{1,N}$.

Consider now the mth elementary unit. Applying Eq. (7) progressively to all N atomic planes in the elementary unit, one finds

$$\begin{bmatrix} u_{mN+1} \\ u_{mN} \end{bmatrix} = \underline{M} \begin{bmatrix} u_{(m-1)N+1} \\ u_{(m-1)N} \end{bmatrix} , \tag{10}$$

where

$$\underline{M} = \underline{M}_N \underline{M}_{N-1} \cdots \underline{M}_2 \underline{M}_1 . \tag{11}$$

We have made use here of the fact that $\underline{M}_{mN+n} = \underline{M}_n$ ($n = 1,2,\ldots,N$) due to the periodicity along the stacking direction. From Eq. (9) it follows that

$$\det \underline{M} = 1 . \tag{12}$$

In the following sections we will use the matrix $\underline{M}$ to describe the spin-wave modes in infinite, semi-infinite, and finite layered systems, as well as in structures which contain some layered defects. Before this, however, we will analyze the eigenvalues and eigenvectors of the matrix $\underline{M}$.

III. EIGENVALUE PROBLEM FOR THE MATRIX $\underline{M}$

Consider now the eigenvalue equation for the matrix $\underline{M}$:

$$\underline{M}\underline{u} = \lambda \underline{u} . \tag{13}$$

Taking into account Eq. (12), one obtains the following equation for the eigenvalues λ:

$$\lambda^2 - \lambda(M_{11} + M_{22}) + 1 = 0 . \tag{14}$$

If λ obeys Eq. (14), then λ^{-1} fulfills it, too. Consequently, both eigenvalues of the matrix $\underline{M}$, denoted in the following as λ_+ and λ_-, fulfill the conditions

$$\lambda_+ \lambda_- = 1 , \tag{15a}$$

$$\lambda_+ + \lambda_- = M_{11} + M_{22} . \tag{15b}$$

Since the elements of the matrix $\underline{M}$ are real, one can distinguish the following three possibilities.

(i) If $-2 \le M_{11} + M_{22} \le 2$, then the eigenvalues λ_+ and λ_- are complex and $\lambda_- = \lambda_+^*$ (with $\lambda_+ \neq \lambda_-$), except for $M_{11} + M_{22} = 2$ when $\lambda_+ = \lambda_- = 1$ and for $M_{11} + M_{22} = -2$ when $\lambda_+ = \lambda_- = -1$. Both eigenvalues can be written in the form

$$\lambda_\pm = \exp(\pm iQL) , \tag{16}$$

with Q real and with $L = Na$ being the thickness of the elementary unit. The upper limit $M_{11} + M_{22} = 2$ corresponds to $Q = 0$, whereas the lower one $M_{11} + M_{22} = -2$ corresponds to $QL = \pm\pi$.

(ii) For $M_{11} + M_{22} > 2$ the eigenvalues λ_+ and λ_- are real and positive with $\lambda_+ \neq \lambda_-$. One may write them as

$$\lambda_\pm = \exp(\pm \kappa L) , \tag{17a}$$

with κ real and $\kappa > 0$ by definition.

(iii) For $M_{11} + M_{22} < -2$ the eigenvalues are real and negative with $\lambda_+ \neq \lambda_-$. One may write them in the form

$$\lambda_\pm = \exp[\pm(i\pi + \kappa L)] , \tag{17b}$$

with κ real and $\kappa > 0$ by definition.

Region (i) corresponds to the bands of bulk waves, whereas regions (ii) and (iii) are the regions where the surface *acoustic* and *optic* modes can propagate. It is worth noting that the eigenvalues of the matrix $\underline{M}$ are degenerate only at the boundary of region (i), i.e., for $M_{11} + M_{22} = \pm 2$.

The parameter Q or κ can be found from the following explicit expression for $\lambda_\pm$:

$$\lambda_\pm = \tfrac{1}{2}\{M_{11} + M_{22} \pm \xi(M_{11} + M_{22}) \times [(M_{11} + M_{22})^2 - 4]^{1/2}\} , \tag{18}$$

where $\xi(z) = 1$ for $|z| \le 2$ and $\xi(z) = \mathrm{sgn}(z)$ for $|z| > 2$.

If $\underline{M}$ is nondiagonal, then the eigenvectors $\underline{u}_\pm$ corresponding to $\lambda_\pm$ can be taken in the form

$$\underline{u}_\pm = \begin{bmatrix} 1 \\ (\lambda_\pm - M_{11})/M_{12} \end{bmatrix} , \tag{19a}$$

if $M_{12} \neq 0$, or

$$\underline{u}_\pm = \begin{bmatrix} (\lambda_\pm - M_{22})/M_{21} \\ 1 \end{bmatrix} , \tag{19b}$$

if $M_{21} \neq 0$. The case of diagonal $\underline{M}$ is a trivial one. For $M_{11} + M_{22} \neq \pm 2$ the eigenvectors span the space in which the matrix $\underline{M}$ is defined.

IV. BULK MODES

The normal modes which can propagate in the infinite layered structures under consideration are described by the eigenvalues $\lambda_\pm = \exp(\pm iQL)$ [region (i)]. To find the appropriate dispersion equation, one can use Eq. (15b), which gives

$$\cos(QL) = \tfrac{1}{2}(M_{11} + M_{22}) = \tfrac{1}{2}\mathrm{Tr}\underline{M} . \tag{20}$$

The above relation is the general dispersion equation for the bulk modes propagating in the layered structures under consideration. The bulk waves are simply Bloch-type waves with the wave-vector component along the stacking direction in the range of the first Brillouin zone of the superlattice, $-\pi/L \le Q \le \pi/L$. In other words, the envelope function of a bulk solution in superlattices varies oscillatorylike along the superlattice direction. However, the corresponding solutions inside constituting materials do not necessarily have to behave oscillatorylike. For example, the bulk modes in a two-component layered structure can be composed of bulk-type (oscillatorylike) solutions in one material and surface-type (exponentially decaying) solutions in the other one.

