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Anomalous behavior of torque at high angles in high-temperature superconductors

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The deviations of experimental results for torque in $YBa_2Cu_3O_7$ at high angles from predictions of the anisotropic Ginzburg-Landau model are explained in the framework of the Lowrence-Doniach model taking into account the two-dimensional pancake structure of the normal cores of the vortices. Measurements of the torque at high angles allow us to obtain the core energy of the vortex and the parameter that characterizes the interlayer coupling.

Magnetization or torque measurements gives important and accurate information on superconducting anisotropy. Such experiments have been performed to study high-temperature superconductors.[1-4] The longitudinal M_L and transverse M_T components of the magnetization, as well as torque, usually are studied as functions of temperature T and the angle θ between the magnetic field H and the anisotropy axis c (perpendicular to the layers). To evaluate the anisotropy ratio of the effective masses $\gamma^2=m_\perp/m_\parallel$, the Ginzburg-Landau (GL) or London model for anisotropic superconductors[5] was used,[6] and it gives a surprisingly precise description for the magnetization, except for the narrow interval of high angles at low enough temperatures in $YBa_2Cu_3O_7$.[3]

The London model for an anisotropic superconductor predicts the following dependence of torque τ on θ in a system with a dense vortex lattice:

$$\tau(\theta)=\frac{A\gamma\sin\theta\cos\theta}{\varepsilon(\theta)}\ln\left[\frac{\gamma\eta H_{c2,\perp}}{H\varepsilon(\theta)}\right],$$
$$\varepsilon(\theta)=(\sin^2\theta+\gamma^2\cos^2\theta)^{1/2},\quad \gamma=\lambda_\perp/\lambda_\parallel\,, \tag{1}$$

where $\lambda_\parallel$ and $\lambda_\perp$ are parallel and perpendicular London penetration depths, A is a parameter independent of angle, η is numerical parameter of order of unity, and $H_{c2,\perp}$ is a perpendicular (to the layers) upper critical field. The magnetic field H is supposed to be much larger than the lower critical field $H_{c1}(\theta)$. Actually, $\eta H_{c2,\perp}$ and γ are fitting parameters.

The expression (1) follows from the minimization of the Gibbs energy for a vortex lattice in the presence of an external field:

$$G(B,\phi)=\frac{B^2}{8\pi}+\frac{\Phi_0\beta\gamma^{-1}}{(4\pi\lambda_\parallel)^2}\varepsilon(\phi)\ln\left[\frac{\gamma\eta H_{c2,\perp}}{B\varepsilon(\phi)}\right]-\frac{BH}{4\pi}\cos(\phi-\theta)\,, \tag{2}$$

where B is the induction, Φ_0 is the quantum flux, and ϕ is the angle between the vortex lines and the c axis. Parameters B and ϕ should be obtained by minimization of the Gibbs energy with respect to these values at given H and θ. As was mentioned above, the magnetic field is supposed to be much larger than the lower critical field, and under such a condition $B\approx H$. The torque is proportional to $BH\sin(\phi-\theta)/4\pi$, where B and ϕ are equilibrium values. The logarithmic factor comes from integration of the magnetic energy of the vortices over space. In the Fourier representation, the summation gives a logarithmic function of the squared momentum with the lower limit of summation Φ_0/B and the upper limit given by the inverse cross section of the normal core $\xi_\parallel^2\varepsilon(\theta)/\lambda$. Here, $\xi_\parallel$ is the parallel superconducting correlation length.

The deviations from London-model predictions (1) were observed in $YBa_2Cu_3O_7$ ($T_c=90.5$ K, $\gamma\approx 8$) in the interval of angles $\theta>88°$ at temperatures below 80 K, while such deviations were absent at higher temperatures.[3] According to the London theory, the torque should vanish proportionally to $\cos\theta$ as θ approaches $\pi/2$. Such behavior was observed above 80 K, but at 75 K and very high angles up to 89.75° the torque value was remarkably different from zero and the normalized torque $\tau(\theta)/\tau_{\max}$ increases with H in this interval of angles. In $Bi_2Sr_2CaCu_2O_8$ ($T_c=85$ K) no deviations were found at $T_c>77.5$ K.[2]

