

Mean-field solution of a nonequilibrium random-exchange Ising-model system

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We report first-order mean-field results for *nonequilibrium* random-exchange lattice systems, namely, Ising-like models whose kinetics involve a simultaneous, random competition between ferromagnetic and antiferromagnetic interactions that generally induces nonequilibrium steady states. We consider the competition between symmetric bonds, $\pm J_0$, symmetric bonds competing with broken ones, $\pm J_0$ and $J=0$, nonsymmetric bonds, $|J_1|$ and $-|J_2|$, and some of those systems under an external magnetic field h . The time evolution and steady-state properties, including phase diagrams, are obtained for several lattice coordination numbers, e.g., the case of simple-cubic lattices of dimension $d \leq 3$. A comparison is made with existing exact results for $d=1$, $h=0$, and $\pm J_0$.

I. INTRODUCTION AND DEFINITION OF MODELS

There is a consensus that the so-called spin glass behavior detected in diluted metallic alloys¹ is a consequence of frustration. In fact, some unusual macroscopic observations in a class of magnetic materials at low temperature seem largely determined by the impossibility of satisfying all the interactions when the exchange energies are capable of both positive and negative values.²⁻⁶ While this idea has certainly produced fundamental progress, it may be said that the understanding of spin glasses is still an open problem.⁵⁻⁷ In addition to the fact that exact results remain scarce, one may note, for example, that nonequilibrium effects seem here to play a fundamental role. For example, under some conditions, frustration may prevent a system from reaching a single (equilibrium) steady state and cause dynamics to determine relevant macroscopic behavior. Therefore, the fact that there is at present, no solvable microscopic model that is accepted to capture all the essential features of ideal spin glasses, thus serving as a reference to studying real materials, has several origins. Namely, this is not only due to the resistance demonstrated by familiar equilibrium models of disordered systems to admit a precise analytical treatment, but also a consequence of the present relatively poor development of nonequilibrium statistical mechanics, which is probably relevant to the understanding of those materials. Hence there is significant interest in studying simple nonequilibrium model systems incorporating microscopic disorder. This belief has recently motivated a nonequilibrium version^{8,9} of the model studied by Edwards and Anderson.² That system, to be designated here as the nonequilibrium spin-glass model (NSGM), allows investigation of nonequilibrium steady states (eventually undergoing phase transitions and critical phenomena), which is an active area of research at present,⁸⁻²⁰ and might also allow the examination of the possible relation between nonequilibrium states and spin-glass behavior.

A simple version of the NSGM consists of a simple-hypercubic d -dimensional lattice Ω , whose sites sit interacting Ising spins in contact with a thermal bath at

temperature T . The latter, together with the action of some undetermined non-Hamiltonian agent, induces stochastic changes in a way that the probability $P(\mathbf{s}, t)$ of any spin configuration, $\mathbf{s} \equiv \{s_r = \pm 1, \mathbf{r} \in \Omega\}$, at time t satisfies a (Markovian) master equation,²¹⁻²³ i.e.,

$$\partial P(\mathbf{s}; t) / \partial t = \sum_{\mathbf{s}^r} [c(\mathbf{s}|\mathbf{s}^r)P(\mathbf{s}^r; t) - c(\mathbf{s}^r|\mathbf{s})P(\mathbf{s}; t)] . \quad (1.1)$$

Here, \mathbf{s}^r represents the configuration obtained from \mathbf{s} after flipping the spin at site \mathbf{r} , i.e., $s_r \rightarrow -s_r$, and $c(\mathbf{s}^r|\mathbf{s})$ stands for the corresponding transition rates per unit time. Unlike in more familiar cases, however, the latter describes here a competing dynamics involving different interactions. This peculiarity has two main effects; namely, it induces some extra randomness and dynamical frustration during the evolution, and the spin system may thus be asymptotically driven to a nonequilibrium steady state. Formally, the transition rates in this paper are defined^{8,9}

$$c(\mathbf{s}^r|\mathbf{s}) = [c(\mathbf{s}^r|\mathbf{s}; J)] \equiv \int_{-\infty}^{+\infty} dJ f(J) c(\mathbf{s}^r|\mathbf{s}; J) . \quad (1.2)$$

Here, J stands for a random variable, with (normalized) distribution $f(J)$, which represents the interaction strengths or bonds between pairs of spins, and $c(\mathbf{s}^r|\mathbf{s}; J)$ describe independent spin-flip or Glauber mechanisms²³ satisfying individually a detailed balance condition, i.e.,

$$c(\mathbf{s}^r|\mathbf{s}; J) = c(\mathbf{s}|\mathbf{s}^r; J) \exp(-\beta \Delta H_J) \quad (1.3)$$

where $\beta \equiv (k_B T)^{-1}$ and $\Delta H_J \equiv H(\mathbf{s}^r; J) - H(\mathbf{s}; J)$. This involves a set of Hamiltonians which, for simplicity, we will take to be of the nearest-neighbor (NN) Ising type with no field, i.e., $H(\mathbf{s}; J) = -J \sum_{\text{NN } \mathbf{s}_r, \mathbf{s}_r'} s_r s_r'$, where the sum is over all pairs of NN sites. A simple and natural choice for the elementary transition rates appearing in definition (1.2), is

$$c(\mathbf{s}^r|\mathbf{s}; J) = \phi(\beta \Delta H_J) , \quad (1.4)$$

where $\phi(X)$ is an arbitrary function except that it satisfies $\phi(X) = e^{-X} \phi(-X)$, $\phi(0) = 1$, and $\phi(X) \rightarrow 0$ as $X \rightarrow \infty$, in order to fulfill both (1.3) and appropriate boundary conditions.

Several different choices for the distribution $f(J)$ of bonds in (1.2) produce a number of interesting cases. The simplest situation occurs for $f(J)=\delta(J-J_0)$, where δ represents the Dirac delta function. Then, when J_0 is a positive (negative) constant, a single ferromagnetic (antiferromagnetic) Hamiltonian is involved, and any spin-flip rate satisfying (1.3) drives the system to the equilibrium state for temperature T and energy $H(\mathbf{s};J_0)$, whose nature is well known.²¹ The crossover situation one may imagine between those two limits is more intriguing. In fact, the (kinetic) competition of J 's will then induce the system to go asymptotically towards a steady state which is a nonequilibrium one in general, as if it is acted on by some external agent, whose nature depends on T , $f(J)$, $\phi(X)$, and $H(\mathbf{s};J)$. More precisely, when the bond distribution is

$$f(J)=p\delta(J-J_0)+(1-p)\delta(J+J_0), \quad J_0>0, \quad (1.5)$$

a superposition of two independent spin-flip mechanisms ensues, each one with its own coupling constant chosen at random from distribution $f(J)$. This means that kinetics attempts to change the interaction strength at each step to take the same value ($\pm J_0$ with respective probabilities p and $1-p$) all over the system. Note, however, that, in so far as the elementary spin-flip processes are local and the interactions in $H(\mathbf{s};J)$ are restricted to NN, so that only a small neighborhood of the spin s_r , involved by the attempted transition enters $\phi(\beta \Delta H_J)$, kinetics attempts to change interactions (only) around s_r . This justifies the name of NSGM given to the system.⁵ Actually, starting from an arbitrary spatial distribution of bonds (every one equal to each other, for instance) kinetics will soon establish a random spatial distribution which is a realization of $f(J)$ at any given time during the evolution. That is, instead of remaining frozen in, as in the Edwards-Anderson model,² the spatial distribution of bonds keeps changing with time in the NSGM system as if it were locally driven by a kind of very fast infinite-temperature mechanism. (This may be interpreted⁹ as a first approach to the actual situation in natural systems where atomic mobility might in practice induce a change with time of the spatial distribution of impurities.) The NSGM also differs from the annealed version of the spin-glass model,²⁴ where the bond disorder is instead in equilibrium at temperature T with the spin degrees of freedom. Another case of interest corresponds to the bond distribution

$$f(J)=q\delta(J)+(1-q)[p\delta(J-J_1)+(1-p)\delta(J+J_2)], \quad (1.6)$$

where $J_1, J_2 > 0$. Kinetics then involve a simultaneous competition between broken bonds and nonsymmetric ferro- and antiferromagnetic interactions which, in particular, is expected¹⁰ to induce a sort of percolation phenomena. Finally, the rectangular distribution

$$f(J)=\begin{cases} (2xJ_0)^{-1}, & \text{when } J_0(1-x) \leq J \leq J_0(1+x) \\ 0, & \text{otherwise.} \end{cases} \quad (1.7)$$

where J_0 and x are both constant positive-definite param-

eters, is also interesting; this corresponds to a random-exchange situation which involves a competition between ferro- and antiferromagnetic interactions (only) for $x > 1$.

