Spin precession and weak localization

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Spin effects play an important role in weak-localization theory. In addition to the well-studied influence of spin-orbit and spin-flip scattering, electron-spin precession can be important when a magnetic field is present. A simple derivation of the spin-precession effect is given, which reproduces the results of impurity-diagram calculations by Maekawa and Fukuyama. Spin precession becomes important when the precession rate ω_p satisfies $\omega_p > |1/\tau_{\rm so} - 1/\tau_{\rm sf}|$, where $1/\tau_{\rm so}$ and $1/\tau_{\rm sf}$ are the spin-orbit and spin-flip scattering rates. In this case, the ac conductivity will show a resonance at $\omega = \omega_p$. The resonant line shapes are examined for the ac magnetoconductivity in quasi-one- and quasi-two-dimensions, and the feasibility of experiments is considered.

I. INTRODUCTION

The now-standard theory of weak localization, which deals with quantum corrections to the conductivity of weakly disordered systems, has explained a large body of data, particularly low-field magnetoresistance measurements.[1] An important role in the theory is played by spin-orbit and spin-flip (magnetic impurity) scattering, the effects of which were discussed in the works of Hikami, Larkin, and Nagaoka[2] and Maekawa and Fukuyama.[3] In magnetoresistance measurements the precession of the electron spin in the applied magnetic field can also be important.[3] However, the precession effect has only rarely been included in descriptions of data,[4] and its experimental consequences have not been fully explored. As will be discussed below, spin precession becomes important when the precession rate $\omega_p = g^* \mu_B B/\hbar$ satisfies $\omega_p > |1/\tau_{\rm so} - 1/\tau_{\rm sf}|$, and this must occur at a magnetic field that is not so large that weak localization is suppressed by orbital effects. Spin precession does not have a dramatic influence on the dc conductivity, but the ac conductivity $\sigma(\omega)$ may develop a resonant peak at $\omega = \omega_p$. The precession effect may therefore be relevant to experiments now being conducted on weak localization in disordered metals at microwave frequencies.[5]

Below I first derive the effects of spin evolution on weak localization, using a direct method related to that of Bergmann[6] and Chakravarty and Schmidt,[7] but including spin precession. I then consider the specific predictions for the longitudinal ac magnetoconductivity in quasi-one- and quasi-two-dimensions, and discuss the behavior of the resonant peak in the conductivity.

II. DERIVATION OF THE SPIN-PRECESSION EFFECT

Weak localization is a coherent backscattering effect, due to interference between pairs of diffusive paths which are related by time reversal. The classical velocity correlation function for an electron at the Fermi surface, which decays with the elastic-scattering time τ, is corrected at times $t \gg \tau$ by the coherent backscattering. The corresponding quantum correction to the conductivity is given by

$$\delta\sigma(\omega) = -4\frac{e^2}{h} D\tau \int_\tau^\infty e^{i\omega t} C_o(t;\mathbf{x},\mathbf{x})\,dt \ , \tag{1}$$

where C_o is the orbital part of the cooperon propagator, found by solving the equation

$$\left[D \left[-i\nabla - \frac{2e}{\hbar}\mathbf{A}(\mathbf{x}) \right]^2 + \frac{\partial}{\partial t} + \frac{1}{\tau_\phi} \right] C_o(t;\mathbf{x},\mathbf{x}') = \delta(t)\delta(\mathbf{x}-\mathbf{x}')/\tau \ , \tag{2}$$

where $D = \frac{1}{3} v_F^2 \tau$ is the diffusion coefficient, and v_F is the Fermi velocity.[8] When the vector potential and the inelastic dephasing rate $1/\tau_\phi$ vanish, C_o satisfies a simple diffusion equation, and the conductivity depends on the probability that the electron returns to the origin by diffusion. The dephasing rate $1/\tau_\phi$ has the effect of suppressing the contribution of those pairs of time-reversed paths which take longer than τ_ϕ to traverse, while the vector potential suppresses the contributions of paths which enclose more than a flux quantum, except in multiply connected geometries, where it can lead to Aharonov-Bohm oscillations of the conductivity.

