PHYSICAL REVIEW B VOLUME 44, NUMBER 16 15 OCTOBER 1991-II

Orbital magnetism of mesoscopic systems

S. Oh, A. Yu. Zyuzin,* and R. A. Serota
Department of Physics, University of Cincinnati, Cincinnati, Ohio 45221-0011
(Received 28 January 1991; revised manuscript received 29 April 1991)

We study the orbital magnetic response of mesoscopic metallic systems. The response in singly (Landau) and multiply (Aharonov-Bohm) connected geometries is considered. It is argued that the absence of phase and energy relaxation at the sample boundaries in a typical magnetic measurement has dramatic consequences for the magnitude and the scale of variation of the response with the magnetic flux. We identify several regimes, depending on the strength of the magnetic field and the degree of disorder, and investigate the response in each of them. In particular, we discuss the mesoscopic fluctuations of the response and its average properties including the interaction-localization correction.

I. INTRODUCTION

Quantum phenomena in disordered electronic systems have been a topic of intense scrutiny in the past decade. Their effect on electronic transport attracted most of the effort. Initially, researchers studied the ensemble-average properties of the systems, such as the weak-localization corrections to the conductivity.[1] More recently, it has also been realized that the same kind of quantum-interference effects can lead to the sample-specific phenomena, such as conductance fluctuations,[2] which were called mesoscopic. Unlike kinetic properties, the quantum effects on thermodynamic properties of disordered conductors have only been of occasional interest, especially in the experimental area. Nonetheless, in the theoretical area the subject has been addressed quite extensively by Altshuler, Aronov, and Zyuzin,[3] who evaluated the interaction-localization corrections to the magnetic susceptibility and specific heat, and by Altshuler, Khmelnitskii, and Spivak,[4,5] who investigated the supercurrents in *SNS* junctions. For a long junction, the latter problem is in one-to-one correspondence with that of Aharonov-Bohm (AB) response. This problem can be traced back to the pioneering works of Altshuler, Aronov, Spivak, and Khmelnitskii,[6,7] who pointed out the sensitivity of quantum-interference effects to the perturbations at the sample boundaries, such as the AB flux and the phase difference between the superconductors in an *SNS* junction. Another seminal contribution, the theory of level statistics in disordered metals, forms a foundation for the understanding of mesoscopic thermodynamic effects and is due to Altshuler and Shklovskii.[8] In the past few years, interest in orbital magnetic response of disordered metals has been revived in part by new experimental effort.[9] For instance, some of the results derived in Refs. 4 and 5 for the interaction-localization correction and mesoscopic fluctuations have been reproduced in the context of the AB response of thin metallic rings.[10,11] Of greater interest, in our view, was the realization of the canonical nature of the system in a typical magnetic measurement. The striking effect it can have on the AB response has been first investigated numerically[12,13] and subsequently analytically.[14,15] Attempts to evaluate the mesoscopic fluctuations of the Landau response to the flux through metal (FM), as opposed to the AB case, should also be mentioned.[16]

The subject of this paper is the mesoscopic aspects of orbital magnetic response of small metallic systems. We point out the crucial distinction between transport and magnetic measurements: while in the former case the sample is in contact with electron reservoirs, in the latter case it typically is not. This has major implications. Indeed, a mechanism whereby the electron phase and energy relaxation take place in a transport measurement—via inelastic collisions in ideal metallic leads (electron reservoirs), is not ordinarily invoked in a magnetic measurement. In other words, provided that scattering off the sample boundaries is elastic, sample size L is no longer a valid scale for the breakdown of phase coherence. To emphasize this circumstance, in what follows we shall consider the extreme quantum coherence limit, $L \lesssim \min\{L_T, L_\varphi\}$, or $E_c \equiv \hbar D/L^2 \gtrsim \max\{T, \tau_\varphi^{-1}\}$, where $L_T = \sqrt{\hbar D/T}$ is the thermal length, L_φ is the phase coherence length, and D is the diffusion coefficient. For definiteness, it is assumed that $\min\{L_T, L_\varphi\} = L_T$ or $\max\{T, \tau_\varphi^{-1}\} = T$, which is believed to be true in metals. Mathematically, E_c appears as the effective level broadening in the calculations for samples with open boundaries,[2] meaning that at low temperatures sample size would be the minimal effective phase coherence length. This may not be the case for isolated samples.[17] Whereas the fixed number of electrons, coupled with the absence of energy relaxation in the system, has been emphasized[12–15] by means of the canonical ensemble treatment of orbital magnetic response, our emphasis is on its intimate relationship to the issue of phase coherence of an isolated sample. In particular, we establish the significance of the magnetic field of strength H_c such that $H_c L L_T \simeq \phi_0 = hc/e$, which corresponds to the flux through the sample, $H_c L^2$, of order $\phi_c \simeq \phi_0 (L/L_T) \ll \phi_0$.

It should be mentioned that the notion of the importance of the exact nature of the phase coherence breaking mechanism in an isolated system versus the system attached to electron reservoirs had been previously invoked

 ©1991 The American Physical Society

in the context of absorption fluctuations by Serota *et al.*[17] They argued that the lack of phase relaxation at the boundaries would lead to the order-of-magnitude absorption fluctuations, in contrast with the relatively small fluctuations of the conductance (transmission). Furthermore, they showed that if the sample is weakly coupled to the reservoirs via thin leads, the typical period of the fluctuations could be much less than the flux quantum ϕ_0.

In what follows, we study the Landau (FM) and the AB orbital magnetic responses. The model for the former will be a disk and for the latter a narrow ring. Both are taken to be of the same circumference L and are placed in the uniform perpendicular magnetic field. We restrict our discussion of the simply connected geometry (disk) to weak magnetic field such that $\phi \simeq BL^2 \lesssim \phi_0$. Due to gauge considerations, the AB response is periodic with the period $\sim\phi_0$. It turns out that, aside from the periodicity of the AB response and certain geometrical factors, in quantum coherent samples the Aharonov-Bohm and the Landau responses are of the same order of magnitude. Clearly, this is because the orbital magnetic response is the response to the vector potential, underscoring its quantum-mechanical nature. However, the AB effect is more sensitive to the destruction of phase coherence; the absence thereof along the paths surrounding the AB flux leads to the exponential decay of the response as $\exp(-L/L_T)$. In contrast, the FM response decays as a power of (L_T/L).

