### Period of oscillatory exchange interactions in Co/Cu and Fe/Cu multilayer systems

R. Coehoorn

Philips Research Laboratories, P.O. Box 80000, 5600 JA Eindhoven, The Netherlands

(Received 17 April 1991)

A simple expression is presented for the period  $\Lambda$  of oscillatory interlayer exchange interactions in Co/X and Fe/X magnetic multilayer systems (X= a nearly-free-electron metal).  $\Lambda$  is shown to be much longer than the period  $\lambda$  of the Ruderman-Kittel-Kasuya-Yosida-like spin-density oscillations in the X layers, which are induced by the coupling with the ferromagnetic layers, if  $\lambda$  is close to an integral fraction of the interlayer spacing in the X layers. The dependence of  $\Lambda$  on the growth direction of the multi-layers is explained. From the available experimental data on fcc Co/Cu(111), fcc Co/Cu(100), and fcc Fe/Cu(100), and from results of recently published band-structure calculations for bcc Fe/Cu(100) effective values of  $\lambda$  have been derived. In some cases small but significant deviations were found from the value of  $\lambda$  which is predicted from the Ruderman-Kittel-Kasuya-Yosida theory.

## I. INTRODUCTION

In several Co/X and Fe/X multilayer systems, where X is a nonferromagnetic element, the exchange coupling across the X layers oscillates as a function of the thickness of the X layers. After the discovery of this phenomenon by Parkin *et al.* for Fe/Cr, Co/Cr, and Co/Ru multilayer systems<sup>1</sup> such oscillatory exchange interactions have also been found in Fe/Cu (Refs. 2–4) and Co/Cu.<sup>5–8</sup> In thickness intervals in which the coupling is antiferromagnetic the saturation magnetoresistance is strongly enhanced, <sup>1,4,7,9–11</sup> which is related to the change in the magnetic structure upon applying a magnetic field. From magnetoresistance measurements, and from various other (more direct) determinations of the magnetic structure, it follows that the oscillations in these systems are approximately periodic, with periods between 5 and 21 Å.

One of the unresolved issues concerning the oscillatory interlayer coupling is its period  $\Lambda$ . Several authors have suggested a relation with the period  $\lambda$  of the spin density oscillations which are induced by a layer of magnetic atoms in a nonferromagnetic host metal. In the limit of large interlayer distances the Ruderman-Kittel-Kasuya-Yosida (RKKY) theory yields a period  $\lambda = \pi/k_F$ , if a spherical Fermi surface with a Fermi wave vector  $k_F$  is assumed for the interlayer material.<sup>12-14</sup> However, recent experimental results for Co/Cu multilayers reveal much larger values of  $\Lambda$  than  $\pi/k_F$ . For Cu,  $\pi/k_F = 2.31$ Å (assuming a spherical Fermi surface), whereas for fcc Co/Cu(111)  $\Lambda$  is about 12-15 Å.<sup>7</sup> Moreover, the combination of experimental results from different groups indicates that for fcc Co/Cu(100) the period  $\Lambda$  is different, about 5-7 Å (see the analysis below), although again much larger than  $\pi/k_F$ . So far, no explanation has been given for the period of the interlayer coupling in (111) and (100) grown Co/Cu.

The main purpose of this paper is to show that  $\Lambda$  is much longer than  $\lambda$  if  $\lambda$  is close to an integral fraction of the interplanar spacing d in the X layers. We present a very simple relation between  $\Lambda$ ,  $\lambda$ , and d, and derive effective values of the period  $\lambda$  for Co/Cu and Fe/Cu multilayer materials from the available data on  $\Lambda$ . One of the conclusions is that under certain conditions  $\Lambda$  depends on the growth direction and the crystal structure of the interlayer metal, even if the electron structure of this metal is described well in terms of a spherical Fermi surface. Relations are given with the models by Yafet for Gd/Y multilayers,<sup>15</sup> Wang *et al.* for Fe/Cr multilayers,<sup>16</sup> and Edwards *et al.*<sup>17</sup> In particular, we show that our result can be viewed as a generalization of the conclusions concerning the period  $\Lambda$  which were obtained by Edwards *et al.*<sup>17</sup> for one specific model system.