To include spin effects, we consider an initial spin state $|s\rangle$, and a rotation operator $U(1 \to N)$ under which the spin state evolves along the forward path from the first scatterer to the Nth, and another rotation operator $U(N \to 1)$ under which it evolves along the time-reversed path. The interference between the final states will then involve the matrix element $\langle s | U^\dagger(N \to 1) U(1 \to N) | s \rangle$. Assuming that the orbital and spin parts are uncorrelated, the full cooperon is given by the product

$$\begin{aligned} &C(t;\mathbf{x},\mathbf{x}) = C_o(t;\mathbf{x},\mathbf{x}) C_s(t), \\ &C_s(t) = \tfrac{1}{2} \sum_s \overline{\langle s | U^\dagger(N \to 1) U(1 \to N) | s \rangle} \ , \end{aligned} \tag{3}$$

where the overbar denotes the average over all pairs of

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paths of duration t, and the dependence of C_s on t is through N, the number of scattering events. The equally weighted sum over initial spin states is appropriate for the limit $kT \gg \hbar\omega_p$. However, the two terms in the spin sum turn out to be equal, so that the final result is valid at all temperatures. The normalization is chosen so that $C_s = 1$ when there is no spin rotation.

The operator $U(1\to N)$ is composed of a sequence of small rotations

$$U(1\to N) - e^{-(i/2)\hat{\mathbf{b}}\cdot\sigma\omega_p\tau} e^{-(i/2)\varepsilon_N\cdot\sigma} e^{-(i/2)\delta_N\cdot\sigma}\cdots \times e^{-(i/2)\hat{\mathbf{b}}\cdot\sigma\omega_p\tau} e^{-(i/2)\varepsilon_1\cdot\sigma} e^{-(i/2)\delta_1\cdot\sigma} . \quad (4)$$

From the right, the first three factors are a spin-orbit rotation from the first scatterer, a spin-flip rotation, and the spin precession during the first time interval τ with $\hat{\mathbf{b}}$ a unit vector in the direction of the applied magnetic field. We assume that the average over paths can be handled by taking the components of the spin-orbit and spin-flip-rotation vectors to be mean-zero independent random variables satisfying

$$\overline{\delta_{ix}^2} = \overline{\delta_{iy}^2} = \overline{\delta_{iz}^2} \equiv \overline{\delta^2}, \quad \overline{\varepsilon_{ix}^2} = \overline{\varepsilon_{iy}^2} = \overline{\varepsilon_{iz}^2} \equiv \overline{\varepsilon^2} . \quad (5)$$

Each individual rotation is infinitesimal because we require that the spin-precession rate and the two spin scattering rates are much smaller than the elastic-scattering rate, as is always the case in experiments. The reversed operator involves the same set of infinitesimal rotations but in the opposite sequence, and with the sign of the spin-orbit rotations changed, because the electron velocity has changed sign:

$$U(N\to 1) = e^{+(i/2)\delta_1\cdot\sigma} e^{-(i/2)\varepsilon_1\cdot\sigma} e^{-(i/2)\hat{\mathbf{b}}\cdot\sigma\omega_p\tau}\cdots \times e^{+(i/2)\delta_N\cdot\sigma} e^{-(i/2)\varepsilon_N\cdot\sigma} e^{-(i/2)\hat{\mathbf{b}}\cdot\sigma\omega_p\tau} . \quad (6)$$

To evaluate $C_s(t)$ it is convenient to introduce a direct-product space $|ss'\rangle$, with operators subscripted by a operating on the first spin index and those subscripted by b operating on the second:

$$C_s(t) = \tfrac{1}{2}\sum_{s,s'} \overline{\langle s|U^\dagger(N\to 1)|s'\rangle\langle s'|U(1\to N)|s\rangle} = \tfrac{1}{2}\sum_{s,s'} \overline{\langle ss'|U_a^\dagger(N\to 1)U_b(1\to N)|s's\rangle} . \quad (7)$$

We then have

$$U_a^\dagger(N\to 1)U_b(1\to N) = e^{+(i/2)\hat{\mathbf{b}}\cdot(\sigma_a-\sigma_b)\omega_p\tau} e^{+(i/2)\varepsilon_N\cdot(\sigma_a-\sigma_b)} e^{-(i/2)\delta_N\cdot(\sigma_a+\sigma_b)}\cdots \times e^{+(i/2)\hat{\mathbf{b}}\cdot(\sigma_a-\sigma_b)\omega_p\tau} e^{+(i/2)\varepsilon_1\cdot(\sigma_a-\sigma_b)} e^{-(i/2)\delta_1\cdot(\sigma_a+\sigma_b)} . \quad (8)$$

