A-B interface of superfluid ^{3}He in a magnetic field

E. V. Thuneberg
Research Institute for Theoretical Physics, University of Helsinki, Siltavuorenpenger 20C, SF-00170 Helsinki, Finland
(Received 5 November 1990; revised manuscript received 31 May 1991)

The A-B interface of superfluid ^{3}He was studied with use of Ginzburg-Landau theory. The effect of a magnetic field on the interface structure was calculated. The surface tension of the interface was found to vanish rapidly when approaching the weak-coupling limit. By doing perturbation theory around this limit, an analytic expression for the interface tension was found that presumably is accurate for arbitrary physical values of the strong-coupling parameters. The effect of the dipole interaction was studied in both low and high magnetic fields. The rotation axis of the B phase was found to deviate substantially from the direction of the magnetic field near the interface.

I. INTRODUCTION

The interface between the A and B phases of superfluid ^{3}He is unique in the sense that it can be analyzed theoretically in great detail. Several papers have been devoted to the dynamical properties of the interface. Here we will concentrate on the equilibrium properties instead. The stationary interface has been studied theoretically by Osheroff and Cross,[1] by Kaul and Keinert,[2] and most accurately by Schopohl.[3] Some calculations have also been made by Salomaa.[4] The effect of the dipole-dipole interaction has been included by Yip.[5] All these studies are limited to zero magnetic field, in which case the interface exists only at pressures above the tricritical one. The present paper studies the interface in a nonzero magnetic field. We have solved the order parameter in the interface using the Ginzburg-Landau theory. We especially address two questions. Firstly, what is the equilibrium orientation of the two phases relative to the magnetic field and to the interface? Because the magnetic field and dipole interaction compete with each other, the phases take different orientations in low and high fields. Secondly, what is the structure of the interface at low pressures? The interface behaves singularly and its surface tension vanishes in the weak-coupling limit (which might approximately be realized at low pressures). This might be used to obtain information about strong-coupling parameters if a measurement of the surface tension[1] could be made at low pressures. An analytic solution for the interface tension is found by expanding around the singular point. The accuracy of the result seems to be good at arbitrary physical values of the strong-coupling parameters.

This work was initially motivated by the experiment of Kyynäräinen *et al.*[6] who studied the attenuation of ultrasound under a strong magnetic field. They observed a reversible extra attenuation during the A-B transition. *A priori*, this might be explained if, near the A-B interface, the $\hat{\boldsymbol{l}}$ vector of the A phase were deflected from its equilibrium orientation, which is perpendicular to the direction of sound propagation. (In the experiment, the field is in the direction of the sound propagation.) On the contrary, we found that $\hat{\boldsymbol{l}}$ is essentially perpendicular to the field for arbitrary orientation of the interface. It is also unlikely that the attenuation could arise from the B phase near the interface, although no calculation of attenuation has been done in the strongly distorted B phase. Therefore, it seems more probable that the attenuation arises from phase cancellation due to different sound velocities in the two phases, as has been suggested very recently.[7]

Some necessary background information for the rest of the paper is given in Sec. II. The numerical solution is discussed in Sec. III and the analytic one in Sec. IV. The effect of the dipole-dipole interaction is considered in Sec. V.

II. UNITS AND THE BULK PHASES

Our notation closely follows the presentation of the Ginzburg-Landau (GL) theory given in Ref. 8. Instead of reproducing sections of this reference here, we generally refer to it for all quantities not properly defined here. Note especially that the coherence length used here $[\xi(T)=\sqrt{K/\alpha}]$ is by a factor of $\sqrt{3/5}$ shorter than that used in Refs. 1 and 2. This also makes the unit of the interface energy different from Refs. 1–3.

We are going to study the A-B interface in the bulk liquid far from any surfaces. The energy of the A-B interface is on the order of $f_c^B(0)\xi(T)$, where $\xi(T)$ is the GL coherence length and $f_c^B(H)$ the (field-dependent) superfluid condensation energy of the bulk B phase. In comparison, the contribution of the dipole energy to the interface energy is smaller by the factor $\xi(T)/\xi_D \approx 10^{-3}$ and we will neglect it until Sec. V.

The interaction of the order parameter with the magnetic field $\mathbf{H}$ is described by two energy terms, one linear and the other quadratic in the field. We recall some properties of the bulk phases neglecting the linear term. The A phase has the order parameter

$$\underline{A}=\Delta_A\hat{\mathbf{d}}(\hat{\mathbf{u}}_1+i\hat{\mathbf{u}}_2)\ , \tag{1}$$

where $\hat{\mathbf{u}}_1$, $\hat{\mathbf{u}}_2$, and $\hat{\mathbf{d}}$ are arbitrary real unit vectors except that $\hat{\mathbf{u}}_1\cdot\hat{\mathbf{u}}_2=0$. A unit vector $\hat{\boldsymbol{l}}$ is defined as $\hat{\boldsymbol{l}}=\hat{\mathbf{u}}_1\times\hat{\mathbf{u}}_2$. The phase angle is equivalent to the rotation angle of $\hat{\mathbf{u}}_1$ and $\hat{\mathbf{u}}_2$ around $\hat{\boldsymbol{l}}$. The magnetic field orients $\hat{\mathbf{d}}$ perpendicular to the field, but the order parameter amplitude

$\Delta_A=\sqrt{\alpha/4\beta_{245}}$ and the condensation energy $f_c^A=\alpha^2/4\beta_{245}$ are not affected.

