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Absence of fractional giant Shapiro steps in diagonal Josephson-junction arrays

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When rf and dc bias currents are injected *diagonally* into an $N\times M$ Josephson-junction array and a perpendicular magnetic field is applied, *fractional* giant Shapiro steps are unexpectedly absent, although giant Shapiro steps are still observed. This absence is in direct contrast to observations in the usual case of in-line current injection and to theoretical predictions. Numerical simulations confirm these experimental results. A theoretical argument is presented to explain why diagonal Josephson-junction arrays cannot support fractional giant Shapiro steps.

When a radio-frequency (rf) current, $i_{rf}\sin(2\pi\nu t)$, is applied to an $N\times M$ array of superconducting-normal-superconducting (SNS) Josephson junctions, *giant* Shapiro steps occur[1,2] at voltages

$$V_n = n\left[\frac{Nh\nu}{2e}\right],\quad n=0,1,2,\ldots. \tag{1}$$

When a perpendicular magnetic field, corresponding to a strongly commensurate number of flux quanta per unit cell, $f=p/q$ (where p and q are small integers), is also applied to the system, Benz, Rzchowski, Tinkham, and Lobb[2] (BRTL) found that *fractional* giant Shapiro steps occur at voltages

$$V_n = n\left[\frac{Nh\nu}{q2e}\right],\quad n=0,1,2,\ldots \text{ and } q=1,2,3,\ldots. \tag{2}$$

BRTL attribute these fractional giant Shapiro steps to the driven motion, in a direction perpendicular to the macroscopic current, of a superlattice of *field-induced* vortices commensurate with the underlying array lattice. The motion of *current-induced* vortices is thought to be responsible for *subharmonic* steps occurring at voltages

$$V=\frac{n}{m}\left[\frac{Nh\nu}{q2e}\right],\quad n=0,1,2,\ldots \text{ and } m=1,2,3,\ldots \tag{3}$$

in these arrays.[3,4] In addition to these features characteristic of arrays in commensurate fields, minima in dV/dI often appear at voltages corresponding to half-integer n in Eq. (1) which simply reflect the presence of an inflection point in the I-V curve midway between strong integer steps. These features occur in single junctions as well, and, in arrays, can occur at voltages corresponding to subharmonic steps [Eq. (3)].

In this paper, we describe our investigation of a system similar to that of BRTL. Rather than injecting the current into an array along the [10] direction of the array unit cell, as BRTL have done, we inject the current along the [11] direction [see insets of Figs. 1(a) and 1(b)]. (We shall hereafter refer to the BRTL configuration as [10]; ours, as *diagonal*.) Surprisingly, and contrary to theoretical predictions,[5] we find that although integer giant