V. SURFACE MODES

Consider now the semi-infinite structure which occupies the half-space $x > 0$. Let the surface atomic plane correspond to $l = n = 1$ and the elementary unit at the surface to $m = 1$. The parameters at the surface differ, in general, from the appropriate ones in the bulk. We assume that the exchange constant for NN's lying in the

surface atomic plane is $J_s \neq J_1$. Apart from this we assume that the anisotropy constant D_s for the surface atoms differs from the corresponding bulk value.

To describe surface solutions of Eq. (2), we will make use of Eq. (10). For semi-infinite structures this equation, however, does not apply directly to the case of $m=1$. To make it applicable also to $m=1$, we replace all surface parameters by the corresponding bulk ones and introduce a fictitious atomic layer at the surface, which corresponds to $l=0$ and $n=N$ (i.e., described by the parameters corresponding to the Nth atomic plane in the elementary unit). The $l=1$ atomic plane of the fictitious system is then described by the same parameters as the corresponding atomic planes inside the structure. The equivalence of the systems with and without the fictitious layer will be set up by appropriate boundary conditions as described later.

Applying now Eq. (10) to the system with the fictitious atomic plane, one can write

$$\begin{bmatrix} u_{N+1} \\ u_N \end{bmatrix} = \underline{M} \begin{bmatrix} u_1 \\ u_0 \end{bmatrix} , \tag{21}$$

where u_0 is the amplitude at the fictitious layer.

Inside the regions where the surface modes can propagate, the eigenvectors $\underline{u}_+$ and $\underline{u}_-$ span the space, and so one can write the general solutions as a linear combination of $\underline{u}_+$ and $\underline{u}_-$. It is clear that only $\underline{u}_-$ can enter into the solutions which are exponentially localized at the surface. Thus one can write

$$\begin{bmatrix} u_1 \\ u_0 \end{bmatrix} = A\underline{u}_- , \tag{22}$$

where A is a constant. To consider both *acoustic* and *optic* surface modes simultaneously, we rewrite the eigenvalues in the regions (ii) and (iii) (see Sec. III) in the form

$$\lambda_\pm = \exp(\pm\beta L) , \tag{23}$$

where real and positive β,

$$\beta = \kappa , \tag{24a}$$

corresponds to the region where *acoustic* surface modes can exist [region (ii)], whereas β of the form

$$\beta L = i\pi + \kappa L , \tag{24b}$$

with κ real and positive, describes region (iii) where *optic* surface modes can propagate.

Applying Eq. (10) to Eq. (22), one obtains

$$\begin{aligned} \begin{bmatrix} u_{mN+1} \\ u_{mN} \end{bmatrix} &= A \exp(-\beta L m)\underline{u}_- \\ &= A(\pm 1)^m \exp(-\kappa L m)\underline{u}_- , \end{aligned} \tag{25}$$

with the upper (lower) sign corresponding to the *acoustic* (*optic*) surface solutions. To be a solution of the problem, expression (25) has to fulfill appropriate boundary conditions. In other words, it has to obey the equation of motion for the surface atomic plane.

Equation (2) applied to the surface atomic plane of the system without the fictitious atomic plane gives

$$(E - E_s)u_1 + S_1 J_{1,2} u_2 = 0 . \tag{26}$$

According to Eq. (3), E_s in the above equation is given by the formula

$$E_s = g_1 \mu_0 \mu_B H_s^{\text{eff}} + 4S_1 J_s (1-\gamma_\parallel) + S_2 J_{1,2} , \tag{27}$$

where H_s^{eff} is the effective field for the surface atomic plane, given by Eq. (5) for $l=1$, but with D_1 replaced by D_s.

On the other hand, Eq. (2) applied to the $l=1$ atomic plane of the system with the fictitious atomic layer leads to the relation

$$(E - E_1)u_1 + S_1 J_{1,2} u_2 + S_1 J_{1,N} u_0 = 0 , \tag{28}$$

where E_1 is given by Eq. (5) for $l=1$.

Equations (26) and (28) are equivalent, provided the following condition is imposed on the amplitudes u_1 and u_0:

$$\begin{aligned} S_1 J_{1,N} u_0 = [&J_{1,N} S_N - g_1 \mu_0 \mu_B \Delta H_s^{\text{eff}} \\ &- 4S_1 \Delta J_s (1-\gamma_\parallel)] u_1 , \end{aligned} \tag{29}$$

where ΔH_s^{eff} and ΔJ_s describe the deviations of the surface parameters from the corresponding bulk ones:

$$\Delta H_s^{\text{eff}} = H_s^{\text{eff}} - H_1^{\text{eff}} , \tag{30}$$

$$\Delta J_s = J_s - J_1 . \tag{31}$$

It is convenient to rewrite the condition (29) as

$$\begin{bmatrix} u_1 \\ u_0 \end{bmatrix} = C \begin{bmatrix} 1 \\ p \end{bmatrix} \equiv C\underline{\mathcal{P}} , \tag{32}$$

where C is a constant and

$$p = \frac{J_{1,N} S_N - g_1 \mu_0 \mu_B \Delta H_s^{\text{eff}} - 4S_1 \Delta J_s (1-\gamma_\parallel)}{J_{1,N} S_1} . \tag{33}$$