The validity of the London theory for highly anisotropic layered compounds is limited by the condition $\xi_\perp(T)\gg d$, where $\xi_\perp$ is the perpendicular correlation length and d is the interlayer spacing in the compounds with one layer in the unit cell or the distance between the neighboring layers with weakest coupling in the more complicated lattice. If $\xi_\perp(T)\leq d$, the Lowrence-Doniach (LD) model[7] should be used, which takes into account the Josephson character of interlayer currents. The parameter that characterizes such crossovers is $r=2\xi_\perp^2(0)/d^2$. If $r\ll 1$, then at temperatures $T<T_r=(1-r)T_c$ the London theory can be invalid. In $YBa_2Cu_3O_7$ with anisotropy $\gamma\approx 8$ the parameter r can be estimated as 0.1 by use of the expression $r=2\xi_\parallel^2(0)/d^2\gamma^2$ at $\xi_\parallel(0)=16$ Å and $d=8$ Å for the distance between CuO_2 planes separated by a plane with CuO chains. In such a model the crossover temperature T_r is approximately 80 K, and below T_r the deviations from the results of the London model can appear at high angles. In fact, at such an orientation of the vortices the interlayer currents are important, and they are restricted by some critical value due to the Josephson coupling of the layers, while such a restriction is absent in the London model.

To make the necessary modification of London results for a vortex structure due to the Josephson nature of the interlayer coupling, it is useful to understand where (in

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real space) the interlayer currents $j_\perp(\mathbf{r})$ exceed the Josephson critical value $j_c = c\Phi_0/8\pi^2 d\lambda_\perp^2$. The current $j_\perp(\mathbf{r})$ as a function of distance from the core center may be calculated in the framework of the London anisotropic model using, for example, the results from Ref. 8. The answer is that the inequality $j_\perp \ll j_c$ is violated at distances of order d from the center of the vortex in the direction perpendicular to the layers and at distances γd in a parallel direction at high angles between the vortex line and the c axis. The same result was obtained also for a strictly parallel orientation of the vortex along the layers.[9,10] So for magnetic fields H far enough from H_{c2} the difference between the anisotropic London model and the LD model for the vortex structure is in the core only.

The structures of the cores in the London and LD models are different. In the anisotropic London model the normal core is a tube with cross section $\xi_\parallel^2 \varepsilon(\phi)/\lambda$, as was mentioned above in connection with expression (2). In the LD model at $T > T_r$ the same picture of the core as a normal tube is also valid because $\xi_\perp(T) \gg d$ here, and the energy of the interlayer coupling is of the same order of magnitude as the intralayer condensation energy. However, below T_r the normal core of the vortex consists of two-dimensional (2D) isotropic pancakelike cores with diameters of order of $\xi_\parallel$ arranged along the vortex "line" because weak Josephson currents cannot significantly change the superconducting ordering inside the layers (the energy of interlayer coupling is much less than intralayer condensation energy at temperatures below T_r). Now the energy of each pancake core is $(\Phi_0/4\pi\lambda_\parallel)^2\alpha$, where α is a numerical coefficient of the order unity. As a result, the core energy of the vortex lattice in the LD model is given by the expression

$$E_c = \frac{\Phi_0 B}{(4\pi\lambda_\parallel)^2}\alpha(\cos^2\phi + \gamma^{-2}\sin^2\phi)^{1/2}, \quad T > T_r , \tag{3}$$

$$E_c = \frac{\Phi_0 B}{(4\pi\lambda_\parallel)^2}\alpha\cos\phi, \quad T < T_r . \tag{4}$$

The crossover from the London dependence (3) to the LD dependence (4) takes place near the temperature T_r. The expression (4) is valid if $\cos\phi > d/L$, where L is the size of the sample along the vortex lines; otherwise $E_c = 0$ because the normal core is absent at such orientations.[9,11] Practically, (4) is valid if $\phi \neq 0$.

Now the energy of the normal cores given by expressions (3) and (4) should be added to (2) to obtain the equilibrium values of ϕ and B by minimization of the Gibbs energy. In the London model or in the Ginzburg-Landau region of the LD model ($T > T_r$) accounting for the core energy is unimportant because it changes the fitting parameter η only. However, addition of (4) and (2) modifies the behavior of the system at high angles significantly because $\partial E_c/\partial\phi$ is nonzero in the limit $\phi \to \pi/2$, while in the London model the corresponding term vanishes in this limit. For the angles $\theta > \theta_p$ the value $\phi \leq \pi/2$, where

$$\cos\theta_p = \frac{\Phi_0 \alpha}{4\pi\lambda_\parallel^2 H} . \tag{5}$$

In the interval of angles $\theta \geq \theta_p$ the values of torque is

$$\tau(\theta) = A\left(\frac{\gamma\sin\theta\cos\theta}{\varepsilon(\theta)}\ln\frac{H_0}{H} + \alpha\right) , \tag{6}$$

where $H_0 = \eta\Phi_0\gamma/2\pi\xi_\parallel^2\varepsilon(\theta)$ if $\xi_\parallel\cos\theta > d$ and $H_0 \approx \eta\Phi_0/2\pi d\xi_\parallel$ if $\xi_\parallel < d$. According to (6) the deviations from the result (1) given by the London model start to be noticeable at angles $\theta \geq \tilde\theta$, where $\cos\tilde\theta = \alpha/\gamma\ln(H_0/H)$, and $\tilde\theta$ is the angle where the torque is two times larger than that given by the GL model. So in the interval $\tilde\theta < \theta \leq \theta_p$ the normalized value of torque $\tau/\tau_{\max}$ is $[\gamma\cos\theta + \alpha/\ln(H_0/H)]$, where $\tau_{\max}$ is the maximal value of the torque with respect to θ.