The conventions used in this paper are as follows. For simplicity, we shall consider only the two-dimensional samples and three-dimenstional slabs of thickness $a \ll L$. We shall always refer to the total magnetic moment of the sample and the magnetic susceptibility per unit volume (area). Denoting $N(0)$ the single-particle density of states at the Fermi level and $\mathcal{V}$ the volume (area) of the sample, we shall make use of the following parameters defining the energy and length scales in the system in addition to E_c, temperature T, and sample size L: the elastic-scattering time τ, the average interlevel spacing $\Delta=[N(0)\mathcal{V}]^{-1}$, the cyclotron frequency $\omega_c=eH/mc$, the electron mean free path $l=v_F\tau$ (v_F is the Fermi velocity), the Landau orbit size $L_H=(\hbar c/eH)^{1/2}$, and the cyclotron radius $L_C=v_F/\omega_C$.

This paper is organized as follows. In Sec. II we give a qualitative outline of our central results. In Sec. III we derive the gauge-invariant perturbative expressions for the diffusion propagators of an isolated system subject to the magnetic field. In Sec. IV, we evaluate the rms fluctuation of the response. Sec V is devoted to the average response of an isolated system. In Sec. VI we calculate the interaction-localization correction to the average response. Discussion in Sec. VII includes a simple representation of our results in terms of the dimensionless conductance and addresses their experimental implications.

II. QUALITATIVE CONSIDERATIONS

Consider first the FM response. In strong magnetic fields, $\omega_C\tau \gg 1$ or $L_C \ll l$, energies are grouped into Landau levels, leading to the de Haas–van Alphen (dHvA) oscillations of the magnetic moment. In two dimensions (2D), for instance, the differential susceptibility and the envelope of the dHvA oscillations of the sample moment are given by[18]

$$\chi_{\rm dHvA} \simeq \chi_L(\varepsilon_F/\hbar\omega_C)^2 \text{ and } M_{\rm dHvA} \simeq \mu_B(\varepsilon_F/\Delta) , \quad (1)$$

respectively. Here $\mu_B=e\hbar/2mc$ is the Bohr magneton and $\chi_L=-\frac{1}{3}\mu_B^2 N(0)$ is the Landau susceptibility. Physically, the above expression for the magnetic moment indicates that each state contributes $\sim\mu_B$ to the magnetic moment. Averaging over the period of the dHvA oscillations yields[18] $\langle\chi_{\rm dHvA}\rangle=\chi_L$. Recently, predictions of mesoscopic fluctuations (Akkermans and Shapiro[16]) and the AB oscillations[19] of the magnetic moment, superimposed on the dHvA oscillations, have been made. However, only the case of a few Landau levels, $\hbar\omega_C \simeq \varepsilon_F$, was discussed so that presently it is not entirely clear how these effects match the mesoscopic fluctuations at $\omega_C\tau \lesssim 1$ (see next paragraph), at which point the dHvA oscillations fall off exponentially.

In the opposite limit, $\omega_C\tau \lesssim 1$ or $L_C \gtrsim l$, the structure of energy levels is determined by the impurity configuration. As a result, the orbital response will fluctuate from sample to sample. At present, the fluctuation pattern is not fully understood in the case of $\phi \gg \phi_0$, or $L \gg L_H$. However, the result for the disorder-averaged susceptibility,[20] $\langle\chi\rangle=\chi_L$, is well established and applies equally to canonical and grand canonical ensembles.

In the linear-response regime for the sample-specific response, identified as $\phi \lesssim \phi_0$, or $L \lesssim L_H$, the evaluation of the rms fluctuation of the magnetic susceptibility yields

$$\delta\chi \simeq |\chi_L|(\kappa_F l)\begin{cases}[-\ln(\phi_c/\phi_0)]^{1/2}, & \phi \lesssim \phi_c \\ [-\ln(\phi/\phi_0)]^{1/2}, & \phi \gtrsim \phi_c ,\end{cases} \quad (2)$$

which translates to $\delta M \simeq \mu_B(\kappa_F l)(\phi/\phi_0)$ for the rms fluctuation of the sample moment. The significance of the flux scale ϕ_c, mentioned in the Introduction, is already clearly seen in the above equation, albeit through a weak logarithmic dependence. The existence of such a scale in an isolated sample is manifested even more dramatically in the average response. This is because on the average the response is due to a single level, closest to the average chemical potential, whose dependence on the flux is directly related to the repulsive level correlations trying to avoid crossings.[12–15] The latter can be expressed in terms of the correlations of the chemical potential (an intensive quantity), in contrast with the correlations of the free energy (an extensive quantity) which determines the fluctuations of the response. Consequently, the dependence on the length scale $L_T \gg L$ is much more pronounced.

Translating these ideas into an analytical calculation, we show below that the average magnetic moment grows linearly *along* the magnetic field *(paramagnetically)* for $\phi \lesssim \phi_c$ until it peaks at $\phi \simeq \phi_c$. The average susceptibility in this regime is found as

$$\langle\chi\rangle \simeq |\chi_L|(\kappa_F l)(\Delta/T) , \quad (3)$$

which saturates to $\chi_L(\kappa_F l)$ at $T \simeq \Delta$. The maximal moment is then $\langle M\rangle_{\rm max} \simeq \langle\chi\rangle\phi_c$. For the magnetic field such that $\phi \gtrsim \phi_c$, the moment falls off universally as $\langle M \rangle \simeq \mu_B(\phi_0/\phi)$. It will match $\langle M \rangle \simeq \chi_L \phi$ at $\phi \simeq \phi_0$, indicating a crossover to the diamagnetic Landau susceptibility (see above). The paramagnetic sign of the susceptibility can be attributed to the ability of an electron to take advantage of the potential energy of impurities (or the confining potential at the sample boundaries of clean samples). It should be emphasized that the fluctuation being larger than the average response points to the random sign of the response as a function of the impurity configuration.

An interesting question of the "clean-limit" behavior arises when $l \simeq L$. Clearly, this condition applies to samples with a great deal of variety in sample surfaces, including rough surfaces considered by Gor'kov and Eliashberg,[8] smooth surfaces with chaotic scattering,[21] such as elliptic billiards, and smooth surfaces with integrable underlying classical dynamics such as the round disk. Since the latter have zero measure among all possible surfaces, the condition $l \simeq L$ does not by itself have any bearing on the applicability of Dyson statistics to the electron energy levels in the presence of random (chaotic) scattering. On the other hand, it is the energy-level correlations[8] in random systems, described by Dyson statistics, which are responsible for the mesoscopic effects in discussion, both average and sample specific. As a result, we find that $\delta\chi \sim |\chi_L|(k_F L) \sim |\chi_L| N^{1/2}$ and $\langle \chi \rangle \sim |\chi_L| N^{1/2}(\Delta/T)$, where N is the total number of electrons in the system. At zero temperature, $\langle \chi \rangle \sim |\chi_L| N^{1/2}$.