Equations (22) and (32) have to be satisfied simultaneously. The condition for the existence of a nontrivial solution gives then the dispersion equation

$$\lambda_- = pM_{12} + M_{11} . \tag{34a}$$

This equation can also be written in an equivalent form as

$$\lambda_- = \frac{1}{p} M_{21} + M_{22} . \tag{34b}$$

These equations can be expressed in terms of the transfer-matrix elements only by eliminating λ_- from them. Finally, one can write the general dispersion equation for surface modes in the form

$$p^2 M_{12} - M_{21} + p(M_{11} - M_{22}) = 0 . \tag{35}$$

From Eq. (35) one obtains a set of its solutions $E = E^s$. However, not all of them, in general, describe the surface modes. To find true surface solutions, one has to verify the localization conditions [or equivalently Eq. (34a) or (34b)]. For each E^s one can calculate the corresponding eigenvalue $\lambda = \lambda^s$ from one of the relations

$$\lambda^s = pM_{12} + M_{11}\Big|_{E=E^s} = \frac{1}{p}M_{21} + M_{22}\Big|_{E=E^s} . \quad (36)$$

If the condition

$$|\lambda^s| < 1 \quad (37a)$$

is fulfilled, then, according to Eqs. (23) and (24), the corresponding solution describes a surface mode—*acoustic* for positive λ^s and *optic* for negative λ^s. If the above condition is not fulfilled, the corresponding solution does not describe a surface wave. Taking into account Eqs. (23) and (24), one can rewrite the condition (37a) in the equivalent form

$$\mathrm{Re}\beta^s > 0 , \quad (37b)$$

where β^s is defined by the relation $\lambda^s = \exp(-\beta^s L)$. If this condition is fulfilled, then the case with $\mathrm{Im}\beta^s = 0$ describes the *acoustic* surface wave, whereas with $\mathrm{Im}\beta^s = \pi/L$ corresponds to the *optic* surface mode.

The surface solutions discussed above are the solutions for which the envelope function vanishes exponentially toward the interior of the structure. However, the corresponding solution inside a particular material does not have to be of surface type. For example, the surface waves in a semi-infinite two-component superlattice can be composed of bulk-type solutions in one material and surface-type solutions in the other.

VI. SUPERLATTICES WITH DEFECTS

In this section we will consider infinite structures in which one elementary unit differs from the remaining ones, breaking this way the translational symmetry along the stacking direction. Let the complete elementary units which are adjacent to the layered defect correspond to the index $m_d - 1$ (on the left side) and $m_d + 1$ (on the right side). By a complete unit we mean here the one for which the transfer matrix acquires the same form as inside the structure. The modes which are localized at the layered defect can propagate region (ii) or (iii), where the eigenvalues of the matrix $\underline{M}$ are of the form $\lambda_\pm = \exp(\pm\beta L)$ with $\beta = \kappa$ or $\beta = i\pi/L + \kappa$ (κ real). For the localized solutions one may then write

$$\begin{bmatrix} u_{(m_d-1)N+1} \\ u_{(m_d-1)N} \end{bmatrix} = A\underline{u}_+ \quad (38)$$

and, similarly,

$$\begin{bmatrix} u_{(m_d-1)N+N_d+1} \\ u_{(m_d-1)N+N_d} \end{bmatrix} = B\underline{u}_- , \quad (39)$$

where N_d is the number of atomic planes in the m_dth unit (i.e., the defected one) and A and B are constants. One can easily verify that the wave functions decay exponentially away from the defect if Eqs. (38) and (39) are fulfilled.

Taking into account Eq. (7), one may write

$$\begin{bmatrix} u_{(m_d-1)N+N_d+1} \\ u_{(m_d-1)N+N_d} \end{bmatrix} = \underline{M}^d \begin{bmatrix} u_{(m_d-1)N+1} \\ u_{(m_d-1)N} \end{bmatrix} , \quad (40)$$

where $\underline{M}^d$ is the transfer matrix for the defected unit,

$$\underline{M}^d = \underline{M}^d_{N_d}\underline{M}^d_{N_d-1}\cdots\underline{M}^d_2\underline{M}^d_1 , \quad (41)$$

and the matrices $\underline{M}^d_n$ ($n = 1,2,\ldots,N_d$) correspond to the N_d atomic planes in the m_dth (defected) unit.

Equations (38)–(40), together with the condition for the existence of a nontrivial solution, lead to the following simple dispersion equation for the modes localized at the layered defect:

$$(\lambda_- - M_{22})M^d_{22} - (\lambda_+ - M_{22})M^d_{11} - M_{12}M^d_{21} - M_{21}M^d_{12} = 0 . \quad (42)$$

The above dispersion equation for the spin-wave modes in structures containing layered defects is a counterpart to Eq. (34a) [or (34b)], which was derived for surface waves. As previously, one can eliminate λ_- and λ_+ from Eq. (42) and rewrite it in a form which is analogous to Eq. (35). In other words, one can express the dispersion equation (42) in terms of the transfer matrix only. This form, however, is not so simple as Eq. (42), and so it will not be given here explicitly. In further numerical calculations, we will use Eq. (42).