At angles $\theta_p \leq \theta \leq \pi/2$ the vortices are locked in the orientation parallel to the layers and $E_c = 0$. The value of torque is proportional to $BH\cos\theta/4\pi$ here and is given by the expression

$$\tau(\theta) = A\frac{4\pi\lambda_\parallel^2 H}{\Phi_0}\cos\theta . \tag{7}$$

It is this interval of angles where $\tau(\theta) \to 0$ as $\theta \to \pi/2$.

The behavior of the torque at high angles and experimental data[3] for $YBa_2Cu_3O_7$ is shown in Fig. 1. The proposed model explains all observed features of torque behavior in $YBa_2Cu_3O_7$ at high angles.

(1) The appearance of deviations below the crossover temperature $T_r = 80$ K, which corresponds to the value $r \approx 0.1$.

(2) The correct order of angles where deviations were observed as well as amplitude of deviations for $T = 75$ K and $H = 1$ T. In Ref. 3 the dependence $\tau(\theta)/\tau_{\max} = 9.12(\cos\theta + 0.011)$ was obtained at $\theta > 88°$, which gives $\alpha/\gamma\ln(H_0/H) = 0.011$. Thus $\alpha \approx 0.4$ at $\ln(H_0/H) \approx 4.6$. Theoretical prediction is $\alpha \approx 0.5$ near T_c, according to the variational calculations in the framework of the GL theory.[11] The data[3] show the remarkable increase of α as the temperature lowers below 75 K.

(3) The growth of deviations with H resulting from the decrease of logarithmic factor in (6).

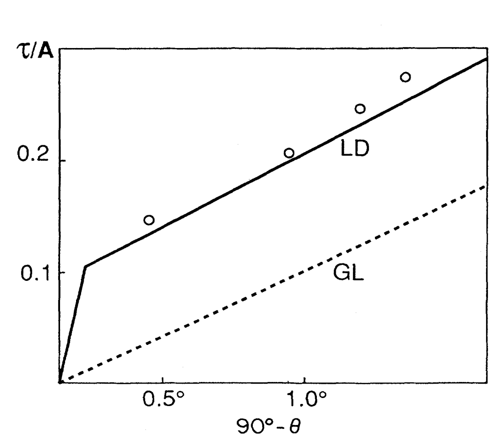

FIG. 1. The dependence of torque τ/A on the angle θ between the magnetic field **H** and the c axis at high angles in the Lowrence-Doniach model (solid curve) and in the GL model (dashed line). The experimental data (Ref. 3) for $\tau/\tau_{\max}$ in $YBa_2Cu_3O_7$ at 75 K are also shown.

The value $r=0.1$ obtained from $T_r=80$ K gives $\xi_\parallel(0)/d \approx 2$ and $d \approx 8$ Å confirming that the weakest interlayer coupling is in between CuO_2 planes separated by the plane with CuO chains. Therefore one can assume that chain layers are weakly superconducting at least down to 75 K. Otherwise the transfer integral between CuO_2 and the chain layers should be smaller than the transfer integral in between neighboring CuO_2 layers. This seems to be unlikely due to the presence of the bridge oxygen atoms, which enhance the coupling of CuO_2 and chain planes.

At the angle θ_p the second-order phase transition occurs between the phases with inclined and parallel vortices as the temperature (or external field) changes. The critical lines $T_p(\theta)$ at given H or $H_p(\theta)$ at given T of such an orientational lock-in transition are described by Eq. (5). However, the specific-heat jump at T_p is very small; it is smaller than the jump at T_c by a factor of $(\xi_\parallel/\lambda_\parallel)^2$. The lock-in effect in the orientation of vortices slightly changes the dependence of the lower critical field H_{c1} on θ in comparison with the London model, where such an effect is absent. In the anisotropic London model the orientation of the vortex at H_{c1} is given by the expression $\tan\phi=\gamma^{-2}\tan\theta$ at all θ; see Ref. 8. In the LD model the Gibbs energy given by the sum of the expressions (2) and (4) should be used to determine the orientation of vortex at lower critical field and the value of $H_{c1}(\theta)$. The result is that below T_r the vortex is parallel to the layers ($\phi=\pi/2$) at $H_{c1}(\theta)$ for the angles $\theta>\theta_p$, where

