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Quantum theory of infrared absorption in a grating-coupled two-dimensional electron gas

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A calculation of the experimentally measurable transmission coefficient for infrared light through a planar grating and the underlying two-dimensional electron gas is carried out using classical electrodynamics and quantum-mechanical linear-response theory. Si inversion layer and GaAs heterostructure systems are studied, both with and without an external magnetic field. We find the difference between the classical Drude and the quantum-mechanical results to be rather small for currently accessible grating spacings. We discuss the effect of a finite relaxation time on the transmission spectra and the higher-harmonic magnetoplasmon resonances in the presence of a magnetic field.

The study of optical properties of two-dimensional (2D) electron systems in terms of its far-infrared (FIR) transmission spectra has been an active area of research for close to twenty years.[1–10] Usually the far-infrared radiation field is coupled to the 2D electron gas through a metallic grating coupler above the electron gas, so that different (discrete) values of the probe wave vector parallel to the 2D plane can be created to excite the collective modes in the 2D electron gas which show up as structures in the transmission spectrum at characteristic frequencies. The configuration of the grating and the 2D electron gas is shown in Fig. 1. The infrared light is incident along the surface normal. The wavelength of the light is much larger than the period a of the grating, which in turn is much larger than the distance d between the grating and the electron layer. The 2D electron gas (2DEG) has a density N, an effective mass m^*, and a Drude relaxation time τ. Note that the presence of the grating enables one to create discrete 2D probe wave numbers $q \equiv G_n = 2\pi n/a$ with $n = 1,2,3,\ldots$.

In this paper, we develop a quantum-mechanical linear-response theory for calculating the FIR transmission spectra of such a grating-coupled 2D system for electronic excitation along the 2D plane (i.e., leaving out intersubband transitions). The basic electrodynamics theory for calculating the transmission spectra already exists in the literature[4,5] and will not be repeated here. The main ingredient needed to obtain the transmission spectra is the dynamical conductivity of the 2D electron gas. Our main additional contribution is a calculation of the differential transmission for the geometry of Fig. 1(a) by using the quantum-mechanical dynamical conductivity of the 2D electron gas. Earlier theories[1–5] have treated the electron-gas response semiclassically by using the Drude conductivity. We calculate the quantum-mechanical dynamical conductivity of the 2D electron gas within a linear-response formalism in the random-phase approximation (RPA). For theoretical details showing the actual (rather long and cumbersome) expression for the transmission coefficient we refer to the existing literature.[4,5] In the following we present our calculated results for Si(100) inversion layer and GaAs-$Al_xGa_{1-x}As$ heterostructure (both with and without an external magnetic field), particularly emphasizing the difference (if any) between the quantum and the (already existing) classical Drude theory. We find that for currently accessible grating wave vectors (i.e., the values of G_n for small n) quantum corrections are generally rather small because at small wave vectors (long wavelengths) $q \ll k_F$ (where k_F is the 2D Fermi wave vector) the quantum and the Drude dynamical conductivities are almost the same. In Fig. 1(b) we show our classical Drude and quantum RPA results for an experimental situation (not yet practically

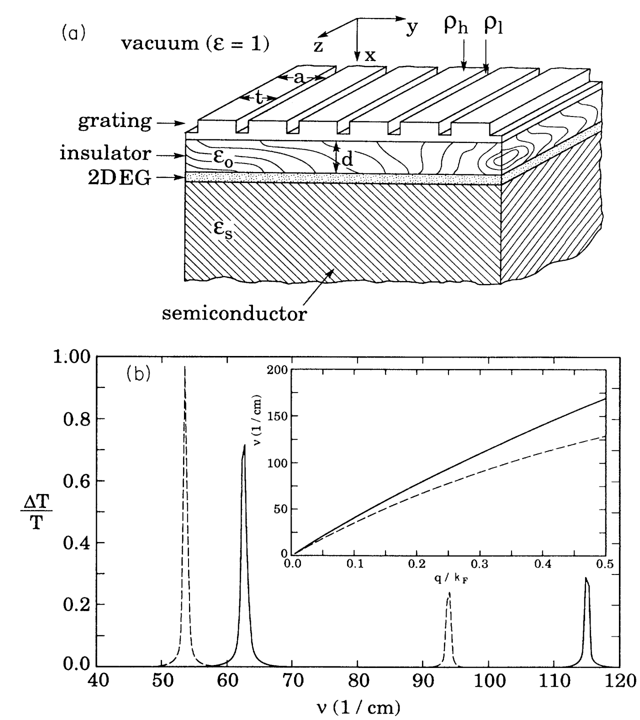

FIG. 1. (a) Qualitative sketch of the interface region showing the configuration of the 2D grating and the 2D electron gas. ρ is the resistivity. (b) Calculated $\Delta T/T$ for $G_1/k_F = 0.16$ in GaAs with $N = 4\times 10^{11}$ cm^{-2}, $\tau = 40$ ps, $a = 0.25$ μm, and $d/a = \frac{1}{20}$. Inset shows the calculated plasmon dispersion (solid lines: quantum; dashed lines: classical).

