Fluctuation conductivity of layered high-T_c superconductors: A theoretical analysis of recent experiments

L. Reggiani and R. Vaglio
Dipartimento di Fisica, Università di Salerno, 84081, Baronissi, Salerno, Italy

A. A. Varlamov
Moscow Institute of Steel and Alloys, Leninskij p., d.4, 117936 Moscow, U.S.S.R.
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The theory of fluctuation conductivity in two-dimensional clean superconductors is expanded over a wider range of temperatures with respect to the conventional Aslamazov-Larkin theory. The results of this modified theory are discussed in relation to recent experiments on the paraconductivity of Ba-Sr-Ca-Cu-O compounds.

I. INTRODUCTION

In a number of recent experimental papers,[1-5] the excess conductivity of high-T_c superconductors has been experimentally studied. It is worth mentioning that the theory of fluctuation conductivity is anisotropic structures was developed a rather long time ago,[6-8] but now some new features, especially in high-T_c superconductors, appeared in the experiments, and they deserve more discussion and some additional theoretical consideration, which we propose in this article.

We shall restrict ourselves here to the discussion of two-dimensional fluctuation behavior found in Ba-Sr-Ca-Cu-O and related compounds, and we shall extend over a wider range of temperatures the region of applicability of the previous theory of fluctuation conductivity for clean superconductors.

We will especially concentrate our attention on the analysis of recent experiments where the excess conductivity of 85- and 110-K phases of Ba-Sr-Ca-Cu-O was studied in a sufficiently wide range of temperatures.[4,5] The authors succeeded in interpreting the obtained results in the framework of the Aslamazov-Larkin theory of paraconductivity[6] in the range of temperatures $-4 \lesssim \ln[(T-T_c)/T_c)] \lesssim -2$. In this range the excess part of the conductivity, extracted from the experiments, σ_{fl}, was described by the formula

$$\sigma_{\text{fl}} = \frac{e^2}{16\hbar a}\left[\frac{T_c}{T-T_c}\right]^n , \quad (1)$$

where a is the interlayer distance and $n = 0.9 \pm 0.1$, in good agreement with the appropriate theoretical expression $n=1$. The agreement between theory and experiments in the temperature range considered allows one to conclude some facts and to formulate some hypotheses.

(1) The fluctuation term, in the vicinity of T_c, does not include the anomalous Maki-Thompson term (which in quasi-two-dimensional dirty superconductors, in the absence of pair breaking, must be dominant). This fact suggests two possible circumstances. The first is the presence of some pair-breaking sources, for example, localized magnetic moments of Cu^{2+} ions or even thermal phonons, the density of which at temperatures $T \simeq T_c \simeq 100$ K is sufficiently high. The second is the possibility of the absence of any Maki-Thompson contribution. Indeed, as is well known,[9] the Maki-Thompson term is tightly connected with the coherent scattering of electrons involved in fluctuation pairing on impurities. This is why this contribution is characteristic of dirty superconductors ($\xi_0 \gg l$) only. The value of ξ_0 for high-T_c superconductors is estimated to be extremely small: $\xi_0 \simeq 10$ Å. The value of the mean free path for high-quality samples may be estimated by the extrapolation of the normal conductivity at zero temperature: it gives $l(0) \simeq 70-90$ Å, so that it is reasonable to assume $l \gtrsim \xi_0$, implying that these superconductors have to be treated as clean systems.

(2) If we assume that the movement of the electrons in high-T_c Ba-Sr-Ca-Cu-O compounds has a quasi-two-dimensional character and that the Fermi surface has the form of a modulated cylinder (Fig. 1), we have

$$\xi(\mathbf{p}) = \varepsilon_{\mathbf{p}} - \epsilon_F = v_0(|\mathbf{p}_{\parallel}| - p_0) + w\cos(p_{\perp}a) , \quad (2)$$

where v_0 is the Fermi velocity in the layers, a is the interlayer distance, and w is the hopping integral which has the sense of a probability of electron hopping between layers. Then, in the considered range of temperatures, the coherence length in the direction perpendicular to the layers, $\xi_{\perp}(T) \simeq v_{\perp}/\sqrt{T_c(T-T_c)} \simeq wa/\sqrt{T_c(T-T_c)}$, remains less than a and the fluctuative Cooper pairs are moving in the layers only. This justifies that the temperature dependence of fluctuation conductivity follows the two-dimensional law[6] $\sigma_{\text{fl}} \simeq T_c/(T-T_c)$.

