Dynamics of granular superconductors at zero and large magnetic fields: Glassy behavior

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We calculate the transport properties of a model granular superconductor in zero magnetic field and in a strong random magnetic field. The Hamiltonian is taken as the sum of two terms: a Josephson interaction coupling neighboring phases and a diagonal charging energy. The noncommutativity of charge and phase is neglected (classical, or large-grain, limit), as is dissipation occurring through shunt resistances connecting neighboring grains. The dynamical properties, including current-current correlation functions, conductivities, and voltage noise, are calculated as a function of temperature via molecular-dynamics techniques. For a simple-cubic lattice in zero magnetic field, the model exhibits the expected insulator-to-superconductor transition near $T_c = 2.2E_J/k_B$, where E_J is the Josephson coupling energy. The frequency-dependent conductivity has a strong fluctuation peak near T_c. Its behavior above T_c is qualitatively reproduced by a simple Aslamasov-Larkin-like model. The voltage noise has a time correlation function, which oscillates below T_c but falls off nearly monotonically above T_c. Its low-temperature behavior is explained analytically in terms of a model of voltage fluctuations due to propagating phase waves. A disordered system in a strong magnetic field is modeled by grains on a simple-cubic lattice with random magnetic phase factors on each bond, following Huse and Seung. Evidence for a vortex-fluid-to-vortex-glass transition is found from the increasingly slow decay of the current-current correlation function above $T_g \approx 0.45E_J/k_B$, and from its nonergodic behavior below T_g. The behavior is similar to that found by Leutheusser from an analytic model of a structural liquid-glass transition.

I. INTRODUCTION

The physics of composite or granular superconductors has been extensively studied for many years.[1,2] Many potentially practical superconducting materials are composites of superconducting and nonsuperconducting constituents. Among low-T_c materials, $A15$-structure superconductors embedded in Cu are the materials of choice for high-field magnet wire, while among the high-T_c materials, composites of Ag-sheathed $YBa_2Cu_3O_{7-\delta}$ (Ref. 3) have shown promise as useful materials at $T > 77$ K. On the other hand, reliable or even useful models for estimating the transport properties of such materials as a function of temperature are still very much needed.

In this paper, we describe a simple model to describe transport in a model granular superconductor consisting of Josephson-coupled superconducting grains embedded in an insulating matrix. The model neglects dissipative (one-electron) tunneling between grains, and hence may be quantitatively applicable to real superconducting materials only in very exceptional circumstances. Nevertheless, it has the advantage of being numerically soluble. It may therefore serve as a starting point for more realistic treatments of granular materials which include dissipative tunneling and low-temperature quantum effects.

The model consists of a collection of Josephson-coupled grains. The charging energy of the grains is included in the large-capacitance limit where this energy may be treated classically. The omission of dissipative effects corresponds to the assumption $\beta \gg 1$, where β is the analog of the well-known McCumber parameter[4] introduced from single junctions. We treat this model by standard molecular-dynamics techniques in two regimes: zero magnetic field, and strong random magnetic fields. In the former case, we find an insulator-to-superconductor transition with a conspicuous fluctuation conductivity on either side of the transition. Below T_c we find a significant high-frequency voltage noise which can be treated analytically. In a strong random magnetic field, which we treat by an extension of a model due to Huse and Seung,[5] we find evidence of a vortex-fluid-to-vortex-glass transition,[6-10] signaled by a very slow decay of the current-current correlation functions above the glass transition temperature T_g, and a nonergodic, history-dependent behavior below T_g.

The remainder of this paper is organized as follows. The model is described in Sec. II. Our numerical techniques are presented in Sec. III. The results for zero and strong magnetic field are given in Sec. IV and V, followed by a brief discussion in Sec. VI. An Appendix describes the limits of applicability of this model to real composite superconductors.

II. MODEL

We consider a system of N superconducting grains. The superconducting order parameter on the ith grain is assumed to be

$$\psi_i = |\psi_i| \exp(i\theta_i) , \tag{1}$$

and the system is assumed to be described by the Hamiltonian

$$H=\tfrac{1}{2}\sum_{ij} C_{ij}V_iV_j-\tfrac{1}{2}\sum_{ij} E_{J;ij}\cos(\theta_i-\theta_j)\ , \tag{2}$$

where V_i is the voltage on the ith grain; C_{ij} is the capacitive coupling between grains i and j; $E_{J;ij}$ is the Josephson coupling energy between grains i and j; and the sums run over all grains i and j.

The physics behind the terms in Eq. (2) is straightforward. The first term represents the Coulomb energy of the charges distributed on the various grains, as determined by the capacitance matrix C_{ij}.[11] The second term is the Josephson coupling energy between grains i and j. It is related to the critical current $I_{c;ij}$ between those grains by $E_{J;ij}=2eI_{c;ij}/\hbar$.

With the help of the Josephson equation, $\dot{\theta}_i=2eV_i/\hbar$, Hamiltonian (2) can be rewritten as

$$H=\frac{1}{2}\frac{\hbar^2}{4e^2}\sum_{ij}\dot{\theta}_iC_{ij}\dot{\theta}_j-\frac{1}{2}\sum_{ij}E_{J,ij}\cos(\theta_i-\theta_j)\ . \tag{3}$$

Introducing a generalized momentum p_i by $p_i=-\partial H/\partial\dot{\theta}_i=-(\hbar^2/4e^2)\sum_j C_{ij}\dot{\theta}_j$ and substituting into Eq. (3), we obtain H in the canonical form

$$H=\frac{2e^2}{\hbar^2}\sum_{ij}p_iU_{ij}p_j-\frac{1}{2}\sum_{ij}E_{J;ij}\cos(\theta_i-\theta_j) \tag{4}$$

where the charging energy matrix U is the inverse of the capacitance matrix, $U_{ij}=(C^{-1})_{ij}$.