In the B phase the order parameter has the form ($\hat{\mathbf{H}}=\mathbf{H}/H$)

$$\underline{A}=e^{i\phi}[\Delta_\perp+(\Delta_\parallel-\Delta_\perp)\hat{\mathbf{H}}\hat{\mathbf{H}}\cdot]\underline{R}(\hat{\mathbf{n}},\theta)\ . \tag{2}$$

Here ϕ is an arbitrary phase angle and $\underline{R}$ is an arbitrary rotation matrix parametrized by an angle θ and a rotation axis $\hat{\mathbf{n}}$. The amplitudes $\Delta_\perp$ and $\Delta_\parallel$ depend on the magnetic field[9]

$$\begin{aligned}\Delta_\perp^2&=\Delta_B^2\left[1+\frac{\beta_{12}}{\beta_{345}}\frac{g_Z H^2}{\alpha}\right],\\ \Delta_\parallel^2&=\Delta_B^2\left[1-\frac{2\beta_{12}+\beta_{345}}{\beta_{345}}\frac{g_Z H^2}{\alpha}\right],\\ \Delta_B^2&=\alpha/(6\beta_{12}+2\beta_{345})\ .\end{aligned} \tag{3}$$

The condensation energy is given by

$$f_c^B(H)=\frac{3\alpha^2}{4(3\beta_{12}+\beta_{345})}\times\left[1-\frac{2}{3}\frac{g_Z H^2}{\alpha}+\frac{2\beta_{12}+\beta_{345}}{3\beta_{345}}\left[\frac{g_Z H^2}{\alpha}\right]^2\right]\ . \tag{4}$$

In high field, the B phase can be deformed so much that $\Delta_\parallel=0$. This version of the B phase is called the planar state. We note for later use that there is a continuous transformation between the planar and the A phases via the so-called axi-planar states.[10] Setting the magnetic field parallel to $\hat{\mathbf{z}}$, the most general axi-planar state has the order parameter

$$\underline{A}=\Delta_{\rm ap}(\chi)[(\hat{\mathbf{x}}-i\hat{\mathbf{y}})(\hat{\mathbf{u}}_1+i\hat{\mathbf{u}}_2)+(\hat{\mathbf{x}}+i\hat{\mathbf{y}})(\hat{\mathbf{u}}_1'+i\hat{\mathbf{u}}_2')]\ , \tag{5}$$

where $(\hat{\mathbf{u}}_1,\hat{\mathbf{u}}_2,\hat{\boldsymbol{l}})$ and $(\hat{\mathbf{u}}_1',\hat{\mathbf{u}}_2',\hat{\boldsymbol{l}}')$ are two arbitrary orthonormal vector basis and 2χ is the angle between $\hat{\boldsymbol{l}}$ and $\hat{\boldsymbol{l}}'$. The A phase (1) is obtained when $\hat{\boldsymbol{l}}$ and $\hat{\boldsymbol{l}}'$ are parallel and the planar phase when they are antiparallel. The condensation energy equals

$$f_c^{\rm ap}(\chi)=\alpha\Delta_{\rm ap}^2(\chi)=\frac{\alpha^2}{2[2\beta_{245}-2\beta_{45}\sin^2\chi+(2\beta_1+\beta_{345})\sin^4\chi]}\ . \tag{6}$$

For the β_i coefficients in the numerical calculations, we used the values suggested by Sauls and Serene.[11] They tabulated the β_i's at a number of pressures above 12 bar, and at intermediate pressures it is reasonable to use linear interpolation. These coefficients give the tricritical point $[f_c^B(H=0)=f_c^A]$ at 28.5 bar, which differs from the true tricritical pressure. To avoid confusion, we have used the symbol $P_{\rm SS}$ to denote the pressure according to the Sauls-Serene coefficients. In addition, we arbitrarily assigned $P_{\rm SS}=0$ to the weak-coupling values of β_i. These assignments are consistent with the standard identification of the bulk phases (1) and (2) at any $P_{\rm SS}>0$.

In order to study the equilibrium interface, the free energies of the two phases must be equal $[f_c^B(H)=f_c^A]$. This condition fixes the field as a function of pressure and temperature. In a strict sense we can study the interface only at pressures below the tricritical one because the interface cannot be stabilized in the GL region at higher pressures.

III. THE INTERFACE ON THE CONDENSATION ENERGY SCALE

We studied the A-B interface by numerical solution of the Ginzburg-Landau equations in one dimension.[12,8] The resulting order parameter is shown in Fig. 1(a) in zero magnetic field. We agree with previous calculations[1–3] that the minimum interface energy is achieved when $\hat{\boldsymbol{l}}$ is prependicular to the interface normal $\hat{\mathbf{s}}$, and