It is clear that, approaching T_c, $\xi_{\perp}(T)$ will become larger than a and in fluctuative pairing electrons of different layers are involved, so that the picture starts to be three dimensional. From the expression for $\xi_{\perp}(T)$, it is easy to see that this crossover between two- and three-dimensional regimes takes place at a temperature $T_{\text{cr}} \simeq T_c + w^2/T_c$.

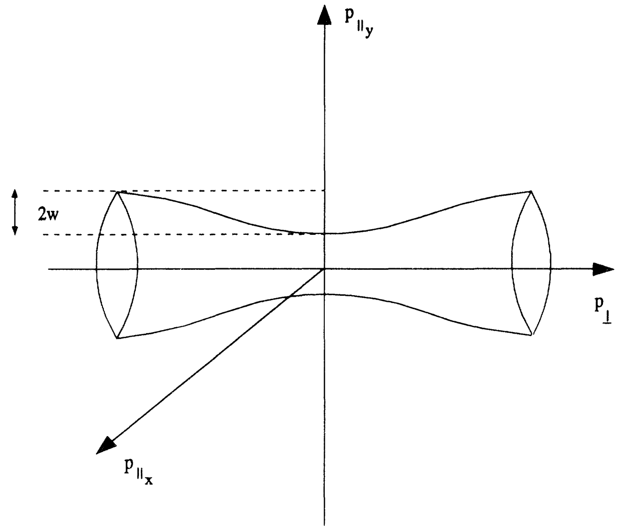

FIG. 1. Model of the Fermi surface for a layered superconductor.

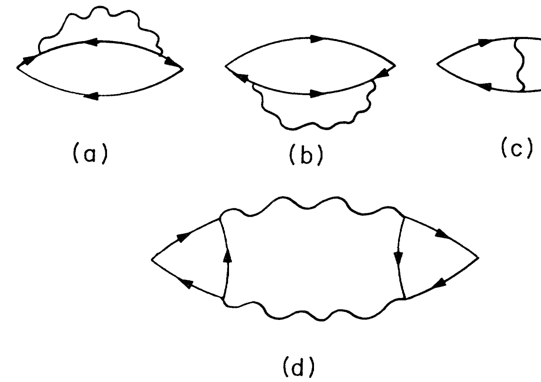

FIG. 2. Diagrams of the operators of the electromagnetic response in the first order of the perturbation theory.

But in the experiment under discussion,[4,5] a good two-dimensional behavior was observed in a wide range of temperatures, and only for $\ln[(T-T_c)/T_c]\lesssim -4$ [i.e., $(T-T_c)\lesssim 2$ K] did the temperature dependence of $\sigma_{\rm fl}$ begin to deviate from the two-dimensional law. Unfortunately, on the basis of experimental results, it is difficult to consider that this deviation is directly connected with the passage to a three-dimensional regime (where $\sigma_{\rm fl}\simeq[T_c/(T-T_c)]^{1/2}$) since close to T_c the sample inhomogeneities and the precise choice of the mean-field transition temperatures T_c could influence the results. So we can only estimate the upper limit for the crossover temperature in Ba-Sr-Ca-Cu-O to be $T_{\rm cr}-T_c\lesssim 2$ K, and hence for the hopping integral w one can find $w\simeq\sqrt{T_c(T_{\rm cr}-T_c)}\lesssim 12$ K. Taking into account that $\sigma_\perp/\sigma_\|\simeq(v_\perp/v_\|)^2\simeq(wa/v_0)^2$, one can estimate from the above consideration that $\sigma_\perp/\sigma_\|\lesssim 2.5\times10^{-4}$ in good agreement with direct measurements.

(3) It seems in any case interesting to study the opposite region of the experimental curve, i.e., the range of "high" temperatures, $\ln[(T-T_c)/T_c]\gtrsim -2$, where the Aslamazov-Larkin approach begins to break down. For dirty layered superconductors this question was discussed previously.[7]

The scope of the present paper is to discuss this high-temperature region in the clean case.