realizable) where the two differ markedly. In the main figure 1(b) we show our calculated differential transmission for FIR experiments where in an inset we show the corresponding plasmon dispersion (including screening by the gate) up to the wave vector where the plasmon enters the electron-hole continuum and consequently becomes damped. It is clear that for larger wave vectors (with narrower gratings) quantum nonlocal corrections are substantial. (In fact, for still larger q values, the quantum peak in $\Delta T/T$ essentially disappears due to Landau damping with the classical peak remaining qualitatively unchanged.) In the rest of the paper we discuss the currently experimentally accessible situations where one is probing essentially $q/k_F \ll 1$ region and quantum corrections are consequently small.

The curve of the calculated zero-field relative change of transmission

$$\Delta T/T \left(= \frac{T\,(\text{without 2DEG}) - T\,(\text{with 2DEG})}{T\,(\text{without 2DEG})} \right)$$

versus the wave number of the incident infrared light is shown in Fig. 2 for both the quantum and the Drude theory. We investigate two cases in which the semiconductor is Si or GaAs and the insulator is SiO_2 or $Al_xGa_{1-x}As$, respectively. The peaks in Fig. 2 appear at the plasma frequencies including screening by the gate.[1,2] There are small blueshifts for the quantum resonance frequencies compared with the classical result. Quantum correction for GaAs is somewhat larger than that for Si, but overall the Drude theory works quite well. We point out that the peaks in the calculated FIR spectra agree well with the long wavelength 2D plasmon dispersion relation[1–5] given by

$$\omega_n^2 = \omega_p^2 = (2\pi N e^2/m^*) G_n [\varepsilon_s + \varepsilon_0 \coth(G_n d)]^{-1}$$

with the quantum result showing small nonlocal corrections arising from the dynamical polarizability function.

Results for different relaxation times (τ) are shown in the insets in Fig. 2. Relaxation time has little effect on the resonance peak position, but significantly affects the resonance width. Since the relaxation time is related to the mobility of the 2DEG by $\mu = e\tau/m^*$, the sharpness of the resonance peaks characterizes the mobility of the sample. Our basic conclusion from Fig. 2 is that in general quantum corrections on the FIR spectra are small, being somewhat larger for GaAs-based 2DEG than for Si inversion layer. This can be understood as a consequence of the characteristic q values associated with the external radiation being much smaller than the Fermi wave vector.