VII. FINITE STRUCTURES

Consider now structures which contain a finite number m_0 of the complete elementary units. In a general case we assume an additional incomplete unit at one of the surfaces, say, the (m_0+1)st unit at the right surface, which consists of N_c atomic planes with $N_c < N$. The method we used for surface modes can be easily adapted to the present problem. For the same reasons as in Sec. V, we introduce two fictitious atomic planes—one at the left and the other one at the right surface of the system—replacing simultaneously all the surface parameters by the corresponding ones inside the structure. The parameters which we assign to the fictitious planes are those which follow from the periodicity of the structure along the superlattice direction. As in Sec. V, the equivalence of the systems with and without the fictitious atomic planes will be set up by appropriate boundary conditions. Such a condition for the left surface of both complete and incomplete structures is given by Eq. (32), in which we replace (to distinguish both surfaces) C, p, and $\underline{\mathcal{P}}$ by C_L, p_L, and $\underline{\mathcal{P}}_L$, respectively. The parameter p_L is given by Eq. (33) with $\Delta H_s^{\mathrm{eff}}$ and ΔJ_s (and, consequently, H_s^{eff}, J_s, and D_s) replaced by $\Delta H_L^{\mathrm{eff}}$ and ΔJ_L (H_L^{eff}, J_L, and D_L). In further consideration we will distinguish the complete and incomplete cases explicitly. Let us consider first the complete case.

A. Complete finite structures

The boundary condition for the right surface can be written in the form

$$\begin{bmatrix} u_{m_0N+1} \\ u_{m_0N} \end{bmatrix} = C_R \begin{bmatrix} p_R \\ 1 \end{bmatrix} \equiv C_R \underline{\mathcal{P}}_R \ , \tag{43}$$

where C_R is a constant and

$$p_R = \frac{J_{1,N}S_1 - g_N\mu_0\mu_B\Delta H_R^{\text{eff}} - 4S_N\Delta J_R(1-\gamma_{\parallel})}{J_{1,N}S_N} \ , \tag{44}$$

with

$$\Delta J_R = J_R - J_N \ , \tag{45}$$

$$\Delta H_R^{\text{eff}} = H_R^{\text{eff}} - H_N^{\text{eff}} \ , \tag{46}$$

and J_R and H_R^{eff} the exchange constant and effective field for the atomic plane at the right surface.

For arbitrary energy, except those particular values at which the eigenvalues of the matrix $\underline{M}$ are degenerate (i.e., with the exception of the values corresponding to the band edges of the bulk modes in infinite structures), one can write

$$\begin{bmatrix} u_1 \\ u_0 \end{bmatrix} + A\underline{u}_- + B\underline{u}_+ \ , \tag{47}$$

where A and B are constants. Applying now Eq. (10) to the above relation, one obtains

$$\begin{bmatrix} u_{m_0N+1} \\ u_{m_0N} \end{bmatrix} = A(\lambda_-)^{m_0}\underline{u}_- + B(\lambda_+)^{m_0}\underline{u}_+ \ . \tag{48}$$

Equations (32), (43), (47), and (48) have to be satisfied simultaneously. The condition for a nontrivial solution gives then the following equation for the allowed spin-wave modes:

$$\begin{aligned} &p_Lp_R[(\lambda_+)^{m_0-1} - (\lambda_-)^{m_0-1}] + [(\lambda_+)^{m_0+1} - (\lambda_-)^{m_0+1}] \\ &\quad + [p_LM_{12} - p_RM_{21} - (1+p_Lp_R)M_{22}] \\ &\quad \times [(\lambda_+)^{m_0} - (\lambda_-)^{m_0}] = 0 \ . \end{aligned} \tag{49}$$

The above expression is the general dispersion equation for spin-wave modes in finite structures and will be employed in the further numerical calculations. An equivalent form of the dispersion equation can be obtained by eliminating the eigenvalues λ_+ and λ_- from Eq. (49). Instead of this, however, we rewrite it in another form which allows direct comparison with some results available for uniform systems. To do this we write the eigenvalues of the matrix M in the form $\lambda_{\pm} = \exp(\pm ik_{\perp}L)$, where $k_{\perp}$ is real ($k_{\perp} = Q$) for region (i), $k_{\perp} = -i\kappa$ for region (ii), and $k_{\perp} = \pi/L - i\kappa$ for region (iii). Equation (49) can be then rewritten in the equivalent form

$$\begin{aligned} &p_Lp_R\{\sin[k_{\perp}L(m_0-1)] - M_{22}\sin(k_{\perp}Lm_0)\} \\ &\quad + (p_LM_{12} - p_RM_{21})\sin(k_{\perp}Lm_0) \\ &\quad + \sin[k_{\perp}L(m_0+1)] - M_{22}\sin(k_{\perp}Lm_0) = 0 \ . \end{aligned} \tag{50}$$

B. Structures with an incomplete unit at one surface

Consider now structures with the incomplete elementary unit at the right surface. Let this unit consist of N_c atomic planes with $0 < N_c < N$. The boundary condition at the right surface leads now to the equation

$$\begin{bmatrix} u_{m_0N+1} \\ u_{m_0N} \end{bmatrix} = C_R(\underline{M}^c)^{-1} \begin{bmatrix} p_R \\ 1 \end{bmatrix} = C_R(\underline{M}^c)^{-1}\underline{\mathcal{P}}_R \ , \tag{51}$$

where the parameter p_R is now equal,

$$p_R = \frac{J_{N_c,N_c+1}S_{N_c+1} - g_{N_c}\mu_0\mu_B\Delta H_R^{\text{eff}} - 4S_{N_c}\Delta J_R(1-\gamma_{\parallel})}{J_{N_c,N_c+1}S_{N_c}} \ , \tag{52}$$

with

$$\Delta J_R = J_R - J_{N_c} \ , \tag{53}$$

$$\Delta H_R^{\text{eff}} = H_R^{\text{eff}} - H_{N_c}^{\text{eff}} \ , \tag{54}$$

and J_R and H_R^{eff} the exchange constant and effective field for the atomic plane at the right surface. The matrix $\underline{M}^c$ in Eq. (51) is defined as

$$\underline{M}^c = \underline{M}_{N_c}\underline{M}_{N_c-1}\cdots\underline{M}_2\underline{M}_1 \ . \tag{55}$$

Equation (51) describes the boundary condition for the incomplete structure. It has the same form as for the complete case, but with $\underline{\mathcal{P}}_R$ replaced by $(\underline{M}^c)^{-1}\underline{\mathcal{P}}_R$. In other words, one may say that the incomplete case has been replaced by a complete one with some effective boundary condition which includes the dynamical properties of the atomic planes in the incomplete unit.