The NSGM cases (1.5) and (1.6) have been previously solved exactly, under some conditions, for one-dimensional lattices.^{9,10} That already revealed some striking behavior which, as argued above, differs essentially from the more standard (quenched) spin-glass models and from its corresponding annealed versions. That solution was feasible because the steady solution of (1.1), i.e., the limit of $P(\mathbf{s};t)$ as $t \rightarrow \infty$, may be written as $P^{\text{st}}(\mathbf{s}) \propto \exp(-E(\mathbf{s}))$, and it follows in some cases that $E(\mathbf{s})$ exists and it has only a finite number of items. Namely, for $d=1$ in the absence of any external field, and for certain choices of the function $\phi(X)$, one simply gets $E(\mathbf{s}) = -K_e \sum_r s_r s_{r+1}$, with

$$K_e = -\frac{1}{4} \ln [[\phi(4K)] [\phi(-4K)]^{-1}];$$

here, $K \equiv \beta J$ and $[\]$ denotes the average defined by (1.2) and (1.4). Some partial exact results have also been reported for $d > 1$;⁹ namely, a simple (cellular-automaton) representation of the evolution of the system at zero temperature, and bounds for the location of a possible phase transition. Those studies for $d \geq 1$ have suggested, and demonstrated some times, very rich phase diagrams and a variety of critical behavior. This compels one to look for more general results when $d > 1$, and also to consider more complex one-dimensional versions than in previous studies. In particular, a great deal of interest exists in cases where a simple, short-ranged global or effective Hamiltonian $E(\mathbf{s})$ does not exist or cannot be found by using the techniques now at hand.¹¹

We report in this paper analytical and numerical results for NSGM systems obtained by means of a kinetic mean-field method in the pair approximation.¹² We are specifically concerned here about d -dimensional systems under the presence of a field h , i.e.,

$$H(\mathbf{s};J) = -J \sum_{\text{NN}} s_r s_{r'} + h \sum_r s_r \quad \text{for all } J, \quad (1.8)$$

where h is a constant, and we consider several lattice coordination numbers corresponding, for instance, to simple-cubic lattices with $1 \leq d \leq 3$, for which no effective Hamiltonian $E(\mathbf{s})$ is known. This generalizes the exact solutions found previously^{9,10} for $h=0$ and $d=1$. We investigate here the influence of kinetics on (nonequilibrium) steady states by considering different transition rates. In fact, while most of our results are for the algorithm by Metropolis *et al.*,²⁵

$$\phi(X) = \min(1, e^{-X}), \quad (1.9)$$

we also consider the cases^{26,13,22}

$$\phi(X) = 2(1 + e^X)^{-1} \quad (1.10)$$

and

$$\phi(X) = e^{-X/2}. \quad (1.11)$$

The influence of the distribution $f(J)$ on the steady-state properties is more conveniently evaluated when the model admits an exact solution; consequently, we shall re-

strict ourselves in this paper to the most familiar cases, i.e., to distributions (1.5)–(1.7). Two different versions will be considered for (1.6), i.e., the symmetric case $J_2 = J_1$ and the asymmetric case corresponding to the choice $J_2 = \frac{1}{2}J_1$.

II. DETAILS OF THE METHOD

Our method of solution is based on a kinetic mean-field approximation which has been successful in the study of various lattice problems involving spin flips, interchange processes, or both.^{12,15,16} The same method of solution has been applied recently²⁷ to a kinetic Ising model, whose dynamics involves a competition between two temperatures,²⁸ which essentially corresponds to one of the versions we consider here. Note that mean-field approximations are exceptionally convenient here because they are believed to be more realistic in the description of nonequilibrium phenomena than in equilibrium; actually, there are some recent indications in that sense.^{12–14,17–20,29}

The resulting description in this paper has a mean-field nature because it rests upon specific equations for the time evolution of mean local quantities obtained from the master equation (1.1) under well-defined hypotheses, which amount to neglecting some correlations. In order to make this explicit in the present case, where the system evolves via spin-flip processes, let $A = A(\mathbf{s}_D)$ represent a microscopic dynamical function which depends on the spin variables at a small compact set of sites or local domain D whose configuration is \mathbf{s}_D . The relevant averages are

$$\langle A \rangle_t = \sum_{\mathbf{s}} P(\mathbf{s}; t) A(\mathbf{s}_D) = \sum_{\mathbf{s}_D} Q(\mathbf{s}_D; t) A(\mathbf{s}_D),$$

where (using an obvious notation) $Q(\mathbf{s}_D; t) \equiv \sum_{\mathbf{s}_{S-D}} P(\mathbf{s}; t)$ represents the probability that D has configuration \mathbf{s}_D at time t . For any given domain D , define the set of interior sites to be denoted I , and the set of surface sites, S , as follows: $D = I \cup S$, $I \cap S = \emptyset$, and $I = \{\mathbf{r} \in D; \text{ such that if } \mathbf{r}' \text{ is an NN of } \mathbf{r} \text{ then } \mathbf{r}' \in D\}$. That is, I contains all the lattice sites in D which have all NN sites as elements of D , while any $\mathbf{r} \in S$ will have at least a NN site outside D . Then, one may conclude from the master equation (1.1) in the approximation of interest that

$$\partial \langle A(\mathbf{s}_D) \rangle_t / \partial t = \sum_{\mathbf{r} \in I} \sum_{\mathbf{s}_D} \delta A(\mathbf{s}_D; \mathbf{r}) c(\mathbf{s}_D^{\mathbf{r}} | \mathbf{s}_D) Q(\mathbf{s}_D; t). \quad (2.1)$$

Here, $\delta A(\mathbf{s}; \mathbf{r}) \equiv A(\mathbf{s}^{\mathbf{r}}) - A(\mathbf{s})$, and it is assumed that, given the function $c(\mathbf{s}^{\mathbf{r}} | \mathbf{s})$ with \mathbf{r} belonging to I , every other site involved by $c(\mathbf{s}^{\mathbf{r}} | \mathbf{s})$ belongs to D ; this condition is fulfilled by any familiar transition probability and, in particular, by rates (1.9)–(1.11). It should be noted that the temporal evolution of the system will proceed by performing only flips on the spins belonging to I , while the spins in S contribute to the energy and to the probability

of the configuration within D . In order to obtain (2.1) from (1.1) one also needs to assume that

$$\sum_{\mathbf{r} \in S} \sum_{\mathbf{s}} \delta A(\mathbf{s}_D; \mathbf{r}) c(\mathbf{s}^{\mathbf{r}} | \mathbf{s}) P(\mathbf{s}; t) = 0. \quad (2.2)$$

This is the fundamental approximation in the method; it amounts to destroying correlations between the spins in D and the rest of the system. Consequently, the effect of assumption (2.2) may become negligible as D tends to equal the whole lattice; in fact, varying the size of D modifies the order of the approximation insofar as one deals consequently (in a more detailed way) with the correlations inside the domain D .