A recent numerical study[21] of the diamagnetic response of an elliptic billiard with a fixed aspect ratio clearly points towards Dyson statistics for the level structure in the system in a sufficiently strong magnetic field. It was found that the magnetic moment shows large fluctuations in magnitude and sign with the number of electrons. We point out that even for integrable systems breaking of rotational symmetry leads to the paramagnetic[22] (second-order term in perturbation series) term in the response known as the Van Vleck polarization paramagnetism. The diamagnetic term is the usual Langevin response[18] which is due to the shrinkage of orbits in the magnetic field. In disordered metals, the relative magnitude of the Van Vleck and Langevin responses varies from sample to sample and the already large response, $\propto N^{1/2}$, of a typical sample is a result of near compensation of these two contributions.

Turning to the AB case, we note that for $\phi \lesssim \phi_0$ the magnitude of the response is of the same order of magnitude as the FM response aside from the geometrical factors, which can be traced to the conductance and will be discussed later, and a greater sensitivity to the destruction of quantum coherence, which was already mentioned above. On the other hand, it is well known that the AB flux can be gauged away from the integer number of flux quanta. This leads to the periodic oscillations of the persistent current. The mesoscopic nature of the response in quantum coherent rings is reflected in this case by the random phase shift[5] from sample to sample, in addition to the fluctuations of amplitude. The average persistent current oscillates with the period of half the flux quantum since it is described by the Cooper propagator with the charge $2e$. It should be noted that if the rings are attached to electron reservoirs (leads), the average AB response will fall off exponentially.[23] An important observation concerning the experimental situation has to do with the magnetic field penetrating the annulus (metallic part) of the ring. In this case, one should take into account the FM response of the annulus, adapted to its particular geometry.

Finally, in what follows we shall also consider the interaction-localization correction to the average response. We show that the magnetic moment of singly connected samples of oblonged shapes does not scale with the area of the sample. The physical reason behind this phenomenon is the sensitivity of the Cooper-propagator eigenvalues to the boundary conditions in a quantum coherent sample. It can have remarkable consequences such as the larger magnetic moment in a sample of smaller area subject to the same flux. Alternatively, the two samples of unequal areas can have equal moments in the magnetic fields of equal strength. It must be stressed that this is the average effect and as such is amenable to experimental verification on a large number of identically prepared samples. We also point out that whereas the average response of noninteracting electrons peaks at the flux of order ϕ_c, and is repeated periodically in the AB case, the interaction-localization correction depends very weakly on ϕ_c, suggesting that the two effects can be separated experimentally at sufficiently low temperatures.

III. COOPER AND DIFFUSION PROPAGATORS IN ISOLATED SAMPLES

In this section we present the perturbative derivation of the particle-hole propagator in the presence of magnetic field, which reduces to the evaluation of the shift of the energy eigenvalue. In zero field, the energy is $[\omega_m + \mathcal{E}(\{n_i\})]$, where $\omega_m = 2\pi m T$ is the Matsubara frequency and the function $\mathcal{E}(\{n_i\})$ depends on the sample geometry.[2] In a rectangular, for instance, it is given by

$$\mathcal{E}(\{n_i\}) = \hbar D \pi^2 (n_x^2/L_x^2 + n_y^2/L_y^2) .$$

As was mentioned in the Introduction, a typical magnetic measurement does not involve sample contact with electron reservoirs. Therefore phase and energy relaxation will not necessarily take place at the sample boundary, as would be the case in a typical transport measurement. In particular, the electron may still retain its quantum coherence after collision with the sample boundary. Mathematically, this fact is reflected by the existence of the "zero mode" of the propagator,[17] $\{n_i\} = \{0\}$. Moreover, at very low temperatures the zero mode gives the largest contribution to the response. Accordingly, our calculation is limited here to the evaluation of the zero-mode eigenvalue shift by the magnetic field.

Consider the Cooper channel as it appears in a localization correction.[1] The crucial observation for the evaluation of the zero-mode energy shift by the weak magnetic field is that the usual stationary state perturbation theory has to be modified here to account for the boundary condition of zero covariant derivative for the Cooper propagator.[5] The latter circumstance ensues the following consequences. First, in integrations by parts, one has to keep track of the surface terms and, second, the higher-order corrections cannot be expanded in terms of the unperturbed Cooper propagators which satisfy the Neumann boundary conditions. As a result, we obtain that the first-order correction to the energy of the zero-mode Cooper propagator is zero, as is expected on the basis of time-reversal symmetry invariance. For the sake of the argument, we express the second-order correction in the Schrödinger equation notation:

$$\delta\mathcal{E}_2=\left[\frac{e\hbar}{2mc}\right]^2\frac{2m}{\hbar^2}\int dV|\psi_0|^2\mathcal{A}^2\ , \tag{4}$$

where $\psi_0=\mathcal{V}^{-1/2}$ is the normalized zero-mode "eigenfunction," $\mathcal{A}=\mathbf{A}+(ic/e\psi_0)\nabla\psi_1$, $\mathbf{A}$ is the vector potential, ψ_1 is the first correction to the zero-mode "eigenfunction," and $\mathcal{A}$ is such that $\nabla\cdot\mathcal{A}=0$ and $\mathbf{n}\cdot\mathcal{A}|_{\text{boundary}}=0$. Since ψ_0 is a constant in space, we conclude that $\mathcal{A}$ is actually the vector potential in the Coulomb gauge tangential to the boundary. This, and the fact that $\nabla\times\mathcal{A}=\mathbf{H}$, will define $\mathcal{A}$ uniquely as a curl of some function.[24] It should be pointed out that ψ_1 is all but irrelevant for this derivation which should be contrasted with an ordinary case of a Schrödinger equation with the Dirichlet boundary conditions, where the first-order correction to the lowest zeroth-order eigenstate admixes with all zeroth-order eigenstates and the eigenvalue correction is actually evaluated in a selected gauge. Here, the gauge $\mathcal{A}$ "selects" itself by forcing ψ_1 into a gauge transformation and no actual calculation is performed with the gauge $\mathbf{A}$. The significance of the gauge in which the vector potential is tangential to the surface is tied to the condition of zero current through the boundary. The same conclusion is drawn via minimization of the energy with respect to the gauge.

