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# Subharmonic Shapiro steps in Josephson-junction arrays

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Subharmonic Shapiro step structure has been observed in the current-voltage characteristics of 300×300 Nb-Au-Nb superconducting arrays. This subharmonic structure does not appear in the current-voltage characteristics of individual junctions, but appears to reflect a unique dynamical mode of the coupled system. A qualitative model is proposed that attributes the structure to the dynamical response of quantized loops of flux that are created by the antisymmetric self-magnetic field of the array bias current. Application of an external asymmetric magnetic field modifies the structure, which lends strong support to our model.

This paper reports the observation of subharmonic Shapiro step structure in the current-voltage ($I$-$V$) characteristics of superconducting arrays of Nb-Au-Nb proximity-effect junctions. This subharmonic structure, which appears to be a unique feature of arrays and is not present in individual junctions, is distinct from both the zero magnetic field "giant" Shapiro steps and the nonzero-field "fractional" giant steps recently reported by Benz *et al.*[1] We attribute the subharmonic Shapiro structure to dynamical behavior of an ordered vortex lattice which is nucleated at opposite edges of the array by the antisymmetric self-field of the dc bias current.

It has long been known that a Josephson junction (or proximity-effect junction) subjected to rf currents of frequency $\nu$ will display steplike structure (Shapiro steps[2]) on its dc $I$-$V$ characteristics at voltage intervals of $h\nu/2e$. These Shapiro steps are accounted for by the resistively shunted Josephson-junction (RSJ) model of McCumber,[3] and are a consequence of the intrinsic sinusoidal current-phase relationship of the Josephson effect. Very recently, Benz *et al.* observed Shapiro steps in $N\times N$ ($N=1000$) square arrays of Nb-Cu-Nb junctions with voltage spacing of $Nh\nu/2e$, indicating a collective mode in which the individual junctions were tending to phase lock to the applied rf current. Furthermore, Benz *et al.* found that a weak perpendicular magnetic field induced additional fractional Shapiro steps onto the $I$-$V$ characteristics at voltages given by

$$V_n=n[Nh\nu/q2e],\quad n=0,1,2,\ldots, \tag{1}$$

where $q$ is related to $f$, the average number of flux quanta per unit cell of the array, according to $f=p/q$, where $p$ and $q$ are integers.

Benz *et al.* proposed a phenomenological model for their observations based on the motion of the field-induced vortices, which are believed to arrange themselves into a superlattice that is commensurate with the underlying array lattice. This model was confirmed in subsequent calculations by Lee, Stroud, and Chung[4] and by Free *et al.*[5] who numerically solved the coupled RSJ equations for a network of proximity-coupled junctions in the limit of zero capacitance and zero self-inductance. These simulations produced theoretical $I$-$V$ characteristics which are in general accord with both the work of Benz *et al.* and this work.

Single Josephson junctions do not display subharmonic Shapiro steps in zero magnetic field so long as the junction current-phase relationship is purely sinusoidal. It is known,[6] however, that nonsinusoidal behavior can give rise to subharmonic structure of the form $V_{nm}=(n/m)h\nu/2e$, where $m$ is an integer greater than one. For arrays, Halsey has suggested[7] that subharmonic structure might be observed at both zero and nonzero magnetic field even though individual junctions display no such behavior. However, the simulations do not corroborate this prediction for zero field, although they do suggest the possibility of subharmonic structure in certain circumstances in nonzero field.[8] As described below, we observe subharmonic structure at *both* zero and nonzero field in our arrays, provided the amplitude of the rf current is large enough. Our results do not appear to result from the mechanism proposed by Halsey, however, but rather from the creation of current-induced vortices at the sample edges.