To examine the meaning of (2.2) more deeply, one may note that the microscopic dynamical variables of interest may be written for all practical purposes as $A(\mathbf{s}_D) = \alpha(\mathbf{s}_{r,D}) + s_r \beta(\mathbf{s}_{r,D})$, for all $\mathbf{r} \in D$, where $\alpha(x)$ and $\beta(x)$ stand for arbitrary functions and $\mathbf{s}_{r,D}$ represents \mathbf{s}_D when the spin variable s_r is excluded. Thus, $\delta A(\mathbf{s}_D; \mathbf{r}) = -2s_r \beta(\mathbf{s}_{r,D})$ and (2.2) reduces to $\sum_{\mathbf{r} \in S} \langle s_r c(\mathbf{s}^{\mathbf{r}} | \mathbf{s}) \beta(\mathbf{s}_{r,D}) \rangle = 0$. For an arbitrary $A(\mathbf{s}_D)$, implying that $\beta(\mathbf{s}_{r,D})$ is also arbitrary, the latter condition indicates that $s_r c(\mathbf{s}^{\mathbf{r}} | \mathbf{s})$ is zero on the average for any \mathbf{r} belonging to the surface of D . That is, (2.2) is equivalent to neglecting some fluctuations in the local magnetization surrounding I . This (nonequilibrium) approximation is essentially analogous to the one in the familiar Bethe-Peierls theory for equilibrium (in particular, it is a first-order mean-field description) when D has a minimum size, while the consideration of larger domains may successively lead to higher-order approximations, as indicated above. The minimum size for D when kinetics consists of spin flips is such that I contains a single spin for any lattice dimension. Nevertheless, given that the system here is expected to present antiferromagneticlike order, one needs to consider clusters whose interior I contains (at least) two NN spins surrounded by $2(d-1)$ spins belonging to the surface S . This allows for the necessary definition of two sublattices; namely, each spin within I then belongs to a different sublattice.

In addition to the two mentioned assumptions, the explicit use of equation (2.1) and the need of consistency of the ensuing description requires further simplifications concerning $Q(\mathbf{s}_D; t)$. Namely, one may write (exactly)

$$Q(\mathbf{s}_D; t) = 1 + \langle s_{\mathbf{r} \in D} \rangle_t \sum_{\mathbf{r} \in D} s_{\mathbf{r}} + \langle s_{\mathbf{r} \in D} s_{\mathbf{r}' \in D} \rangle_t \sum_{\mathbf{r}, \mathbf{r}' \in D} s_{\mathbf{r}} s_{\mathbf{r}'} + \cdots \langle \prod_{\mathbf{r} \in D} s_{\mathbf{r}} \rangle_t \prod_{\mathbf{r} \in D} s_{\mathbf{r}},$$

but the averages here, which are defined as taken with $P(\mathbf{s}; t)$, are to be written in terms of (only) a few low-order correlation functions, thus introducing a new approximation in the description of the system. (Note that, in any case, this approximation needs to be consistent with the previous ones.) In the present case, we will approximate the average $\langle \prod_{\mathbf{r} \in D} s_{\mathbf{r}} \rangle_t$ above by a function of only $\langle s_{\mathbf{r} \in D} \rangle_t$ and $\langle s_{\mathbf{r} \in D} s_{\mathbf{r}' \in D} \rangle_t$, where \mathbf{r} and \mathbf{r}' are NN, in order to remain within a first-order mean-field description. This means in practice that

$$Q(\mathbf{s}_D; t) = Q(\mathbf{s}_D; \langle s \rangle, \langle ss \rangle_{\text{NN}}; t)$$

is to be written in terms of the following quantities only: the densities of up spins in sublattice i , to be denoted by x_i , with $i=1,2$ referring to the sublattices 1 and 2, respectively, and the independent densities of different kinds of NN pairs. In the present case, with two sites in I , if we indicate state up by $+$ and state down by $-$, one may have four different types of NN pairs, namely $++$, $--$, $+-$, and $-+$, where the two symbols always correspond to sublattices 1 and 2, respectively, whose densities will be designated in the following as z , w , v_1 , and v_2 , respectively; note that $w = 1 + z - x_1 - x_2$ and $v_1 = x_i - z$.

$$Q(\mathbf{s}_D; x_1, x_2, z; t) = p(s_1, s_2) p(+|s_1)^{N_1^+} p(-|s_1)^{2d-1-N_1^+} p(+|s_2)^{N_2^+} p(-|s_2)^{2d-1-N_2^+}. \quad (3.1)$$

Here, $p(s, s')$ represents the joint probability for the NN pair of spin variables (s, s') , $p(s|s') = p(s, s') p(s')^{-1}$ is a conditional probability, and $p(s)$ is the probability of s . Then, $p(+, +) = z$, $p(-, -) = w$, $p(+, -) = v_1$, and $p(-, +) = v_2$ as a consequence of the definitions in the preceding section. It is also convenient to refer to the density of down spins at each sublattice, to be denoted by $y_i (= 1 - x_i)$, and define the variables $m = \frac{1}{2}(m_1 + m_2)$ and $\Delta = \frac{1}{2}(m_1 - m_2)$, where $m_i = x_i - y_i$ stands for the magnetization density at each sublattice, $i=1,2$.

Combining Eqs. (3.1) and (2.1), it follows that the main independent equations describing the time evolution of the system in this approximation are

$$dx_i/dt = A_{ij}, \quad i, j = 1, 2, \quad i \neq j, \quad (3.2)$$

and

$$(4d-1)(dz/dt) = 2d(B_{12} + B_{21}), \quad (3.3)$$

where A_{ij} and B_{ij} depend strongly on the system dimension and on the form assumed for the elementary transitions rates, namely,

$$A_{ij} = \sum_{n=0}^{2d} \binom{2d}{n} [w^{2d-n} v_j^n y_i^{1-2d} \Phi_-(d-n) - z^{2d-n} v_i^n x_i^{1-2d} \Phi_+(d-n)] \quad (3.4)$$

and

$$B_{ij} = \sum_{n=0}^{2d} \binom{2d}{n} [nw^{2d-n} v_j^n y_i^{1-2d} \Phi_-(d-n) - (2d-n)z^{2d-n} v_i^n x_i^{1-2d} \Phi_+(d-n)], \quad (3.5)$$

where

$$\Phi_{\pm}(X) = \int dJ f(J) \phi(4X\beta J \pm 2\beta h), \quad (3.6)$$

and $f(J)$ and $\phi(X)$ are the functions introduced in (1.2) and (1.4), respectively.