Expanding each factor, averaging, and retaining only the lowest order in the infinitesimals yields N identical factors and gives the result

$$C_s(t) = \tfrac{1}{2}\sum_{ss'} \left\langle ss' \left| \exp\left[+\frac{i}{2}\hat{\mathbf{b}}\cdot(\boldsymbol{\sigma}_a-\boldsymbol{\sigma}_b)\omega_p t - \tfrac{1}{6}(\boldsymbol{\sigma}_a-\boldsymbol{\sigma}_b)^2 t/\tau_{\mathrm{sf}} - \tfrac{1}{6}(\boldsymbol{\sigma}_a+\boldsymbol{\sigma}_b)^2 t/\tau_{\mathrm{so}} \right] \right| s's \right\rangle . \quad (9)$$

Here we have used $t = N\tau$, and the spin scattering rates have been defined as

$$\frac{N}{8}\overline{\delta^2} = \frac{1}{6}\frac{t}{\tau_{\mathrm{so}}} , \quad \frac{N}{8}\overline{\varepsilon^2} = \frac{1}{6}\frac{t}{\tau_{\mathrm{sf}}} , \quad (10)$$

with the numerical factors chosen to agree with previous definitions in the literature.[3]

Equation (9) is easily evaluated if $\omega_p = 0$, for then the spin operator is diagonal in the coupled basis $|j\,m\rangle$,

$$(\boldsymbol{\sigma}_a+\boldsymbol{\sigma}_b)^2|jm\rangle = 4j(j+1)|jm\rangle , \quad (\boldsymbol{\sigma}_a-\boldsymbol{\sigma}_b)^2|jm\rangle = [12-4j(j+1)]|jm\rangle , \quad (11)$$

and we find a standard result

$$C_s(t) = \frac{3}{2}\exp\left[-\left[\frac{4}{3}\frac{1}{\tau_{\mathrm{so}}} + \frac{2}{3}\frac{1}{\tau_{\mathrm{sf}}} \right] t \right] - \tfrac{1}{2}e^{-(2/\tau_{\mathrm{sf}})t} . \quad (12)$$

The first term comes from the triplet ($j = 1$) diagonal matrix elements, and the second term from the singlet.

When the precession term is retained, evaluating Eq. (9) is more involved because the argument of the exponential is not diagonal in the $|jm\rangle$ basis. However, it may of course be diagonalized by a similarity transformation, and then exponentiated. After some algebra we find the result

$$C_s(t)=\exp\left[-\left[\frac{4}{3}\frac{1}{\tau_{so}}+\frac{2}{3}\frac{1}{\tau_{sf}}\right]t\right]+\frac{1}{2}\frac{1}{\sqrt{1-\gamma}}\left[\exp\left\{-\left[\frac{2}{3}\left[\frac{1}{\tau_{so}}+\frac{2}{\tau_{sf}}\right]+\frac{2}{3}\left[\frac{1}{\tau_{so}}-\frac{1}{\tau_{sf}}\right]\sqrt{1-\gamma}\right]t\right\}-\exp\left\{-\left[\frac{2}{3}\left[\frac{1}{\tau_{so}}+\frac{2}{\tau_{sf}}\right]-\frac{2}{3}\left[\frac{1}{\tau_{so}}-\frac{1}{\tau_{sf}}\right]\sqrt{1-\gamma}\right]t\right\}\right], \tag{13}$$

where

$$\gamma\equiv\left[\frac{\frac{3}{2}\omega_p}{1/\tau_{so}-1/\tau_{sf}}\right]^2. \tag{14}$$

For $\omega_p \ll |1/\tau_{so}-1/\tau_{sf}|$ we have $\gamma \ll 1$ and Eq. (13) reduces to Eq. (12), while for $\omega_p \gg |1/\tau_{so}-1/\tau_{sf}|$ the spin part of the cooperon becomes oscillatory:

$$C_s(t)=\exp\left[-\left[\frac{4}{3}\frac{1}{\tau_{so}}+\frac{2}{3}\frac{1}{\tau_{sf}}\right]t\right]-\frac{(1/\tau_{so}-1/\tau_{sf})}{\frac{3}{2}\omega_p}\times\exp\left[-\left[\frac{2}{3\tau_{so}}+\frac{4}{3\tau_{sf}}\right]t\right]\sin(\omega_p t). \tag{15}$$

According to Eq. (1), the conductivity is proportional to the Fourier transform of the cooperon, so there will be a peak in the conductivity at $\omega=\omega_p$.