Instead of Eq. (49), one obtains now the following dispersion equation for the spin-wave modes:

$$p_L(p_R M^c_{22}-M^c_{12})[(\lambda_+)^{m_0-1}-(\lambda_-)^{m_0-1}]+(-p_R M^c_{21}+M^c_{11})[(\lambda_+)^{m_0+1}-(\lambda_-)^{m_0+1}]$$
$$+\{p_L(-p_R M^c_{21}+M^c_{11})M_{12}-(p_R M^c_{22}-M^c_{12})M_{21}-[(-p_R M^c_{21}+M^c_{11})+p_L(p_R M^c_{22}-M^c_{12})]M_{22}\}$$
$$\times[(\lambda_+)^{m_0}-(\lambda_-)^{m_0}]=0\ , \quad (56)$$

which can also be written in the equivalent form

$$(p_R M^c_{22}-M^c_{12})(p_L\{\sin[k_\perp L(m_0-1)]-M_{22}\sin(k_\perp L m_0)\}-M_{21}\sin(k_\perp L m_0))$$
$$+(-p_R M^c_{21}+M^c_{11})\{\sin[k_\perp L(m_0+1)]-M_{22}\sin(k_\perp L m_0)+p_L M_{12}\sin(k_\perp L m_0)\}=0\ . \quad (57)$$

In this and preceding sections, we have derived some general analytical expressions which determine the spin-wave spectrum in infinite, semi-infinite, finite, and defected layered periodical structures. They are valid for a large class of systems. In the following sections we will consider explicitly some of them. The simplest case the derived formulas apply to is the uniform system, which in our notation corresponds to $N=1$. Let us begin with this simple case.

VIII. UNIFORM SYSTEM

There is a rich literature on the spin-wave spectrum in uniform systems. It seems, however, to be advisable to discuss this particular case briefly and generalize some of known results.

It is convenient now to introduce dimensionless units for the spin-wave energy and effective fields according to the definitions

$$\tilde{E}=E/JS\ , \quad (58)$$

$$h^{\rm eff}=g\mu_0\mu_B H^{\rm eff}/JS\ . \quad (59)$$

The atomic plane index at all material parameters is now meaningless and has been dropped in the above formulas. Apart from this we define the parameters

$$\epsilon_{\|s}=J_s/J\ , \quad (60)$$

$$\Lambda=1-\gamma_\|\ . \quad (61)$$

In this notation the transfer matrix $\underline{M}$ has the form

$$\underline{M}=\begin{bmatrix}2+4\Lambda+h^{\rm eff}-\tilde{E} & -1\\ 1 & 0\end{bmatrix}\ . \quad (62)$$

Equation (20) gives the well-known expression for the spectrum of bulk waves. Following Sec. V, one can also easily determine the spectrum of surface modes. The appropriate formula for the spin-wave energy $\tilde{E}^s$ acquires now the form

$$\tilde{E}^s=h^{\rm eff}+\frac{4\Lambda[1+4\Lambda\epsilon_{\|s}(1-\epsilon_{\|s})+\Delta h_s^{\rm eff}(1-2\epsilon_{\|s})]-(\Delta h_s^{\rm eff})^2}{1+4(1-\epsilon_{\|s})\Lambda-\Delta h_s^{\rm eff}}\ , \quad (63)$$

where $\Delta h_s^{\rm eff}=h_s^{\rm eff}-h^{\rm eff}$. For $h_s^{\rm eff}=h^{\rm eff}=0$, Eq. (63) reduces to the one derived by Wolfram and DeWames[30] [their Eq. (2.21)]. The surface *acoustic* waves exist if the following condition is fulfilled:

$$4(1-\epsilon_{\|s})\Lambda-\Delta h_s^{\rm eff}>0\ , \quad (64)$$

whereas the existence condition for the surface *optic* modes takes the form

$$4(1-\epsilon_{\|s})\Lambda-\Delta h_s^{\rm eff}<-2\ . \quad (65)$$

The dispersion equation (50) for the spin-wave spectrum of finite structures takes now the form

$$p_L p_R\sin[k_\perp(m_0-1)a]-(p_L+p_R)\sin(k_\perp m_0 a)$$
$$+\sin[k_\perp(m_0+1)a]\ . \quad (66)$$

In this simple case m_0 is the number of atomic planes in the system. Solutions of the above equation for symmetrical and asymmetrical boundary conditions have been intensively studied by Puszkarski.[31]

Let us pay now a little more attention to spin-wave modes in an infinite ferromagnet in which a number of impurity atomic planes is embedded. A similar problem has been recently studied by Chen and Cottam,[32] who analyzed the spin-wave spectrum of a semi-infinite ferromagnet in which a single atomic plane of impurity spins is embedded close to the surface. Here we consider an infinite system, but with a few atomic planes of impurity spins, which are in direct contact or are separated by a few atomic planes of the host spins. To apply the formulas derived in Sec. VI, we treat the whole block of atomic planes, which includes all impurity planes, as a single layered defect embedded in the uniform system.