It is clear that for a round disk, subject to the field perpendicular to the surface, the correction given by Eq. (4) will be expressed in terms of the radial gauge. Returning to the diffusion equation notations, we obtain the following expression for the eigenvalue shift of the Cooper propagator ($\hbar=1$):

$$\delta\mathcal{E}_2\equiv\tau_H^{-1}=2\pi^2E_c(2\phi/\phi_0)^2\ , \tag{5}$$

where $E_c=D/L^2$ and L is the circumference of the disk. The generalization to an ellipse with the axes $L_\parallel$ and $L_\perp$ is obvious upon realization that $\mathcal{A}=H\{L_\parallel^{-2}y,-xL_\perp^{-2}\}/(L_\perp^{-2}+L_\parallel^{-2})$, whereof Eq. (5) is recovered if the relation $L\simeq[2\pi^2(L_\perp^2+L_\parallel^2)]^{1/2}$ for the disk circumference is used. In a similar fashion, we can evaluate the eigenvalue shift for the Cooper and diffusion propagators encountered in the cumulants[2] describing the correlations of the mesoscopic fluctuations. As a result, we find the following expression for the eigenvalues:

$$\omega_m+\tau_H^{-1},\quad \tau_H^{-1}=2\pi^2E_c(2\phi/\phi_0)^2\ , \tag{6}$$

$$\omega_m+\tau_{H_\pm}^{-1},\quad \tau_{H_\pm}^{-1}=2\pi^2E_c(\phi_\pm/\phi_0)^2\ , \tag{6$'$}$$

where[2] $\phi_\pm=\phi_1\pm\phi_2$ for the Cooper and diffusion propagators, respectively. Unlike the FM case, the problem of the eigenvalue shift by the AB flux is exactly solvable for a thin metallic ring.[6] Neglecting the contribution of the transverse mode, the eigenvalue is defined by the energy of the longitudinal mode, $Dq_\parallel^2$, where $q_\parallel=(2\pi/L)(n+2\phi/\phi_0)$ or $q_\parallel=(2\pi/L)(n+\phi_\pm/\phi_0)$:

$$\omega_m+4\pi^2E_c(n+2\phi/\phi_0)^2\ , \tag{7}$$

$$\omega_m+4\pi^2E_c(n+\phi_\pm/\phi_0)^2\ . \tag{7$'$}$$

As was mentioned in the Introduction, the problem of the response of a long *SNS* junction to the phase difference φ between the superconductors can be exactly mapped to that of the AB response of a thin metallic ring via the substitution $\varphi\leftrightarrow2\pi\phi/\phi_0$, $2L\leftrightarrow L$.

IV. MESOSCOPIC FLUCTUATIONS OF ORBITAL MAGNETIC RESPONSE

The problem of mesoscopic supercurrent fluctuations in a disordered *SNS* junction has been solved in Ref. 5. For a long junction, this problem is equivalent to the problem of the AB persistent current fluctuations in a thin metallic ring. In this section we evaluate the fluctuations of the Landau response in a singly connected geometry. The calculation is based on the following expression for the correlation function of the thermodynamic potential:[5]

$$\langle\delta\Omega(H_1)\delta\Omega(H_2)\rangle=\mathcal{V}^2\int\int d\varepsilon_1d\varepsilon_2[\varepsilon_1\varepsilon_2f(\varepsilon_1)f(\varepsilon_2)\times\langle\nu(\varepsilon_1)\nu(\varepsilon_2)\rangle]\ , \tag{8}$$

where $f(\varepsilon)$ is the Fermi distribution function and the contribution of the zero mode to the correlation function of the density of states is given by[8]

$$\langle\nu(\varepsilon_1)\nu(\varepsilon_2)\rangle=\frac{s^2}{2\pi^2\mathcal{V}^2}\sum_\pm\mathrm{Re}[-i(\varepsilon_1-\varepsilon_2)+\tau_{H_\pm}^{-1}]^{-2}\ , \tag{9}$$

where $\tau_{H_\pm}^{-1}=2\pi^2E_c(\phi_\pm/\phi_0)^2$ is taken from Eq. (6$'$), and s is the degeneracy, for instance, of spin degrees of freedom. Equation (9) should be compared with the similar expression[5] for the AB case. Indeed, with the help of Eq. (7$'$) we obtain

$$\langle\nu(\varepsilon_1)\nu(\varepsilon_2)\rangle=\frac{S^2}{2\pi^2\mathcal{V}^2}\times\sum_{n=-\infty\pm}^{\infty}\sum_\pm\mathrm{Re}[-i(\varepsilon_1-\varepsilon_2)+\pi^2E_c(n+\phi_\pm/\phi_0)^2]^{-2}\ . \tag{9$'$}$$

Converting the integral in Eq. (8) into Matsubara sums, we obtain with the help of Eq. (9)

$$\langle \delta\Omega(H_1)\delta\Omega(H_2)\rangle = \frac{1}{12\pi^4}2\pi T \times \sum_{m=1}^{\infty} \omega_m^3 \left[\frac{1}{(\omega_m+\tau_{H_-}^{-1})^2} + \frac{1}{(\omega_m+\tau_{H_+}^{-1})^2} \right], \quad (10)$$

where we have kept only the sum which gives the largest contribution at low temperatures. The expressions for the correlation functions of the magnetic moment and the susceptibility are obtained by successive differentiations on H_1 and H_2. The variances of these quantities are obtained by setting $H_1 = H_2$ after the differentiation.[2] Following this scheme, we find

$$\langle \delta M^2 \rangle = \frac{3s^2}{\pi^2}\mu_B^2(\varepsilon_F\tau)^2 \left[\frac{\phi}{\phi_0}\right]^2 \ln(E_c/T^*) ,$$

$$T^* = \max\{T, (2\pi\tau_H)^{-1}\} , \quad (11)$$

where E_c was used as the upper cutoff of the logarithmic divergence. This result applies to 2D samples and 3D slabs. From Eq. (11), we obtain for the variance of the susceptibility,

$$\langle \delta\chi^2 \rangle = \frac{27s^2}{\pi^2}\chi_L^2(\varepsilon_F\tau)^2 \ln(E_c/T^*) . \quad (12)$$

For a 3D slab the above result should be multiplied by $(\pi/k_F a)^{-2}$. Notice that the logarithmic dependence changes its form for the flux given by

$$\phi_c \approx \phi_0(T/E_c)^{1/2} = \phi_0 L/L_T \ll \phi_0 . \quad (13)$$

The significance of this flux scale is due to the absence of phase relaxation at the boundary of an isolated sample, which is mathematically represented by the zero-mode contribution. It corresponds to the field strength such that $H_c L L_T \approx \phi_0$. Its role is especially dramatic for the disorder-averaged susceptibility (see next section).