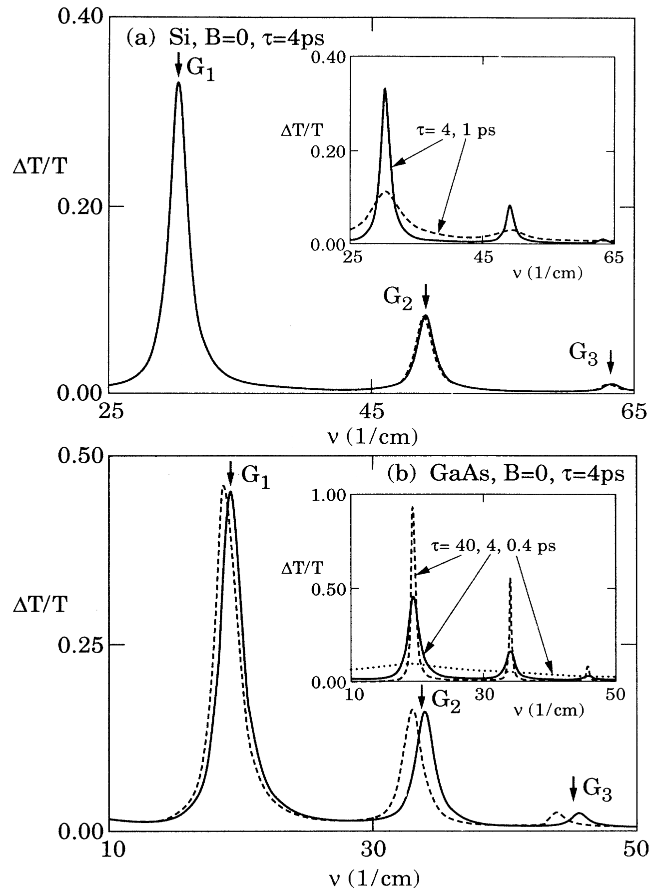

FIG. 2. The fractional change in transmission $\Delta T/T$ vs the wave number $\nu(=\omega/2\pi c)$ of infrared light in the absence of a dc magnetic field. Both quantum (solid lines) and classical (dashed lines) results are shown. Here $a = 2\ \mu$m, $d = 1000$ Å, $\rho_h/\rho_l = 1000$, $t/a = 0.25$, $\tau = 4$ ps. (a) Si: $\varepsilon_S = 11.5$, $\varepsilon_0 = 3.9$, $m^* = 0.19$ m, $N = 1.5 \times 10^{12}/\text{cm}^2$. (b) GaAs: $\varepsilon_S = 10.92$, $\varepsilon_0 = 10.92$, $m^* = 0.067$ m, $N = 4 \times 10^{11}/\text{cm}^2$. In the insets we show the effect of changing τ in the quantum theory to (a) $\tau = 1$ and 4 ps in Si and (b) $\tau = 0.4$, 4, and 40 ps in GaAs.

Next we consider the case of a strong dc magnetic field

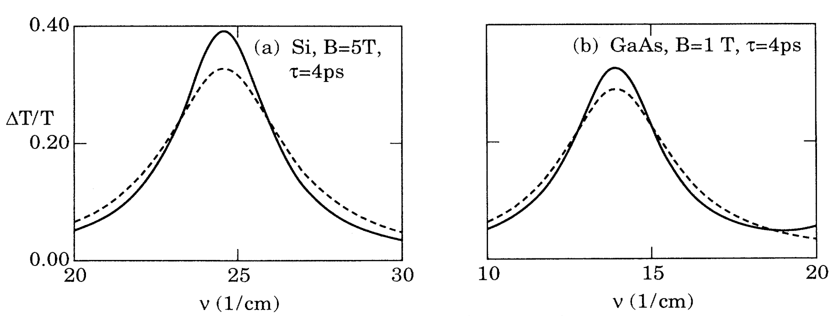

FIG. 3. The CR transmission spectra in the quantum theory with (solid line) and without (dashed line) the grating. (a) Si: $B = 5$ T; (b) GaAs: $B = 1$ T.

applied parallel to the direction of the propagation of the infrared light (i.e., normal to the 2DEG). The magnetoconductivity of the 2D electron gas can also be calculated[11,12] by the quantum-mechanical linear-response theory within RPA. In a strong magnetic field, the energy spectrum of the 2D electron gas is quantized into Landau levels. The energy spectrum is similar to that of a 1D quantum harmonic oscillator with discrete energy levels at $(n+1/2)\hbar\omega_c$ where $\omega_c=eB/m^*c$ is the cyclotron frequency.

In the presence of the magnetic field, even without a grating, one should be able to observe the cyclotron resonance (CR) in the FIR spectrum at long wavelengths. In Fig. 3 we compare our calculated CR spectra with and without the grating. We can see that the presence of the grating somewhat sharpens the CR peak. In the presence of the grating, we show in Fig. 4 the calculated relative change of transmission versus the wave number of the incident infrared light in the presence of a magnetic field. Again we investigate the cases for both Si and GaAs and compare quantum results with the Drude results.

Generally, in the presence of the B field, the first resonance peak corresponds to the long wavelength CR. Two types of resonances show up in the calculated spectra. The large peaks at wave vectors G_n with $n=1,2$ (and, sometimes, 3) are the magnetoplasmon excitations of the system which are shifted from the cyclotron frequency due to the plasma shift (the $n=0$ magnetoplasmon mode is the CR mode of Figs. 3 and 4). There are additional smaller peaks around $2\omega_c$ which are the higher harmonic magnetoplasmon excitations, allowed in the quantum theory at finite wave vectors.[6,10] The higher harmonics (e.g., $\omega\simeq 2\omega_c$), however, do not show up as classical Drude resonances. The reason is that $2\omega_c$ is a pole for the quantum magnetoconductivity of the 2D electron gas, but not for the semiclassical Drude magnetoconductivity. For the high harmonics, the inter-Landau-level transition rule $\Delta n=\pm 1$ is broken because of the dynamical spatial modulation of the charge density of the plasmon excitation. We note that for the main CR the quantum and the classical results are almost the same. For even higher wave numbers (larger values of G_n), we would find quantum resonances at $3\omega_c$, $4\omega_c,\ldots,$ but the resonance strengths are going to be negligibly weak. We investigate the GaAs case for three different magnetic fields. It turns out that lower magnetic fields give larger quantum corrections, which follows from the fact that quantum corrections go as $(ql)^2$ where $l=(\hbar c/eB)^{1/2}$ is the magnetic length which increases with decrease of B. We should mention that the main magnetoplasmon peaks in the calculated FIR spectra are described accurately by the plasma-shifted CR formula $\omega_n^2=\omega_c^2+\omega_p^2$ where ω_p describes the corresponding peaks in Fig. 2 at $B=0$.