# **II. THEORETICAL MODEL**

In pseudo-1D theoretical models for the exchange coupling J(L) across a nonferromagnetic interlayer, the thickness L of these layers is often viewed as a continuously variable parameter. However, in epitaxial multilayers the local interlayer thickness L cannot be varied continuously, but only in steps which are equal to one monolayer thickness d. The implications of this notion can most easily be illustrated by a discussion of the RKKY theory for J(L). The RKKY theory for the exchange interaction between two uniformly magnetized, infinitely thin parallel sheets at a distance L and placed in a free electron gas which fills the whole space, gives a coupling that oscillates as  $\sin(2k_F L)/L^2$  for  $L \gg \lambda$ .<sup>12-14</sup> The period is the same as in the case of the coupling between two magnetic impurities in a free electron host metal, but in the planar (pseudo-1D) case the coupling decays more slowly than in the impurity case (decay like  $1/L^3$ ). As an example we show in Fig. 1 the coupling function J(L) for a monovalent fcc (100) metal. The full curve shows the function J(L) that would be expected in a continuum model. The oscillations reflect the oscillations in the spin density which are induced in the free electron gas by one of the two magnetic layers. The broken curve shows that it is better to represent the exchange interaction at the



FIG. 1. Full curve: the coupling function J(L) for a monovalent fcc (100) metal (arbitrary units), as calculated within the continuum version of the RKKY model. Broken curve: the actual coupling function, with the experimentally measured periodicity, for L = Nd. In the case of partially occupied atomic planes at the interfaces J(L) is also defined for nonintegral values of L/d, and could be expressed as a weighted sum of values J(Nd).

actual distances L = Nd, where N is an integer and where d is the thickness of one monolayer, by a coupling function with a period  $\Lambda$  much longer than the period  $\lambda$  of the spin density oscillations.

If J(L) oscillates with a period  $\lambda$ , one can obtain  $\Lambda$  by reducing the wave number  $2\pi/\lambda$  to the first Brillouin zone of the one-dimensional lattice (with lattice parameter d) which is the projection on an axis parallel to the growth direction of the three-dimensional lattice.  $\Lambda$  is given by the expression

$$\Lambda = \frac{1}{|1/\lambda - n/d|} , \qquad (1)$$

where the non-negative integer *n* is such that  $\Lambda \ge 2d$ . A is at least 2*d*, because the wave vector  $2\pi/\Lambda$  lies in the first Brillouin zone. In Fig. 2 the period  $\Lambda$  (in units of *d*) is given as a function of  $\lambda/d$ . A goes to infinity if  $\lambda$  approaches d/n ( $n=0,1,2,\ldots$ ), whereas  $\Lambda=2d$  for  $\lambda=d/(n+\frac{1}{2})$ . In most cases Eq. (1) should be used with n=1, as can be concluded from Table I. The table gives values of  $\lambda$  and  $\Lambda$  for monovalent (N=1), divalent (N=2) and trivalent (N=3) fcc and bcc free electron metals. The period  $\lambda$  has been calculated within the RKKY model ( $\lambda=\pi/k_F$ ). In several cases  $\lambda$  is close to the interplanar distance *d*, so that  $\Lambda$  is much larger than  $\lambda$ . Within the RKKY model, the largest enhancement is predicted for divalent fcc (100) grown metals:  $\Lambda/\lambda=64$ .

Models for the oscillatory exchange interaction across nonferromagnetic layers, which go beyond the RKKY theory, have been proposed by several authors. In the work by Yafet for Gd/Y multilayers<sup>15</sup> the discrete nature of the interlayer distance was explicitly taken into account in the expression for the coupling function [Eq. (3) of Ref. 14). However, for the specific system studied the final result was not affected by the discreteness of L, because the coupling was shown to exhibit variations on a



FIG. 2. Graphical representation of Eq. (1). For the meaning of  $\Lambda$ ,  $\lambda$ , and d, see text. The vertical broken lines at  $\lambda/d = 1/n$ are asymptotes, where  $\Lambda$  approaches infinity. For reasons of clarity the figures has not been extended to the shaded area.

scale larger than 2d. For the same reason the functions J(L) calculated by Wang *et al.*<sup>16</sup> for Fe/Cr multilayers would not be altered.

Edwards *et al.*<sup>17</sup> have calculated the coupling energy for a simple cubic (100) superlattice, using a single-band tight-binding description of the electronic structure. They retained thereby the discrete nature of the inter-

TABLE I. Oscillation periods  $\Lambda$  of exchange interactions across free electron metals, using the RKKY model for  $\lambda$ , different crystal structures, growth directions, and valences (N); *a* is the lattice parameter and *d* is the spacing between planes normal to the growth direction;  $\lambda/d$  is given between parentheses. For hcp (0001) systems with the ideal c/a ratio  $\Lambda/d$  and  $\lambda/d$  are equal to the values for fcc (111) systems.