Our sample consisted of an array of 300×300 Nb crosses deposited in a square lattice configuration onto a 1500-Å thick Au film ($N=299$). The Nb film thickness was nominally 2000 Å, with the arms of the crosses 1.2 $\mu$m wide and the spacing between arms (the junction spacing) 0.4 $\mu$m. The lattice constant of the array was 10 $\mu$m. The array showed a Kosterlitz-Thouless transition temperature (the temperature below which the resistance vanished) of $T_{KT}=7.26$ K. Measurements on a single Nb-Au-Nb junction at $T=1.6$ K showed it to have the following properties: critical current $i_c=24$ $\mu$A, normal-state resistance $r_n=10$ m$\Omega$, characteristic frequency $\nu_c\equiv 2er_ni_c/h=116$ MHz, and damping factor $\beta_c\equiv 2\pi r_n C\nu_c$

 

$=2.7\times10^{-6}$. Comprehensive measurements taken on an individual junction showed strong, well-defined Shapiro steps but no evidence of subharmonic structure.

Figure 1 shows the rf power dependence of the amplitude of the $n=0$ and $n=1$ Shapiro step for a single junction for $\nu_{rf}=104$ MHz. The solid line is a fit of the RSJ model to the data using the measured junction parameters and shows that the individual junctions in our array are extremely well behaved with nearly prototypical Josephson characteristics.

Figure 2 shows representative data for the array at $T=4.2$ K ($I_c=1.2$ mA and $R_n=23.3$ m$\Omega$) in zero external magnetic field. The curve plots the dynamic resistance $dV/dI$ of the entire array as a function of the normalized voltage $v_n=2eV/Nh\nu$ for $\nu_{rf}=90$ MHz, and $I_{rf}=3I_c$. The data clearly show giant Shapiro steps at $v_n=1$, 2, and 3. Pronounced steps were observed to $v_n=6$ and $\nu_{rf}=500$ MHz; larger values of $v_n$ and $\nu_{rf}$ could not be observed because of instrument limitations. The curve of Fig. 2 also shows subharmonic structure of $v_n=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$. The strengths of the subharmonic peaks show an almost identical dependence on the amplitude of the rf bias current as the integer giant Shapiro steps. Qualitatively, the peaks were most pronounced when $I_{rf}\cong 1-5$ mA, comparable to the array dc critical current $I_c\cong 1.2$ mA. The peaks rapidly disappear at higher or lower rf currents, and over the current range of our rf signal generator the $v_n=\frac{1}{2}$ peak was modulated periodically by the rf current (in form similar to that shown for a single junction in Fig. 1). The maximums in the strength of the subharmonic peaks (i.e., $v_n=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$) occurred at progressively higher values of $I_{rf}$. The subharmonic peaks have a temperature dependence very similar to that of the integer steps; both increase uniformly in amplitude with decreasing temperature, with the integer steps forming before the subharmonic structure appears. No abrupt appearance of the subharmonic peaks was observed. Detailed investigations into the rf current and temperature dependence of the integer, fractional, and subharmonic giant Shapiro steps are in progress.

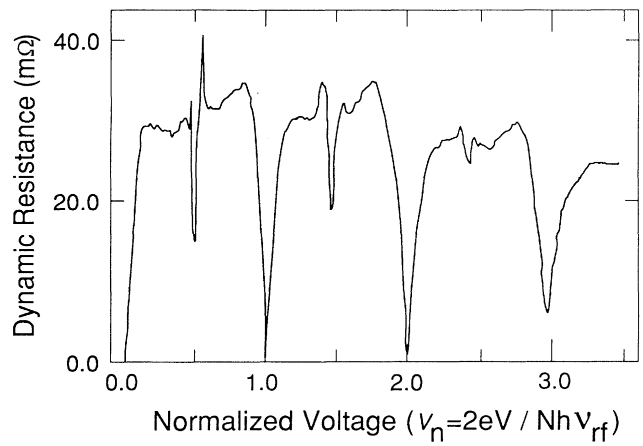


FIG. 2. The dynamic resistance ($dV/dI$) of an array as a function of the normalized voltage $v_n=2eV/Nh\nu_{rf}$, for $\nu_{rf}=90$ MHz, $I_{rf}=3I_c$, and $T=4.2$ K, in zero external magnetic field. The data clearly show giant Shapiro steps at $v_n=1,2,3$. The curve also shows subharmonic structure at $v_n=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$.