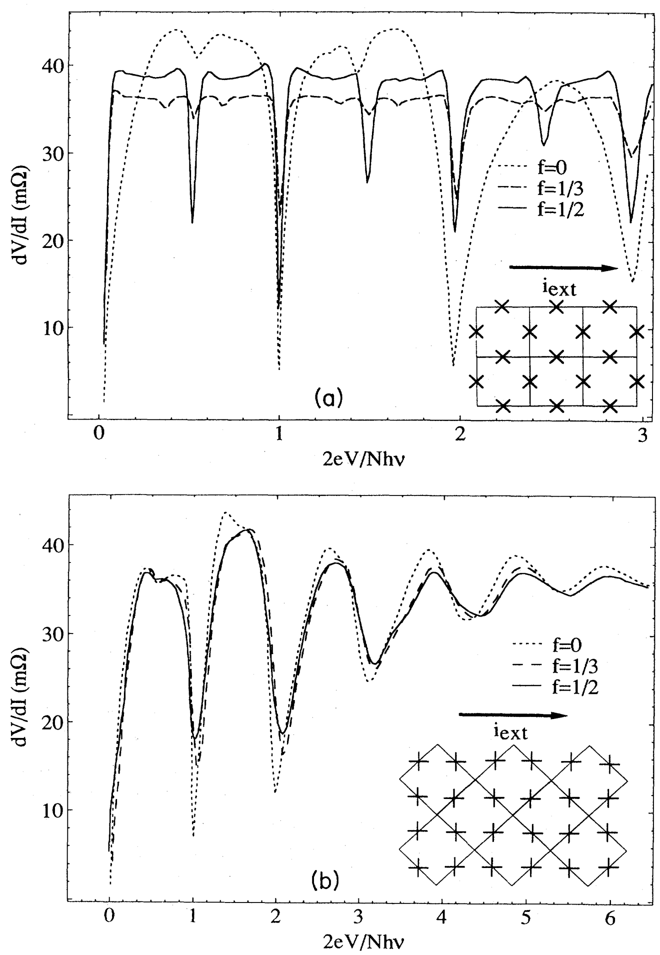

FIG. 1. Dynamic resistance vs normalized voltage in the presence of perpendicular magnetic fields corresponding to $f=0$, $\frac{1}{3}$, and $\frac{1}{2}$. (a) For [10] array at $T=1.39$ K where $I_c=0.26$ mA, rf frequency $\nu=1.5$ MHz ($\Omega=Nh\nu/2eI_cR=0.45$). The rf drive current for $f=0$ was $\frac{2}{3}$ as large as for $f=\frac{1}{2}$ and $\frac{1}{3}$. (b) For diagonal array at $T=2.10$ K where $I_c=0.78$ mA, rf frequency $\nu=4$ MHz ($\Omega=0.58$). The rf amplitude was equal for all curves in (b). The insets show segments of arrays and direction of current.

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Shapiro steps [Eq. (1)] are still observed, *neither* fractional giant [Eq. (2)] *nor* subharmonic [Eq. (3)] Shapiro steps are present in diagonal arrays for any experimentally tested rf frequency or transverse magnetic-field strength. Our numerical simulations of diagonal arrays agree with these experimental results and also show that on a giant step for $f=\frac{1}{2}$ and low rf frequency, the time derivatives of the gauge-invariant phase differences of the array's junctions are all equal throughout an rf cycle. We use the latter result to argue that all the junctions in a diagonal array behave like a single Josephson junction. Since single overdamped Josephson junctions show only integer Shapiro steps,[6] this offers a mathematical explanation for the absence of fractional giant and subharmonic Shapiro steps in diagonal arrays in the fully frustrated case. For general $f=p/q$, we suggest that our surprising results are related to the fact that the bias current feeds equally into all junctions in the diagonal array.

Fabrication and measurement of diagonal arrays are similar to those described by BRTL and will be only briefly described here. A 0.3-μm-thick Cu film is thermally evaporated onto a sapphire substrate. After Ar ion etching the Cu surface, a 0.2-μm-thick Nb film is sputtered onto the Cu. An array of Nb islands is patterned at 45° with respect to the current injection pads and formed by reactive ion etching with sodium hexaflouride (SF_6). The 10 mm×1 mm array contains 1414×141 Josephson junctions with junction length of 2 μm arranged in a square lattice with lattice constant of 10 μm. For comparison, we also fabricated [10] arrays of the same macroscopic size and shape, containing 1000×100 junctions with length and lattice constants identical to those of the diagonal arrays. Both types of arrays exhibit normal-state resistances corresponding to 2.6–5 mΩ per junction and possess Kosterlitz-Thouless (KT) transition tempertures ranging from 1.2 to 3.3 K. Using a lock-in amplifier to perform a four-point measurement, we measured the dynamic resistance dV/dI of our arrays versus dc voltage or current for various external magnetic fields, rf frequencies and amplitudes, and temperatures. Figures 1(a) and 1(b) are representative dV/dI vs V curves of the [10] and diagonal arrays, respectively. (An I-V curve of a simulated diagonal array is shown in Fig. 2 for comparison and will be discussed below.) Note that the voltage axis is normalized to $Nh\nu/2e$ so that $2eV/Nh\nu=n$ when the arrays are on the nth giant step. Note also that both curves were taken well below the KT transition temperature ($T_{KT}\sim 2.8$ K for the [10] array, $T_{KT}\sim 3.3$ K for the diagonal one).