	$\Lambda/d \ (\lambda/d)$			
	d/a	N=1	N=2	N=3
fcc (100)	1/2	4.58	65.55	7.86
		(1.28)	(1.02)	(0.89)
fcc (110)	$\sqrt{2}/4$	2.24	3.29	4.93
		(1.81)	(1.44)	(1.25)
fcc (111)	$\sqrt{3}/3$	10.26	7.29	3.31
		(1.11)	(0.88)	(0.77)
bcc (100)	1/2	2.63	4.58	9.50
		(1.61)	(1.28)	(1.12)
bcc (110)	$\sqrt{2}/2$	8.15	9.49	3.77
		(1.14)	(0.90)	(0.79)
bcc (111)	$\sqrt{3}/6$	2.79	2.22	2.07
		(2.79)	(2.22)	(1.94)

layer thickness. This led to the conclusion that the period  $\Lambda$  approaches infinity if the Fermi surface approaches the edge of the Brillouin zone  $(k_F = \pi/a)$  whereas  $\Lambda$  was found to attain a minimum for  $k_F = (\pi/2)a$ . For the specific case considered this conclusion agrees with Eq. (1), if  $\lambda$  is equal to  $\pi/k_F$ , as in the RKKY theory. Equation (1) makes clear how this conclusion can be generalized to other crystal structures and growth directions. For cubic crystals, Eq. (1) can be rephrased as the statement that  $\Lambda$  approaches infinity if the Fermi surface approaches the wave vector

$$\mathbf{K} = \frac{n\pi}{a} (h\hat{\mathbf{p}} + k\hat{\mathbf{q}} + l\hat{\mathbf{r}}) , \qquad (2)$$

where n=0,1,2,..., where  $\hat{\mathbf{p}}$ ,  $\hat{\mathbf{q}}$ , and  $\mathbf{r}$  are the orthonormal unit vectors of the reciprocal space, and where h, k, and lare the Miller indices of the growth planes. These indices should reflect the interplanar distance. For example, one should use (200), (220), and (111) for the growth of fcc layers along the [100], [110], and [111] directions, respectively. The Fermi surface should be represented within the extended zone scheme. In a few simple cases the  $\mathbf{k}$ vector, which is most relevant, viz. the vector found for n=1, lies on the edge of the first Brillouin zone. These cases are (100), (110), and (111) growth for a sc lattice, (100) and (111) growth for an fcc lattice, and (100) and (110) growth for a bcc lattice. For all other growth planes K lies outside the first Brillouin zone, and the approach of the Fermi surface to the edge of the first Brillouin zone does not result in an approach of  $\Lambda$  to infinity. In the case of *c*-axis oriented growth of an hcp lattice (hkl) = (002), because d = c/2, and  $\mathbf{k} = (2\pi/c)\hat{c}$ . The situation in which the Fermi surface coincides with the edge of the Brillouin zone [the A point at  $\mathbf{k} = (\pi/c)\hat{c}$ ]) then leads to a minimum in the period:  $\Lambda = 2d = c$ .

#### **III. COMPARISON WITH EXPERIMENT**

We have investigated to what extent the experimental data on interlayer exchange interactions in Co/Cu and Fe/Cu multilayer systems can be understood from Eq. (1). In Table II the values of d,  $\Lambda$  and  $\lambda$  for fcc Co/Cu(111), fcc Co/Cu(100), bcc Fe/Cu(100) and fcc Fe/Cu(100) are summarized. The interplanar spacing dhas been assumed to be the same as in bulk Cu. Possible tetragonal distortions were neglected. In the case of bcc Fe/Cu(100), d was derived using the assumption that the atomic volume in bcc Cu is the same as in fcc Cu. The period  $\lambda$  was obtained from  $\Lambda$  using Eq. (1) with n=1. These values of  $\lambda$  were found to be fairly close to the value of 2.31 Å, which is expected from models within which the electronic structure of the interlayer material is described in terms of a free electron model, such as, for example, the RKKY model. However, in some cases significant differences were found. In the next section we will discuss possible explanations for these deviations. In this section we will briefly discuss the experimental (and in one case the theoretical) data on  $\Lambda$  which are available for the four systems studied.

For fcc Co/Cu(111) multilayers Mosca *et al.*<sup>7</sup> studied the exchange coupling over a Cu thickness range 6-35 Å.

TABLE II. Oscillation periods  $\Lambda$  of the exchange interaction across Cu in Co/Cu and Fe/Cu, and values of  $\lambda$  that follow from  $\Lambda$  using Eq. (1) with n=1.  $\Lambda$  has been taken from experimental work, except in the case of bcc Fe/Cu(100), for which  $\Lambda$ was derived from the results of band-structure calculations. All distances are in Å.

System	d	Λ	λ
Co/Cu(111), fcc	2.08	12-15 <sup>a</sup>	2.40-2.55
Co/Cu(100), fcc	1.80	$\simeq 5.4^{\rm b}, 5.4-7.2^{\rm c}$	2.40 - 2.70
Fe/Cu(100), bcc	1.43	3.3-4.5 <sup>d</sup>	2.10 - 2.50
Fe/Cu(100), fcc	1.80	13-14 <sup>e</sup>	2.06-2.08

<sup>a</sup>Reference 7.