Equations (3.2)–(3.5) may also be written in terms of

III. TIME EVOLUTION EQUATIONS FOR A d -DIMENSIONAL SYSTEM

When one refers to a d -dimensional system, the domain of interest (as defined above) may be characterized by the spin variables at the two sites in I , say s_1 and s_2 , and by the number of up spins within D_1 and D_2 , say N_1^+ and N_2^+ . D_1 and D_2 are defined such that $D_1 \cap D_2 = \emptyset$, $D_1 \cup D_2 = S$, and D_i contains the $2d-1$ sites (around the interior site i) which belong to S and are NN of i . Thus, the associated probabilities for the occurrence of a given domain configuration are, within the (pair) approximation of interest,

m , Δ , and z . Then, in the absence of a field ($h=0$) the leading terms have the following structure:

$$dm/dt = mg_1(z; \beta J), \quad (3.7)$$

$$d\Delta/dt = \Delta g_2(z; \beta J), \quad (3.8)$$

and

$$dz/dt = g_3(z; \beta J) \quad (3.9)$$

to first order in m and Δ . Here,

$$g_1(z; \beta J) \equiv \sum_{n=0}^{2d} \binom{2d}{n} z^{2d-n} \left(\frac{1}{2} - z\right)^n \times \left[2(2d-1) - \frac{2d-n}{z} \right] \Phi(d-n), \quad (3.10)$$

$$g_2(z; \beta J) \equiv \sum_{n=0}^{2d} \binom{2d}{n} z^{2d-n} \left(\frac{1}{2} - z\right)^n \times \left[2(d-1) - \frac{n}{v} \right] \Phi(d-n), \quad (3.11)$$

and

$$g_3(z; \beta J) \equiv \sum_{n=0}^{2d} \binom{2d}{n} (d-n) z^{2d-n} \left(\frac{1}{2} - z\right)^n \Phi(d-n), \quad (3.12)$$

and we are naturally dropping the index \pm in the function defined by (3.6) when it refers to $h=0$.

IV. STEADY-STATE PROPERTIES FOR $h=0$ AND $d \leq 3$

The search for stationary solutions of the system (3.7)–(3.12) may proceed by noting that, at high enough temperatures, the stationary state is expected to be characterized by $m = \Delta = 0$ and z given by $g_3(z; \beta J) = 0$; these solutions are paramagnetic ones whose stability re-

quires that $g_1(z; \beta J) < 0$ and $g_2(z; \beta J) < 0$. In addition, one may expect, at least, ferromagneticlike states characterized by $m \neq 0$ and $\Delta = 0$ and antiferromagneticlike states characterized by $m = 0$ and $\Delta \neq 0$. Thus, the transition points between disordered and ordered states may be associated with solutions of $g_3(z; \beta J) = 0$ showing up an incipient instability condition, i.e., those producing either $g_1(z; \beta J) = 0$ or else $g_2(z; \beta J) = 0$. The former case leads to

$$z = \frac{1}{2}d(2d-1)^{-1}, \quad (4.1)$$

$$\sum_{n=0}^{2d} \binom{2d}{n} (d-n)(d-1)^n d^{-n} \Phi(d-n) = 0.$$

which corresponds to the transition between paramagnetic and ferromagneticlike states. Actually, the equilibrium, noncompeting condition occurs for $\Phi(n) = \phi(4n\beta J_0)$, whose substitution into (4.1) leads to the familiar Bethe-Peierls result, i.e., $T_c = 2J_0/k_B \ln[d/(d-1)]$. On the other hand, the condition $g_2(z; \beta J) = g_3(z; \beta J) = 0$ leads to

$$z = \frac{1}{2}(d-1)(2d-1)^{-1}, \quad (4.2)$$

$$\sum_{n=0}^{2d} \binom{2d}{n} (d-n)[(d-1)/d]^{2d-n} \Phi(d-n) = 0.$$

which corresponds to transitions between paramagnetic and antiferromagneticlike phases. Given that we have required arbitrarily small values for m and Δ above, both (4.1) and (4.2) will in general define lines of critical points; first-order phase transitions only occur under some special conditions, as revealed below.

The simplest case corresponds to a chain under zero field. It then follows from (4.1) and (4.2) that $\Phi(1) = 0$ and $z = \frac{1}{2}$, and $\Phi(-1) = 0$ and $z = 0$, respectively. This implies in particular that, as it is also exhibited by the exact one-dimensional solution of the same model,⁹ the competition (1.5), i.e., the case

$$\Phi(n) = p\phi(4n\beta J_0) + (1-p)\phi(-4n\beta J_0),$$

precludes the existence of the pure zero- T critical point for any p differing from either 0 or 1 and for any transition rate of the form (1.4). We also find that $z^{-1} = 2[1 + \Phi(1)^{1/2}\Phi(-1)^{-1/2}]$ for any $T > 0$.

Let us consider now the case $d = 2$, $h = 0$, and distribution (1.5). The critical line which follows from (4.1), occurs for $z_c = \frac{1}{3}$ and $T_c = T_c(p)$, which is given by

$$g(p, \beta_c J) \equiv (17p-1)\phi(8\beta_c J_0) + (20p-4)\phi(4\beta_c J_0) \\ - (20p-16)\phi(-4\beta_c J_0) \\ - (17p-16)\phi(-8\beta_c J_0) = 0, \quad (4.3)$$

$\beta_c \equiv (k_B T_c)^{-1}$, and the critical line which follows from (4.2) occurs for $z'_c = \frac{1}{6}$ and for $T'_c = T'_c(p)$ obtained from $g(1-p, \beta'_c J) = 0$, $\beta'_c \equiv (k_B T'_c)^{-1}$. To be more explicit, when the elementary rates are of the Metropolis type, (1.9), the transition line separating the ferromagneticlike phase occurs at

$$(17p-1)\chi_c^2 + (20p-4)\chi_c + (32-37p) = 0, \quad z_c = \frac{1}{3} \quad (4.4)$$

with $\chi_c \equiv \exp(-2\beta_c J_0)$, and one obtains a similar equation from (4.3), namely $z'_c = \frac{1}{6}$ and (4.4) with p replaced by $1-p$ and χ_c replaced by $\chi'_c \equiv \exp(-2\beta'_c J_0)$, for a line separating the paramagnetic- and antiferromagneticlike phases. By requiring $T_c = 0$ (or $T'_c = 0$), the solution p_0 (or $1-p_0 = \frac{32}{37}$) follows. Summing up, the steady states for $d = 2$ with Metropolis rates are characterized by $m = \Delta = 0$ for any temperature T when $1-p_0 < p < p_0$, are ferromagneticlike (in the sense that they are characterized by $m \neq 0$ and $\Delta = 0$) for $p \geq p_0$ and $T < T_c(p)$, are antiferromagneticlike (characterized by $m = 0$ and $\Delta \neq 0$) for $p \leq 1-p_0$ and $T < T'_c(p)$, and the transitions occurring at both $T_c(p)$ and $T'_c(p)$ are always of second order.

It seems worthwhile to note that, in the same approximation, the corresponding annealed version of the equilibrium model, i.e., assuming that an initial (spatial) bond distribution (1.5) has finally reached equilibrium with the spin system, is characterized by transition lines satisfying²²

$$(2p-1)\tanh(\beta_c J_0) = \pm(2d-1)^{-1}, \quad (4.5)$$

which implies $p_0 = \frac{2}{3}$ for $d = 2$, and $p_0 = 1-p_0 = \frac{1}{2}$ as $d \rightarrow \infty$. The comparison with the corresponding result above, suggests that the dynamical competition between bonds in the nonequilibrium system is more effective in destroying magnetization than the, say, "correlated competition" between bonds that characterize the annealed case, as one should expect.