For convenience we change now a little the notation for all parameters which describe the uniform system: $S\rightarrow S_A$, $J\rightarrow J_A$, $h^{\rm eff}\rightarrow h_A^{\rm eff}$, and $g\rightarrow g_A$. To describe the impurity planes, we define the parameters

$$\eta = S_B / S_A \ , \tag{67}$$

$$\epsilon = J_B / J_A \ , \tag{68}$$

$$\alpha = J_{AB} / J_A \ , \tag{69}$$

where S_B is the spin number for impurities, J_B is the exchange constant between two impurities, and J_{AB} denotes the exchange constant between the impurity and host spins. Consequently, Eqs. (58) and (59) will be replaced by

$$\tilde{E} = E / J_A S_A \ , \tag{70}$$

$$h^{\rm eff}_{A(B)} = g_{A(B)} \mu_0 \mu_B H^{\rm eff}_{A(B)} / J_A S_A \ . \tag{71}$$

The spin-wave modes are described by Eq. (42) with the transfer matrix $\underline{M}^d$ for the layered defect given by Eq. (41). In Fig. 1 some numerical results are presented for the spin-wave spectrum of a single impurity plane (solid line), two adjacent impurity planes (dashed lines), and three adjacent atomic planes of impurity spins (dotted lines). The spin-wave energy is plotted there against the parameter Λ for vanishing external and anisotropy fields. For the parameters assumed there, one finds, respectively, one, two, and three branches of the localized modes, which split off from the top of the band of bulk modes of the host system (hatched region in Fig. 1).

In Fig. 2 the spin-wave spectrum for two impurity atomic planes in direct contact (dashed lines) and separated by a single atomic plane of the host spins (solid lines) is shown for the same parameters as in Fig. 1. As one could expect, the splitting of both localized modes is smaller in the case of separated impurity planes than in the case of adjacent ones. Generally, one can find that the modes localized at well-separated similar impurity planes do not couple and are degenerate.

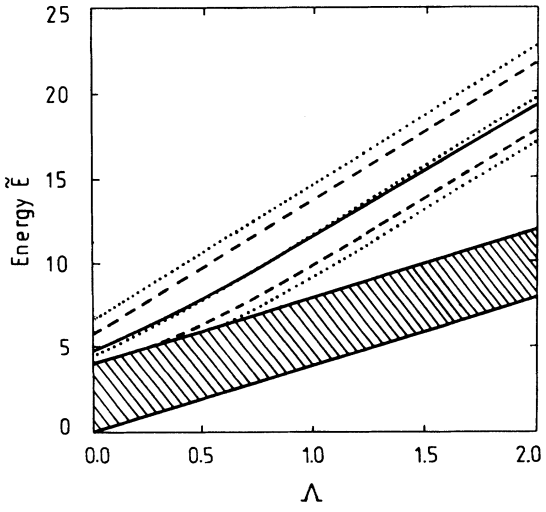

FIG. 1. Dispersion curves of the spin-wave modes localized at the single impurity plane (solid line), two adjacent impurity planes (dashed line), and three impurity planes in direct contact (dotted line) in an infinite uniform system. The other parameters assumed here are $\eta = 1$, $\epsilon = 2$, $\alpha = 1.4$, and $h^{\rm eff}_A = h^{\rm eff}_B = 0$.

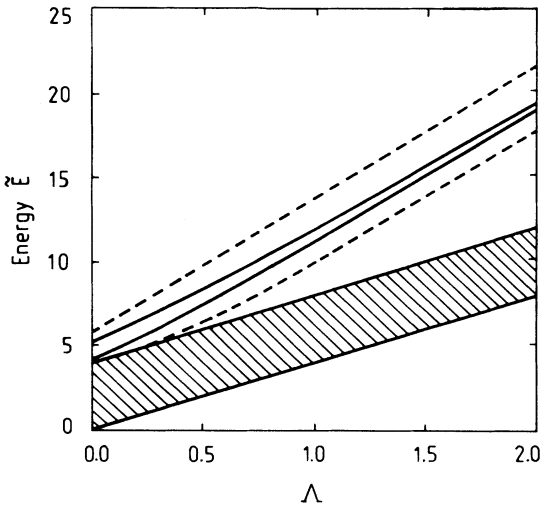

FIG. 2. Spin-wave modes localized at two adjacent impurity planes (dashed line) and two impurity planes separated by a single plane of host spins (solid line) in an infinite uniform system. The other parameters are the same as in Fig. 1.

IX. TWO ATOMIC PLANES IN THE ELEMENTARY UNIT

Consider now layered structures with two different atomic planes in the elementary unit, which correspond to two different materials A and B ($n = 1 \rightarrow A$ and $n = 2 \rightarrow B$). As in the preceding section, we introduce the dimensionless units for energy and effective fields according to the definitions (70) and (71) and the dimensionless parameters given by Eqs. (67)–(69) with J_{AB} being the exchange constant between spins of kind A and B. The corresponding transfer matrix $\underline{M}$ takes now the form

$$\underline{M} = \begin{bmatrix} \frac{1}{\eta\alpha^2}(\tilde{E}-\tilde{E}_2)(\tilde{E}-\tilde{E}_1) & \frac{1}{\eta\alpha}(\tilde{E}-\tilde{E}_2) \\ -\frac{1}{\alpha}(\tilde{E}-\tilde{E}_1) & -1 \end{bmatrix} , \tag{72}$$

with

$$\tilde{E}_1 = h^{\rm eff}_A + 4\Lambda + 2\alpha\eta \ , \tag{73a}$$

$$\tilde{E}_2 = h^{\rm eff}_B + 4\epsilon\eta\Lambda + 2\alpha \ . \tag{73b}$$

According to Eq. (20), the spectrum of bulk modes is given by the equation

$$(\tilde{E}-\tilde{E}_1)(\tilde{E}-\tilde{E}_2) = 2\eta\alpha^2(1+\cos QL) \ , \tag{74}$$

for $-\pi/L \le Q \le \pi/L$ with $L = 2a$.