<sup>b</sup>References 5 and 6 (see text).

°Reference 8.

<sup>d</sup>Reference 18.

<sup>e</sup>Reference 4.

A lies between 12 and 15 Å, which leads to  $\lambda = 2.40 - 2.55$  Å.

For Co/Cu(100) we have estimated  $\Lambda$  by combining the published experimental results by Cebollada *et al.*<sup>5</sup> and Pescia *et al.*<sup>6</sup> Their results are summarized in Fig. 3. As shown schematically in the figure it is possible to explain all results if  $\Lambda$  is about 3 ML (5.4 Å). Recently, Kirschner *et al.*<sup>8</sup> extended the work by Cebollada *et al.* and found oscillations in the coupling with a period between 3 and 4 ML (5.4–7.2 Å). These data for  $\Lambda$  can be explained from an effective value of  $\lambda$  in the range 2.4–2.7 Å.

Heinrich et al.<sup>2</sup> and Cochran et al.<sup>3</sup> found that in epitaxial bcc Fe(100)/Cu(100)/Fe(100) trilayer systems the coupling is ferromagnetic at low Cu thicknesses, and changes to antiferromagnetic between 9 and 10 ML Cu. Above 10–11 ML Cu the bcc Cu structure was no longer stable. The system exhibited a structural transition, which made the system less suitable for measurements of



FIG. 3. Analysis of experimental data on the exchange interaction across Cu layers in fcc Co/Cu(100) systems, in terms of an oscillatory interaction function J(n). Open circles: Ref. 5; closed circles: Ref. 6. F, AF, and — denote ferromagnetic, antiferromagnetic, and no (experimentally detectable) coupling, respectively. Parentheses denote that the coupling is relatively weak.

the coupling at larger Cu thicknesses. Using the period  $\lambda \simeq 2.5$  Å found for the Co/Cu systems we would expect oscillations with a period of  $\Lambda \simeq 3.3$  Å. This short period has not been observed experimentally. It is of interest to compare the experimental results with the results of first-principles band-structure calculations of the coupling in bcc Fe/Cu(100) carried out by Herman et al.<sup>18</sup> They studied superlattices with 2 or 3 ML thick Fe layers and with Cu thicknesses up to 11 ML. As far as we know, this is the only system with Cu interlayers for which such calculations of the coupling have been carried out. As shown in Fig. 4, the coupling was found to be ferromagnetic in the entire thickness range, decreasing in strength from 2 to 6 ML Cu, but fluctuating above 6 ML. The results suggest that there is an oscillatory contribution to the coupling energy superimposed on a ferromagnetic contribution which decreases monotonically with the Cu layer thickness. The calculations explain why oscillations in the coupling have not been observed experimentally for thicknesses below 9 ML Cu. The monotonically decreasing component of the coupling energy cannot be understood from the RKKY model. We have tentatively analyzed the calculated coupling energy J(n), where n is the number of Cu layers, in terms of a contribution which oscillates as  $\sin(2\pi n/\Lambda)/n^2$ , superimposed on a contribution which decreases monotonically as  $n^{-2}$ . The latter contribution is indicated in Fig. 4 by the broken curve. It includes a small offset (0.15 mRy). We stress that we have no physical basis from which the  $n^{-2}$ dependence can be justified. The dotted curve is the total energy, which results from a fit to the calculated data using  $\Lambda = 2.6$  ML (3.7 Å). The effective value of  $\lambda$  that follows from this period is about 2.3 Å, with an uncertainty of about 0.2 Å.



FIG. 4. Calculated exchange interaction energy J(n) (per antiferromagnetic unit cell) for bcc Fe/Cu(100) superlattices, as a function of the Cu layer thickness *n*, after Herman *et al.*<sup>18</sup> Circles and triangles refer to Fe layers composed of 2 and 3 Fe monolayers, respectively. The broken and dotted curves are explained in the text.

Bennett *et al.*<sup>4</sup> have observed oscillatory exchange interactions in fcc Fe/Cu/Fe(100) systems with Fe thicknesses of 3 ML. An oscillatory coupling with a period of about 13.5 Å was found, resulting in  $\lambda \simeq 2.07$  Å.