The canonical equations of motion for this system are $\dot{\theta}_i=\partial H/\partial p_i$, $\dot{p}_i=-\partial H/\partial\theta_i$. Evaluation of these equations using the Hamiltonian in the form (4) gives

$$\dot{\theta}_i=\frac{4e^2}{\hbar^2}\sum_j U_{ij}p_j\ , \tag{5}$$

$$\dot{p}_i=-\sum_j E_{J;ij}\sin(\theta_i-\theta_j)\ , \tag{6}$$

which are the classical equations of motion determining the evolution of the phases in the granular superconductor.

Equations (5) and (6) involve, of course, a number of approximations. First, they omit quantum effects associated with the noncommutativity of θ_i and p_i. Such quantum effects are small in the regime

$$E_c\equiv e^2/(2C)\ll E_J\ , \tag{7}$$

where E_c is a charging energy associated with a typical element C of the capacitance matrix, and E_J is a typical Josephson coupling. Secondly, they leave out dissipative transport of charge through shunt resistances. The current through such a shunt resistance R_{ij} in the ijth junction has the form $I_{ij}^{\rm shunt}=\hbar(\dot{\theta}_i-\dot{\theta}_j)/(2eR_{ij})$, which could readily be included, if needed, in the classical equations of motion.[12] But this dissipative term can be neglected when the dimensionless McCumber parameter

$$\beta\equiv\frac{4e^2R^2CE_J}{\hbar^2}\gg 1\ , \tag{8}$$

where R, C, and E_J represent typical shunt resistances, capacitances, and intergranular couplings.

In order for Eqs. (5) and (6) to be applicable, both of the inequalities (7) and (8) must be satisfied. The applicability of these conditions in a real granular superconductor is further discussed in the Appendix. Although the range of rigorous applicability may be limited, the predictions of this model are still of interest as the limiting case of a more realistic treatment which includes both dissipation and capacitive tunneling through shunt resistances. An analogous calculation, involving dissipation but no capacitive kinetic energy, has been presented elsewhere.[13]

In the present calculations, we consider a cubic lattice of grains with diagonal charging energy matrix $U_{ij}=U\delta_{ij}$, and only nearest-neighbor coupling. The Hamiltonian (3) then becomes

$$H=\frac{\hbar^2C}{8e^2}\sum_i(\dot{\theta}_i)^2-\frac{1}{2}E_J\sum_{ij}\cos(\theta_i-\theta_j) \tag{9}$$

where E_J is the nearest-neighbor coupling.

In this model, it is convenient to adopt natural units, i.e., energy in units of E_J and time in units of $\tau_0=(\hbar/2e)\sqrt{C/E_J}$. The equations of motion then take the form

$$\ddot{\theta}_i=-\sum_j\sin(\theta_i-\theta_j)\ , \tag{10}$$

where the sum runs over nearest neighbors to i.

The dynamical model (10) is suitable for an ordered composite in zero magnetic field. To introduce a magnetic field, we replace the phase difference $\theta_i-\theta_j$ by the *gauge-invariant phase difference* $\gamma_{ij}=\theta_i-\theta_j-A_{ij}$ where $A_{ij}=(2\pi/\Phi_0)\int_i^j\mathbf{A}\cdot\mathbf{dx}$, $\mathbf{A}$ being the vector potential and $\Phi_0=hc/2e$ the flux quantum.

To treat a *disordered* granular system in a field, we adopt a simplified model,[5] according to which the lattice of grains is ordered, with nearest-neighbor coupling E_J, but the A_{ij}'s are taken as a random variables uniformly distributed on the interval $[-\pi,\pi]$, such that the phase factors A_{ij} on different bonds are uncorrelated. The flux Φ through a given plaquette is given by the sum of the phase factors A_{ij} around a plaquette, i.e.,

$$\Phi=\frac{\Phi_0}{2\pi}\sum A_{ij}\ . \tag{11}$$

Hence, this model corresponds to having a random flux passing through each plaquette of nearest-neighbor bonds of the cubic lattice. Although the lattice of grains is ordered, the model may still be reasonable for a positionally disordered collection of coupled grains in a strong but uniform magnetic field ("strong" meaning a field much larger than one flux quantum per typical plaquette), since in both cases the phase factors A_{ij} will be random variables. In fact, it has been shown numerically that the I-V characteristics of a such a positionally disordered array of grains coupled by resistively-shunted Josephson junctions is very similar to that of the corresponding Huse-Seung glass, in the limit of strong magnetic fields.[13]

The variables A_{ij} are taken to be "quenched," that is, determined by an externally applied field. This implies a neglect of the fields produced by the screening currents

themselves. The model is therefore appropriate only when the Josephson screening length is large compared to the size of the sample considered in the simulation.

III. METHOD OF CALCULATION

We have treated the model by standard finite-temperature molecular dynamics techniques, using an $Ld\times Ld\times Ld$ cubic lattice ($d=$ lattice constant) with periodic boundary conditions and generally $L=7$. We use the Verlet leap-frog algorithm[14,15] to solve the equations of motion. The generalized velocity $\dot{\theta}_i$ is scaled so that the total kinetic energy term is equal to $Nk_BT/2$, where $N=L^3$ is the number of grains. For each temperature, we have run 2×10^5 time steps Δt. In general, we used a time interval $\Delta t=0.01\tau_0$.