The linear dependence of the rms fluctuations on the flux, predicted by Eq. (11), ends when the flux penetrating the sample becomes of order of the flux quantum, at which point $\tau_H^{-1} \simeq E_c$ and $\langle \delta M^2 \rangle \simeq (3s^2/\pi^2)\mu_B^2(\varepsilon_F\tau)^2$. Further calculation requires taking higher modes into account, as well as the use of a modified form of the propagator, and has not been fully completed at present. Thus it is not entirely clear whether the rms fluctuation saturates at the above value or scales as a power of the sample area.

Comparing our results and the results of Ref. 5 for the supercurrent fluctuations is an *SNS* junction, as translated to the persistent current fluctuations in a ring, we emphasize that in Ref. 5 the Cooper channel was omitted under the assumption of time-reversal symmetry breaking. Its contribution is easily recovered in a general situation from the expression for the diffusion channel via the substitution $\varphi - \varphi' \to \varphi + \varphi'$. Converting the expression for the current into the expression for the magnetic moment, we arrive at Eqs. (11) and (12) up to a numerical coefficient.

In concluding this section we would like to comment on the previous attempts of evaluation of the fluctuations of the FM (Landau) response.[16] In the first two papers on the subject, the reciprocal space formalism has been used. It leads to erroneous results in a mesoscopic situation since the q vector, appearing in the cross product for the magnetic field with the vector potential, couples to the electron momenta which are quantized by the sample size. In addition, the zero-mode contribution has been omitted in these calculations as well. Serota and Oh[16] have realized the importance of the zero-mode contribution when there is no sample contact with electron reservoirs. However, since they used the reciprocal space formalism, they found the variance to be $(\varepsilon_F\tau)^2$ larger than the correct answer in Eq. (12). Akkermans and Shapiro[16] have correctly used the real-space formalism. If evaluated properly, the zero-mode contribution (suggested by Serota and Oh[16]) in their paper would be in complete agreement with Eq. (12).

V. AVERAGE RESPONSE OF A CANONICAL ENSEMBLE

In recent papers[12–15] it was shown that in a canonical ensemble the AB (persistent) currents have, on the average, the paramagnetic sign. This phenomenon has been attributed to electron level correlations (Dyson statistics) in a disordered metal.[8] The physical picture emerging from these papers can be summarized as follows: Whereas a standard derivation of the response function assumes a fixed chemical potential, it would be appropriate only in a circumstance of a conductor connected to electron reservoirs with well-defined chemical potentials. However, in a magnetic response measurement the sample is typically detached from electron reservoirs thereupon leading to quite dramatic consequences in a mesoscopic situation. The difference between the grand canonical and canonical ensembles is that in the latter the total number of electrons in the system remains constant, while in the former the electron at the Fermi level can leave or enter the sample once its energy crosses the average chemical potential with the change of the AB flux. Consequently, whereas the average AB current in a grand canonical ensemble is exponentially small,[10] in a canonical ensemble its maximal value will be of order[14] $(e\Delta/\hbar)(E_c/T)^{1/2}$ when $T \gg \Delta$, or $(e\Delta/\hbar)(E_c/\Delta)^{1/2}$ when $T \lesssim \Delta$. Obviously, on the average it is a single level current. It has the same (paramagnetic) sign for the levels crossing the chemical potential from above and below since those levels have slopes of the opposite sign as a function of the AB flux.[25] Since the average interlevel spacing is of order Δ, the maximal current is $\sim \Delta/\phi_c$ ($\hbar = c = 1$), where ϕ_c is given by Eq. (13). The absence of energy relaxation at the boundary of an isolated sample, emphasized in Refs. 12–15, parallels the absence of phase relaxation,[17] provided the reflection from the sample boundary is of elastic nature. As a result, the average current assumes its maximal value when the flux becomes of order $\phi_c \ll \phi_0$ (see Secs. I and IV). This translates into the magnetic moment $M \simeq \mu_B(2\phi/\phi_c)(E_c/T)^{1/2} \simeq \mu_B(2\phi/\phi_0)(E_c/T)$ and the susceptibility $\chi \simeq \chi_L(E_c/T)$. The same argument applies, of course, to the FM response as well.

In this section we derive the exact expressions for the average magnetic moment in both the AB and FM cases. As was discussed above, for an isolated sample the derivation in the FM case can be limited to the zero-mode contribution. As a result of our calculation, we find that above ϕ_c the magnetic moment falls off as ϕ^{-1}. In the AB case, it is driven to zero at $\phi_0/2$ due to periodicity imposed by gauge invariance. In the FM case, it falls to the value of order χ_L between ϕ_c and $\phi_0/2$ and saturates to exactly the Landau diamagnetic[20] value for $\phi \gg \phi_0$. Since the average susceptibility is Δ/T times smaller than the root-mean-square fluctuation of the susceptibility, found in the preceding section, we conclude that for a given sample, that is, specific impurity configuration, the susceptibility can have either the diamagnetic or paramagnetic sign, with the average tendency towards the latter. However, the flux scale of ϕ_c is not pronounced in the sample-specific response, the typical distance between peaks and valleys being of order ϕ_0. Although the effect of having a fixed number of particles was not considered in the preceding section, its contribution to the fluctuation is of the order of the average susceptibility and is therefore smaller than the one found there.