II. FLUCTUATION CONDUCTIVITY OF A CLEAN SUPERCONDUCTOR AT ARBITRARY TEMPERATURES

Having in mind the description of the behavior of the excess conductivity of a superconductor in the intermediate range of temperatures, we shall discuss the usual diagrams for the operator of electromagnetic response which determines the fluctuation conductivity in the first order of perturbation theory (see Fig. 2). Since we discuss now the case of clean superconductors, we shall not accomplish the averaging of these diagrams over the positions of scattering centers. But, on the other hand, in the case under discussion of arbitrary temperatures, we cannot restrict our consideration to the most singular terms in $1/\epsilon=\ln^{-1}T/T_c$ only, as is usually done in the vicinity of T_c (when $\epsilon\ll1$).[10] This means that we have to take into consideration all boson frequencies Ω_k and moments $\mathbf{q}$ in the fluctuation propagators $L(\mathbf{q},\Omega_k)$ (the wavy lines in Fig. 2). There are no reasons now to omit the first three diagrams of Fig. 2 either. Usually, they give a less singular contribution in comparison with the Aslamazov-Larkin paraconductive contribution (the fourth diagram of Fig. 2), but now, since ϵ may not be small, their contributions have to be taken into account.[11]

In contrast to the Aslamazov-Larkin consideration of paraconductive contribution,[6] the frequency and momentum dependence of the Green-function blocks in the fourth diagram of Fig. 2 have to be taken into account (as was done previously for the dirty case[7]). All these circumstances make the calculations more complicated. As an example, the first diagram may be written as

$$Q_1=e^2v_0^2T\sum_{\Omega_k}\int\frac{d^dq}{(2\pi)^d}L(\mathbf{q},\Omega_k)T\sum_{\epsilon_n}\int\frac{d^dp}{(2\pi)^d}G^2(\mathbf{p},\epsilon_n)G(\mathbf{q}-\mathbf{p},\Omega_k-\epsilon_n)G(\mathbf{p},\epsilon_n-\omega_\nu)\ , \tag{3}$$

where $\epsilon_n=2\pi T(n+\frac{1}{2})$, $\Omega_k=2\pi Tk$ $(n,k=0,\pm1,\pm2,\ldots)$ are fermion and boson Matsubara frequencies, $G(\mathbf{p},\epsilon_n)=[i\epsilon_n-\xi(\mathbf{p})]^{-1}$ is the one-electron Green function of a normal metal, d = dimensionality of the electron spectrum, and $\omega_\nu=2\pi T\nu$ $(\nu=0,1,2,\ldots)$ is the boson frequency of the external electromagnetic field. The function

$$L(\mathbf{q},\Omega_k)=-\frac{1}{\rho}\,\frac{1}{\ln(T/T_c)+\psi\{\frac{1}{2}+|\Omega_k|/4\pi T+[7\zeta(3)/8\pi^4T^2]\langle(\mathbf{v}\cdot\mathbf{q})^2\rangle\}-\psi(\frac{1}{2})} \tag{4}$$

denotes the fluctuation propagator of a clean metal at temperatures above T_c (here ρ is the density of states, $\langle\ \rangle$ means the averaging over the Fermi surface, $\psi(x)$ is the digamma function, and $\zeta(3)=1.202$ is the Riemann zeta function of argument (3).

There are no major problems in calculating expression (3) with nonzero Ω_k and $vq\le T$. In accordance with the argument presented in the Introduction, we shall now restrict ourselves to the two-dimensional case ($d=2$) only. Transforming the integral over $\mathbf{p}$ to one over $\xi(\mathbf{p})$ and accomplishing the integration for four nontrivial poles in the complex plane, one can turn out the summation over ε_n and then, after the analytical continuation over the external frequency $i\omega_\nu\to\omega$, find for the contribution from the first two diagrams in frequency-dependent fluctuation conductivity:

$$\frac{\sigma_{\rm fl}^{(1+2)}}{\sigma_n}=\frac{iT}{8\omega^3\tau}\int_0^\infty q\,dq\int_{-\infty}^{+\infty}d\Omega\coth\left[\frac{\Omega}{2T}\right]\mathrm{Im}L^R(q,-i\Omega)$$
$$\times\left\{\psi\left[\frac{1}{2}+\frac{i\omega}{4\pi T}-\frac{i\Omega}{4\pi T}\right]+\psi\left[\frac{1}{2}+\frac{i\Omega}{4\pi T}\right]+\psi\left[\frac{1}{2}-\frac{i\omega}{4\pi T}-\frac{i\Omega}{4\pi T}\right]+\psi\left[\frac{1}{2}-\frac{i\omega}{2\pi T}+\frac{i\Omega}{4\pi T}\right]\right.$$
$$\left.-2\psi\left[\frac{1}{2}-\frac{i\Omega}{4\pi T}\right]-2\psi\left[\frac{1}{2}-\frac{i\omega}{4\pi T}+\frac{i\Omega}{4\pi T}\right]-\frac{i\omega}{2\pi T}\left[\psi'\left[\frac{1}{2}+\frac{i\Omega}{4\pi T}\right]-\psi'\left[\frac{1}{2}-\frac{i\omega}{4\pi T}+\frac{i\Omega}{4\pi T}\right]\right]\right\}, \tag{5}$$

where $\sigma_n=\frac{1}{2}e^2v_0^2\rho\tau$ is two-dimensional normal conductivity and the meaning of Ω is obvious.

The calculation of the third diagram is even more troublesome, but one can evaluate it too. The so-called anomalous Maki-Thompson contribution in the case under discussion of a clean superconductor naturally does even not appear, and so the final result for the frequency-dependent contribution in the fluctuation conductivity from the third diagram is

$$\frac{\sigma_{\rm fl}^{(3)}}{\sigma_n}=-\frac{iT}{8\omega^3\tau}\int_0^\infty q\,dq\int_{-\infty}^{+\infty}d\Omega\coth\left[\frac{\Omega}{2T}\right]\mathrm{Im}L^R(q,-i\Omega)$$
$$\times\left[\psi\left[\frac{1}{2}+\frac{i\omega}{4\pi T}+\frac{i\Omega}{4\pi T}\right]-\psi\left[\frac{1}{2}+\frac{i\Omega}{4\pi T}\right]+\psi\left[\frac{1}{2}-\frac{i\omega}{4\pi T}+\frac{i\Omega}{4\pi T}\right]-\psi\left[\frac{1}{2}+\frac{i\Omega}{4\pi T}\right]\right.$$
$$\left.-\psi\left[\frac{1}{2}+\frac{i\omega}{4\pi T}-\frac{i\Omega}{4\pi T}\right]+\psi\left[\frac{1}{2}-\frac{i\Omega}{4\pi T}\right]+\psi\left[\frac{1}{2}-\frac{i\Omega}{4\pi T}\right]-\psi\left[\frac{1}{2}-\frac{i\omega}{4\pi T}-\frac{i\Omega}{4\pi T}\right]\right]. \tag{6}$$

From expressions (5) and (6), it is easy to see that in the limit $\omega\to 0$ the total contribution of the first three diagrams goes to zero as ω. Hence, for the direct current conductivity $\sigma_{\rm fl}(0)$, we have to take into account the last, fourth diagram of Fig. 2 only, which determines the paraconductivity contribution ($\sigma_{\rm fl}=\sigma_{\rm fl}^{(4)}$). The appropriate explicit analytical expression for it may be written in the form

$$Q_{\rm fl\parallel}^{\rm AL}=-2e^2T\sum_{\Omega_k}\int\frac{d^dq}{(2\pi)^d}L(\mathbf{q},\Omega_k)L(\mathbf{q},\Omega_k+\omega_\nu)\mathbf{B}_\parallel^2(\mathbf{q},\Omega_k,\omega_\nu)\ , \tag{7}$$

where the Green-function blocks (see Fig. 3) are of the form

$$\mathbf{B}_\parallel(\mathbf{q},\Omega_k,\omega_\nu)=T\sum_{\varepsilon_n}\int\frac{d^dp}{(2\pi)^d}\mathbf{v}_\parallel G(\mathbf{p},\epsilon_n+\omega_\nu)G(\mathbf{p},\epsilon_n)G(\mathbf{q}-\mathbf{p},\Omega_k-\epsilon_n)\ , \tag{8}$$

and $\mathbf{v}_\parallel=\partial\varepsilon(\mathbf{p})/\partial\mathbf{p}_\parallel$. Having in mind the application of the present theory to layered superconductors [see Eq. (2)] we write in (7) the longitudinal (in layer) part of $Q_{\rm fl}^{\rm AL}(\omega_\nu)$ only.