The annealed spin-glass system also differs from the NSGM in that the steady states of the latter may depend, even strongly, on the distribution $f(J)$ and on the rate function (1.4). In order to illustrate the latter fact, consider first the Kawasaki type of rate (1.10) which transforms (4.1) into

$$\frac{17p-1}{1+\chi_c^{-2}} + \frac{20p-4}{1+\chi_c^{-1}} - \frac{20p-16}{1+\chi_c} - \frac{17p-16}{1+\chi_c^2} = 0, \quad z_c = \frac{1}{3}. \quad (4.6)$$

As shown in Fig. 1(a), even though some significant deviations occur, the behavior implied by (4.6) is qualitatively similar to the one for the Metropolis case (4.4). It may be mentioned that both cases are characterized by (1) $p_0 = \frac{32}{37}$, (2) a symmetry around $p = \frac{1}{2}$, which implies, in particular, that $\Delta(T, 1-p)$ behaves identically to $m(T, p)$, and (3) curves $m = m(T)$ and $\Delta = \Delta(T)$, which, due to the competition, do not tend towards saturation as $T \rightarrow 0$ when $p < 1$. The fact that the states are not self-similar, i.e., no common behavior is depicted by curves such as $m = m(T)$ when one measures T in units of the transition temperature and scales m arbitrarily, is also noticeable. The qualitative (even semiquantitative) similarity found between cases (1.9) and (1.10) does not hold, however, for other transition rates. For example, the case (1.11), which happens to produce the same results as the choice¹⁴ $\phi(X) = e^{-X/2}[\cosh(\beta J_0)]^{-2d}$ when one considers the distribution (1.5), leads to

$$(17p-1)\chi_c^2 + (20p-4)\chi_c - (20p-16)\chi_c^{-1} - (17p-16)\chi_c^{-2} = 0, \quad z_c = \frac{1}{3}. \quad (4.7)$$

Even though the symmetry around $p = \frac{1}{2}$ with $\Delta(T, 1-p) = m(T, p)$ still exists, the situation implied by (4.7) is more involved (and interesting) than those following from either (4.4) or (4.6). This is illustrated by Figs. 1(b) and 2, which indicate that the solutions of (4.7) are always stable (corresponding to second-order phase transitions, as discussed above for the other rates) when $T > T_2$, while unstable solutions representing first-order phase transitions may occur when $T < T_2$. Thus, (T_2, p_2) is a sort of (nonequilibrium) tricritical point separating first- from second-order phase transitions. Moreover, as depicted by the phase diagram in Fig. 2, two different phase transitions may occur for $p_1 > p > p_3$: The high-temperature phase transition corresponds to the familiar equilibrium one, i.e., it separates ordered states from states which are thermally disordered, while the low-temperature phase transition, which is first order for

$p_4 > p > p_3$ and continuous for $p_1 > p > p_4$, is to be associated with dynamics. That is, long-range order may be suppressed at low temperatures by rates (1.11), which are not normalized, given that they favor low-energy configurations very strongly, and the effectiveness of the competition is enhanced in such a way that any segregation is avoided in practice. Note, however, the essential differences between this effect and the one reported for some annealed systems,²⁴ where some sort of low-temperature suppression of order occurs that is independent of the dynamical process and comes associated instead with the properties of a certain bond distribution; moreover, all the transitions therein are of second order and there is no tricritical point consequently.

The transition line following from (4.1) when $d = 3$ (and $h = 0$) is defined via $z_c = \frac{3}{10}$ and

$$729\Phi(3) + 1944\Phi(2) + 1620\Phi(1) - 720\Phi(-1) - 384\Phi(-2) - 64\Phi(-3) = 0. \quad (4.8a)$$

For distribution (1.5), (4.8a) reduces to

$$(793p - 64)\phi(12\beta_c J_0) + (2328p - 384)\phi(8\beta_c J_0) + (2340p - 720)\phi(4\beta_c J_0) - (2340p - 1620)\phi(-4\beta_c J_0) - (2328p - 1944)\phi(-8\beta_c J_0) - (793p - 729)\phi(-12\beta_c J_0) = 0. \quad (4.8b)$$

As one might expect, the ensuing picture is qualitatively similar to the one for $d = 2$ (cf. Fig. 2); actually, we only expected singular the case $d = 1$, where the equilibrium critical point occurs at zero temperature.

We find it interesting to remark that the existence of a region of the phase diagram, which is defined by $1 - p_0 < p < p_0$, where steady states are characterized by $m = \Delta = 0$ at any temperature, is an attribute of the model for any d when $h = 0$. As is known to occur in the standard (quenched) spin-glass model, this does not ex-

clude the existence of a condensed phase whose description requires an order parameter which is independent of m and Δ .² Such a possibility cannot be analyzed within the present mean-field approach, however. This becomes evident after realizing that the steady states with $m = \Delta = 0$ are characterized by the value of z obtained from condition $g_3(z; \beta J) = 0$. This implies the existence of lines in the phase diagram characterized by $z = \text{const}$, which connect nonequilibrium self-similar (p, T) states with the equilibrium state corresponding to $p = 1$ (or 0)

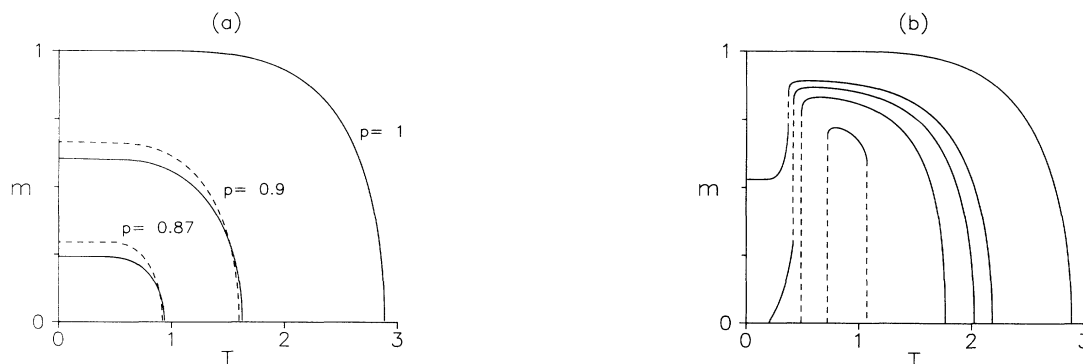


FIG. 1. (a) Temperature dependence of the order parameter m for $d=2$, $h=0$, and distribution (1.5), as one varies p as indicated. The solid lines are for these rates (1.9), and the dashed lines are for rates (1.10); they correspond to Eqs. (4.4) and (4.6) respectively. $m \equiv 0$ for all T when $p < p_0 = 0.8649$ in both cases. The curves $\Delta = \Delta(T, 1-p)$ are identical to those for $m = m(T, p)$. The temperature is given everywhere in units of J_0/k_B . (b) The same as (a) except that transition rates are (1.11), i.e., the behavior implied by Eq. (4.7). The curves here correspond to $p = 1, 0.95, 0.94, 0.93$, and 0.912 from top to bottom. The solid lines represent stable solutions. Unstable solutions (dashed lines) corresponding to discontinuous behaviors occur for $p \leq p_4 = 0.9396$. Two discontinuities appear for $p < p_2 = 0.9240$ which finally coalesce at $T = T_3 = 0.87$ for $p = p_3 = 0.9095$.

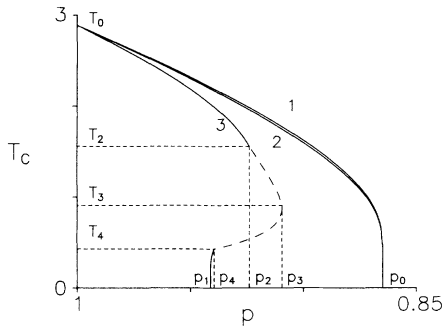


FIG. 2. Mean-field phase diagram for d -dimensional simple-cubic lattice, $h=0$, (1.5) and different rates: curves 1, 2, and 3 are for (1.9), (1.10), and (1.11), respectively. The solid curves indicate lines of critical points. Dashed curves correspond to first-order phase transitions occurring for rates (1.11) below a tricritical point at (T_2, p_2) . The curves are qualitatively similar for $d=2$ and 3. Here $p_0 = \frac{32}{37}$, $p_1 = 0.9412$, $T_0 = 2.8854$, $T_2 = 1.5574$, and $T_4 = 0.431$ for $d=2$.

and temperature T' . For instance, the solutions of (4.3) correspond to states which are self-similar to the equilibrium state ($p=1$, $T'=2/\ln 2$), and the line $z = \frac{1}{4}$ corresponds to $p = \frac{1}{2}$, which is self-similar to $(p=1, T' = \infty)$. A Monte Carlo analysis might be very helpful in detecting possible new qualitative kinds of order in the NSGM.