Equation (35) for the surface modes takes now the explicit form

$$p^2(\tilde{E}-\tilde{E}_2)\alpha + (\tilde{E}-\tilde{E}_1)\eta\alpha + p(\tilde{E}-\tilde{E}_1)(\tilde{E}-\tilde{E}_2) = 0 \ , \tag{75}$$

where

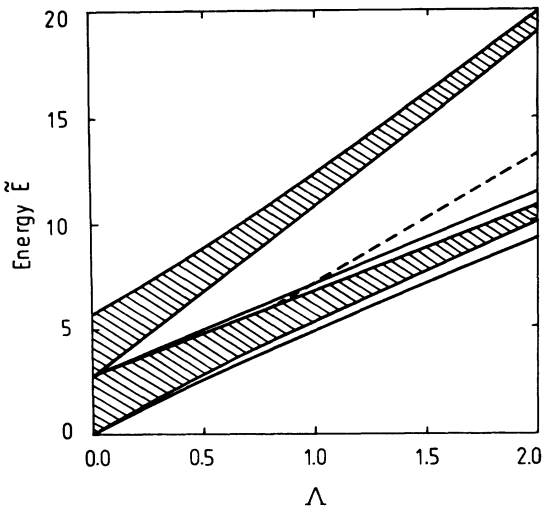

FIG. 3. Spectrum of bulk (hatched regions) and surface (dashed line) modes in a semi-infinite structure containing two different atomic planes in the elementary unit. The solid lines describe the localized modes which occur when a sequence of two adjacent atomic planes of spins A appears in the structure. The other parameters are $\eta=1$, $\epsilon=2$, $\alpha=1.4$, $h_A^{\rm eff}=h_B^{\rm eff}=h_s^{\rm eff}=0$, and $\epsilon_{\|s}=1.5$.

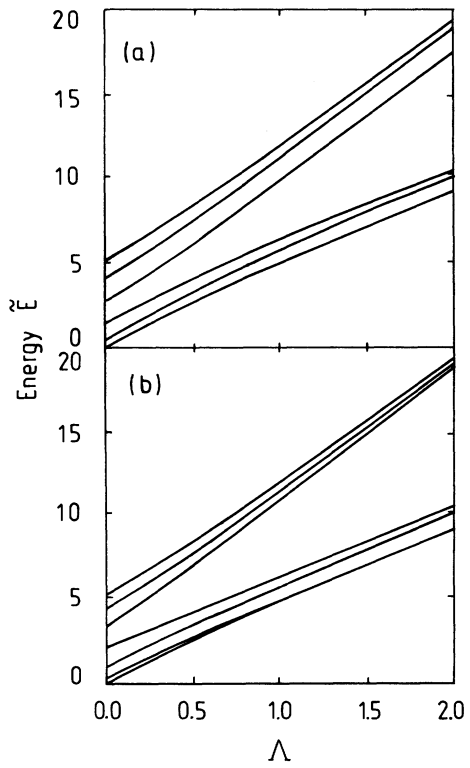

FIG. 4. Spectrum of spin-wave modes in a structure composed of (a) three complete elementary units and (b) three complete elementary units followed by the first atomic plane of the fourth unit. The parameters assumed here are $\eta=1$, $\epsilon=2$, $\alpha=1.4$, $h_A^{\rm eff}=h_B^{\rm eff}=h_L^{\rm eff}=h_R^{\rm eff}=0$, and $\Delta J_L=\Delta J_R=0$.

$$p=\eta-\Delta h_s^{\rm eff}/\alpha-4\Lambda(\epsilon_{\|s}-1)/\alpha\ , \tag{76}$$

with $\Delta h_s^{\rm eff}=h_s^{\rm eff}-h_A^{\rm eff}$ and $\epsilon_{\|s}=J_s/J_A$. Only those solutions of Eq. (75), which fulfill appropriate localization conditions as described in Sec. V, correspond to surface modes.

An exemplary spin-wave spectrum of bulk and surface modes is shown in Fig. 3, where the hatched regions correspond to the two bands of the bulk modes and the dashed line represents the surface modes. For the parameters assumed in Fig. 3, the surface mode is of the *optic* type.

Modes in structures with one defected unit are determined by Eq. (42) with the transfer matrix $\underline{M}^d$ corresponding to the layered defect. The solid lines in Fig. 3 represent the localized modes for the case when two adjacent planes of A spins occur once in the structure.

If the structure consists of a finite number of atomic planes, the appropriate spin-wave spectrum is determined by Eqs. (49) and (56), respectively, for complete and incomplete cases. In Fig. 4(a) the spectrum is shown for structures containing three complete units (structure *ABABAB*). The corresponding spectrum in the incomplete case is shown in Fig. 4(b) for the system containing an additional A atomic plane (structure *ABABABA*). In the first case there are six different modes, whereas in the second one seven different spin-wave modes are obtained.