We have calculated several relevant quantities. The first is the current-current correlation function, defined as $\langle J_\alpha(0)J_\beta(t)\rangle$, where $\langle\ \rangle$ denotes a canonical average. $J_\alpha(t)$ is the αth Cartesian component of the current density multiplied by volume at time t. It is given by

$$J_\alpha(t)=d\sum_{ij;\alpha} I_c\sin\gamma_{ij} \tag{12}$$

where $I_c=(2e/\hbar)E_J$ is the critical current between grains i and j, and we have used the Josephson current-phase relation

$$I_{ij}=I_c\sin\gamma_{ij}\ ; \tag{13}$$

the sum runs over bonds oriented in the α direction. The current-current correlation function is related to the frequency-dependent conductivity $\sigma_{\alpha\beta}(\omega)$ by the Kubo formula,[16] which in the classical limit takes the form

$$\sigma_{\alpha\beta}(\omega)=\frac{1}{(Ld)^3k_BT}\int_0^\infty\langle J_\alpha(0)J_\beta(t)\rangle\cos(\omega t)dt \tag{14}$$

at temperature T. In the present model, the conductivity is isotropic, so that $\sigma_{\alpha\beta}(\omega)=\sigma(\omega)\delta_{\alpha\beta}$. To reduce numerical fluctuations, we calculate $\sigma(\omega)$ as an average over three directions, i.e., $\sigma(\omega)=\frac{1}{3}\sum_{\alpha=1}^{3}\sigma_{\alpha\alpha}(\omega)$. Likewise, we calculate

$$\langle J(0)J(t)\rangle\equiv\tfrac{1}{3}\sum_{\alpha=1}^{3}\langle J_\alpha(0)J_\alpha(t)\rangle\ .$$

We have also computed a measure of the voltage noise between grains i and j, namely

$$S_{VV}(t)\equiv\frac{1}{N_b}\sum_{\langle ij\rangle}\langle(V_i(0)-V_j(0))(V_i(t)-V_j(t))\rangle \tag{15}$$

and its Fourier transform

$$S_{VV}(\omega)=\int_0^\infty S_{VV}(t)\cos(\omega t)dt\ , \tag{16}$$

where N_b is the number of bonds and the sum (15) runs over distinct nearest-neighbor pairs.

IV. ZERO MAGNETIC FIELD

In zero magnetic field, since kinetic and potential energies commute, the present model has static properties equivalent to the three-dimensional $XY(n=2)$ model, which is known from Monte Carlo simulations and from high-temperature series expansions to have a phase transition at $T_c\approx2.21E_J/k_B$.[17] This transition may be interpreted as a superconductor-to-insulator (SI) transition from a state with long range to short range phase coherence.

The SI transition is reflected in the current-current correlation function $\langle J(0)J(t)\rangle$, shown in Fig. 1 at several temperatures, and in the corresponding Fourier transform, $\mathrm{Re}\sigma(\omega)$ [Eq. (14)], shown in Fig. 2. There is a strong peak in $\sigma(0)$ near temperature $T=2.4E_J/k_B$, the maximum value falling off on both sides of the transition. A similar peak is also evident in $\langle J(0),J(t)\rangle$ at $t=0$.

The behavior of $\mathrm{Re}\sigma(\omega)$ for $T>T_c$ can be qualitatively understood from a crude fluctuation model, analogous to the Aslamasov-Larkin picture used to understand homogeneous superconductors.[18] We write the fluctuation conductivity, for $T>T_c$, as a sum of Drude-like contributions,

$$\mathrm{Re}\sigma(\omega)=\mathrm{Re}\sum_{\mathbf{k}\epsilon\mathrm{BZ}}\frac{n_\mathbf{k}q^2}{1/\tau_\mathbf{k}+i\omega}\ , \tag{17}$$

where $n_\mathbf{k}$ is the Fourier component of a superfluid density of wave vector $\mathbf{k}$, $\tau_\mathbf{k}$ is the corresponding relaxation time, q is the charge of a charge carrier, and the sum runs over the first Brillouin zone of the grain lattice. Next, we assume

$$n_\mathbf{k}=\frac{1}{a\epsilon+bk^2}\equiv\frac{1}{b(\xi_p^{-2}+k^2)} \tag{18}$$

$$\frac{1}{\tau_\mathbf{k}}=c(a\epsilon+bk^2)\ , \tag{19}$$

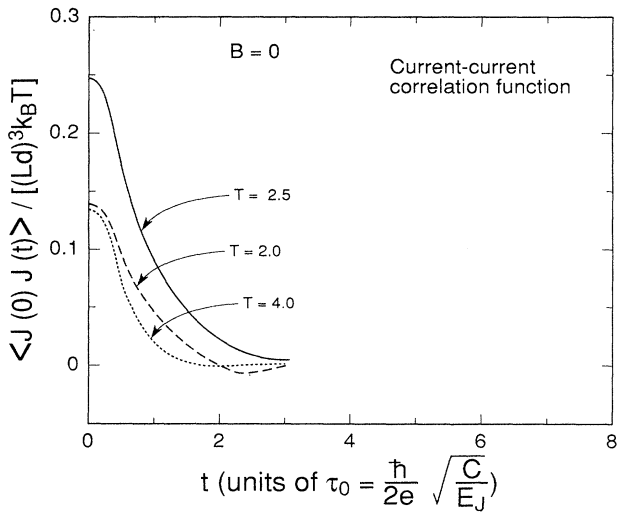

FIG. 1. Current-current correlation function $\langle J(0)J(t)\rangle/((Ld)^3k_BT)$ for a simple cubic $7\times7\times7$ granular superconductor ($L=7$) with no magnetic field, plotted as a function of time for several temperatures. T_c for the infinite lattice is $2.21E_J/k_B$. Units are such that $d=1$ and $I_c=1$. Temperature is in units of E_J/k_B; time in units of $\tau_0=(\hbar/2e)\sqrt{C/E_J}$.

where $\epsilon=(T-T_c)/T_c$, a, b, and c are constants and ξ_p is the phase coherence length in the Gaussian approximation. Substituting (18) and (19) into (17), and making a Debye approximation by transforming the sum into an integral over a spherical volume of radius d, where d is the grain lattice constant, we obtain

$$\mathrm{Re}\sigma(\omega)=C_0\int_0^{1/d}\frac{k^2}{c^2(a\epsilon+bk^2)^2+\omega^2}dk \tag{20}$$

where C_0 is a temperature-independent constant. It is readily verified that, for $\omega=0$ and $\epsilon\ll 1$, this form predicts $\mathrm{Re}\sigma(0)\propto\epsilon^{-1/2}$, whereas for sufficiently high frequency $\mathrm{Re}\sigma(\omega)=A\omega^{-2}$ with a constant of proportionality A which is independent of ϵ. This latter prediction is in agreement with our simulations; the former cannot be confirmed without approaching T_c more closely using larger molecular-dynamics (MD) cells. Equation (20) does disagree with the simulations in one respect: it predicts a conductivity which decreases monotonically with ω at any positive ϵ.