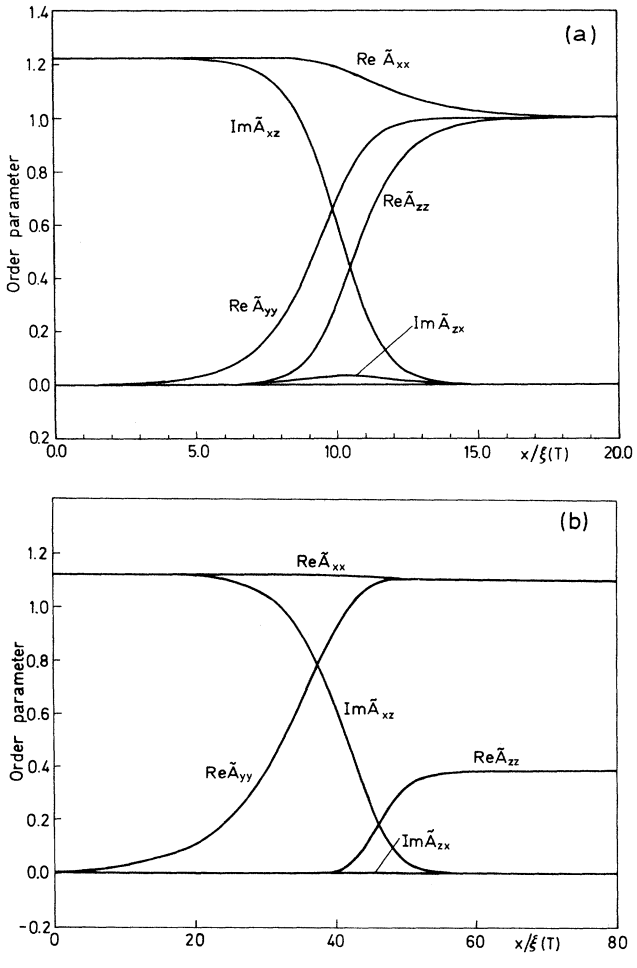

FIG. 1. The order parameter in the A-B interface at (a) the tricritical pressure and (b) at $P_{\rm SS}=1$ bar. The order parameter is represented in the form $\underline{A}=\Delta_B\underline{R}\cdot\tilde{\underline{A}}$, and all the nonzero components of $\tilde{\underline{A}}$ are displayed. The field is in the direction of $\underline{R}\cdot\hat{\mathbf{z}}$ in (b). There is the A phase on the left and the B phase on the right, and the x axis is chosen perpendicular to the interface. Note the different scales of x in the two figures. In the weak-coupling limit ($P_{\rm SS}\to 0$) we get that $\tilde{A}_{zz}$ and $\tilde{A}_{zx}$ vanish everywhere and $\tilde{A}_{xx}^2(x)=[{\rm Im}\,\tilde{A}_{xz}(x)]^2+\tilde{A}_{yy}^2(x)=\frac{5}{4}$.

$$e^{i\phi}\underline{R}\cdot\hat{\mathbf{s}}=\hat{\mathbf{d}}(\hat{\mathbf{u}}_1+i\hat{\mathbf{u}}_2)\cdot\hat{\mathbf{s}}\ .$$

The latter condition is equivalent to $\hat{\mathbf{d}}=\pm\underline{R}\cdot\hat{\mathbf{s}}$ and the continuity of the phase angle. Because the phase angles across the interface are strongly coupled, so are the circulations. Therefore, we conclude that, on the scale $\xi(T)$, all vortices must continue through (or at oblique angles within) the interface, contrary to Ref. 13. However, considerable changes in the vortex structure have to take place further away from the interface because the vortices are rather different in the two bulk phases.

There is a spontaneous mass current within the interface.[4] It has the magnitude $0.29\hbar\rho_s^B/2m_3$ and direction $\hat{\mathbf{s}}\times\hat{\mathbf{l}}$, where $\hat{\mathbf{s}}$ is taken to point towards the B phase and ρ_s^B is the superfluid density of the B phase. (The direction is opposite to that one would naively expect for a surface current resulting from a local angular momentum $\propto\hat{\mathbf{l}}$ in the A phase.) Also spontaneous spin currents flow along the interface. The spin-current tensor is given by

$$\underline{j}^{\rm spin}=\frac{\hbar\rho_s^B}{10m_3^2}[0.68(\underline{R}\cdot\hat{\mathbf{l}})(\hat{\mathbf{s}}\times\hat{\mathbf{l}})-0.70(\underline{R}\cdot\hat{\mathbf{s}}\times\hat{\mathbf{l}})\hat{\mathbf{l}}]\ .$$

The magnetic properties in low field are described by a spontaneous magnetic moment[3] and by magnetic susceptibility. The former has the magnitude $0.84g_z'\Delta_B^2\xi(T)$ and direction $\underline{R}\cdot\hat{\mathbf{l}}$. The susceptibility tensor is diagonal in the vector basis formed by $\underline{R}\cdot\hat{\mathbf{s}}$, $\underline{R}\cdot\hat{\mathbf{l}}$, $\underline{R}\cdot\hat{\mathbf{s}}\times\hat{\mathbf{l}}$. The susceptibility is highest in the third direction and it is smaller by $3.2g_Z\Delta_B^2\xi(T)$ in the second direction.

We want to study the distortion of the interface in a magnetic field. Comparing the magnetic energies above, we notice that a sizable distortion requires such high fields that the susceptibility dominates the spontaneous moment except at temperatures very near T_c. Therefore the linear magnetic interaction proportional to g_z' will be neglected in the following. The numerically calculated order parameter is displayed at the pressure $P_{\rm SS}=1$ bar, in Fig. 1b. The boundary conditions mentioned above remain valid at all $P_{\rm SS}>0$. In addition we find a condition for $\hat{\mathbf{H}}\cdot\underline{R}$: the three vectors $\hat{\mathbf{s}}=\pm\hat{\mathbf{d}}\cdot\underline{R},\hat{\mathbf{l}}$ and $\hat{\mathbf{H}}\cdot\underline{R}$ have to be orthogonal to each other. For example, at $P_{\rm SS}=12$ bar the state with $\hat{\mathbf{l}}=\pm\hat{\mathbf{H}}\cdot\underline{R}$ has the energy $0.79f_c^B(0)\xi(T)$ compared to the energy minimum $0.60f_c^B(0)\xi(T)$. The interface was found to change smoothly as a function of pressure. The energy of the interface can be fitted to the formula $2.5\xi(T)\sqrt{f_c^B(0)[f_c^A-f_c^{ap}(\pi/2)]}$. The error here is at most a few per cent. At the tricritical pressure this gives the energy $0.92f_c^B(0)\xi(T)$ in agreement with Ref. 3. (Note the difference in units as discussed in Sec. II.) At low $P_{\rm SS}$, the energy equals $0.15\sqrt{P_{\rm SS}/\text{bar}}$ in units of $f_c^B(0)\xi(T)$.