The paraconductivity contribution in the frequency-dependent conductivity of dirty layered superconductors was carefully studied,[7] and it is well known to what consequences the quasi-two-dimensionality of electron spectrum leads. One of them is the appearance of a crossover between two- and three-dimensional behavior of the fluctuation conductivity at a temperature $T_{\rm cr}-T_c\simeq\tau w^2$. It is not difficult to see how in the clean case the crossover point may be obtained by substituting τ with $1/T_c$. In fact, in the clean case, $T_{\rm cr}-T_c\simeq w^2/T_c$.

As already discussed in the Introduction, here we are interested in the region of temperatures where a two-dimensional behavior of fluctuations takes place, especially for $\epsilon\lesssim 1$. This is why, for the purpose of simplifying the calculations, we shall assume the character of the electron motion to be two dimensional:

$$\int\frac{d^dq}{(2\pi)^d}\to\int\frac{dq_z}{2\pi}\int\frac{d^2q}{(2\pi)^2}=\int_{-\pi/a}^{\pi/a}\int\frac{d^2q}{(2\pi)^2}=\frac{1}{(2\pi)^2a}\int d^2q\ .$$

Then we have that $\mathbf{B}_\parallel=(\mathbf{B}_\parallel\cdot\mathbf{q}_\parallel)\cdot\mathbf{q}_\parallel/q_\parallel^2$ and

$$(\mathbf{B}_\parallel\cdot\mathbf{q}_\parallel)=\frac{1}{2\pi}\rho T\sum_{\epsilon_n}\int_0^{2\pi}(\mathbf{v}_\parallel\cdot\mathbf{q}_\parallel)d\varphi\int_{-\infty}^{+\infty}\frac{d\xi}{[\xi-i(\epsilon_n+\omega_\nu)](\xi-i\epsilon_n)[\xi-\mathbf{v}_\parallel\cdot\mathbf{q}_\parallel-i(\Omega_k-\epsilon_n)]}\ , \tag{9}$$

where $\rho=(1/a)(m/2\pi)$ is the density of states in the case under discussion. The integral over $d\xi$ may be calculated. The result is

$$I=\frac{2\pi}{\omega_\nu}\left[\frac{\theta(\varepsilon_{n+\nu}(\epsilon_{n+\nu}-\Omega_{k+\nu}))\mathrm{sgn}\epsilon_{n+\nu}}{2i\epsilon_{n+\nu}-i\Omega_{k+\nu}-\mathbf{v}_\parallel\cdot\mathbf{q}_\parallel}-\frac{\theta(\epsilon_n(\epsilon_n-\Omega_k))\mathrm{sgn}\epsilon_n}{2i\epsilon_n-i\Omega_k-\mathbf{v}_\parallel\cdot\mathbf{q}_\parallel}\right]\ , \tag{10}$$

where $\omega_\nu=2\pi T\nu$ $(\nu=1,2,\ldots)$. Substituting this expression into (9), one can find

$$(\mathbf{B}_\parallel\cdot\mathbf{q}_\parallel)=\frac{2\rho T}{\omega_\nu}\int_0^{\pi}v_0q_\parallel\cos\varphi\, d\varphi[\Theta(\varphi,\Omega_{k+\nu})-\Theta(\varphi,\Omega_k)]\ , \tag{11}$$

where

$$\Theta(\varphi,\Omega_k)=\sum_{n=-\infty}^{+\infty}\frac{\theta(\epsilon_n(\epsilon_n-\Omega_k))\mathrm{sgn}\epsilon_n}{2i\epsilon_n-i\Omega_k-v_\parallel q_\parallel\cos\varphi}\ .$$

Accomplishing the integration over φ in (11) and then the summation over n, one can find the discouraging result that $(\mathbf{B}_\parallel\cdot\mathbf{q}_\parallel)\equiv 0$ for any $\omega_\nu\neq 0$.