To account for the peak in $\mathrm{Re}\sigma(\omega,T)$ which appears in our simulations at nonzero frequency, we arbitrarily add to the contribution (20) a term arising from a damped Lorentz oscillator:

$$\mathrm{Re}\Delta\sigma(\omega)=\frac{C\omega^2}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}\ . \tag{21}$$

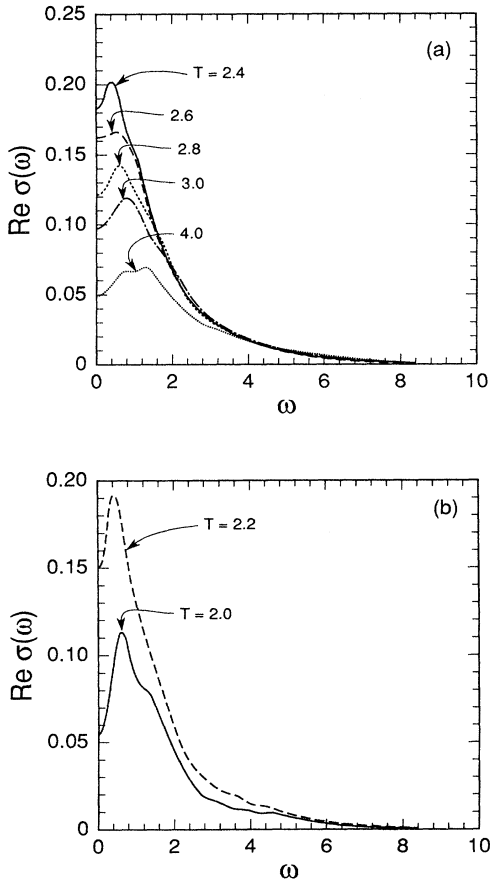

FIG. 2. (a) Real part of the frequency-dependent conductivity, $\mathrm{Re}\sigma(\omega)$, plotted as a function of frequency for the temperatures $T=2.4$, 2.6, 2.8, 3.0, and 4.0 (in units of E_J/k_B). (b) Same as (a) but for temperatures $T=2.0$ and 2.2. Frequency in units of $1/\tau_0$.

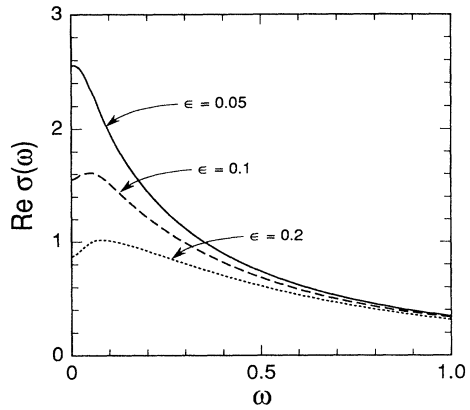

FIG. 3. Model fluctuation conductivity [Eqs. (20) plus (21)] at temperatures $\epsilon\equiv(T-T_c)/T_c=0.05$, 0.1, and 0.2. Frequency in units of $1/\tau_0$.

Figure 3 shows $\mathrm{Re}\sigma(\omega,T)$ calculated as the sum of Eqs. (20) and (21), with the arbitrary assumptions $a=b=c=1$, $C_0=1$, $C=0.6$, and $\omega_0=0.2$. No effort has been made to fit these parameters to the computer data, but the simple analytic model shows all the essential features of this data. Namely, all curves collapse onto the same $(1/\omega^2)$ behavior at high frequencies, and there is a peak at finite frequency, which moves to smaller and smaller frequencies and eventually disappears as T_c is approached from above.

Figure 4 shows $\mathrm{Re}\sigma(\omega,T=2.35E_J/k_B)$, very close to the infinite-lattice T_c, for several lattice sizes. Evidently, the larger lattices behave as if they are effectively nearer T_c than the smaller lattices. This can also be understood from our analytic model, eqs. (17)–(21). First, note that

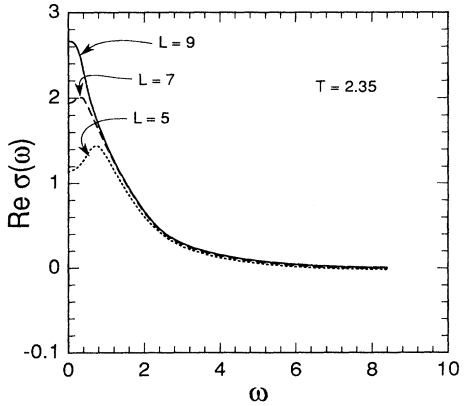

FIG. 4. Real part of the MD conductivity $\sigma(\omega)$ for an $L\times L\times L$ cubic lattice at temperature $T=2.35J/k_B$, for $L=5$, 7, and 9. Frequency in units of $1/\tau_0$.

the phase coherence length is related to the temperature by $\xi_p \propto \epsilon^{-1/2}$. Sufficiently near T_c, ξ_p will exceed the size of the MD cell. Thus, in evaluating expression (20) at $T=T_c$ for a *finite* cell, one should replace ξ_p by L, and hence, replace $a\epsilon$ by L^{-2}. A large L therefore corresponds to an infinite system very close to T_c, with a larger peak in the fluctuation conductivity. This prediction is in qualitative agreement with the computer results shown in Fig. 4.