An interesting point is that the energy of the interface vanishes in the weak-coupling limit. This singularity can be traced back to the following conditions:

$$2\beta_1+\beta_3=0,\quad \beta_{45}=0\ , \tag{7}$$

which are satisfied in the weak-coupling approximation. It follows from these that $\Delta_\parallel=0$ (3) in the field that is required to reach equilibrium between the A and B phases, i.e., the B phase reduces to the planar phase. Moreover, all the axi-planar states (6) are degenerate.[10] Thus, there exists a continuous transformation between the A and B phases such that all the intermediate states have the same energy. It follows that only gradient energy is lost in the interface, and that can be reduced to an arbitrary small value by increasing the thickness of the interface. Thus, the A-B interface has zero energy, infinite thickness, and its structure becomes undetermined.

It would be interesting to know how well the weak-coupling approximation describes the superfluid at low pressures. Some information has been obtained from measurements of the specific heat,[14] magnetization,[15] and phase diagram.[16–18] While the intrinsic properties of the B phase seem to be in fair agreement with the weak-coupling approximation at zero pressure, deviations appear in other properties. We want to point out that the surface tension of the A-B interface, which is a measurable quantity,[1] is simply equal to the interface energy calculated above. Measurement of the surface tension at low pressures could be a sensitive test of the strong-coupling effects because of the vanishing of the tension at small values of the parameter $P_{\rm SS}$.

Let us briefly discuss the weak-coupling limit at arbitrary temperature. The GL calculations should be accurate only at temperatures near T_c. However, it can be shown that the axi-planar states remain degenerate at all temperatures.[19] Thus the interface tension remains at zero in a finite-temperature interval below T_c. Only below temperatures $\approx 0.8T_c$ does the tension become finite, because in this region the parallel gap of the B phase ($\Delta_\parallel$) becomes nonzero.[19] There the interface is made up of two parts: a finite-thickness interface between the B and planar phases and an infinite-thickness interface between the planar and A phases. We must point out that, experimentally, a finite tension may also result from magnetic field gradients or small geometries because they limit the thickness of the interface.

IV. ANALYTIC SOLUTION AT THE SINGULAR LIMIT

Let us study in more detail the behavior of the interface near the singular limit. We will show below that an analytic solution of the interface is possible near this limit. This has the great advantage that the dependence on all the five β_i's can be seen immediately. This may be important in setting experimental constraints on the β_i's.

We want to study the interface at some arbitrary values of β_i. For that purpose we choose another set of values $\beta_i^{(0)}$, which satisfy the condition (7). (These are not necessarily the weak-coupling values.) Our aim is to calculate the energy density of the interface to linear order in $\delta\beta$, where $\delta\beta$ measures the deviation of β_i's from $\beta_i^{(0)}$'s. Usually, this is achieved by calculating the free-energy change δF with the unperturbed order parameter $\underline{A}^{(0)}$. This is because the response $\delta\underline{A}$ does not contribute to the energy in first order due to stationarity of functional $F(\underline{A})$. The first complication in the present case is that the unperturbed state is vastly degenerate, and we have to select the state giving the lowest energy in response to $\delta\beta$. The second complication is that, in order to keep the A and B phases in equilibrium, the field has to be changed

by $\delta H \propto \sqrt{\delta\beta}$. Because δH is large, the response of the order parameter has to be taken into account in calculating δF. What makes the whole problem manageable is that these complications appear in different regions: the field affects only the nearly planar state, not the general axi-planar states. The calculation is presented below in more detail.

In the zeroth approximation, the problem consists of the bulk energy with $\beta_i^{(0)}$ and a field energy just enough to stabilize the planar phase among the B phases. No gradient energy is included in the zeroth order. The solution of the zeroth-order problem is an arbitrarily varying axi-planar state (5) with $\Delta_{ap}(\chi)=\Delta_0=\sqrt{\alpha/(4\beta_2^{(0)})}$. The energy of the interface vanishes in this approximation. The deviations $\delta\beta_i=\beta_i-\beta_i^{(0)}$ produce, in the first order, the energy

$$F_1=2\Delta_0^4\int dx\,[2\delta\beta_{245}-2\beta_{45}\sin^2\chi + (2\beta_1+\beta_{345})\sin^4\chi]\ . \tag{8}$$

A second perturbation is the gradient term of the GL energy functional. It restricts the spatial dependence of the zeroth-order solution within the states (5). Examination of the various possibilities reveals that the lowest energy is obtained with the form

$$\underline{A}^{(0)}(x)=\Delta_0\begin{bmatrix}1 & 0 & i\cos\chi\\ 0 & \sin\chi & 0\\ 0 & 0 & 0\end{bmatrix}\ , \tag{9}$$

where χ depends on the coordinate x perpendicular to the interface. This form is also obtained from the numerical solution, see Fig. 1(b). The gradient energy of (9) is simply

$$F_2=K\Delta_0^2\int dx\left[\frac{d\chi}{dx}\right]^2\ . \tag{10}$$

We express the energy of the B phase as a functional of the parallel gap $\Delta_\parallel=\Delta_0\psi$. Near the planar limit ($\Delta_\parallel\approx 0$) we get

$$F_3=\alpha\Delta_0^2\int dx\left[-1+\frac{g_Z}{\alpha}\delta(H^2)\psi^2 + \frac{\beta_3^{(0)}(\beta_1^{(0)}+3\beta_2^{(0)})}{8(\beta_2^{(0)})^2}\psi^4\right]\ . \tag{11}$$

The condition that the A and B phases should be in equilibrium gives, from (8) and (11), the condition

$$\frac{g_Z}{\alpha}\delta(H^2)=-\left[\frac{\beta_3^{(0)}(\beta_1^{(0)}+3\beta_2^{(0)})}{4(\beta_2^{(0)})^3}(2\beta_1+\beta_3-\beta_{45})\right]^{1/2}\ . \tag{12}$$

It follows from (11) that the B phase develops a finite parallel gap proportional to $\delta\beta^{1/4}$. The parallel gap appears as the component $A_{zz}=\Delta_0\psi$ in our axi-planar ansatz for $\underline{A}^{(0)}$ (9). In order to study the parallel component in the axi-planar states ($\chi\neq\pi/2$), we have to generalize the calculation leading to (11). Doing this we find that the coefficient of the term proportional to ψ^2 in (11) is nonzero and positive even if $\delta H=0$. It means that ψ has to decay near the junction of the planar and axi-planar states. This is in agreement with Fig. 1(b). Thus, it is enough to keep only the leading correction to (11) arising from $\chi\neq\pi/2$. It has the form

$$F_4=4\beta_{34}^{(0)}\Delta_0^4\int dx\ \psi^2\cos^2\chi\ . \tag{13}$$

Finally, we have to include the gradient energy of the parallel component

$$F_5=K\Delta_0^2\int dx\left[\frac{d\psi}{dx}\right]^2\ , \tag{14}$$

whereas the gradients of the perpendicular components can be neglected in the present approximation.