For the [10] arrays [see Fig. 1(a)] in zero field, we see giant Shapiro steps whose voltages agree with Eq. (1). As expected, in the presence of a strongly commensurate field, i.e., $f=\frac{1}{2}$ or $\frac{1}{3}$, both giant and fractional giant Shapiro steps are observable at voltages agreeing with Eqs. (1) and (2), respectively. In addition, weak subharmonic steps at voltages corresponding to Eq. (3), where $m=2$ or 3, are present in the [10] arrays at $f=0$, $\frac{1}{2}$, and $\frac{1}{3}$. These subharmonic steps begin to disappear at low rf frequencies ($\Omega=Nh\nu/2eI_cR<0.3$ where I_c is the critical current[7] and R is the normal-state resistance of the entire

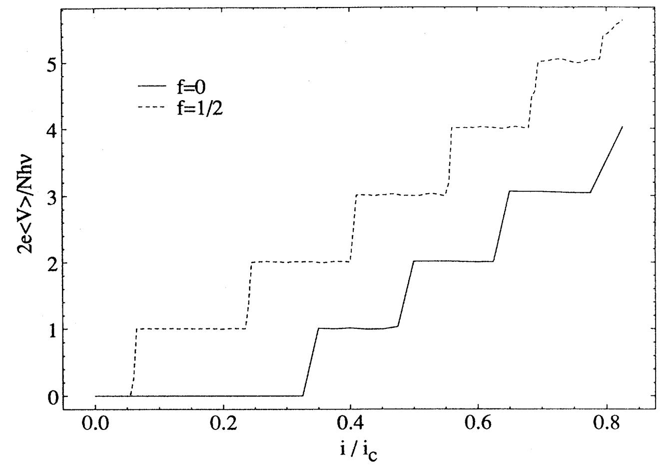

FIG. 2. I-V curves of a simulated 8×8 junction diagonal array at magnetic-field strengths corresponding to $f=0$ and $\frac{1}{2}$ (solid and dashed lines, respectively). $\Omega=0.1$ and $i_{rf}/i_c=0.75$ for both these curves. Increments of i/i_c were 0.025 for $f=0$ and 0.005 for $f=\frac{1}{2}$.

array).[8]

In the diagonal arrays [see Fig. 1(b)], we also see giant Shapiro steps whose voltages correspond to Eq. (1). More importantly, however, we *do not* observe fractional giant *or* subharmonic Shapiro steps at any transverse field strength ($f=0$, 0.07, 0.18, 0.29, 0.33, and 0.50 were tried) in these arrays, apart from a small feature seen at the half-integer position. We tentatively attribute this to the inflection point feature mentioned at the end of the first paragraph. Changes in the rf frequency, such that Ω ranges from 0.18 to 0.73, have no effect on this absence of fractional steps. Measurements made at different rf amplitudes and at different temperatures also consistently fail to show fractional giant or subharmonic Shapiro steps in diagonal arrays. This unexpected absence is the principal result of this paper.

We stress that the surprising absence of fractional giant and subharmonic Shapiro steps in diagonal arrays is not due to any type of boundary effects[5,9,10] that might have been introduced as a result of our arrays being macroscopically long and narrow. Although the two types of arrays have identical macroscopic geometry, only the [10] arrays—and not the diagonal ones—exhibit fractional giant and subharmonic Shapiro steps. It is also unlikely that our results are due to inhomogeneities[5] in our arrays, as our results are reproducible from array to array. Finally, we discount the possibility that the exact configuration of the boundaries provide critical pinning effects since the simulations described below use *periodic* boundary conditions and give results that confirm the experiments.

Numerical simulations were performed on 4×4, 6×6, and 8×8 junction diagonal arrays, and are identical to those performed by Free *et al.*[11] with the obvious exception that the arrays are now diagonal, not [10]. Using the Landau gauge, $\mathbf{A}=Hx\hat{\mathbf{y}}$ and choosing periodic boundary conditions perpendicular to the transport current direction ($\hat{\mathbf{x}}$), we allow the phases of the islands to relax 400 rf periods before time averaging the voltage for an addition-al 400 rf periods. For a wide range of rf frequencies $(0.1 < \Omega < 1.5)$ and amplitudes, we find that the resulting I vs V curves for 8×8 and 4×4 junction diagonal arrays at $f=\frac{1}{2}$ (see Fig. 2 for a representative curve) and for 6×6 junction diagonal arrays at $f=\frac{1}{3}$ show only giant Shapiro steps whose widths varied with rf amplitude. Nowhere do we see fractional giant or subharmonic Shapiro steps in any of the computed curves.