Several authors[7,19] have proposed a scaling form for the conductivity of a superconductor near T_c in zero magnetic field. This prediction takes the form

$$\mathrm{Re}\sigma(\omega,\xi_p)=\xi_p^{z+2-d}F_{\pm}(\omega\xi_p^z)\ , \tag{22}$$

where ξ_p is a correlation length, $F_{\pm}(x)$ is a scaling function, which may have different forms above and below T_c, and z is a dynamical critical exponent. In the vicinity of the phase ordering temperature T_c, ξ_p is expected to vary as a power of $\epsilon \equiv |T-T_c|/T_c$: $\xi_p \propto \epsilon^{-\nu}$, where ν is the correlation length exponent for the $d=3$ xy model and is expected to be around $\frac{2}{3}$.[20] At $\omega=0$, therefore, $\mathrm{Re}\sigma(\omega)$ is expected to diverge as $\epsilon^{-\nu(z-1)}$.

The analytic form (20) is consistent with the scaling form (22) with $\nu=0.5$, $z=2$, and

$$F_+(x)=C_0\int_0^\infty \frac{u^2}{c^2b^2(1+u^2)^2+x^2}du\ . \tag{23}$$

In principle, our simulations should allow us to test scaling at a level beyond mean-field theory, and hence, to extract a non-mean-field value for the dynamical exponent z. In practice, this would require simulations with much larger cells than those we have used, at temperatures very close to T_c. Thus, we have *not* been able to verify the scaling form numerically.

Figure 5 shows the behavior of the voltage noise function $S_{VV}(t)$, as defined in Eq. (15). The Fourier transform of this quantity [Eq. (16)] is shown in Fig. 6 for several

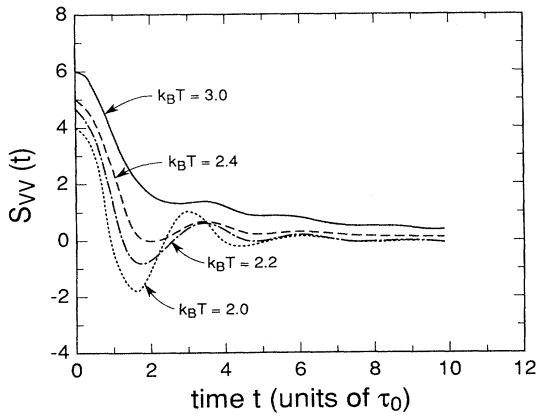

FIG. 5. Voltage noise $S_{VV}(t)$ [Eq. (15)], plotted as a function of time for temperatures $k_BT/E_J=2.0$, 2.2, 2.4, and 3.0, where $E_J=\hbar I_c/(2e)$ is the coupling energy.

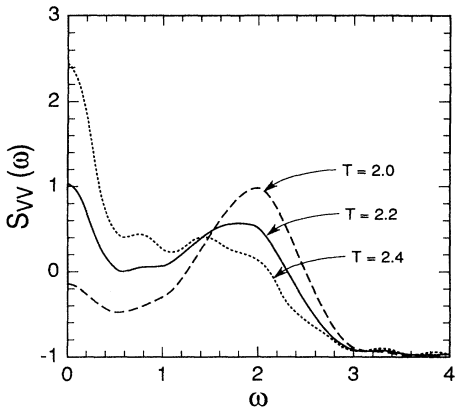

FIG. 6. Fourier transform $S_{VV}(\omega)$ of the voltage noise for temperatures $k_BT/E_J=2.0$, 2.2, and 2.4. ω is in units of $1/\tau_0$.

temperatures both below and above T_c. At low temperatures, $S_{VV}(t)$ shows a clear oscillatory behavior as a function of time. These oscillations correspond to the propagation of "phase waves," in this model—that is, of excitations involving small fluctuations in both θ and $\dot\theta$ which are analogous to lattice vibrations in a conventional crystal. The frequency of these oscillations is of the order of $(2e/\hbar)\sqrt{E_J/C}$. At higher temperatures, the phase arrangement in the granular lattice "melts," and the time-dependence of $S_{VV}(t)$ resembles a decaying exponential with some oscillatory residue. $S_{VV}(t)$ behaves rather like the velocity autocorrelation functions of conventional crystals,[21] which oscillate in the crystal but tend to fall off monotonically in the liquid. However, in contrast to conventional melting, the present phase "melting" is a second-order transition.

The behavior of $S_{VV}(t)$ and of its Fourier transform $S_{VV}(\omega)$ can be deduced analytically at low temperatures, where the harmonic approximation is expected to apply. In that regime, the Hamiltonian (9) in natural units is

$$H=\tfrac{1}{2}\sum_i \dot\theta_i^2+\tfrac{1}{2}\sum_{\langle ij\rangle}(\theta_i-\theta_j)^2\ . \tag{24}$$

The noise [Eq. (15)] is a sum over factors of the form

$$\begin{aligned}S_{VV;ij}(t)&=\langle\dot\theta_i(t)\dot\theta_j(0)\rangle\\&=\sum_{\mathbf{k}\in BZ}e^{i\mathbf{k}\cdot(\mathbf{R}_i-\mathbf{R}_j)}\langle\dot\theta_k(t)\dot\theta_{-k}(0)\rangle\ .\end{aligned} \tag{25}$$

In the harmonic approximation, each $\dot\theta_k$ propagates independently with frequency $\omega_k\tau_0=\sqrt{U_k}$ where $U_k=\sum_{\mathbf{R}}(1-\cos(\mathbf{k}\cdot\mathbf{R}))$, the sum running over nearest neighbor lattice vectors $\mathbf{R}$. Since by the equipartition theorem $\langle|\dot\theta_k(t=0)|^2\rangle/2=k_BT/2N$ where N is the number of grains, it follows that

$$\langle\dot\theta_k(t)\dot\theta_{-k}(0)\rangle=\frac{k_BT}{2N}\exp(-i\omega_kt)\ . \tag{26}$$