The problem now consists of finding the functions $\chi(x)$ and $\psi(x)$ that minimize the sum of the energies $F_1\cdots F_5$. This problem can be divided into two parts. In the first part we solve the problem consisting only of F_1+F_2. The Euler-Lagrange equations for this part have the first integral

$$\frac{d\chi}{dx}=\Delta_0\sin\chi\sqrt{(2/K)[-2\beta_{45}+(2\beta_1+\beta_{345})\sin^2\chi]}\ . \tag{15}$$

Here the constant of integration is fixed by requiring $d\chi/dx=0$ in the bulk A phase. The contribution to the energy from the first part can now be calculated from

$$F_1+F_2=2K\Delta_0^2\int_0^{\pi/2}d\chi\frac{d\chi}{dx}\ , \tag{16}$$

where we have cut off the x integration at point $\chi(x)=\pi/2$.

The second part of the solution consists of minimizing the sum of all the energies $F_1\cdots F_5$ in the region $\chi\approx\pi/2$. In order to avoid counting the same energy twice, we have to subtract the energy

$$F_{dc}=\sqrt{8K\Delta_0^6(2\beta_1+\beta_3-\beta_{45})}\int dx\frac{d\chi}{dx} \tag{17}$$

that is already included in (16). Interestingly, the second part is mathematically identical to the well-known problem of the normal-superconductor interface.[20] The GL parameter κ in this problem is given by

$$\kappa^2=\frac{\beta_3^{(0)}(\beta_1^{(0)}+3\beta_2^{(0)})}{4\beta_2^{(0)}\beta_{34}^{(0)}}\ . \tag{18}$$

Since we could not easily find a published numerical solution, we have calculated it here. Let I_2 denote the energy of the superconductor-normal interface in units of the GL coherence length times condensation energy density. We find that, below the critical value ($\kappa<1/\sqrt{2}$), the results can be well fitted by

$$I_2\approx 1.89-1.98\sqrt{\kappa}-0.31\kappa\ . \tag{19}$$

We can now write the final result for the surface tension (=free energy) of the A-B interface in superfluid ^{3}He as

$$F_{AB}=\frac{\xi(T)\alpha^2}{4\beta_2^{(0)}}\left[\frac{I_1}{\sqrt{2\beta_2^{(0)}}}+\frac{I_2}{2}\left[\frac{4a^3}{\beta_2^{(0)}\beta_3^{(0)}(\beta_1^{(0)}+3\beta_2^{(0)})}\right]^{1/4}\right], \tag{20}$$

where

$$I_1=\begin{cases}\sqrt{a+c}+\dfrac{a}{\sqrt{c}}\ln\left[\dfrac{\sqrt{a+c}+\sqrt{c}}{\sqrt{a}}\right] & \text{if } c>0\\ \sqrt{a+c}+\dfrac{a}{\sqrt{-c}}\arcsin(\sqrt{-c/a}\,) & \text{if } c<0\,,\end{cases} \tag{21}$$

$a=2\beta_1+\beta_3-\beta_{45}$ and $c=-(2\beta_1+\beta_{345})$. We have implicitly assumed that $a>0$ and $a+c>0$, which guarantees that the A phase (1) has the lowest energy among all axiplanar states (5).

The two terms in (20) can be associated in Fig. 1 as follows. The first term comes from the changeover of $\mathrm{Re}\,A_{yy}$ and $\mathrm{Im}\,A_{xz}$ and the latter from the foot of $\mathrm{Re}\,A_{zz}$. They have the orders of magnitude of $\sqrt{\delta\beta}$ and $\delta\beta^{3/4}$, respectively. The difference arises because the former has the length scale $\propto\delta\beta^{-1/2}$ and the latter $\propto\delta\beta^{-1/4}$. The difference in the length scales justifies the separation made in the calculation above.

We note that only four combinations of the five parameters β_i/α^2 appear in the thermodynamics of bulk A, A_1, and B phases in magnetic field. (Here we assumed that the linear magnetic interaction can be neglected in the B phase.) The surface tension (20) provides, at least in principle, a fifth combination of these parameters.

Let us discuss the accuracy of the expansion (20). Naturally, the accuracy should be better the nearer the expansion point ($\beta_i^{(0)}$) is the point we want to calculate. As far as we know, there is no unique way of defining concepts like "distance" and "orthogonality" in the space of β_i's, and thus there is no unique way of defining the best point of expansion. Therefore, we have tested a couple of reasonable choices. In the different choices we have varied the overall magnitude of $\beta_i^{(0)}$'s but have kept the relative magnitudes the same as in the weak-coupling theory, i.e., $(\beta_1^{(0)},\beta_2^{(0)},\beta_3^{(0)},\beta_4^{(0)},\beta_5^{(0)})\propto(-1,2,2,2,-2)$. [It follows from (18) that $\kappa=0.559$ for all choices. This is rather near the critical value 0.707, which partly explains the fact that the latter term in (20) seems to contribute only about 5%.] At $P_{SS}=1$ bar, none of the choices differs from the exact value more than the numerical uncertainty ($\approx1\%$) of the exact result. The differences appear at higher pressures and are compared below at the tricritical point (on the P_{SS} scale). Using for $\beta_i^{(0)}$ the absolute weak-coupling values gives a result that is 42% too small. Looking for better choices, we noticed from (20) that the most crucial parameter is $\beta_2^{(0)}$. Assuming $\beta_2^{(0)}=\beta_2$ gives only slightly better result than the absolute weak-coupling point. The crucial effect of $\beta_2^{(0)}$ is that it determines the amplitude of the order parameter (9) in the zeroth order. Therefore, it seems to us that the most reasonable choice for $\beta_2^{(0)}$ would be such that it produces either the amplitude of the planar phase or the average (squared) amplitude of the A and planar phases. These lead to the conditions $\beta_2^{(0)}=\beta_{12}+\beta_{345}/2$ and