### **IV. DISCUSSION**

The effective values of  $\lambda$  derived from the experimental (and in one case theoretical) data fall in the range 2.0-2.7 Å. So, roughly speaking, there is a reasonable agreement with the period  $\pi/k_F=2.31$  Å, which would be expected from models in which the electronic structure of the interlayer material is described in terms of a free electron model (such as, for example, the RKKY model). However, for some of the systems  $\lambda$  and  $\pi/k_F$  are significantly different. For fcc Co/Cu(111), for example, the period  $\Lambda$ which would follow from  $\lambda=2.31$  Å is 20.8 Å, whereas the experimental period is 12-15 Å. The upper limit of  $\Lambda$  leads to a lower limit for  $\lambda$  of 2.4 Å. Another interesting observation is that for fcc Co/Cu(100) and fcc Fe/Cu(100) the effective periods  $\lambda$  are significantly different, viz. 2.4-2.7 Å and 2.06-2.08 Å, respectively.

In this section we discuss some factors that might contribute to the difference between  $\lambda$  and  $\pi/k_F$  and to the dependence of  $\lambda$  on the type of magnetic atom: (i) Deviations of the interplanar spacing d from the bulk value. (ii) Nonfree electronlike electronic structure of Cu. (iii) Effects on  $\lambda$  related to the coupling with the magnetic layers. (iv) Interface roughness and interlayer thickness variations. (v) Dependence of the internal magnetic structure in the magnetic layers on the magnetic structure of the multilayer. The discussion is of a rather qualitative nature. In part, this is due to the lack of some essential experimental data [points (i) and (iv)]. Furthermore, a quantitative discussion on points (ii), (iii), and (v) would require a fairly complete model of the electronic structure of the multilayer systems, including the effects related to the formation of a superlattice band structure. Presently, such models are not available.

If  $\lambda$  is close to the interplanar spacing *d* the period  $\Lambda$  is very sensitive to variations in *d*. Deviations from the bulk value of *d* could be caused by stress, induced, for example, by the lattice mismatch.<sup>19</sup> For Co/Cu the lattice mismatch is 2%. A deviation in *d* of about 2% would also lead to a deviation of about 2% in the value of  $\lambda$  derived from the experimentally determined period  $\Lambda$ . Experimental studies of *d* are required if one would like to determine  $\lambda$  very precisely from  $\Lambda$ .

The effects on J(L) due to deviations from a free electron band structure of the interlayer material have been included in the models by Yafet, <sup>15</sup> Wang *et al.*, <sup>16</sup> and Edwards *et al.*<sup>17</sup> We will follow the approach by Yafet, who expressed J(L) in the form of a Fourier integral. Analogous to his approach we write

$$J(L) = \frac{c}{2\pi} \int_0^\infty |j(q_z)|^2 \chi_{\rm Cu}(q_z) \cos(q_z L) dq_z , \qquad (3)$$

where c is the repeat distance parallel to the growth (z) direction in the Cu layers. The function  $\chi_{Cu}$  is the generalized susceptibility of bulk Cu, calculated along the z direction. The coupling function  $j(q_z)$  characterizes the

magnetic interaction between the exchange split 3d orbitals of the Fe or Co atoms at the interface and the Cu conduction electrons. The function is squared because this interaction takes place at two interfaces. We remark that the RKKY results could be recovered from Eq. (3) by using the generalized susceptibility of a free electron gas, and by taking into account that in the case of an infinitely thin magnetic sheet the coupling function  $j(q_z)$  is the Fourier transform of a delta function at z=0, which is independent of  $q_z$ . The oscillations in J(L) result from the singularity in  $\chi(q_z)$  for  $q_z = 2k_F$ . In the case of a nonspherical Fermi surface, the relevant wave vector is given by the extremal size of the Fermi surface, measured in the direction perpendicular to the layers. For the [100] direction in fcc and bcc Cu this leads to corrections of +8.1% and -1.6%, respectively, on  $k_{F}$ .<sup>20</sup> For the [111] direction in fcc Cu the situation is more complicated, due to the necks of the Fermi surface around the L points of the Brillouin zone. As far as we know, no calculation of  $\chi(q_z)$  has been performed for this direction, from which the effective period  $\lambda$  could be derived. The agreement between the periods  $\lambda$  derived for the Co/Cu (111) and (100) systems suggests that, in spite of the nonspherical shape of the Fermi surface around the L points, the effective period  $\lambda$  is rather isotropic.

A second aspect of the nonfree electron-like electronic structure of Cu is the presence of the 3d band, about 1.5-5.5 eV below the Fermi level. At present we do not know to what extent the inclusion of the resulting superexchange interaction in a more complete theory of J(L) would alter  $\lambda$ .