Figure 3 shows the effects on $dV/dI$ of a weak external magnetic field applied perpendicular to the plane of the array. For these data, the external field was chosen so that $f=\frac{1}{3}$, resulting in discernable fractional steps at $v_n=\frac{1}{3}$ and $\frac{2}{3}$. Analogous steps have been observed at other fields, including $f=\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$. Note also that the figure shows subharmonic structure at $v_n=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$, which appears to coexist on the $I$-$V$ characteristics with the field-induced fractional structure. The magnitude of the external magnetic field does influence the strength of the subharmonic structure, however, as shown in Fig. 4, which plots the amplitude of the $dV/dI(v_n=\frac{1}{2})$ structure as a function of $f$. For this curve, the rf bias current was chosen to enhance the strength of $dV/dI(v_n=\frac{1}{2})$ at $f=0$. A lower value of rf current maximized the feature at $f=\frac{1}{2}$, but produced a

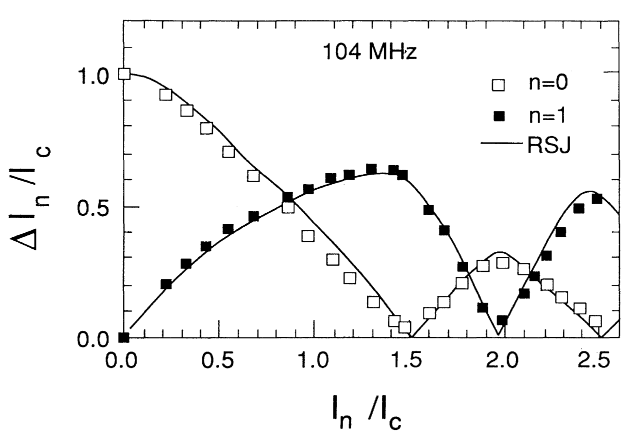


FIG. 1. The rf power dependence of the amplitude of the $n=0$ and $n=1$ Shapiro step for a single junction for $\nu_{rf}=104$ MHz. The solid line is a fit of the RSJ model to the data using the measured junction parameters given in the text.

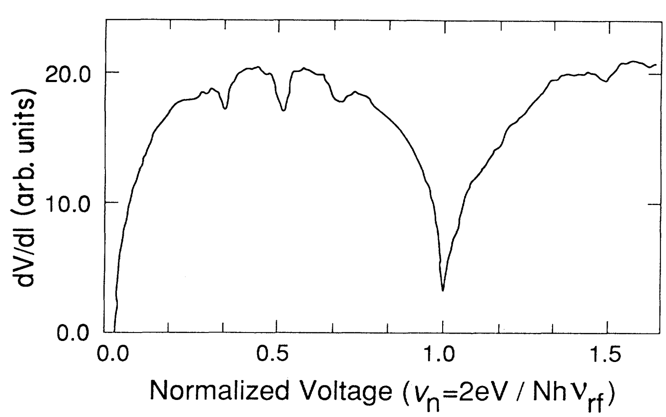


FIG. 3. $dV/dI$ of an array in a weak external magnetic field $f=\frac{1}{3}$. Fractional steps are discernable at $v_n=\frac{1}{3}$ and $\frac{2}{3}$. Note also that the figure shows subharmonic structure at $v_n=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$, which appears to coexist on the $I$-$V$ characteristics with the field-induced fractional structure.