X. STRUCTURES WITH SIX ATOMIC PLANES IN THE ELEMENTARY UNIT

For small N one can derive some more or less explicit analytical expressions which describe the appropriate spin-wave spectrum. For large N, however, these expressions become cumbersome and the problem is well fitted

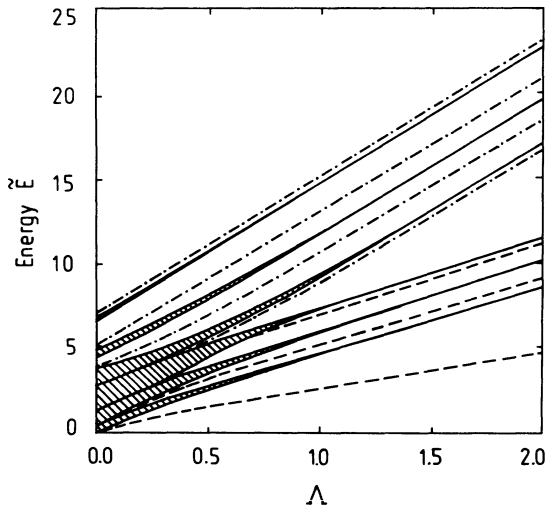

FIG. 5. Bulk (shaded regions) and surface (dashed line) spin-wave modes of the semi-infinite structures with $N=6$ atomic planes in the elementary unit (elementary units of the type *AAABBB*). The dot-dashed lines describe the localized modes in infinite structures in which one elementary unit contains four atomic planes of spins B. The other parameters are $\eta=1$, $\epsilon=2$, $\alpha=1.4$, $h_A^{\rm eff}=h_B^{\rm eff}=h_s^{\rm eff}=0$, and $\epsilon_{\|s}=0.5$.

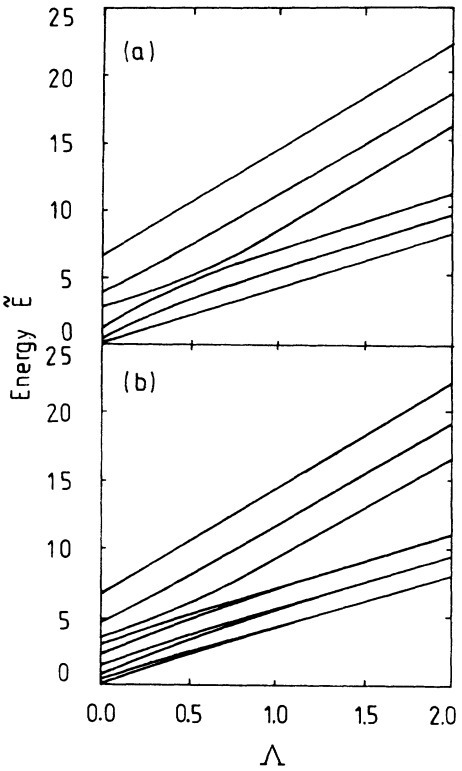

FIG. 6. Spin-wave modes in finite structures of the types (a) $AAABBB$ and (b) $AAABBBAAA$ for the parameters $\eta=1$, $\epsilon=2$, $\alpha=1.4$, $h_A^{\rm eff}=h_B^{\rm eff}=h_L^{\rm eff}=h_R^{\rm eff}=0$, and $\Delta J_L=\Delta J_R=0$.

for numerical calculations. Results of such calculations for structures with $N=6$ atomic planes in the elementary unit are shown in Figs. 5 and 6. The elementary unit is assumed there to consist of three atomic planes of the material A followed by three atomic planes of the material B. The shaded areas in Fig. 5 (limited by the solid lines) correspond to the bands of bulk modes, whereas the dashed lines represent the surface modes. For the parameters assumed there, one finds three different surface modes. The lower and upper branches are of *acoustic* type, whereas the one in between is of *optic* type. The remaining curves in Fig. 5 represent the modes localized at a defected unit which contains a sequence of four atomic planes (instead of three) of the material B.

The spin-wave spectrum in finite complete and incomplete structures is shown in Fig. 6. The complete structure consists of one elementary unit (six atomic planes), whereas the incomplete one consists of one elementary unit and the first three atomic planes of the next unit. Thus the incomplete case is equivalent to a symmetrical sandwich structure. For large values of Λ, some of the modes are degenerate for the parameters assumed for numerical calculations. This is consistent with Fig. 5, where the bands become very narrow for increasing Λ.

XI. CONCLUDING REMARKS

The transfer-matrix approach to layered structures seems to be very effective one. Apart from the applications described in this paper, it can also be used in many other situations such as, for instance, for the description of interface modes between two bulk materials or for the description of excitations in disordered layered structures.

One may also look at the transfer-matrix approach from another point of view. It is easy to note that Eq. (10) gives the relation

$$u_{(m+1)N+1}+(M_{11}+M_{22})u_{mN+1}+u_{(m-1)N+1}=0 . \quad (77)$$

This equation has a similar form to that for a uniform system:

$$u_{l+1}+(\tilde{E}-\tilde{E}_0)u_l+u_{l-1}=0 , \quad (78)$$

where $\tilde{E}_0=2+4\Lambda+h^{\rm eff}$. Comparison of the above two equations leads to the conclusion that the layered structure with N atomic planes in the elementary unit has been effectively replaced by a uniform system with one "effective atomic plane" in the elementary unit and with $E-\tilde{E}_0$ replaced by $(M_{11}+M_{22})=\mathrm{Tr}\underline{M}$. The transfer matrix $\underline{M}$ then allows one to consider layered structures as uniform systems which are described by Eq. (78), but with $E-\tilde{E}_0$ replaced by $\mathrm{Tr}\underline{M}$. The dynamical properties of the atomic planes in the elementary unit are contained in the transfer matrix $\underline{M}$. This is very similar to the problem we mentioned while considering the modes in incomplete finite structures, where the system with an incomplete last unit was replaced by a complete structure and the influence of the remaining atomic planes was included into some effective boundary conditions.

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