Hence in the harmonic approximation

$$S_{VV;ij}(t)=\frac{k_BT}{2N}\sum_{\mathbf{k}\in BZ}e^{i\mathbf{k}\cdot(\mathbf{R}_i-\mathbf{R}_j)}\exp(-i\omega_kt) \tag{27}$$

and the sum $S_{VV}(t)$ takes the form

$$S_{VV}(t)=\frac{k_BT}{2N}\sum_k U_k\exp(-i\omega_k t)\ . \tag{28}$$

Similarly, the frequency transform is

$$S_{VV;ij}(\omega)=\frac{k_BT}{2N}\sum_{k\epsilon BZ} e^{i\mathbf{k}\cdot(\mathbf{R}_i-\mathbf{R}_j)}\delta(\omega-\omega_k) \tag{29}$$

and the corresponding $S_{VV}(\omega)$ takes the form

$$S_{VV}(\omega)=\frac{k_BT}{2N}\sum_k U_k\delta(\omega-\omega_k)\ , \tag{30}$$

where we have used Eq. (16).

Equations (28) and (30) are plotted in Fig. 7 for temperature $T=2.0J/k_B$. Evidently $S_{VV}(t)$ of the simulations (Fig. 5) is reproduced very well by this harmonic approximation at temperatures as high as $T=2.0J/k_B$, quite near the melting temperature, including the positions of the first dip and first peak. The ratio of the height of the first peak to $S_{VV}(t=0)$ is smaller than predicted by the harmonic approximation,[22] presumably because the harmonic modes are damped. $S_{VV}(\omega)$ shows only a qualitative agreement with the computer experiments (Fig. 6)

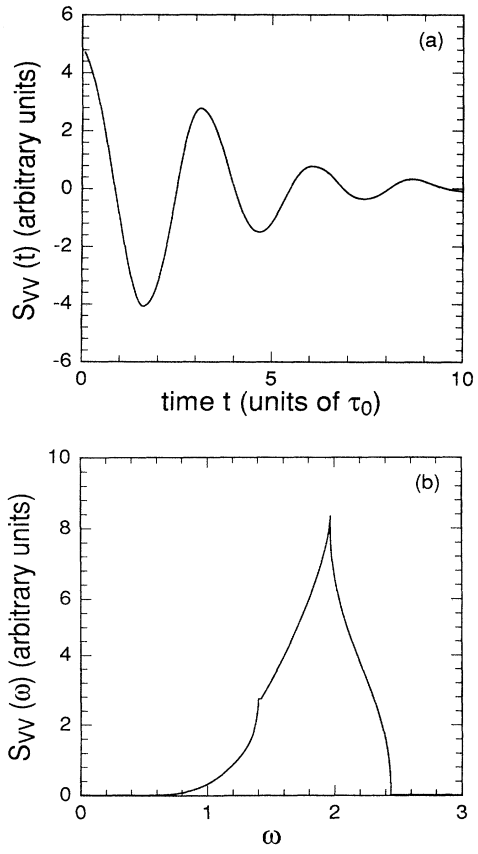

FIG. 7. (a) $S_{VV}(t)$ as calculated in the harmonic approximation, Eq. (28). (b) $S_{VV}(\omega)$ as calculated in the harmonic approximation, Eq. (30); units of frequency as in Fig. 6.

even at $T=2.0J/k_B$, which have a central peak absent from the harmonic approximation. Note also that the "phonon peak" in the computer $S_{VV}(\omega)$ shifts to lower frequencies at higher temperatures. This is consistent with the softening of the phase waves near T_c, as would be expected near a second-order phase transition.

The extremely small values of the harmonic $S_{VV}(\omega)$ at small ω follow from density of states arguments. At small frequency, the harmonic modes satisfy $\omega=ck$, while the matrix element U_k varies as k^2. These factors combine to give $S_{VV}(\omega)\propto\omega^4/c^3$ at low frequencies. The extra factor of ω^2 appears only in nearest-neighbor voltage noise and would be absent in the harmonic expression for the noise between more distant grains.

V. STRONG RANDOM MAGNETIC FIELD

A strong random magnetic field is expected to lead, not to a conventional superconducting transition (analogous to the ordered three-dimensional XY model), but instead to a glass transition corresponding to a randomly frustrated XY model.[5–9] Such a glass transition is signaled by unusual behavior in the current-current correlation, as we discuss in this section.

The results involve several types of averages. A thermal average of the quantity $F(0)G(t)$ for a particular choice of the bond variables A_{ij} and of the initial phases of the order parameters (i.e., of the initial conditions) is denoted by triangular brackets, i.e., $\langle F(0)G(t)\rangle$. Because of the ergodic hypothesis, which implies that an average over a thermal ensemble is equivalent to a time average, $\langle F(0)G(t)\rangle$ is given by

$$\langle F(0)G(t)\rangle=\lim_{\tau\to\infty}\frac{1}{\tau}\int_0^\tau F(t')G(t+t')dt'\ . \tag{31}$$

In the present calculations, we use $\tau\approx 2000\tau_0$.

A disorder average over the variables A_{ij} for a particular choice of initial conditions is denoted [ ]. A thermal and disorder average, assuming particular initial conditions, is denoted $[\langle F(0)G(t)\rangle]$. Finally, an average over thermal ensemble, a disorder ensemble, and initial phase conditions will be denoted $[\langle A(0)B(t)\rangle]_{ic}$. Clearly, it is equivalent to average a quantity over many realizations of the disorder, for a single initial phase configuration, or to average over initial configurations, for a single realization of the disorder.