The case $\omega_\nu=0$ needs a special consideration. Accomplishing the integration over ξ in this case (where only two nontrivial positions of the poles may take place), one obtains

$$\begin{aligned}[\mathbf{B}_\parallel(\mathbf{q},\Omega_k,\omega_\nu=0)\cdot\mathbf{q}_\parallel]&=2i\rho T\sum_{\epsilon_n}\theta(\epsilon_n(\epsilon_n-\Omega_k))\mathrm{sgn}\epsilon_n\int_0^{\pi}\frac{v_0q_\parallel\cos\varphi\, d\varphi}{(2\epsilon_n-\Omega_k+iv_0q_\parallel\cos\varphi)^2}\\&=\frac{v_0^2q_\parallel^2\rho}{16\pi^2T^2}\sum_{n=0}^{+\infty}\frac{1}{[(n+\frac{1}{2}+|\Omega_k|/4\pi T)^2+v_0^2q_\parallel^2/(4\pi T)^2]^{3/2}}\ .\end{aligned} \tag{12}$$

In the simplest case, $\Omega_k=0$ and $q_\parallel=0$, which was adopted by Aslamazov and Larkin in the vicinity of T_c,[6] the expression (12) immediately reproduces their old result for $\mathbf{B}_\parallel$. Hence, finally, for the Green-function block $\mathbf{B}_\parallel$ as a function of the external frequency ω_ν, one obtains the nontrivial result

$$\mathbf{B}_\parallel(\mathbf{q}_\parallel,\Omega_k,\omega_\nu)=\begin{cases}\dfrac{v_0^2\rho\mathbf{q}_\parallel}{16\pi^2T^2}\sum_{n=0}^{+\infty}\dfrac{1}{[(n+\frac{1}{2}+|\Omega_k|/4\pi T)^2+v_0^2q_\parallel^2/(4\pi T)^2]^{3/2}}, & \omega_\nu=0,\\ 0,\quad \omega_\nu\neq 0\ . & \end{cases} \tag{13}$$

Unfortunately, it is impossible to accomplish directly the analytical continuation of such a function from the set of Matsubara frequencies ω_ν in the upper half-plane of the complex variable ω $(\omega=-i\omega_\nu)$.[12] But if we assume that such an analytical continuation $\mathbf{B}^R(\mathbf{q},\Omega,\omega)$ exists, the sum over Ω_k in (7) may be transformed into a contour integral and the dc fluctuation conductivity σ_{fl} will be finally expressed by the means of $\mathbf{B}^R(\mathbf{q},0,0)$ only. Indeed, transforming the sum in (7) into a contour integral in the complex plane $z=-i\Omega_k$, we have

$$\begin{aligned}Q_{\mathrm{fl}\parallel}^{\mathrm{AL}}&=-\frac{2e^2T}{a}\sum_{\Omega_k}\int\frac{d^2q}{(2\pi)^2}L(\mathbf{q},\Omega_k)L(\mathbf{q},\Omega_k+\omega_\nu)\mathbf{B}_\parallel^2(\mathbf{q},\Omega_k,\omega_\nu)\\&=-\frac{e^2}{2a\pi i}\oint_C\coth\left[\frac{z}{2T}\right]\int\frac{d^2q}{(2\pi)^2}\mathbf{B}_\parallel^2(q,z,\omega_\nu)L(\mathbf{q},z)L(\mathbf{q},z+i\omega_\nu)dz\ ,\end{aligned} \tag{14}$$

where the contour integral C is shown in Fig. 4. The cuts in it are separating the domains of different analytical properties of the integral function.

As is usual, the contour integral in (14) may be reduced to the integral along the cuts, and after the final analytical continuation $i\omega_\nu\to\omega$ one can find

$$[Q_{\mathrm{fl}_\parallel}^{\mathrm{AL}}(\omega)]^R=-\frac{e^2}{2\pi i a}\int\frac{d^2q}{(2\pi)^2}\int_{-\infty}^{+\infty}\coth\left[\frac{z}{2T}\right]\{[\mathbf{B}^R(q,z,0)]^2L^R(q,z)L^R(q,z+\omega)$$
$$-[\mathbf{B}^A(q,z,0)]^2L^A(q,z)L^A(q,z+\omega)\}dz$$
$$-\frac{i\omega e^2}{4T\pi a}\int\frac{d^2q}{(2\pi)^2}\int_{-\infty}^{+\infty}\frac{dz}{\sinh^2(z/2T)}[\mathbf{B}^A(q,z,0)]^2[L^R(q,z)-L^A(q,z)]L^A(q,z)\ . \tag{15}$$

Because of the analytical properties

$$L^R(q,z)=L^A(q,-z)\ ,$$
$$\mathbf{B}^R(q,z,0)=\mathbf{B}^A(q,-z,0)\ ,$$

one obtains that the first integral in (15) is of the order of ω^2 and it does not give contribution to the dc conductivity.