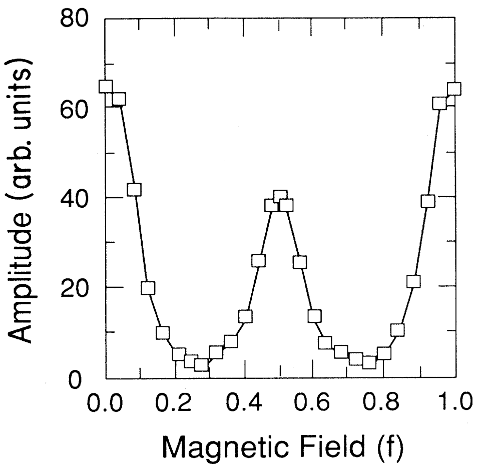


FIG. 4. Plot of the amplitude of the $dV/dI(v_n=\frac{1}{2})$ structure as a function of $f$. The RF bias current was chosen to enhance the strength of $dV/dI(v_n=\frac{1}{2})$ at $f=0$. The line is a guide to the eye.

much smaller subharmonic peak.

We believe the subharmonic (half-integer) Shapiro steps observed in our array may be qualitatively explained by the edge nucleation of vortices induced by the self-field of the array's dc bias current. Unlike the uniform magnetic field of an external electromagnet, the array self-field is strongly peaked at the transverse edges of the array and is antisymmetric with respect to the array center axis. To a good approximation, the self-field perpendicular to the plane of the array is given by

$$B=\frac{\mu_0 I}{2\pi N r}\sum_{k=0}^{N}\left(1+\frac{ka}{r}\right)^{-1}, \qquad (2)$$

where $r$ is measured from the edge of the film and $a$ is the array lattice constant. For our array, a current of $I=30$ mA produces a self-field corresponding to $f=\frac{1}{2}$ at the outer edges of the array. In our array, the subharmonic step at $\frac{1}{2}$ is maximized when the combined dc and rf currents are above 5 mA. In general, this self-field induces a nonuniform fluxoid lattice into the cells of the array which is most dense at the transverse (outer) edges and which falls to zero at the array center line. If the self-field at the edge corresponds to $f=\frac{1}{2},\frac{3}{2},\frac{5}{2},\ldots$, that region of the array is fully frustrated, with a ground state characterized by a checkerboard pattern of alternating clockwise and counterclockwise current loops. Each of these plaquette current loops encloses a $2\pi$ (or $-2\pi$) center of phase. Whether the plaquette also encloses a quantum of magnetic flux depends on the extent to which screening currents are significant, as signified by the perpendicular penetration depth $\lambda_\perp=h/(4\pi e\mu_0 i_c)$, where $i_c$ is the critical current of a single junction in the array.[9]

It is instructive to consider the two limiting cases of weak screening and strong screening. In the weak-screening limit, $\lambda_\perp \gg a$, screening currents are negligible, and both the clockwise and counterclockwise current loops contain identical flux $f$. In the strong-screening limit, by contrast, the plaquettes of one polarity each contain a magnetic-flux quantum, while neighboring plaquettes of opposite circulation are free of flux. The flux pattern on opposite sides of the array are identical, of course, except that one side supports vortices, and the other, antivortices. In intermediate cases, such as for our array, for which $\lambda_\perp \cong 22$ $\mu$m, the screening is not enough to constrain completely the lines of flux to alternate cells, resulting in cell-to-cell flux variation that is something less than a flux quantum.

The dynamical behavior of the $f=\frac{1}{2}$ state corresponds (in both the strong- and weak-screening regimes) to a current-driven back-and-forth alternation of the fluxoid patterns, with the $f=\frac{1}{2}$ Shapiro step occurring when this alternation frequency is locked to the frequency of the applied rf bias current. In the strong-screening regime, this picture has the further physical significance of a lattice of magnetic vortices marching sideways across the array. If the $f=\frac{1}{2}$ state is caused by the antisymmetric self-field, vortex-antivortex pairs will be created at opposing edges of the array and will be drawn to the center of the array where they will annihilate. The dynamical state of the subharmonic $v_n=\frac{1}{2}$ structure corresponds, we believe, to the cyclic motion of concentric quantized rings of flux that are created in phase with the rf current at the transverse outer edges of the sample. The radii of these rings are reduced in step with the rf until they ultimately vanish along the center axis of the array.[10]