We begin with the derivation of the paramagnetic component of the AB (persistent) current in a narrow ring. Our derivation will largely follow those of Refs. 14 and 15. There, it was shown that the average free energy F of the canonical ensemble can be expressed in terms of the average free energy Ω of the identically prepared grand canonical ensemble as

$$F=\Omega-\tfrac{1}{2}\frac{\partial^2\Omega}{\partial\langle\mu\rangle^2}\langle(\delta\mu)^2\rangle=\Omega-\tfrac{1}{2}\Delta^{-1}\langle(\delta\mu)^2\rangle \ . \tag{14}$$

In Eq. (14), $\langle\mu\rangle$ is the average chemical potential of the canonical ensemble, which is also the true chemical potential of the corresponding grand canonical ensemble, $\Delta=[N(0)\mathcal{V}]^{-1}$, and $\langle(\delta\mu)^2\rangle=\langle(\mu-\langle\mu\rangle)^2\rangle$ is the mean variance of the chemical potential of the canonical ensemble given by[15]

$$\langle(\delta\mu)^2\rangle=[N(0)]^{-2}\int\int d\varepsilon_1 d\varepsilon_2\langle v(\varepsilon_1)v(\varepsilon_2)\rangle \times f(\varepsilon_1)f(\varepsilon_2) \ . \tag{15}$$

Combining Eqs. (9′), (14), and (15) and setting $\phi_1=\phi_2$ in Eq. (9′), we obtain, upon differentiation on the flux, the following expression for the persistent current:

$$I=\frac{\Delta s^2}{4\pi^2}\frac{\partial}{\partial\phi}\int\int d\varepsilon_1 d\varepsilon_2 f(\varepsilon_1)f(\varepsilon_2) \times\sum_{n=-\infty}^{\infty}\mathrm{Re}\left[(\varepsilon_1-\varepsilon_2)+i4E_c\pi^2\left[n+\frac{2\phi}{\phi_0}\right]^2\right]^{-2} . \tag{16}$$

Converting the integral into a Matsubara sum and evaluating the sum on n first[26] we find

$$I=e\Delta\frac{s^2}{2\pi^2}\frac{\sin(4\pi\phi/\phi_0)}{\cosh\xi-\cos(4\pi\phi/\phi_0)} , \tag{17}$$

where $\xi=(2\pi T/E_c)^{1/2}\simeq(L/L_T)^{1/2}$ is a small parameter. Multiplying the current in Eq. (17) by the enclosed area $A=L^2/4\pi$, we obtain the magnetic moment of the ring of width $a \ll L$,

$$M=\mu_B\frac{L}{4\pi a}\frac{s^2}{\pi}\frac{\sin(4\pi\phi/\phi_0)}{\cosh\xi-\cos(4\pi\phi/\phi_0)} . \tag{17′}$$

The moment of the disk is evaluated in the same fashion [using Eq. (9)], and is given by

$$M=\frac{s^2\Delta}{4\pi^2}\frac{\partial\tau_H^{-1}}{\partial H}\int_0^\infty dx\left[\frac{\pi x}{\sinh(\pi x)}\right]^2\exp\left[-x\frac{\tau_H^{-1}}{T}\right] \tag{18}$$

$$=\mu_B\frac{s^2}{\pi}\left[\frac{2\pi}{\xi}\right]^2\frac{2\phi}{\phi_0} \times\int_0^\infty dx\left[\frac{\pi x}{\sinh(\pi x)}\right]^2\exp\left[-x\frac{4\pi^3(2\phi/\phi_0)^2}{\xi^2}\right] , \tag{18′}$$

where Eq. (5) is used for τ_H^{-1}. For 3D slabs, the expressions in Eqs. (17′) and (18′) will be multiplied by $(\pi/k_F a)$. Equations (17) and (18) are obtained under the assumption, held throughout, that $T \gg \tau_\varphi^{-1}$, τ_φ being the dephasing time. In general, we have $\xi=[(2\pi T+\tau_\varphi^{-1})/E_c]^{1/2}$ and the exponent in Eq. (18) reads as $\exp[-x(\tau_H^{-1}+\tau_\varphi^{-1})/T]$.

The magnetic moments of the ring and the disk, given by Eqs. (17′) and (18), are plotted in Fig. 1 for $\xi=0.5$, 1, and 2, respectively. Notice that the maximum of the moment shifts towards the origin as the temperature reduces. In fact, the maximum serves as the end point of the linear-response regime and is reached at $\sim\phi_c$. Based on the form of the eigenvalue in Eq. (5), this can be anticipated in the FM case. To gain a better insight in the AB case, we notice that even in the form of Eq. (16) the AB current is clearly periodic with the period of half the flux quantum. Therefore, upon taking the Fourier transform, we can eliminate summation on n by means of extension of the range of integration on ϕ to infinity. Differentiating on the flux, we obtain the following expression for the AB current:

$$I=\sum_{m=1}^{\infty} 2I_m \sin\left[2\pi m\frac{2\phi}{\phi_0}\right], \tag{19}$$

where the current harmonics are given by

$$I_m=-ie\Delta\frac{s^2}{4\pi^3}\int_{-\infty}^{\infty} d\phi \exp\left[i2\pi m\frac{2\phi}{\phi_0}\right]\frac{\partial}{\partial\phi}\int\int d\varepsilon_1 d\varepsilon_2 f(\varepsilon_1)f(\varepsilon_2)\mathrm{Re}\left[(\varepsilon_1-\varepsilon_2)+i4E_c\pi^2\left[\frac{2\phi}{\phi_0}\right]^2\right]^{-2}. \tag{20}$$

Aside from the factor-of-2 difference in the coefficient inside the square brackets, the energy integral in Eq. (20) is identical to that found in the expression for the zero mode of the disk. Neglecting the Cooper-propagator dependence on ϕ for $2\phi \lesssim \phi_c$ and taking the integral,[14] we find

$$I_m=e\Delta(s^2/2\pi^2)\exp(-m\xi), \tag{21}$$

meaning that the sum in Eq. (19) is effectively restricted to ξ^{-1} terms. Consequently, the sine can be expanded in Eq. (19), resulting in the following linear-response expression:

$$\begin{aligned} I &\cong e\Delta\frac{s^2}{2\pi^2}\sum_{m=1}^{\infty} 4\pi m\left[\frac{2\phi}{\phi_0}\right]e^{-m\xi} \\ &= e\Delta\frac{s^2}{2\pi^2}\frac{2}{\xi^2}\left[2\pi\frac{2\phi}{\phi_0}\right] e\Delta\frac{s^2}{2\pi^2}\xi^{-1}\frac{2\phi}{\phi_c}, \end{aligned} \tag{22}$$

which is confirmed by the expansion of Eq. (17). The current peaks to its maximal value of the order of $e\Delta\xi^{-1}\simeq\Delta/\phi_c$ at $2\phi\simeq\phi_c$.