When we study the time evolution of the phases and the supercurrents of the simulated 8×8 or 4×4 junction diagonal array on the first giant Shapiro step at low rf frequencies, i.e., $\Omega < 0.6$, and at $f=\frac{1}{2}$, we find that the supercurrents flow in a sequence of staircase patterns throughout an rf cycle.[12] At the beginning of an rf cycle, when $i_{\rm rf}\sin\omega t = 0$, the phases in the diagonal array resemble the $f=\frac{1}{2}$ ground state, i.e., staircase currents of alternating sign form a checkerboard pattern of clockwise and counterclockwise vortices within the array. Motion of this superlattice of vortices was successfully used to describe phase coherence in [10] arrays.[2] In the [10] case, the Lorentz force drives the vortices straight through the weak links between the Nb islands. In contrast, in the [11] case, the Lorentz force drives the vortices toward the high-energy barrier of the Nb islands. One might imagine that half of the vortices move around an island on one side, and half on the other side. If incoherent, such a motion of the vortex lattice across the array might be thought to explain the absence of fractional giant and subharmonic steps in diagonal arrays. In our simulations, however, the actual vortex motion does not follow this scenario. As the rf drive advances in its cycle, only staircases of supercurrents flowing in the same direction as the drive current exist in the array, and the aforementioned vortices completely disappear. At the peak of the rf drive cycle, at which time the transport current is carried largely as normal current with an accompanying pulse of voltage (as seen in Fig. 3), the pattern of vortices momentarily reappears, but with the positions of the vortices now interchanged. This new pattern, however, quickly disappears and subsequently, the previous state reappears, remaining until the end of the rf cycle. The motion described above corresponds to a $2\pi N$ phase slip of the array per rf cycle. Because the vortices disappear and reappear, their motion cannot be traced continuously in a simple way and a more general model than that used for the [10] arrays must be adopted.

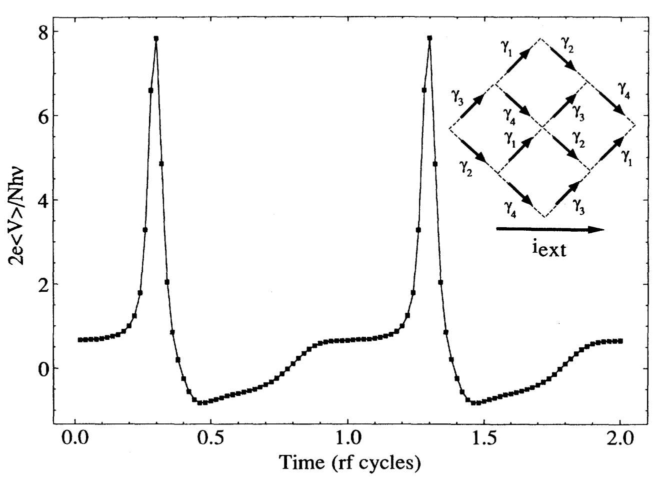

FIG. 3. Normalized instantaneous voltage vs time (in rf cycles) across each junction in a plaquette of the simulated 8×8 junction diagonal array on the first giant Shapiro step at $i/i_c = 0.15$ under the drive conditions in Fig. 2. Data points representing all junctions lie on top of each other and therefore cannot be distinguished in this figure. Inset: 2×2 section of the diagonal array where the γ's are the gauge-invariant phase differences of the corresponding junctions.