This qualitative picture is supported by Fig. 5, which plots the amplitude of the $dV/dI(v_n=\frac{1}{2})$ structure as a function of the dc current, $I_a$, through a planar conducting film[11] the same size as the array and which was placed directly over the array so that their current axes were parallel. This arrangement made it possible to approximately null (or enhance) the antisymmetric dc self-field of the array by varying the direction and magnitude of the current through the conducting film. As shown in the figure, the amplitude of the subharmonic step is strongly influenced by the antisymmetric magnetic field produced by the conducting film. For this particular data, the rf

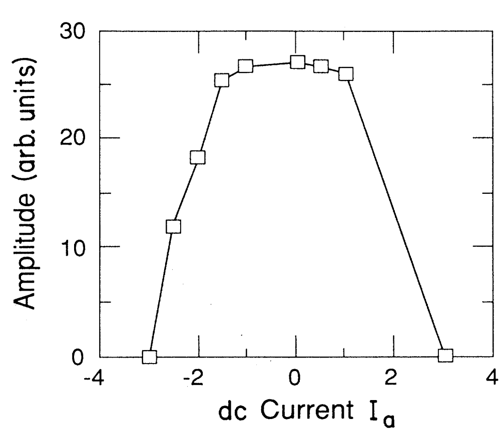


FIG. 5. The amplitude of the $dV/dI(v_n=\frac{1}{2})$ structure as a function of the dc current $I_a$ through a planar conducting film as described in the text. The amplitude of the subharmonic step is strongly decreased by the antisymmetric magnetic field produced by the conducting film. The line is a guide to the eye.

current amplitude was chosen so that the magnitude of the $dV/dI(v_n=\frac{1}{2})$ structure was maximum when $I_a=0$. When $I_{rf}$ was too small (not shown) for the subharmonic peaks to be observed, a limited range of $I_a$ values would make the subharmonic structure appear. Conversely, no value of $I_a$ would generate the subharmonic peaks if $I_{rf}$ was too large. While the origin of the subharmonic structure is not understood in detail, we interpret our ability to create or destroy the subharmonic peaks with an externally applied antisymmetric magnetic field, as indication that the self-field of the dc and rf bias currents are a factor in the dynamical behavior of the subharmonic state and must be included in a full description of this effect.

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[1]S. P. Benz, M. S. Rzchowski, M. Tinkham, and C. J. Lobb, Phys. Rev. Lett. **64**, 693 (1990); see also H. C. Lee, D. B. Mast, R. S. Newrock, L. Bortner, K. Brown, F. P. Esposito, D. C. Harris, and J. C. Garland, Physica B **165&166**, 1571 (1990).

[2]S. Shapiro, Phys. Rev. Lett. **11**, 80 (1963).

[3]D. E. McCumber, J. Appl. Phys. **39**, 3113 (1968).

[4]K. H. Lee, D. Stroud, and J. S. Chung, Phys. Rev. Lett. **64**, 962 (1990).

[5]J. U. Free, S. P. Benz, M. S. Rzchowski, M. Tinkham, C. J. Lobb, and M. Octavio, Phys. Rev. B **41**, 7267 (1990).

[6]J. R. Waldram and P. H. Wu, J. Low Temp. Phys. **47**, 363 (1982).

[7]T. Halsey, Phys. Rev. B **41**, 634 (1990).

[8]The subharmonic structure in the simulations of Lee, Stroud, and Chung (Ref. 4) is a consequence of the use of free boundary conditions. The structure does not appear when periodic boundary conditions are used [J. U. Free *et al.*, Ref. 5, and D. Stroud (private communication)].

[9]C. J. Lobb, D. W. Abraham, and M. Tinkham, Phys. Rev. B **27**, 150 (1983).

[10]This picture can be contrasted with the dynamical state of vortices created by a uniform external magnetic field; in this latter case, vortices of one polarity are nucleated at one edge of the sample and move at constant velocity across the array until they disappear at the other edge.

[11]The conducting film consisted of a 1000-Å-thick layer of gold, sputter deposited on an 8-$\mu$m-thick Kapton film.