$$2/\beta_2^{(0)}=\beta_{245}^{-1}+(\beta_{12}+\beta_{345}/2)^{-1}\,,$$

respectively. These choices lead to energies that are too small by 3% and too high by 9%, respectively. Generalizing slightly, we conclude that the expansion (20) seems to be rather accurate at any physical choice of the strong-coupling parameters provided that the expansion point is properly chosen.

The expansion (20) predicts that the interface tension at the melting pressure is about 12% higher than at the tricritical pressure. This implies that the excellent agreement between the measurement at the melting pressure[1] and the theory at the tricritical pressure[3] is slightly accidental.

V. INCLUSION OF THE DIPOLE ENERGY

The orientations of the bulk phases around the interface are not completely determined by the boundary conditions obtained in Sec. III. Therefore, we also have to consider the effect of the tiny dipole-dipole interaction. The general expression for the interaction energy is

$$f_D=g_D[\,|\mathrm{Tr}\underline{A}\,|^2+\mathrm{Tr}(\underline{A}\,\underline{A}^*)\,]\,. \tag{22}$$

In the bulk A phase (1), it takes the form

$$f_D^A=-2g_D\Delta_A^2(\hat{\mathbf{l}}\cdot\hat{\mathbf{d}})^2\,, \tag{23}$$

which tends to align $\hat{\mathbf{d}}$ and $\hat{\mathbf{l}}$ parallel (or antiparallel). In the B phase (2), the dipole energy (22) gives

$$f_D^B=2g_D\Delta_1^2[\,2(1+\epsilon)\cos\theta+4\cos^2\theta-\epsilon(3+4\cos\theta)\hat{\mathbf{H}}\cdot\hat{\mathbf{h}}+\epsilon^2(\hat{\mathbf{H}}\cdot\hat{\mathbf{h}})^2\,]\,, \tag{24}$$

where $\epsilon=1-\Delta_\parallel/\Delta_\perp$ and $\hat{\mathbf{h}}=\hat{\mathbf{H}}\cdot\underline{R}$. In the bulk B phase it fixes

$$\theta=\arccos\left[-\frac{\Delta_\parallel}{4\Delta_\perp}\right],\quad \hat{\mathbf{n}}=\pm\hat{\mathbf{H}}\,. \tag{25}$$

If, at some point, the dipole energy is not minimized, it will heal towards the minimum on a length scale L determined by minimizing the sum of the gradient energy ($\approx K\Delta^2L^{-1}$) and the dipole energy ($\approx f_DL$). In the A phase and for the angle θ in the B phase, this gives the dipole length $L\approx\xi_D=\sqrt{K/g_D}$. However, as can be seen from Eq. (24), the part of the dipole energy that depends on $\hat{\mathbf{h}}$ (or equivalently on $\hat{\mathbf{n}}$) is smaller by the factor ϵ. It follows that, in small fields, $\hat{\mathbf{n}}$ heals on a considerably longer length scale $\approx\xi_D/\sqrt{\epsilon}$. More exactly, this healing length is either $2\xi_{DZ}$ or $\sqrt{2}\xi_{DZ}$ depending on the way $\hat{\mathbf{n}}$ approaches $\hat{\mathbf{H}}$. The magnetic coherence length ξ_{DZ} varies from approximately 5.4 (melting pressure) to 12 (zero pressure) in units cm G $\sqrt{1-T/T_c}/H$.[8]

In the following we consider the direction of the A-B interface as fixed externally. This is because the small orienting energy (on the order of $g_D\Delta^2\xi_D$ in high fields) on the interface arising from the dipole interaction is, in practice, overwhelmed by the effect of container walls and gradients of the magnetic field. We divide the discussion of the interface in three parts: we first consider zero field, then a weak field, and finally, and most extensively,

the limit of large fields.

In the absence of the magnetic field, the superfluid condensation energy is minimized when $\hat{\mathbf{s}}\cdot\hat{\mathbf{l}}=0$ and $\hat{\mathbf{d}}=\pm\underline{R}\cdot\hat{\mathbf{s}}$. The dipole interaction fixed, in the two phases, $\theta=\arccos(-\frac{1}{4})\approx 104°$ and $\hat{\mathbf{d}}=\pm\hat{\mathbf{l}}$. These can all be satisfied simultaneously if $\hat{\mathbf{n}}$ has any of the following four values:[5] $(\pm 1,\pm 1,\sqrt{3})/\sqrt{5}$ or $(\mp 1,\pm 1,-\sqrt{3})/\sqrt{5}$ expressed in vector basis $\hat{\mathbf{s}}$, $\hat{\mathbf{l}}$, and $\hat{\mathbf{s}}\times\hat{\mathbf{l}}$. This configuration still has the freedom of rotating around $\hat{\mathbf{s}}$.