In more detail, our simulations show that on a giant step at $f=\frac{1}{2}$ and low rf frequencies $(\Omega < 0.6)$ all the time derivatives of the gauge-invariant phases, $d\gamma/dt$ (and hence all normal currents), in a plaquette are equal throughout an rf cycle (a typical wave form is shown in Fig. 3). Thus, normal currents and supercurrents are separately conserved at each node. Supercurrent conservation is accomplished by having the supercurrents always flowing in a staircase pattern with $\gamma_1=\gamma_2$ and $\gamma_3=\gamma_4$ (see inset of Fig. 3). If we also require that the fluxoid in each plaquette must equal $2\pi f=\pi$, since $f=\frac{1}{2}$ in this case, we find that $2\gamma_1 - 2\gamma_4 = \pi$ (mod2π). Using these relationships, we see that the net supercurrents I_S and normal currents I_N per node are

$$I_S = i_c(\sin\gamma_1 + \sin\gamma_4)\,, \tag{4}$$

$$I_N = \frac{\hbar}{e}\frac{1}{r}\frac{d\gamma_1}{dt}\,, \tag{5}$$

where i_c is the critical current per junction and r is the normal-state resistance per junction. If we set the total current through the cell equal to the applied transport current, we find that

$$\frac{\hbar}{2ei_c r}\frac{d\gamma}{dt} + \frac{1}{\sqrt{2}}\sin\gamma = i_{\rm dc} + i_{\rm rf}\sin(\omega t) \tag{6}$$

where $\gamma = \gamma_1 - \pi/4 = \gamma_4 + \pi/4$, and $i_{\rm dc} = I_{\rm dc}/2$ and $i_{\rm rf} = I_{\rm rf}/2$ are the applied dc and rf current per junction, respectively, in units of i_c. From Eq. (6), we see that the phase constraints in this regime cause all junctions in the diagonal array to have the same equation of motion as an isolated single Josephson junction with critical current equal to $i_c/\sqrt{2}$ and resistance equal to r in the resistively shunted-junction approximation. Renne and Polder[6] have shown that single overdamped Josephson junctions produce only integer Shapiro steps. It follows that diagonal arrays cannot support fractional giant or subharmonic Shapiro steps in the fully frustrated case, at least at frequencies low enough for the constraints derived from the simulations to apply.

To gain insight into our experimental results for general $f=p/q$ and higher rf frequencies, we contrast the physical characteristics of the two types of arrays. In [10] arrays, we see that the applied bias current *breaks* the spatial symmetry of the system. The bias current feeds directly into junctions parallel but not perpendicular to it. By contrast, we see that in diagonal arrays, the transport current *preserves* the spatial symmetry of the system by feeding equally into all the junctions. This suggests that all the voltages in a diagonal array will be in phase with one another (see Fig. 3 for a special case). We believe that it is this preservation of the array's symmetry by the bias current feed which accounts for the absence of fractional giant and subharmonic Shapiro steps in diagonal arrays.

We note that our results are not in agreement with the theoretical predictions of Halsey.[5] Based on the assumption of low rf frequencies, the supercurrents flowing in a staircase pattern, and the array being voltage biased, Halsey predicts that fractional giant and subharmonic Shapiro steps will be seen in diagonal arrays. In addition, he predicts that subharmonic steps will be observed in diagonal arrays even when no field is present. Our simulations, which are current biased, confirm Halsey's assumption of staircase currents at low rf frequencies. However, our experimental, numerical, and analytical results do not produce fractional or subharmonic steps. Halsey has suggested that this unanticipated difference might be due to the different bias choices in our two approaches.[13]

We conclude that fractional giant [Eq. (2)] and subharmonic [Eq. (3)] Shapiro steps are *not* seen in either real or simulated diagonal arrays, although they *are* seen in [10] arrays. This absence is surprising given the fact that only the *direction* of the bias current flow is changed. Through simulations, we show that the dynamical properties of the supercurrents and phases of the diagonal arrays are very different from those of the [10] arrays. On a giant step at $f=\frac{1}{2}$ and low rf frequencies, we argue that all the junctions in diagonal arrays behave like a single Josephson junction, thus providing an explanation for the absence of fractional giant and subharmonic Shapiro steps in diagonal arrays in this fully frustrated case. Finally, we suggest that for general $f=p/q$, it is the preservation of the symmetry of the diagonal arrays in the presence of bias current which accounts for the absence of fractional giant and subharmonic Shapiro steps in these arrays.

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