One should expect that the bond distribution will also significantly influence the macroscopic behavior. This is evident, for instance, from the two equations defining the lines of transition points separating paramagnetic from ferromagnetic and antiferromagnetic regions for $d=2$ and $h=0$, namely,

$$16\Phi(2) + 16\Phi(1) - 4\Phi(-1) - \Phi(-2) = 0 \quad (4.9)$$

and

$$\Phi(2) + 4\Phi(1) - 16\Phi(-1) - 16\Phi(-2) = 0, \quad (4.10)$$

respectively, where Φ is defined by (3.6). Restricting ourselves, for simplicity, to the rate function (1.9) and to the ferromagnetic case (4.9), we have

$$27q + (1-q)[(17p-1)\chi_c^2 + (20p-4)\chi_c + (32-37p)] = 0, \quad (4.11)$$

when the distribution is (1.6) with $J_1 = J_2 = J_0$, which represents a diluted (symmetric) spin-glass, and

$$27q + (1-q)[16p\chi_c^2 + (17p-1)\chi_c - 4(1-p)\chi_c^{1/2} + (32-37p)] = 0, \quad (4.12)$$

when the distribution is (1.6) with $J_1 = J_0$ and $J_2 = \frac{1}{2}J_0$, which corresponds to a diluted spin glass with asymmetric interactions. The ensuing magnetization and energy curves are qualitatively similar to those for distribution (1.5), and the transitions given by (4.11) and (4.12), as well as the (antiferromagnetic) ones given by (4.10), are always second order. The corresponding phase diagram is depicted by Fig. 3. By requiring in (4.11) that $T_c = 0$ for any given p , a lack of ferromagnetic order follows

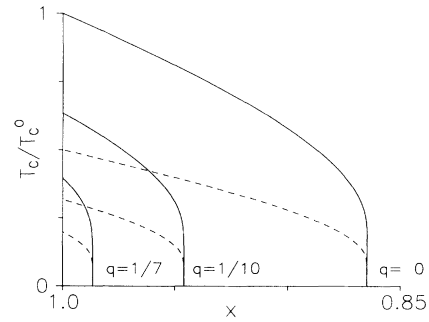


FIG. 3. Phase diagram for $d=2$, $h=0$, Metropolis rates (1.9), and bond distribution (1.6) representing, for several values of the dilution variable q as indicated, the critical temperature T_c (in units of the Bethe-Peierls, equilibrium result) vs x ; the latter corresponds either to p or to $1-p$ according to whether one considers the ferromagnetic case (4.9) or the antiferromagnetic case (4.10). When the distribution (1.6) is asymmetric, i.e., $J_1 = J_0$ and $J_2 = \frac{1}{2}J_0$, the solid lines represent the case (4.9) and the dashed lines represent the case (4.10). The solid lines also represent very accurately both cases (4.9) and (4.10) when the distribution is symmetric, i.e., $J_1 = J_2 = J_0$.

when $q > q_0(p) = (37p - 32)/(37p - 5)$; for $p=1$, this reduces to $q_0(1) = \frac{5}{32}$, which locates the onset of percolation in this system. As expected, the same result follows for $q_0(p)$ in the asymmetric case due to some cancellations occurring in (4.12) at zero temperature. On the other hand, the magnetization curves $m = m(T)$ for the rectangular distribution (1.7) tend to saturate for $0 \leq x \leq 1$, where x measures the width of the bond distribution, while one gets $m(0) < 1$ when one allows for interactions of both signs by setting $x > 1$. Figure 4 illustrates the corresponding phase diagram and compares it with the one for the annealed system in the Bethe-Peierls approximation, i.e.,

$$\frac{2}{3}x = \tau \ln \left\{ \frac{\cosh[(1+x)\tau^{-1}]}{\cosh[(1-x)\tau^{-1}]} \right\}, \quad x \neq 0,$$

where $\tau \equiv k_B T / J_0$. The facts that the indicated phase transition is second order and that the low-temperature states are always ferromagneticlike are also noticeable.

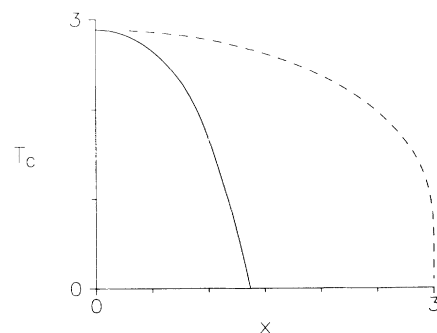


FIG. 4. Phase diagram for $d=2$, $h=0$, Metropolis rates (1.9), and the rectangular bond distribution (1.7) (solid line), and for the corresponding annealed system in the Bethe-Peierls approximation (dashed line), both in a first-order mean-field approximation.

V. STEADY-STATE PROPERTIES WHEN $h \neq 0$

When the system is acted on by an external magnetic field, the analytical treatment becomes more complex. In fact, there is not an exact solution even for $d = 1$. Moreover, the necessary consideration of a domain of $4d$ spins entangles our mean-field analytical description for $h \neq 0$ in such a way that a numerical treatment of approximate equations such as (3.2)–(3.5) is required.

Consider first the case of $d = 1$ with rates (1.9) and a bond distribution given by (1.5). As expected, the stationary solutions are characterized by $m \neq 0$ and $\Delta = 0$. In particular, when $T = 0$, the magnetization is a step function of the applied field, which only depends on the sign of $2J_0 - h$, namely,

$$m(h, p, T = 0) = \begin{cases} m_1(p) & \text{for } h < 2J_0 \\ m_2(p) & \text{for } h = 2J_0 \\ 1 & \text{for } h > 2J_0; \end{cases} \quad (5.1)$$

the variation of m_1 and m_2 with p is depicted by Fig. 5. The situation shown by (5.1) and Fig. 5 reflects the fact that, in the absence of thermal fluctuations, any large enough field (namely, $h > 2J_0$) saturates the system for any bond competition [i.e., for any value of p in (1.5)], while $m(h, p, T = 0)$ significantly depends on both h and p otherwise. Thus, for small fields, one has $m_1 \rightarrow 0$ as $p \rightarrow 0$, when the interaction tends to be fully antiferromagnetic, $m_1 \rightarrow 1$ as $p \rightarrow 1$, and there is a crossover situation for $h = 2J_0$ in which m_2 is observed to increase monotonically with increasing p from a nonzero value towards saturation; cf. Fig. 5.