III. LONGITUDINAL MAGNETOCONDUCTIVITY

To observe the resonance at ω_p, it will be necessary to reduce as much as possible the effect of the applied magnetic field on the orbital part of the cooperon, or else the weak localization correction will no longer be substantial when $\omega_p > |1/\tau_{so}-1/\tau_{sf}|$. For the case of quasi-two-dimensional films, we therefore consider the magnetoconductivity with the applied field in the plane of the film, and for quasi-one-dimensional wires, we orient the field along the length of the wire. For these cases the orbital part of the cooperon is given by

$$C_o^{(1)}(t;\mathbf{x},\mathbf{x})=\frac{1}{\tau}\frac{1}{\sqrt{4\pi Dt}}\frac{1}{A}e^{-(1/\tau_B+1/\tau_\phi)t}, \tag{16a}$$

$$C_o^{(2)}(t;\mathbf{x},\mathbf{x})=\frac{1}{\tau}\frac{1}{4\pi Dt}\frac{1}{s}e^{-(1/\tau_B+1/\tau_\phi)t}, \tag{16b}$$

where A is the cross-sectional area of the wire and s is the thickness of the film. For a wire with a circular cross section, the dephasing rate $1/\tau_B$ due to the applied longitudinal field is given by[9]

$$\frac{1}{\tau_B}=\frac{D}{2\pi}\left[\frac{eB}{\hbar}\right]^2 A, \tag{17a}$$

while in the two-dimensional case, the magnetic dephasing rate is[9]

$$\frac{1}{\tau_B}=\frac{D}{3}\left[\frac{eBs}{\hbar}\right]^2. \tag{17b}$$

The conductivities, computed form Eqs. (1) (with C_o replaced by C), (13), and (16), may be expressed in both cases in the form

$$\delta\sigma^{(d)}=F^{(d)}\left[\frac{1}{\tau_\phi}+\frac{1}{\tau_B}-i\omega+\frac{4}{3\tau_{so}}+\frac{2}{3\tau_{sf}}\right]+\frac{1}{2}\frac{1}{\sqrt{1-\gamma}}F^{(d)}\left[\frac{1}{\tau_\phi}+\frac{1}{\tau_B}-i\omega+\frac{2}{3}\left[\frac{1}{\tau_{so}}+\frac{2}{\tau_{sf}}\right]+\frac{2}{3}\left[\frac{1}{\tau_{so}}-\frac{1}{\tau_{sf}}\right]\sqrt{1-\gamma}\right]-\frac{1}{2}\frac{1}{\sqrt{1-\gamma}}F^{(d)}\left[\frac{1}{\tau_\phi}+\frac{1}{\tau_B}-i\omega+\frac{2}{3}\left[\frac{1}{\tau_{so}}+\frac{2}{\tau_{sf}}\right]-\frac{2}{3}\left[\frac{1}{\tau_{so}}-\frac{1}{\tau_{sf}}\right]\sqrt{1-\gamma}\right], \tag{18}$$

where

$$F^{(1)}(z) = -2\frac{e^2}{h}\frac{1}{A}\sqrt{D}z^{-1/2} ,$$

$$F^{(2)}(z) = +\frac{1}{\pi}\frac{e^2}{h}\frac{1}{s}\ln(\tau z) . \quad (19)$$

In the two-dimensional case the result may be compared with expressions obtained earlier by Maekawa and Fukuyama using the impurity-diagram technique.[3] They are found to agree.

Now taking the limit $\omega_p \gg |1/\tau_{so} - 1/\tau_{sf}|$, we examine the resonant terms. The net scattering rate $1/\tau'$ which determines the linewidths is given by