Beyond the linear-response regime, $2\phi \gg \phi_c$, the magnetic moment falls off as ϕ^{-1} barring the sine dependence in the AB case. For the FM case, for instance, it is given by

$$M=\mu_B\frac{s^2}{2\pi^2}\frac{\Delta}{\mu_B H}=\mu_B\frac{s^2}{2\pi^2}\frac{\phi_0}{\phi}. \tag{23}$$

The result of Eq. (23) applies until the flux through the sample becomes of the order of the flux quantum. To understand what happens in even stronger fields we first note that in the process of derivation of Eqs. (17) and (18) we have neglected $\partial^2\Omega/\partial H^2 \propto \exp(-l/L)$ in the AB case[23] and $\mathcal{V}^{-1}\partial^2\Omega/\partial H^2=\chi_L$ in the FM case,[20] respectively. The latter, however, will be the only surviving contribution to the FM susceptibility for $2\phi \gg \phi_0$ since the Cooper channel will be suppressed and the effects considered in this section no longer survive. As for the persistent current, its dependence on the flux will be periodically repeated with each half the flux quantum, as is seen in Fig. 1.

To stress the role of the flux ϕ_c in the problem of average response, we calculated the correlation function of the chemical potential. For a disk, for instance, it is given by

$$\begin{aligned}\langle[\delta\mu(\phi)-\delta\mu(0)]^2\rangle=\frac{\Delta^2 s^2}{2\pi^2}\{4[C+\psi(\zeta)+\zeta\psi'(\zeta)] \\ -[C+\psi(4\zeta)+4\zeta\psi'(4\zeta)]\}, \end{aligned} \tag{24}$$

where $\zeta=2\pi^2(\phi/\phi_c)^2$, ψ is the digamma function, and C is Euler's constant.[26] Here, the right-hand side scales linearly with ζ for $\phi \ll \phi_c$ and saturates to the $\ln\zeta$ dependence at $\phi\simeq\phi_c$.

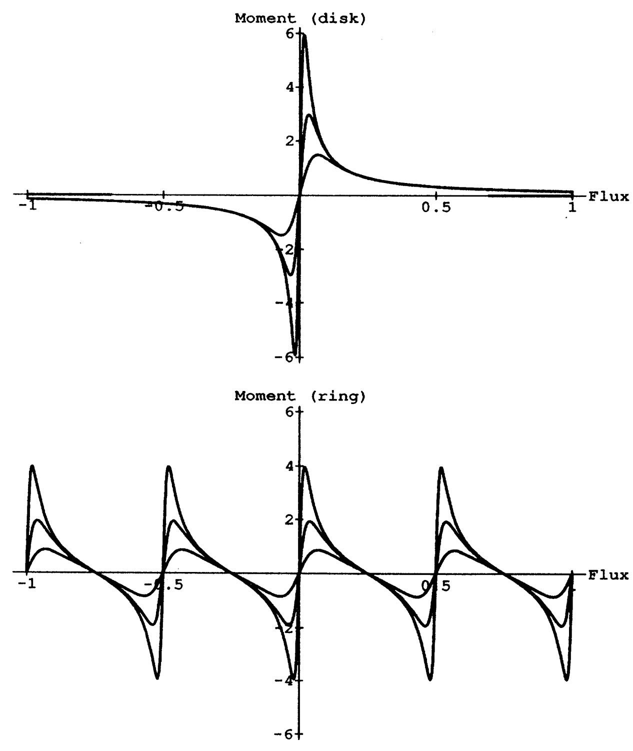

FIG. 1. Leading contribution to the average magnetic moment of a ring of thickness $a \ll L$ and of a disk of equal circumference in units of $\mu_B(s^2/\pi)(L/4\pi a)$ and $\mu_B(s^2/\pi)$, respectively; ξ=0.25, 0.5, and 1, respectively [see Eqs. (17) and (18)]. The moment of the ring is actually bigger due to the smallness of its conductance in the factor $(L/4\pi a)^{-1}$ (see Sec. VIII for the discussion).

VI. INTERACTION-LOCALIZATION CORRECTION

The problem of interaction correction in the Cooper channel to the orbital magnetic response, raised initially in the context of superconducting fluctuations, was first applied to the normal metals by Aslamazov and Larkin.[27] Subsequently, their results were generalized from the point of view of weak-localization theory.[3,4,28] For instance, the evaluation of the magnetic moment in Ref. 3 yields the following result:

$$M=\frac{2s}{9}\mu_B(\varepsilon_F\tau)\frac{2\phi}{\phi_0}\ln[\ln(T_c/T^*)/\ln(T_c\tau)]$$
$$\cong-\frac{2s}{9}\lambda\mu_B(\varepsilon_F\tau)\frac{2\phi}{\phi_0}\ln(T^*\tau)\ , \qquad (25)$$

where $T_c=\varepsilon_F\exp(1/\lambda)$ for $\lambda>0$ (repulsive interaction) and $T_c=\omega_D\exp(1/\lambda)$ for $\lambda<0$ (attractive interaction), ω_D is the Debye frequency, $|\lambda|\lesssim 1$ is the dimensionless interaction strength, $T^*=\max\{T,\Omega_H/2\pi\}$, and[29] $\Omega_H=2eDH$. Equation (25) yields $\chi\simeq\lambda\chi_L(\varepsilon_F\tau)\ln(T^*\tau)$ for the magnetic susceptibility. This can be understood as follows. A localization correction should always have the smallness of $(\varepsilon_F\tau)^{-1}$. On the other hand, the diffusion equation for the Cooper pair is formally equivalent to the Schrödinger equation with the "mass" $(2D)^{-1}$, which is $(\varepsilon_F\tau)$ times lighter than the electron mass. Since the susceptibility is proportional to the squared Bohr magneton, which is inversely proportional to the mass, this factor alone yields the largeness of $(\varepsilon_F\tau)^2$. The overall effect has, therefore, the largeness of $(\varepsilon_F\tau)$, that is, it will be larger in cleaner systems.

Equation (25) and analysis of Ref. 3 in general applies to the case when $L_T\lesssim L$. Turning now to the case of quantum coherent samples, $L\lesssim L_T$, we begin our analysis with the AB response. Converting the results of Ref. 4 for *SNS* junctions to the AB notations, we find, after performing a few simple mathematical manipulations, the following expression for the persistent current:

$$I=\frac{s}{\pi}\lambda eE_c\sin(4\pi\phi_0/\phi)\int_\xi^\infty dx\frac{x^2\sinh x}{[\cosh x-\cos(4\pi\phi/\phi_0)]^2}\ , \qquad (26)$$

or in terms of the magnetic moment,

$$M=\frac{s}{2\pi^2}\lambda\mu_B(\varepsilon_F\tau)\sin(4\pi\phi/\phi_0)$$
$$\times\int_\xi^\infty dx\frac{x^2\sinh x}{[\cosh x-\cos(4\pi\phi/\phi_0)]^2}\ . \qquad (26')$$

In Fig. 2, we plot the magnetic moment for $\xi=0.003$, 0.03, and 0.3, respectively. It is clear that except for very small flux values, the curves are indistinguishable. This should be contrasted with the average response of the noninteracting electrons (see preceding section) where we found dramatic changes at the flux scale of $\phi_c\sim\xi\phi_0$.