To understand the case of finite T and small p , it is convenient to remark that, as illustrated by the inset in Fig. 6, the antiferromagnetic equilibrium system ($p = 0$) has, for not very high temperatures, two qualitatively different behaviors depending on the field value. Namely, for $h > 2J_0$, the magnetization m has a definite tendency towards saturation, which becomes stronger as h increases and/or T is lowered, while there is a region corresponding to small fields (for $h < 2J_0$) where $m \rightarrow 0$ as $h \rightarrow 0$, more decisively as T is decreased. The main graphs in Fig. 6 reveals that two qualitatively similar behaviors occur for $p > 0$. In addition to that, the $\pm J_0$

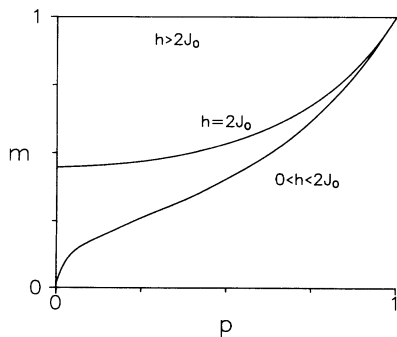


FIG. 5. The zero-temperature isotherm $m(h, p, T = 0)$, cf. Eq. (5.1), as a function of p for $d = 1$, Metropolis rates (1.9), and different range of values for the field, as indicated.

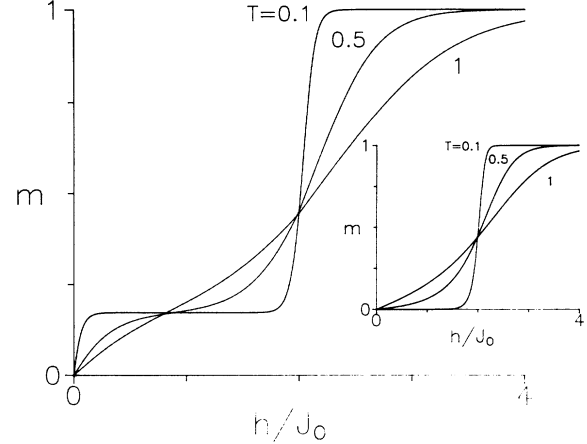


FIG. 6. The field dependence of the isotherms $m(h, p, T)$ for different temperatures, as indicated, $p = 0.1$, Metropolis rates (1.9), and $d = 1$. The inset represents the corresponding antiferromagnetic equilibrium result, i.e., $p = 0$.

competition induces new qualitative effects for small fields; e.g., the system is most affected by the competition, m becomes practically independent of h when $k_B T \ll h < 2J_0$, and m depends on h when $h \ll k_B T$ and $h < 2J_0$ (unlike for $T = 0$). This interesting region of the phase diagram may be investigated by linearizing (3.2) and (3.3) with respect to both m and $h/k_B T$. It then follows that

$$X_I^{-1} \equiv \left. \frac{\partial m}{\partial h} \right|_{h=0} = \frac{1}{2} \tau \frac{(1 + 2\kappa)(P\chi^2 + q) + \kappa^2(2 - p - q\chi^2)}{p\chi^2 + \kappa + q\chi^2\kappa^2}, \quad (5.2)$$

where $\kappa \equiv \Phi(1)/\Phi(-1)$, $\tau \equiv k_B T/J_0$ (as before), and $\chi \equiv \exp(-2\tau^{-1})$, which is singular at $T = 0$ only when one recovers the equilibrium result, $X_I^{-1} \sim T \exp(\pm 2/\tau)$, for $p = 0$ or 1 ; otherwise, $X_I^{-1} \sim \tau[q + (q/p)^{1/2}]$ as $T \rightarrow 0$. It is also interesting to compare X_I with

$$X_{II} \equiv \tau^{-1} \sum_{i,j} G(|i-j|), \quad (5.3)$$

where

$$G(|i-j|) = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle. \quad (5.4)$$

One has in general for $d = 1$ in the present first-order mean-field description:¹⁶

$$G(j) = 4xy[(z - x^2)/xy]^j, \quad j > 0, \quad (5.5)$$

which reduces here to $G(j) = [(1 - \kappa)/(1 + \kappa)]^j$; thus, $X_{II} = (\tau\kappa)^{-1}$. Although X_{II} depicts a behavior which is qualitatively similar to the one shown by X_I in (5.2), their ratio generally differs from unity, implying that a fluctuation-dissipation theorem breaks down for $0 < p < 1$; this is illustrated by Fig. 7.

We have also analyzed equations (3.2) and (3.3) for $d = 2$ and $h \neq 0$ when dynamics is implemented by the Metropolis rate (1.9) and the bond distribution is (1.5).

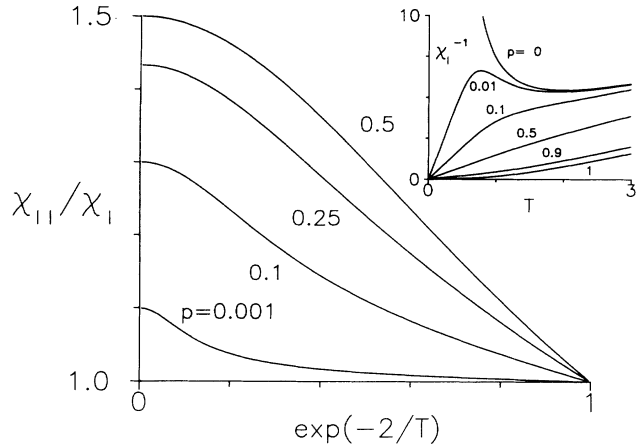


FIG. 7. Temperature dependence of the ratio between the two susceptibilities defined, respectively, by (5.2) and (5.3) for $d=1$ and Metropolis rates. The numbers identifying the curves corresponding to p and to $1-p$, i.e., there is a symmetry around $p=\frac{1}{2}$. The inset represents the behavior of $\chi_{\perp}^{-1}(T)$.

The ensuing behavior may be summarized as follows. When $T=0$, m and Δ behave much as step functions slightly different from (5.1), namely,

$$\mu(h,p, T=0) = \begin{cases} \mu_1(p), & \text{for } h < 2J_0 \\ \mu_2(p), & \text{for } 2J_0 < h < 4J_0 \\ \alpha & \text{for } h > 4J_0, \end{cases} \quad (5.6)$$

where μ stands for either m or Δ and $\alpha=1$ or 0 , respectively. At $h=2nJ_0$ the behavior is singular for $n=0,1,2,\dots$; this is not an artifact to be associated with the singular nature of Metropolis rates, but it also occurs for continuous functions such as (1.10). The variation of $\mu_i, i=1,2$, with p is illustrated in Fig. 8. As expected, this reveals (1) that m_i monotonically increases with p , and (2) a tendency towards saturation ($m_i \rightarrow 1$) as $p \rightarrow 1$, (3) the fact that $m_1 < m_2$ for all p , which only

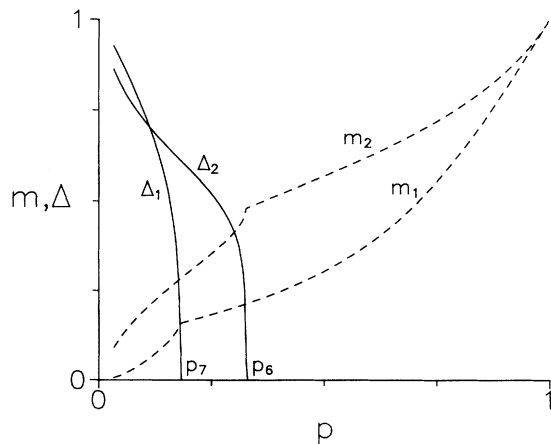


FIG. 8. Variation with p of the parameters in Eq. (5.6) corresponding to zero-temperature isotherms. Here, $p_6=0.3367$ and $p_7=0.1593$.