In this paper we carry out the quantum-mechanical RPA calculation for the measurable transmission coefficient for far-infrared light through the simplest planar grating and 2D electron-gas system. The result turns out to agree well with classical Drude calculations in the experimentally accessible wave-vector regime. One could investigate higher-order quantum corrections by including vertex corrections[13] in the quantum conductivity. Howev-

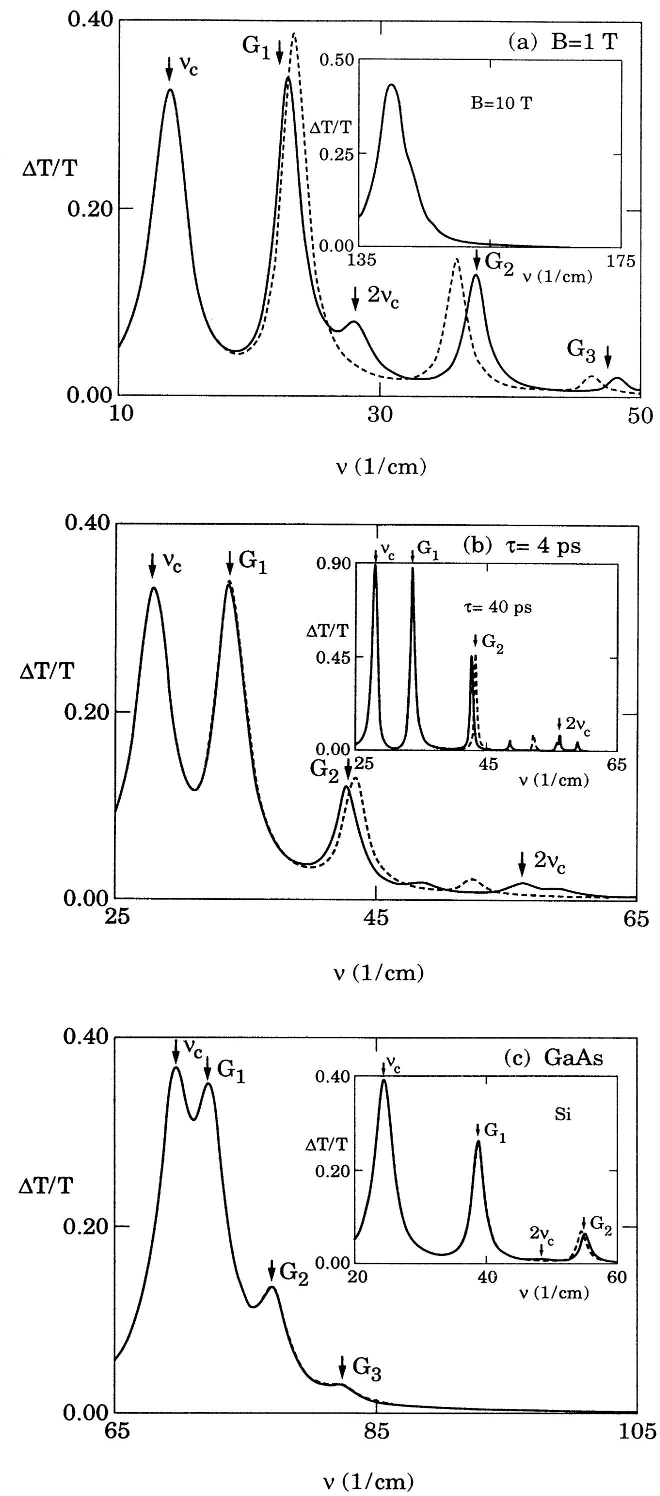

FIG. 4. Calculated $\Delta T/T$ vs the wave number in the presence of a dc magnetic field (solid lines for quantum, dashed lines for classical). All the parameters are the same as in Fig. 2. (a) GaAs: $B=1$ T, $\tau=4$ ps (inset shows $B=10$ T); (b) GaAs: $B=2$ T, $\tau=4$ ps (inset shows $\tau=40$ ps); (c) GaAs: $B=5$ T, $\tau=4$ ps (inset shows Si: $B=5$ T, $\tau=4$ ps). For GaAs at $B=5$ and 10 T, classical and quantum results are essentially identical for these values of wave vector.

er, their effects are expected to be negligibly small because the accessible wave vectors are small. Our formalism is also readily applicable to 1D structures using the known[14] quantum response functions for the 1DEG. Our main result is the verification of the semiclassical Drude theoretical treatment of the FIR spectroscopy which has been in good agreement[1-10] with available experimental results. Quantum corrections are generally small, being somewhat larger in GaAs systems than in Si. We also obtain the higher harmonic magnetoplasmon resonances which are weak, but experimentally observable[6,10] in FIR spectroscopy, particularly at lower magnetic fields in GaAs structures.

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