The effective period  $\lambda$  can also deviate from the free electron value due to the  $q_z$  dependence of the coupling function  $j(q_z)$  [see Eq. (3)]. It is instructive to make a comparison with the problem of spin density oscillations around a 3d transition metal impurity in a Cu matrix. Nuclear magnetic resonance (NMR) Knight-shift measurements have indeed revealed oscillations of the spin density on the Cu neighbor atoms. From a quite realistic model treatment of the hybridization between the exchange-split 3d bands on the impurity atoms and the conduction electrons in a spherical Cu host band, using parameters that gave a good fit between the nearneighbor Knight shifts, the bulk susceptibility, and the high-temperature resistivity, Cohen and Slichter<sup>21</sup> found a significant deviation from the asymptotic oscillation period ( $\lambda = \pi/k_F = 2.31$  Å) for distances below about 50 Å from a 3d impurity. From their figures it can be derived that in the range 4-10 Å from a Mn or Fe impurity the average oscillation period is about 2.45-2.50 Å. Within the RKKY model the deviations from the asymptotic value of  $\lambda$  would be less than 1% in this distance range. The main difference is probably that in the hybridization model by Cohen and Slichter a d-wave centrifugal term was included in the radial Schrödinger equation. In the RKKY model coupling to a state of ssymmetry is assumed. Further investigations of the importance of this effect for multilayer systems are required. One of the implications would be that  $\Lambda$  would depend on L. In the case of fcc Co/Cu (111), for example,  $\Lambda$ 

would become longer with increasing interlayer thickness.

Interface roughness or thickness variations of the Cu layers lead to an additional  $q_z$ —dependence of  $j(q_z)$ . As discussed by Wang et al.<sup>16</sup> this effect can be included in Eq. (3) by multiplying  $j(q_z)$  by the 1D-structure form factor of the magnetic layer, which is the Fourier transform  $f(q_z)$  of the z-dependent function giving the probability of finding a Co or Fe atom in a certain atomic plane. They investigated a simple model for a nonideal monolayer of magnetic atoms, with a fraction of (1-2p)magnetic atoms in the atomic plane at z=0, and fractions p for the atomic planes at z = d and z = -d. We will investigate some consequences of roughness or thickness variations within this model. It is not known to what extent this model is realistic for the Co/Cu and Fe/Cu systems considered. However, the more general, qualitative conclusions are expected to be correct, and independent of the specific-model used. Let us consider a system for which, in the case of perfect interfaces, the exchange coupling is given by the function

$$J(Nd) = \frac{C}{N^2 d^2} \sin\left[\frac{2\pi}{\lambda} Nd\right], \qquad (4)$$

where C is a constant and N is an integer. This function would follow from the RKKY theory for the interaction between two infinitely thin magnetic sheets embedded in a free electron gas, in the limit  $Nd \gg \lambda$  (see Sec. II). If interface roughness or thickness variations are described within the model discussed above, the modified coupling function J''(Nd) is given by

$$J''(Nd) = \frac{C}{N^2 d^2} A \left[ A \sin\left[\frac{2\pi}{\lambda} Nd\right] + \frac{B}{N} \cos\left[\frac{2\pi}{\lambda} Nd\right] \right],$$
(5)

with

$$A = (1 - 2p) + 2p \cos\left[\frac{2\pi}{\lambda}d\right]$$
(6)

and

$$B = 8p \sin\left[\frac{2\pi}{\lambda}d\right] \,. \tag{7}$$

Equation (5) can most easily be derived by calculating first, as an intermediate step, the coupling function

$$J'(Nd) = pJ((N-1)d) + (1-2p)J(Nd) + pJ((N+1)d)$$
(8)

between a rough and a flat interface, and then repeating the operation. Terms of fourth and higher order in 1/Nhave been neglected. Two conclusions follow from the expression for J''(Nd). First, interface roughness or thickness variations can lead to an attenuation of the coupling. In the limit of large N, the attenuation factor is  $A^2$ . Destructive interference between spin-density oscillations is maximal for  $\lambda = d/(n + \frac{1}{2})$ , with n=0,1,2,..., whereas constructive interference (no attenuation) takes place for  $\lambda = d/n$ . Wang *et al.*<sup>16</sup> have used this as an argument to explain the absence of rapid oscillations, with the period  $\Lambda = 2d$  of the spin-density oscillations in bulk Cr, in the interlayer exchange coupling of Fe/Cr multilayers. A comparison with Fig. 2 shows that the attenuation is largest for those values of  $\lambda$  which lead to a small period  $\Lambda$ . So Eq. (1) explains why periods  $\Lambda$  below 2d will never be observed, whereas it follows from Eqs. (3) and (4) that oscillations with  $\Lambda$  slightly higher than 2d are attenuated in the case of rough interfaces or thickness variations. Apart from this attenuation, roughness or thickness variations also affect the phase and the periodicity of the coupling function. The relative change of the period  $\lambda$  is given by