Figure 8 shows $[\langle J(0)J(t)\rangle]$ for an average over twenty different realizations of the phase variables A_{ij}, as obtained using two different random initial phase configurations, shown as full lines and as dotted lines. For $T>0.8$ the two are indistinguishable on the scale of this plot, but below a temperature of about $0.6E_J/k_B$, the difference is noticeable. This is near the temperature at which a glass transition is predicted for a static version of this model (the predictions[5,10] range from about $0.6E_J/k_B$ to $0.45E_J/k_B$). Note also that, compared to the analogous quantity for zero field (cf. Fig. 1), $[\langle J(0)J(t)\rangle]$ decays much more slowly even well above the hypothetical T_g. This is typical of frustrated systems, in which time decays are generally slowed down, in comparison with ordered systems where the decay of order

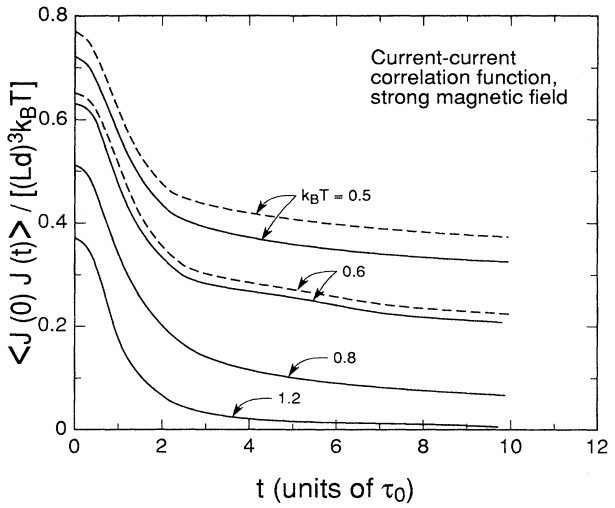

FIG. 8. Current-current correlation function $[\langle J(0)J(t)\rangle]/((Ld)^3k_BT)$ for a dynamical Huse-Seung vortex glass, averaged over twenty different realizations of the random phase factors A_{ij}, for two different initial phase configurations, denoted by full and dashed curves. Units as in Fig. 1.

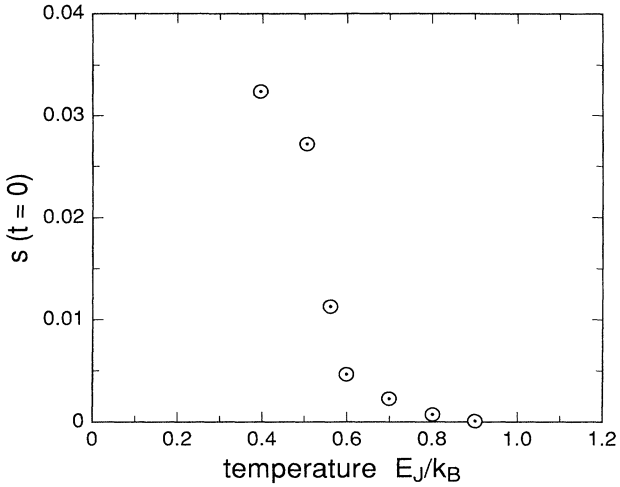

FIG. 9. $s(t)\equiv[(f_1(t)-f_2(t))^2]$, where $f(t)=\langle J(0)J(t)\rangle/((Ld)^3k_BT)$. The square brackets [ ] denote a disorder average; the triangular brackets $\langle\ \rangle$ denotes a thermal average, and the subscripts 1 and 2 refer to two different initial conditions. We plot $s(t=0)$, which denotes the sensitivity of the correlation function $f(0)$ to initial conditions. Units as in Fig. 8.

parameters is not impeded by energy barriers.

In order to make clearer the initial-condition dependence shown in Fig. 8,we have calculated the quantity

$$s(t)\equiv[(f_1(t)-f_2(t))^2]\ , \tag{32}$$

where $f(t)=\langle J(0)J(t)\rangle/[(Ld)^3k_BT]$, and f_1 and f_2 correspond to two different initial phase configurations. The disorder average [ ] is carried out using twenty realizations of the disorder, and f_1 and f_2 correspond to two different initial phase configurations. We find that $s(t)$ is nearly independent of time, a result already suggested by the fact that the full and dashed curves in Fig. 8 differ by constant values at each temperature. Figure 9 shows $s(t=0)$ as a function of temperature. This quantity becomes nonzero (for our choice of τ) near $T=0.55E_J/k_B$, below which the properties of the glass are *history dependent.* This history dependence can be viewed as additional evidence for a glass transition in this granular superconductor.

The explanation for the history dependence is clear. At sufficiently low temperatures, because of the frustration inherent in the model Hamiltonian, there are an enormous number of equally plausible metastable energy minima. Below a temperature of about $0.55E_J/k_B$, the phases tend to be locked in the vicinity of one of the energy minima. This implies that the calculated correlation function $\langle J(0)J(t)\rangle$, for a given realization of the disorder, will depend on the initial phase configuration. This dependence is reflected in $s(t)$, as defined in Eq. (32). Note that $s(t)$ actually depends on the length of the time interval τ over which the time average is evaluated: the longer the time, the smaller the initial-condition-dependent fluctuations, because the frustrated system can more nearly equilibrate over a long time interval.

The behavior of $\langle J(0)J(t)\rangle$ is very similar to that proposed by Leutheusser[23] on the basis of an analytic theory of the glass transition in conventional glasses. Leutheusser's theory is a nonlinear integrodifferential equation for the dynamical correlation functions of a nonlinear oscillator with history dependent response. As in the present calculation, it leads to a $\langle J(0)J(t)\rangle$ which decays more and more slowly as the transition is approached. Exactly at the critical value of the control parameter λ (corresponding to temperature in our case), $\langle J(0)J(t)\rangle$ falls off algebraically with time, $\langle J(0)J(t)\rangle\propto t^{-\alpha}$ where $\alpha=0.395$ in Leutheusser's theory. Below the critical point, $\langle J(0)J(t)\rangle$ becomes nonergodic. While our results are only numerical, it is intriguing that they show many of the features of his model. This seems to be further evidence that this randomly frustrated model might undergo a phase transition near $T_g\approx 0.45E_J/k_B$.