The behavior in the magnetic field is determined by the competition of two magnetic energies [$\approx g_Z'\Delta_B^2 H\xi(T)$ and $\approx g_Z\Delta_B^2 H^2\xi(T)$] and the dipole energy ($\approx g_D\Delta_B^2\xi_D$). Comparing their magnitudes we can distinguish low- and high-field limits. The dipole energy dominates in the low-field limit, which has the range $H\ll 600$ G at the melting pressure, but it practically disappears below the tricritical pressure. The quadratic field term dominates in the high-field limit, which is achieved when $H\gg 600$ G at the melting pressure or $H\gg 50$ G at zero pressure. The field region between the limits is complicated because all three terms are comparable, and it will not be discussed here.

In the low-field limit the zero-field configuration remains valid except that $\hat{\mathbf{d}}$ is fixed perpendicular to $\hat{\mathbf{H}}$, so that the rotation degree of freedom is lost in magnetic field (unless $\hat{\mathbf{H}}=\pm\hat{\mathbf{s}}$). It follows trivially that

$$\hat{\mathbf{n}}\cdot\hat{\mathbf{H}}=(\cos\beta+\sqrt{3}\sin\beta)/\sqrt{5}\ , \tag{26}$$

where β is the angle between $\hat{\mathbf{s}}$ and $\hat{\mathbf{H}}$. Thus, $\hat{\mathbf{n}}$ cannot be in the direction of the field at the interface. Instead, the interface has a tail towards the B phase, in which $\hat{\mathbf{n}}$ heals from its direction at the interface towards $\hat{\mathbf{n}}=\pm\hat{\mathbf{H}}$. The tail is rather long, on the order of the magnetic coherence length ξ_{DZ}. There is no tail on the A-phase side.

In the limit of high fields, the condensation and the field energies have to be minimized first, and the dipole interaction can be treated as a perturbation. The first part was done in Sec. III: $\underline{R}\cdot\hat{\mathbf{s}}=\hat{\mathbf{d}}$, $\underline{R}\cdot\hat{\mathbf{l}}$, and $\hat{\mathbf{H}}$ have to be orthogonal to each other. Considering the dipole energy, we note that the contribution coming from the "hard" interface [length scale $\xi(T)$] can be neglected in comparison to the contribution from the "soft" interface of length scale ξ_D or ξ_{DZ}. [We do not consider the limit

$$P_{SS}<\approx 10^{-3}\ \text{bars}(1-T/T_c)^{-1/4}\ ,$$

where this assumption breaks down because of the expansion of the interface.]

We studied the effect of the dipole interaction using the following simple model. We assume that the *total* dipole energy is given by a linear combination of the expressions (23) and (24):

$$F_D=af_D^A+bf_D^B\ , \tag{27}$$

where the variables ($\hat{\mathbf{d}}$, $\hat{\mathbf{l}}$, θ and $\hat{\mathbf{h}}$) refer to their values just at the interface. We minimize this energy with the constraints set on the condensation-energy scale, and also that the interface normal $\hat{\mathbf{s}}$ has a given angle β with the field $\hat{\mathbf{H}}$. In proper coordinates this leads to the minimization of a function of two real variables without constraints, a problem which can conveniently be solved numerically.

A representative result of the simple model is shown in Fig. 2. This graph is obtained using parameters $a\Delta_A^2=1.4b\Delta_B^2$ and $\epsilon=0.2$. We have used $a>b$ because the average healing length in the A phase ($\sqrt{5}\xi_D/2$) is larger than the healing lengths in the B phase ($2\xi_D/\sqrt{15}$ and $\sqrt{8/15}\xi_D$). Other physical choices (differing by factors of 2) did not produce essentially different results.

It can be seen that $\hat{\mathbf{l}}$ deviates, at most, a few degrees from being perpendicular to the field. It also can be noted that the deviations of $\hat{\mathbf{l}}$ from $\hat{\mathbf{d}}$ and θ from 104° are relatively small for all directions of the interface. The case $\beta=30°$ is a special one where the dipole energy takes its absolute minimum value, and the orientations of the phases are identical low and high fields. Curiously, in this case also, the rotation angle θ exactly equals $\arccos(-\frac{1}{4})$ for any degree of depression ϵ of the B phase.

Figure 2 also displays the deviations of $\hat{\mathbf{h}}$ and $\hat{\mathbf{n}}$ from the field direction. [These are interrelated by the equation

$$\hat{\mathbf{h}}\cdot\hat{\mathbf{H}}=\cos\theta+(1-\cos\theta)(\hat{\mathbf{n}}\cdot\hat{\mathbf{H}})^2\ , \tag{28}$$

which simply follows from the properties of rotation ma-

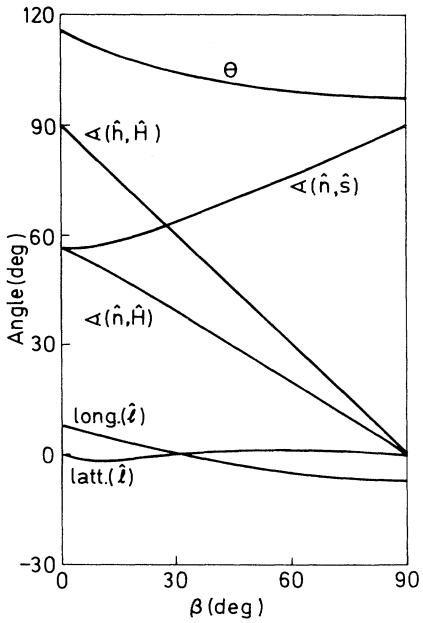

FIG. 2. Approximate orientations of the bulk phases near the *A-B* interface in the limit of high fields. All angles (in degrees) are given as a function of the angle β between the magnetic field and the interface normal $\hat{\mathbf{s}}$. $D_{\text{latt}}(\hat{\mathbf{l}})$ and $D_{\text{long}}(\hat{\mathbf{l}})$ denote the deviation of $\hat{\mathbf{l}}$ from $\hat{\mathbf{d}}$ in the polar and azimuthal angle, respectively, in a coordinate system where $\hat{\mathbf{H}}$ is the pole. All other curves represent angles between vectors indicated on the labels. Note that $\hat{\mathbf{d}}\perp\hat{\mathbf{H}}$ and $\hat{\mathbf{l}}\perp\hat{\mathbf{s}}$ exactly, also at low fields. For comparison, in the limit of low fields the corresponding results are as follows: $D_{\text{latt}}(\hat{\mathbf{l}})$ and $D_{\text{long}}(\hat{\mathbf{l}})$ vanish identically, $\theta\equiv\arccos(-\frac{1}{4})$, $\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}\equiv 1/\sqrt{5}$, and (26) and (28). Both limits of the field give identical results at $\beta=30°$.