$$\frac{\Delta\lambda}{\lambda} = \frac{B}{2\pi A} \frac{1}{N^2} , \qquad (9)$$

for  $\Delta\lambda/\lambda \ll 1$ . In the limit of large N there is only an attenuation, and no change of the period. In the interval  $1 < \lambda/d < 2, \Delta\lambda$  is negative, and at the end points of this interval  $\Delta\lambda$  vanishes. Above L=7 Å ( $N \simeq 4$ ) the correction on  $\lambda$  is very small: even for a fairly rough interface (p=0.2) and for the least favorable case ( $\lambda/d=1.57$ ) it is less than 4%. These results indicate that our conclusions concerning the effective period  $\lambda$  for the four Co/Cu and Co/Fe systems that we studied would probably be altered only slightly if interface roughness or thickness variations were taken into account. We emphasize that for interlayer materials with a nonfree-electron-like interlayer electronic structure the period  $\lambda$  can change due to interface roughness, even in the limit of large L, as follows from the work of Wang *et al.* on the Fe/Cr system.<sup>16</sup>

We have found that for fcc Fe/Cu(100) the value of  $\lambda$ which follows from the experimental period  $\Lambda$  is about 15% smaller than for the corresponding Co system. From studies of the spin density around 3d transition metal impurities in Cu,<sup>21,22</sup> the effective periods  $\lambda$  are expected to be equal within a few percent. The discrepancy might be related to the strong dependence of the internal magnetic structure of fcc Fe to various external conditions (pressure, temperature, interactions with other magnetic layers, etc., Refs. 23–25). This suggestion is rather preliminary, of course, but it should motivate further investigations of fcc Fe/Cu systems, for example of the dependence of the interlayer coupling on the thickness of the Fe layers, and on the internal magnetic structure in the Fe layers.

### V. CONCLUDING REMARKS

We have found that the period  $\Lambda$  of oscillatory exchange interactions across nonferromagnetic layers between Fe or Co layers depends on the ratio between the period  $\lambda$  of spin density oscillations in the nonferromagnetic layers which are induced by the coupling with the Fe or Co layers, and the interplanar spacing d in the nonferromagnetic layers. The proper relation between  $\Lambda$ ,  $\lambda$ , and d is given by Eq. (1). The experimental periods  $\Lambda$  for Co/Cu systems can be explained consistently using  $\lambda = 2.55 \pm 0.15$  Å. Interestingly, this period is not significantly different from the period of spin-density oscillations within 4–10 Å from a transition-metal impurity atom in a Cu matrix, <sup>18</sup> although it is slightly larger than the period expected within the RKKY model. One of the important questions raised in Sec. IV is whether the coupling at the interface with transition-metal d states [instead of states with s symmetry, as is often assumed in models for J(L)] would result in a rather slow convergence of  $\Lambda$  with increasing L.

An extension of the studies of the oscillatory coupling to Fe/Cu and Co/Co systems with other crystal structures and growth directions could be difficult to realize experimentally, but quite well possibly theoretically, by means of band-structure calculations. Of course, such studies also yield very important information about the strength of the interaction, the phase, the dependence of the coupling on the thickness of the magnetic layers, the influence of the interface structure, etc. One of the unresolved issues is the monotonically decreasing component of the coupling, which was found by Herman *et al.*<sup>18</sup> for bcc Fe/Cu(100).

Finally, it would be of interest to look for oscillatory exchange interactions across Ag or Au interlayers. Within the RKKY theory the period  $\lambda$  for Cu, Ag, and Au is simply proportional to the lattice parameter of the cubic unit cell. So if the ratio between the actual effective value of  $\lambda$  and the value of  $\lambda$  expected from the RKKY theory is the same for the three elements,  $\Lambda$  is also proportional to the lattice parameter *a*, leading to 13% larger periods in systems with Ag or Au, compared to systems with Cu interlayers.

Note added in proof. After the submission of this paper we were informed about the work of Chappert and Renard<sup>26</sup> and by Deaven, Rokhsar, and Johnson,<sup>27</sup> who independently developed a similar model for the period of oscillatory interlayer exchange interactions. Recently experimental data on fcc Co/Cu(100) systems have become available, showing a period  $\Lambda = 11-13$  Å.<sup>28-30</sup> In agreement with the predictions given in this paper the period is close to the period in fcc Fe/Cu(100). Furthermore, the corresponding effective values of  $\lambda$  (2.10–2.17 Å) agree well with the value which follows from ASW bandstructure calculations ( $\lambda = 2.13$  Å, see Sec. IV).

# ACKNOWLEDGEMENTS

The author wishes to thank W. Folkerts for very helpful discussions and comments, and P. H. Dederichs for drawing the author's attention to the impurity problem.