Finally, we have[14]

$$\sigma_{\mathrm{fl}}(0)=\lim_{\omega\to0}\frac{[Q_{\mathrm{fl}_\parallel}^{\mathrm{AL}}(\omega)]^R}{-i\omega}=\frac{e^2}{16a}f(\epsilon)\ , \tag{16a}$$

where $\epsilon=\ln(T/T_c)$ and

$$f(\epsilon)=\frac{8\pi^3}{7^2\zeta^2(3)}\int_0^{+\infty}x\,dx\int_{-\infty}^{+\infty}dy\frac{1}{\{[\epsilon+\mathrm{Re}\beta(x,y)]^2+\mathrm{Im}^2\beta(x,y)]\}^2}$$
$$\times\sinh^{-2}(2\pi y)\{\mathrm{Im}^2\beta(x,y)[\mathrm{Re}^2\Sigma^A(x,y)-\mathrm{Im}^2\Sigma^A(x,y)]$$
$$-2\,\mathrm{Im}\beta(x,y)[\epsilon+\mathrm{Re}\beta(x,y)]\mathrm{Re}\Sigma^A(x,y)\mathrm{Im}\Sigma^A(x,y)\}\ , \tag{16b}$$

and the functions $\beta(x,y)$ and $\Sigma(x,y)$ are defined as

$$\beta(x,y)=\psi(\tfrac{1}{2}+x+iy)-\psi(\tfrac{1}{2})\ , \tag{16c}$$

$$\Sigma^A(x,y)=\sum_{n=0}^{+\infty}\frac{1}{[(n+\frac{1}{2}+iy)^2+1.173x^2]^{3/2}}\ . \tag{16d}$$

(The branch of the function Σ^A in the complex plane is chosen in order to obtain $\mathrm{Im}\Sigma^A<0$ when $y>0$, in accordance with the analytical properties of B^A.)

III. COMPARISON WITH EXPERIMENTS

In Fig. 5 the behavior of the function $f(\epsilon)=(16a/e^2)\sigma_{\mathrm{fl}}(\epsilon)$ is reported in a ln-ln scale (continuous line) in the same temperature interval where the experiments are usually performed.[4,5] For $\epsilon\to0$ $(T\to T_c)$, it is sufficient to take into account only the region of small x and y in Eq. (15). This leads to $\Sigma(x,y)\simeq\zeta(3)$ and to $f(\epsilon)=1/\epsilon\simeq T_c/(T-T_c)$ so that the classical Aslamazov-Larkin result is found (dashed line). For $\epsilon\to+\infty$ $(T\gg T_c)$, we find the asymptotic behavior

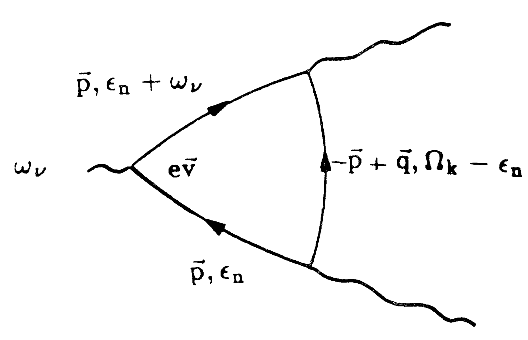

FIG. 3. Graphical representation of the block of Green functions $\mathbf{B}(\mathbf{q},\Omega_k,\omega_\nu)$.

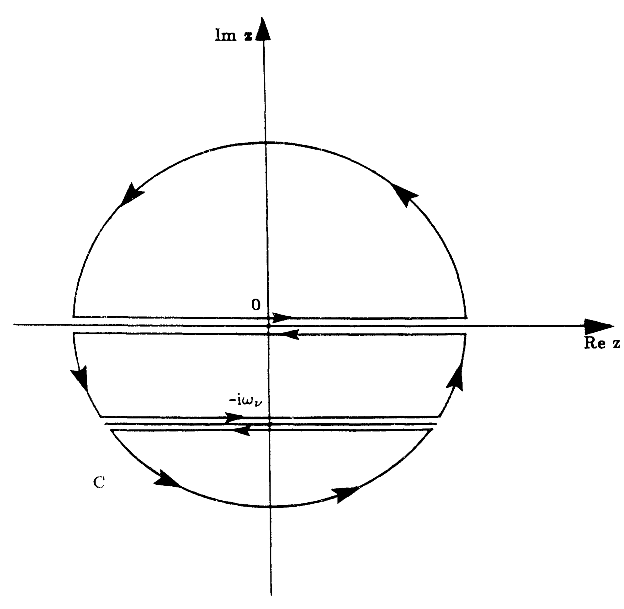

FIG. 4. Contour C in the complex z plane.