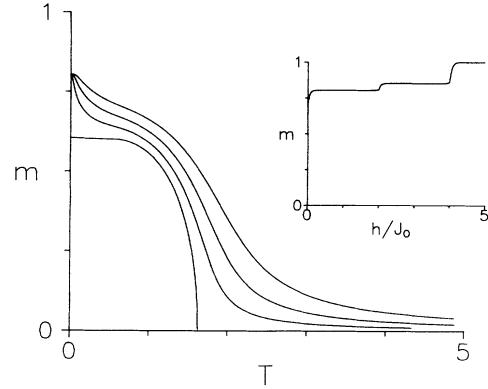


FIG. 9. Solutions $m=m(T)$ for $p=0.9$ and, from top to bottom, $h/J_0=0.1, 0.05, 0.02$, and 0 . The inset represents $m=m(h)$ for $p=0.9$ and $T=0.1$.

reflects that m increases with h , and (4) the existence of solutions $\Delta \neq 0$ for small p , say for $p < p_6$. Note, however, that, contrary to the naive intuition that h tends to destroy antiferromagnetic order, one has here that $p_6 > 1-p_0$ and that $\Delta_2 > \Delta_1$ for $p_5 < p < p_6$ with $p_5=0.1182$.

The insets in Figs. 9 and 10 represent isotherms for $\tau=0.1$ and $p=0.9$ and 0.1 , respectively, where the step shape is still evident. The singular behavior mentioned above of $\mu(h)$ at $h=2nJ_0$ is also clear therein. The main graph in Fig. 9 illustrates the function $m(T)$ for small fields when $p=0.9$; it is noticeable there that $m \rightarrow m_1(p=0.9) \neq m(h=0)$ as $T \rightarrow 0$. The main graph in Fig. 10 represents the situation for relatively larger fields and $p=0.1$, where solutions $\Delta \neq 0$ also exist. Summing up, for a given value of h , one has two types of solutions with $m \neq 0$; namely, $\Delta=0$ and m decreases with T for $T > T^*(h)$, whereas $\Delta \neq 0$ and m increases with T for $T < T^*(h)$. The function $T^*(h)$ is not simple, e.g., it does not decrease monotonically with h as one might naively expect, except for $p=0,1$, which corresponds to equilibrium. The behavior of $T^*(h)$ is illustrated in Fig. 11.

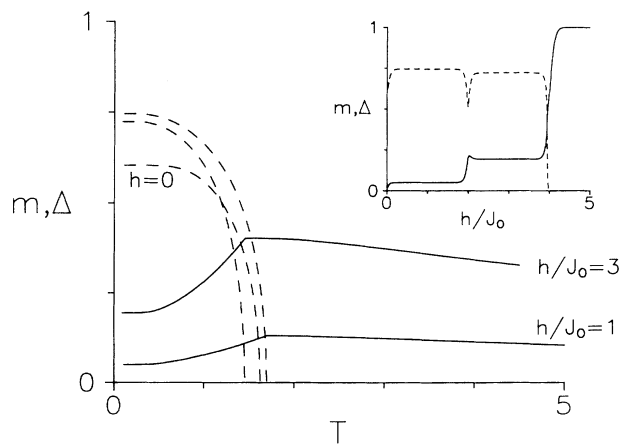


FIG. 10. Solutions $m=m(T)$ (solid lines) and $\Delta=\Delta(T)$ (dashed lines) for $p=0.1$ and $h/J_0=3, 1$, and 0 , as indicated. The inset represents the variation with h when $T=0.1$.

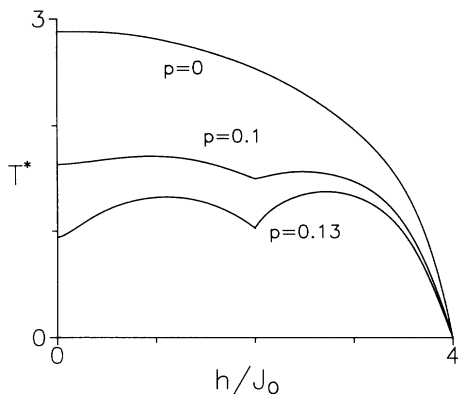


FIG. 11. The function $T^*(h)$ defined at the end of Sec. V for $p = 0, 0.1$, and 0.13 , as indicated.

VI. CONCLUSIONS

We have considered the nonequilibrium spin-glass model (NSGM) introduced in Ref. 9, namely, an Ising-like model system whose dynamics involves a simultaneous competition of exchange energies (bonds) according to distribution $f(J)$. Given that the most elementary dynamical (spin-flip) processes are local, this is essentially equivalent to having the bonds spatially distributed according to $f(J)$, which varies randomly with time independently of the evolution of the spin degrees of freedom; this may perhaps occur in some natural systems. The model is solved by a kinetic mean-field method in the pair approximation which deals with local clusters of two sites and their surrounding nearest-neighbor sites; this allows for the explicit consideration of two sublattices. We have studied several coordination numbers, corresponding to d -dimensional simple-cubic lattices with $1 \leq d \leq 3$, and different transition rates for the elementary spin-flip process, and the case of a system under an external magnetic field h . This treatment leads to the characterization of nonequilibrium steady states, stability conditions, and phase diagrams for situations in which an exact solution is still lacking.

The main specific results may be summarized as follows: When $h = 0$, the present mean-field approach reveals no unexpected behavior; in particular, one recovers the known exact results for $d = 1$ (Refs. 9 and 10) qualitatively. The case $d = 2$, where the symmetric bond distribution (1.5) produces very different phase diagrams according to whether the transition rates are given by (1.9),

(1.10), or (1.11), is more interesting. For example, only (1.11) leads to the existence of both first-order phase transitions and tricritical points separating the latter from second-order phase transitions. Moreover, a competition of rates (1.11) leads to the suppression of long-range order in some cases. Such a strong dependence of the steady-state properties on the details of dynamics, and the breakdown of dissipation-fluctuation theorems in general, are distinct features of the NSGM. The consideration of the bond distribution (1.6), on the other hand, reveals the decisive influence of $f(J)$ on the macroscopic properties of the system. For instance, order is only possible when $q < (37p - 32)/(37p - 5)$ for $J_1 = J_2 = J_0$, and the symmetry around $p = \frac{1}{2}$, which is observed to hold otherwise, is destroyed for $J_1 = 2J_2 = J_0$. The behavior is qualitatively similar for $d = 3$, as expected in a mean-field model.

A most striking behavior is observed for $h \neq 0$. For Metropolis rates, a bond distribution such as (1.6), and $d = 1$ and $d = 2$ the following occurs: (1) the magnetization $m(h, p, T = 0)$ is a step function of (the sign of) $h - 2nJ_0$, (2) the system saturates for any p when $h > 2J_0$, and (3) $m = m(p)$, such that $m \rightarrow 1$ as $p \rightarrow 1$, for $h \leq 2J_0$. For the same rates and bond distribution, the magnetization of the two-dimensional system at zero temperature is independent of p (as for $d = 1$) when $h > 4J_0$, and $\Delta \neq 0$ for $p < 0.3367$. Moreover, Δ increases with h for some values of p , which runs counter to naive intuition that fields tend to destroy antiferromagnetic order. When the temperature is finite, the behavior becomes even richer. In particular, order parameters have a strong dependence on h for $h \approx 2nJ_0$, and a transition occurs at temperature $T^*(h)$, which separates phases with $m \neq 0$ and $\Delta \neq 0$ from phases with $m \neq 0$ and $\Delta = 0$, if p and h are small enough.

The present study generates interest in a numerical study of the NSGM, e.g., a Monte Carlo simulation to characterize the kind of order one might expect (following the trend in quenched models) at very low temperatures for $d \geq 2$ for bond distribution (1.5) and $p \approx \frac{1}{2}$.

ACKNOWLEDGMENTS

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