Finally, in Fig. 10, we show the noise function $S_{VV}(t)$ defined earlier for several temperatures above T_g. Even though $S_{VV}(t)$ is calculated in the "vortex liquid" phase, it shows a very solidlike, ordered behavior over the length and time scales shown in Fig. 10, closely resembling the $B=0$ results for T *below* T_c. Hence even in the "liquid" phase at high fields, there is a great deal of short-range order in the phases, which gives rise to an oscillatory behavior in $S_{VV}(t)$. The melted phase thus shows an almost elastic response on short time scales, similar to what has been envisioned in theories of "entangled flux liquids."[24]

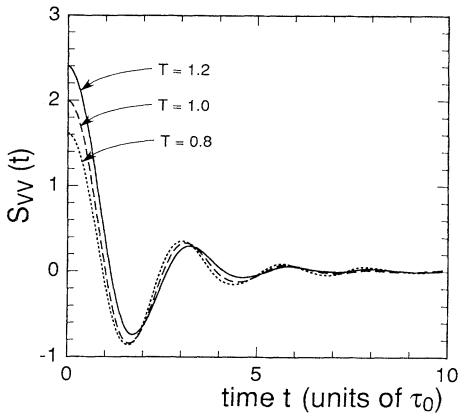

FIG. 10. $S_{VV}(t)$ for a $7\times7\times7$ cubic lattice in a strong magnetic field at temperatures $k_BT/E_J=0.8$, 1.0, and 1.2. Units as in Fig. 5.

VI. DISCUSSION AND SUMMARY

Using molecular dynamics techniques, we have calculated the current-current correlation functions, frequency-dependent conductivity, and a measure of the voltage noise in a model three-dimensional granular superconductor. The model is suitable for a granular superconductor in the large-grain, high-resistivity limit. It might also be viewed as a highly simplified model for the dynamics of a homogeneous superconductor, provided that the "grains" are interpreted as volumes ξ_0^d of superconductor, where ξ_0 is a length scale comparable to the zero-temperature coherence length of the superconductor.

In zero field, our results imply an insulator-to-superconductor transition accompanied by a large fluctuation conductivity which probably diverges at the transition temperature T_c. Below T_c, the voltage noise has a peak at frequencies where there is a high density of states of propagating phase waves. Our results are not adequate to test the scaling behavior predicted for this transition in zero field. However, a simple Aslamasov-Larkin-like theory for the fluctuation conductivity above T_c is in qualitative agreement with the numerical results.

In the presence of a strong random magnetic field, our results are consistent with a vortex-liquid-to-vortex-glass transition. As the transition is approached from above, the current-current correlation function decays more and more slowly with time, and below the presumed transition it becomes nonergodic, as predicted by the analytic theory of Leutheusser[23] for a structural liquid-glass transition. Even above the postulated T_g, the voltage noise is very solidlike, suggesting a great deal of short range order even in the vortex liquid phase.

The present model could be used to treat other transport problems in high-temperature superconductors. For example, with different couplings in the xy plane in the z direction, and with a magnetic field applied along an arbitrary direction, it would be a suitable, if idealized, model, for treating flux motion and resistivity in an anisotropic superconductor in a magnetic field. Disorder can also be readily introduced into the model. Thus one could calculate directly the conductivity as a function of magnetic field and current direction, continuously following the crossover from the $B=0$ (three-dimensional XY) transition to the high-B (vortex glass) transition.

VII. ACKNOWLEDGMENTS

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APPENDIX

In this section, we discuss the experimental conditions where inequalities (7) and (8) may hold in a real granular superconductor.

The Josephson coupling energy between grains is $E_J=\hbar I_c/(2e)$ where I_c is a typical intergranular critical current. At low temperatures limit ($T\ll T_{c0}$, where T_{c0} is the critical temperature of the grains), the Ambegaokar-Baratoff[25] expression for the critical current is $I_c(T)=\pi\hbar\Delta(T)/(4e^2R_n)$ where R_n is the intergranular normal-state tunneling resistance (assumed to be temperature-independent), and $\Delta(T)$ is the superconducting energy gap. Hence, the coupling energy is

$$E_J\approx\frac{\pi\hbar}{4e^2R_n}\Delta(T)\ . \tag{A1}$$

Similarly, we estimate $C=\epsilon d$ where ϵ is an appropriate dielectric constant (comparable to that of the material in which the superconducting grains are embedded) and d is an appropriate length (of the order of the intergrain spacing). If we assume that the tunneling resistance equals the shunt resistance, these estimates can be used to rewrite inequalities (7) and (8) in the form (at $T\ll T_{c0}$)

$$\frac{R}{R_0}\ll\frac{\Delta(0)}{U_c}\ , \tag{A2}$$

$$\frac{R}{R_0}\gg\frac{4}{\pi^2}\frac{U_c}{\Delta(0)}\ , \tag{A3}$$

where

$$U_c=\frac{e^2}{\epsilon d}\ , \tag{A4}$$

$$R_0=\frac{\hbar\pi}{4e^2}\ . \tag{A5}$$

For reasonable estimates appropriate to high-temperature superconductors, e.g. $\Delta(0)\approx200$ K; $\epsilon\approx10$, we find that these inequalities cannot be simultaneously satisfied un-

less $d \approx 500-1000$ Å or greater. The allowed values of R_0 are then in the range of $R_0 \approx 5000\ \Omega$; the corresponding resistivity is very large: $\rho \approx Rd \approx 25\ \text{m}\Omega$. Thus, this model is *quantitatively* appropriate only for a granular superconductor in which the grains are relatively large, and the resistance between the grains is high. For smaller grains, quantum effects enter; for lower-resistivity composites, dissipative effects must be included.

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