trices.] It can be seen that $\hat{\mathbf{h}}$ and $\hat{\mathbf{n}}$ are in the direction of the field only in the case where the field is in the plane of the interface. For all other orientations of the interface there is a tail of length $\approx\xi_{DZ}$ on the B phase side at high fields also.

A special limit is obtained when $\Delta_{\|}=0$. In this case, $\hat{\boldsymbol{l}}$ and $\hat{\mathbf{d}}$ are strictly parallel, but the angle θ takes is maximal variation from 90° (at $\beta=90°$) to 120° (at $\beta=0°$). $\hat{\mathbf{h}}$ and $\hat{\mathbf{n}}$ are essentially unchanged from Fig. 2.

The simple model (27) would be exact if, on both sides of the interface, the healing takes place with a single healing length. Because the vector $\hat{\mathbf{h}}$ has longer healing length than the others, the energy of this degree of freedom appears underestimated in the model (27). This will lead to only a tiny error in our results because, as can be seen from Fig. 2, the vector $\hat{\mathbf{h}}$ is very near its minimum-energy direction that is consistent with it being perpendicular to $\hat{\mathbf{s}}$.

VI. CONCLUSIONS

We have studied the structure of the A-B interface on the energy scales of both the superfluid condensation energy and the dipole energy. We found a simple orthogonality rule for $\hat{\mathbf{s}}$, $\hat{\boldsymbol{l}}$, and $\hat{\mathbf{H}}\cdot\underline{R}$. At low fields, the B phase rotation angle equals 104° and $\hat{\boldsymbol{l}}=\pm\hat{\mathbf{d}}$, but deviations from these appear in higher fields, as shown in Fig. 2. The interface has, on the B-phase side, a tail, where the $\hat{\mathbf{n}}$ vector strongly deviates from its direction in the bulk. The only exception to this rule is the case of a high field parallel to the interface. The dimension of the tail is on the order of the magnetic coherence length and should be observable by nuclear magnetic resonance. The surface tension of the interface is found to vanish steeply in the weak-coupling limit. An analytic expression (20) was found for the tension as a function of general strong-coupling parameters near the singular limit (7). This might be useful in obtaining information about strong-coupling parameters at low pressures.

Note added in proof. The singular behavior of the interface has been studied by G. E. Volovik (Zh. Eksp. Teor. Fiz. **97**, 1198 (1990) [Sov. Phys. JETP **70**, 672 (1990)]), but his conclusions differ from ours.

ACKNOWLEDGMENTS

I wish to thank C. Gould, N. Kopnin, M. Krusius, J. Kyynäräinen, J. Pekola, and N. Schopohl for useful discussions. I am indebted to the Department of Technical Physics and to the Low Temperature Laboratory at Helsinki University of Technology for support.

[1]D. D. Osheroff and M. C. Cross, Phys. Rev. Lett. **38**, 905 (1977).
[2]R. Kaul and H. Kleinert, J. Low Temp. **38**, 539 (1980).
[3]N. Schopohl, Phys. Rev. Lett. **58**, 1664 (1987).
[4]M. M. Salomaa, J. Phys. C **21**, 4425 (1988).
[5]S. Yip, Phys. Rev. B **35**, 8733 (1987).
[6]J. M. Kyynäräinen, J. P. Pekola, A. J. Manninen, and K. Torizuka, Phys. Rev. Lett. **64**, 1027 (1990).
[7]J. M. Kyynäräinen, J. P. Pekola, K. Torizuka, A. J. Manninen, and A. V. Babkin, J. Low Temp. Phys. **82**, 325 (1991).
[8]E. V. Thuneberg, Phys. Rev. B **36**, 3583 (1987).
[9]A. L. Fetter, in *Quantum Statistics and the Many-Body Problem,* edited by S. B. Trickey, W. P. Kirk, and J. W. Dufty (Plenum, New York, 1975), p. 127.
[10]N. D. Mermin, Physica B **90**, 1 (1977).
[11]J. A. Sauls and J. W. Serene, Phys. Rev. B **24**, 183 (1981).
[12]E. V. Thuneberg, Phys. Rev. B **33**, 5124 (1986).
[13]M. M. Salomaa, Nature **326**, 367 (1987).
[14]D. S. Greywall, Phys. Rev. B **33**, 7520 (1986).
[15]R. F. Hoyt, H. N. Scholz, and D. O. Edwards, Physica B **107**, 287 (1981).
[16]J. D. Feder, D. O. Edwards, W. J. Gully, K. A. Muething, and H. N. Scholz, Phys. Rev. Lett. **47**, 428 (1981).
[17]U. E. Israelsson, B. C. Crooker, H. M. Bozler, and C. M. Gould, Phys. Rev. Lett. **53**, 1943 (1984).
[18]Y. H. Tang, I. Hahn, H. M. Bozler, and C. M. Gould (unpublished).
[19]M. Ashida and K. Nagai, Prog. Theor. Phys. **74**, 949 (1985).
[20]V. L. Ginzburg and L. D. Landau, Zh. Eksp. Teor. Fiz. **20**, 1064 (1950); E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics, Part 2* (Pergamon, Oxford, 1980).