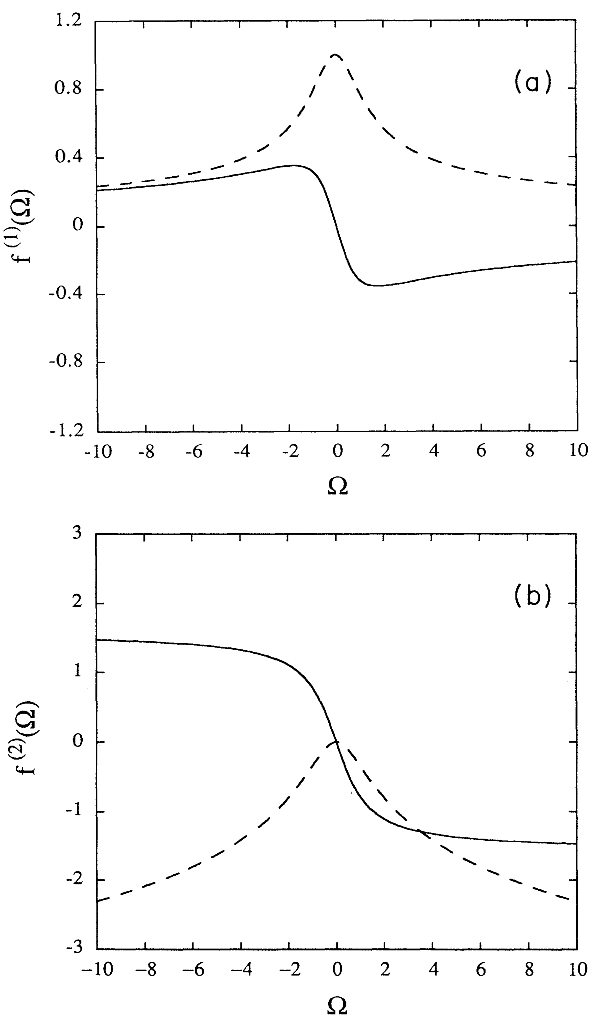

FIG. 1 Real (solid) and imaginary (dashed) parts of the spin precession line shapes for (a) quasi-one and (b) quasi-two-dimensions.

$$\frac{1}{\tau'} \equiv \frac{1}{\tau_\phi} + \frac{1}{\tau_B} + \frac{2}{3\tau_{so}} + \frac{4}{3\tau_{sf}} . \quad (20)$$

In terms of the dimensionless frequency $\Omega \equiv (\omega - \omega_p)\tau'$, the resonant terms in the conductivities are

$$\delta\sigma^{(1)} = \frac{e^2}{h}\frac{1}{A}\sqrt{D\tau'}\frac{(1/\tau_{so} - 1/\tau_{sf})}{\frac{3}{2}\omega_p} f^{(1)}(\Omega) , \quad (21a)$$

$$\delta\sigma^{(2)} = \frac{1}{2\pi}\frac{e^2}{h}\frac{1}{s}\frac{(1/\tau_{so} - 1/\tau_{sf})}{\frac{3}{2}\omega_p} f^{(2)}(\Omega) , \quad (21b)$$

with the line shapes

$$f^{(1)}(\Omega) = i\left[\frac{1}{1-i\Omega}\right]^{1/2} ,$$

$$f^{(2)}(\Omega) = i \ln\left[\frac{1}{1-i\Omega}\right] . \quad (22)$$

In Fig. 1 we plot the real and imaginary parts of the line shapes.

The features of the spin precession resonance may be read from Eqs. (20)–(22). For the resonance to be sharp, in addition to the requirement that $\omega_p \gg |1/\tau_{so} - 1/\tau_{sf}|$, it is also necessary that $\omega_p \gg 1/\tau'$, which means that the precession rate must be greater than all of the other scattering rates: spin-flip, spin-orbit, inelastic dephasing, and field dephasing. The sign and magnitude of the peak depend upon the factor $(1/\tau_{so} - 1/\tau_{sf})/\omega_p$, and the resonance vanishes when there is no spin scattering. This is to be expected because without spin scattering both paths are subject to the same rotation, and $U^\dagger(N \to 1)U(1 \to N) = 1$. A distinctive feature of the resonance line is that the peak occurs in the imaginary part, a consequence of the $\sin(\omega_p t)$ factor in Eq. (15).

IV. CONCLUSIONS

When the spin-precession rate ω_p exceeds all other scattering rates (except the elastic rate $1/\tau$) the ac conductivity will develop a resonance at $\omega = \omega_p$. The question remains as to whether this effect can be observed in realistic experiments.

We assume that one would measure the ac conductivity at a fixed frequency, and sweep the applied field through the resonance. If the frequency chosen is too low, the on-resonance precession rate will not exceed $1/\tau_\phi$, $1/\tau_{so}$, or $1/\tau_{sf}$. If it is too high, then the on-resonance precession rate will be less than $1/\tau_B \propto B^2$. Low-Z materials, such as lithium and magnesium, are attractive because they have long spin-orbit scattering times. Lithium films reported by Sharvin,[10] which have spin-orbit and spin-flip scattering times greater than 10 ns, are an example of a suitable material. With films 20 nm thick, one could obtain $\omega_p \tau' \approx 10$ at $\omega_p = 2\pi \times 1$ GHz and $T = 1$ K. To avoid suppression from transverse magnetoresistance, in the two-dimensional case it would be necessary to align the magnetic field with the plane of the film to $\approx 0.5°$, but in the one-dimensional case accurate alignment is not necessary.

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