$$\tan\theta_p=\frac{1}{\gamma\alpha}\ln\frac{\lambda_\parallel}{d}\,. \tag{8}$$

At $\theta<\theta_p$ the result[8] for the lower critical field is valid, independent of T. So for compounds with $r \ll 1$, the lower critical field below T_r is given by the expressions

$$H_{c1}(\theta)=\frac{\Phi_0}{4\pi\lambda_\parallel^2\varepsilon(\theta)}\ln\frac{\lambda_\parallel}{\xi_\parallel},\quad \theta<\theta_p\,, \tag{9}$$

$$H_{c1}(\theta)=\frac{\Phi_0}{4\pi\lambda_\parallel\lambda_\perp\sin\theta}\ln\frac{\lambda_\parallel}{d},\quad \theta\geq\theta_p\,, \tag{10}$$

while above T_r the expression (9) is valid at any angles.

At temperatures below T_r, the interval of angles θ, where vortices are oriented parallel to the layers, narrows as H increases and at $H \gg H_{c1}(\theta)$ this interval is given by Eq. (5). The phase diagram in the plane (H,θ) at $T<T_r$ is shown in Fig. 2. The line $H_{c1}(\theta)$ separates the Meissner and vortex state, while the line $H_p(\theta)$, given by expression (5) at high fields, separates the state of inclined vortices with normal cores and vortices parallel to the layers without normal cores.

It is worth noting that the magnetic field **H** was used in all expressions obtained above. Usually the applied field $\mathbf{H}_a$ is fixed, so it is useful to obtain the dependence of torque and the critical angle θ_p on $\mathbf{H}_a$. For an ellipsoidal sample with principal axes along the crystal axes, H can be replaced by H_a in expression (6), and in (5) and (7) H should be replaced by $H_a/(1-N_c)$, where N_c is the demagnetization factor along the c axis. Thus for an infinite slab along the (a,b) plane ($N_c=1$), expression (6)

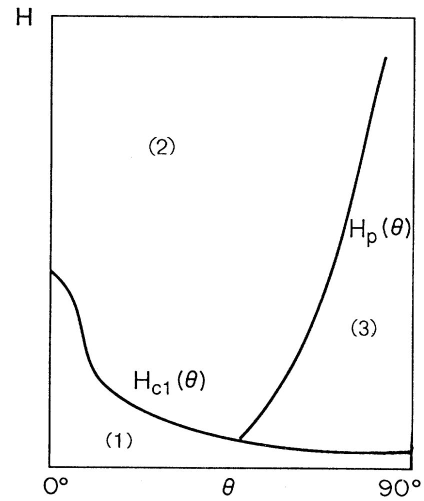

FIG. 2. The phase diagram of (1) the Meissner state, (2) the state with inclined vortices, and (3) the state with vortices oriented parallel to the layers in the (H,θ) plane at $T<T_r$ in the compound with $r \ll 1$.

is valid at all angles because $\theta_p=\pi/2$ in such geometry.

The lock-in transition for the vortex lattice in layered compounds also was obtained in the framework of the continuous model with the core energy varying in the c direction and the London penetration depths and anisotropy independent of coordinates.[12,13] Such a model differs from the LD model and results in a different angular dependence of the torque at high angles [sharp peak instead of the smooth dependence (6)].[13]

In conclusion, the torque (or magnetization) measurements in layered compounds at high angles allow the following.

(1) We may check the validity of the LD model and estimate the parameter r from the crossover temperature T_r. Knowing r and γ, as well as $\xi_\parallel(0)$ (from $H_{c2,\perp}$ measurements), one can obtain the distance d between the layers with weakest interlayer coupling in the compounds with several layers in the unit cell.

(2) We may obtain from experimental data the numerical coefficient $\alpha(T)$, which characterizes the core energy inside the layers. This parameter depends on the microscopic model at low temperatures and thus gives some information on the mechanism of superconducting pairing.

The LD model predicts the lock-in transition at the angle θ_p given by the expression (5) where the right-hand side should be multiplied by the factor $(1-N_c)$ if H is replaced by the applied field H_a. Such a transition is absent in the standard anisotropic 3D London model. The transition can be observed as the break in the angular dependence of the torque at the angle θ_p at temperatures below the crossover temperature T_r; see Fig. 1. The angle θ_p is approximately 0.1° below 90° in the cubic sample of $YBa_2Cu_3O_7$ at the temperature $T=75$ K and in the applied field $H=1$ T.

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