- <sup>1</sup>S. S. P. Parkin, N. More, and K. P. Roche, Phys. Rev. Lett. **64**, 2304 (1990).
- <sup>2</sup>B. Heinrich, Z. Celinski, J. F. Cochran, W. B. Muir, J. Rudd, Q. M. Zhong, A. S. Arrott, and K. Myrtle, Phys. Rev. Lett.

**64**, 673 (1990).

<sup>&</sup>lt;sup>3</sup>J. F. Cochran, J. Rudd, W. B. Muir, B. Heinrich, and Z. Celinski, Phys. Rev. B 42, 508 (1990).

<sup>&</sup>lt;sup>4</sup>W. R. Bennett, W. Schwarzacher, and W. F. Egelhoff, Jr.,

Phys. Rev. Lett. 65, 3169 (1990).

- <sup>5</sup>A. Cebollada, J. L. Martinez, J. M. Gallego, J. J. de Miguel, R. Miranda, S. Ferrer, F. Batallan, G. Fillion, and J. P. Rebouillat, Phys. Rev. B **39**, 9726 (1989).
- <sup>6</sup>D. Pescia, D. Kerkmann, F. Schumann, and W. Gudat, Z. Phys. B **78**, 475 (1990).
- <sup>7</sup>D. H. Mosca, F. Petroff, A. Fert, P. A. Schroeder, W. P. Pratt, Jr., and R. Laloee, J. Magn. Magn. Mater. **97**, L1 (1991).
- <sup>8</sup>J. Kirschner, unpublished.
- <sup>9</sup>G. Binasch, P. Grünberg, F. Saurenbach, and W. Zinn, Phys. Rev. B **39**, 4828 (1989).
- <sup>10</sup>M. N. Baibich, J. M. Broto, A. Fert, F. Nguyen Van Dau, F. Petroff, P. Etienne, G. Creuzet, A. Friederich, and J. Chazelas, Phys. Rev. Lett. **61**, 2472 (1988).
- <sup>11</sup>P. Grünberg, S. Democritov, A. Fuss, M. Vohl, and J. A. Wolf, J. Appl. Phys. **69**, 4789 (1991).
- <sup>12</sup>C. Kittel, in *Solid State Physics*, edited by F. Seitz, D. Turnbull, and H. Ehrenreich (Academic, New York, 1968), Vol. 22, p. 1.
- <sup>13</sup>Y. Yafet, Phys. Rev. B 36, 3948 (1987).
- <sup>14</sup>W. Baltensperger and J. S. Helman, Appl. Phys. Lett. 57, 2954 (1990).
- <sup>15</sup>Y. Yafet, J. Appl. Phys. 61, 4058 (1987).
- <sup>16</sup>Y. Wang, P. M. Levy, and J. L. Fry, Phys. Rev. Lett. 65, 2732 (1990).

- <sup>17</sup>D. M. Edwards, J. Mathon, R. B. Muniz, and M. S. Phan, J. Magn. Magn. Mater. **93**, 85 (1991).
- <sup>18</sup>F. Herman, J. Sticht, and M. Van Schilfgaarde, J. Appl. Phys. **69**, 4783 (1991).
- <sup>19</sup>H. Awano, Y. Suzuki, T. Katayama, and A. Itoh, J. Appl. Phys. 68, 4569 (1990).
- <sup>20</sup>R. Coehoorn, unpublished results from the Augmented Spherical Wave band-structure calculations.
- <sup>21</sup>J. D. Cohen and C. P. Slichter, Phys. Rev. B 22, 45 (1980).
- <sup>22</sup>B. Drittler, H. Ebert, R. Zeller, and P. H. Dederichs, Phys. Rev. B **39**, 6334 (1989).
- <sup>23</sup>V. L. Moruzzi, P. M. Marcus, K. Schwarz, and P. Mohn, Phys. Rev. B 34, 1784 (1986).
- <sup>24</sup>W. A. A. Macedo, W. Keune, and E. D. Ellerbrock, J. Magn. Magn. Mater. **93**, 552 (1991).
- <sup>25</sup>D. Kerkmann, E. Vescovo, and D. Pescia, unpublished.
- <sup>26</sup>C. Chappert and J. P. Renard, Europhys. Lett. (to be published).
- <sup>27</sup>D. M. Deaven, D. Rokhsar, and M. Johnson (unpublished).
- <sup>28</sup>J. J. de Miguel, A. Cebollada, J. M. Gallego, R. Miranda, C. M. Schneider, P. Schuster, and J. Kirschner, J. Magn. Magn. Mater. **93**, 1 (1991).
- <sup>29</sup>G. Giron et al., J. Magn. Magn. Mater. (to be published).
- <sup>30</sup>R. Coehoorn, W. Folkerts, and B. Sachse (unpublished).