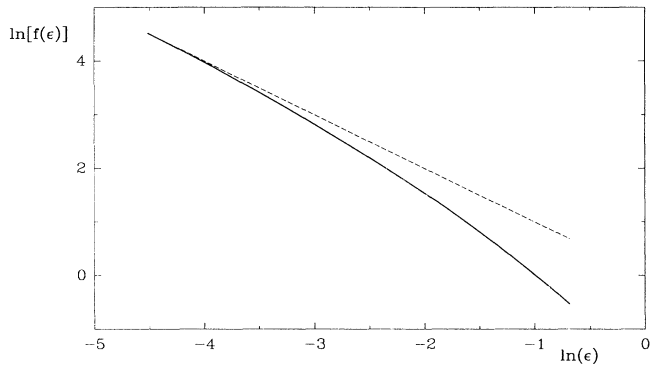

FIG. 5. Behavior of the function $f(\epsilon)=(16a/e^2)\sigma_{fl}(\epsilon)$ in a ln-ln scale (solid line). The dashed line is the Aslamazov-Larkin expression.

$f(\epsilon)\simeq 1/\epsilon^3$ [i.e., $\sigma_{fl}\sim 1/\ln^3(T/T_c)$], which is very different in comparison with the corresponding result for the dirty case where $f(\epsilon)\simeq 1/\epsilon$ [$\sigma_{fl}^{AL}\simeq 1/\ln(T/T_c)$].[15] However, as will be discussed in a moment, this region is not accessible experimentally. For intermediate ϵ values, $f(\epsilon)$ has been computed numerically. The Aslamazov-Larkin behavior is well recovered for $\ln\epsilon < -3$. At high temperatures the present theory predicts a deviation from this behavior with a tendency toward higher scopes. This behavior, always observed in the experiments,[1–5] was previously attributed to a breakdown of the Aslamazov-Larkin theory for short-wavelength fluctuations and treated phenomenologically inserting a cutoff in the momentum spectrum. A qualitative agreement with the experiments is found up to $\ln\epsilon\simeq -1$.[4,5] At even higher temperatures the data usually appear to bend down much faster, but this is simply due to the "technical" reason that the data above $\simeq 2T_c$ are generally used to determine the slope of the linear normal-state conductivity so that at this temperature the fluctuative conductivity is artificially imposed to be 0.

It is worth underling that, in any case, the assumption of a linear temperature dependence of normal-state conductivity, generally made the literature, is somewhat arbitrary, since it does not have a solid theoretical basis.

In conclusion, we have expanded over a wider temperature range the theory of fluctuative conductivity for clean superconductors in the two-dimensional case. The results of the theory are in qualitative agreement with the recent experiments on Ba-Sr-Ca-Cu-O compounds in the region $-4\lesssim\ln\epsilon\lesssim -1$. Higher-temperature regions are not accessible for technical reasons.

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[11]It is worth mentioning that in the case of a dirty superconductor the third diagram from Fig. 2 gives the anomalous Maki-Thompson contribution, which is of the same importance as the Aslamazov-Larkin one.
[12]The fact of the impossibility of a meaningful analytical continuation of the function $\mathbf{B}(\mathbf{q},\Omega_k,\omega_\nu)$ in the framework of Matsubara diagrammatic technique is very unusual and in some sense is occurring accidentally. To convince ourselves we recalculated the block $\mathbf{B}_\parallel$ in the Keldysh diagrammatic technique (Ref. 13) directly for complex ω, and we got the explicit expression for the analytical function, $\mathbf{B}^R(q,\Omega,\omega)$, which at $i\omega=\omega_\nu$ coincided with (13).
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[14]It is worth recalling that in our calculations we adopted $\hbar=1$.
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