Next, we consider the FM case for the disk shaped as an ellipse. Using Eq. (3) and the remark on the ellipse thereafter, we obtain the following expression for the magnetic moment:

$$M=-\frac{s}{\pi}\lambda\mu_B(\varepsilon_F\tau)\frac{2\phi}{\phi_0}\frac{L_\perp L_\parallel}{L_\perp^2+L_\parallel^2}\ln(T^*\tau)\ ,$$
$$T^*=\max\{T,(\pi\tau_H)^{-1}\}\ . \qquad (27)$$

Taking $L_\perp=L_\parallel$, we see that this result matches Eq. (25) at $2\phi\simeq\phi_0$. Comparing Eqs. (26) and (27) we see that the AB and the FM results are of the same order of magnitude. A very interesting situation develops, however, for a highly anisotropic sample, $L_\perp\ll L_\parallel$. In this case, the expression for the magnetic moment is simplified as follows:

$$M\cong-\frac{s}{\pi}\lambda\mu_B(\varepsilon_F\tau)\frac{2\phi}{\phi_0}\frac{L_\perp}{L_\parallel}\ln(T^*\tau)$$
$$=-2s\lambda\mu_B(\varepsilon_F\tau)\frac{HL_\perp^2}{\phi_0}\ln(T^*\tau)\ , \qquad (28)$$

that is, the moment does not scale with the total area of the sample. Clearly, it is a quantum mesoscopic effect which is due to the sensitivity of the magnetic response to the finite-size eigenstate of the Cooper propagator: the eigenstates adjust to the change of the sample area in such a way that total response remains virtually unaffected. This result applies to the response of a thin metallic ring due to the field penetrating the metal: it shows that the ratio of FM to AB response is given by the squared ratio of the ring thickness to its circumferences.[31] Equation (28) is valid at least until the energy

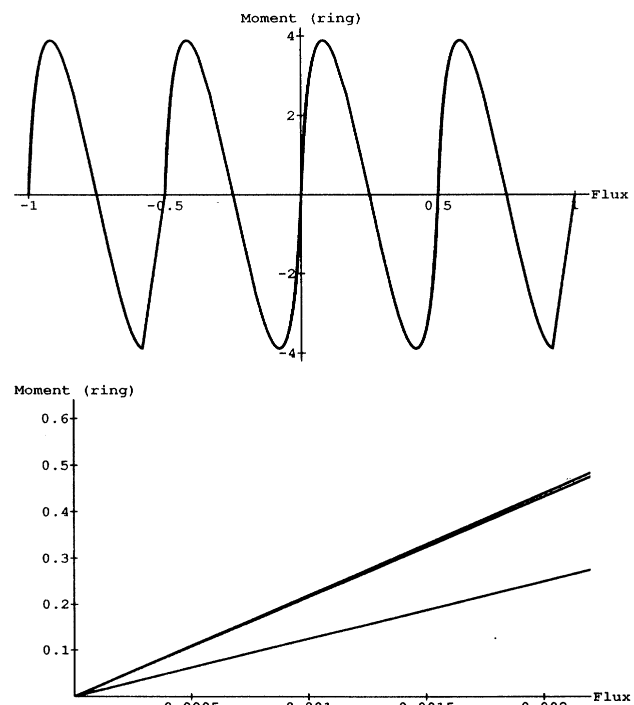

FIG. 2. AB interaction-localization correction to the magnetic moment of a thin ring in units of $\lambda\mu_B(s/2\pi^2)(\varepsilon_F\tau)$; $\xi=0.003$, 0.03, and 0.3, respectively [see Eq. (26')].

shift, $\tau_H^{-1} \cong 4\pi^2 DH^2 L_\perp^2 \phi_0^{-2}$, becomes comparable to the distance to the nearest Cooper-propagator eigenstate, $D/L_\parallel^2$. As anticipated, this condition coincides with $2\phi \simeq \phi_0$.

Historically, τ_H^{-1} in the Cooper propagator was first introduced by Altshuler and Aronov,[30] who studied the localization corrections to the conductivity of a 2D film in the longitudinal magnetic field. A side view of such a film would look like a quasi-1D strip. They assumed, therefore, that the longitudinal spectrum to be continuous, Dq_x^2, whose shift in the field is uniform and is due to the transverse mode. They used the gauge $\mathbf{A} = \{Hy,0,0\}$ and found that in a sample of transverse size $L_\perp$ (from $-L_\perp/2$ to $L_\perp/2$, so that the vector potential is zero in the middle of the sample) the energy shift due to the lowest eigenstate is given by $\tau_H^{-1} = (4\pi^2/3) DH^2 L_\perp^2 \phi_0^{-2}$. This should be compared with τ_H^{-1} for the ellipse. Remarkably, the gauge used in Ref. 12 is just the tangential gauge we study here (see Sec. III). The crossover from 2D to 1D occurs when $L_\parallel$ crosses L_T; thereafter the magnetic moment will scale as[3] $\sqrt{T}$. A rough estimate leading to Eq. (20) is obtained if instead of integration on the longitudinal momentum q_x, one counts only the zero-mode contribution, $q_x = 0$.[31]

VII. CONCLUSIONS

Our results for orbital magnetic response can be summarized in a rather convenient form at very low temperatures, when $T \simeq \Delta$. Indeed, introducing the dimensionless sample conductance, $g \simeq (E_c/\Delta)$, we find that for $\phi \lesssim \phi_0$ the average (paramagnetic) moment of the sample is given by

$$\langle M \rangle \simeq \mu_B (k_F l) g^{-1/2} \begin{cases} (\phi/\phi_c), & \phi \lesssim \phi_c \\ (\phi_c/\phi), & \phi \gtrsim \phi_c \ . \end{cases} \tag{29}$$

It peaks at $\phi \sim \phi_c$, where $\phi_c \sim \phi_0 g^{-1/2}$. The rms fluctuation of the sample magnetic moment, which is also the typical sample-specific value of the moment, is given by

$$\delta M \simeq \mu_B (\kappa_F l)(\phi/\phi_0) \begin{cases} [-\ln(\phi_c/\phi_0)]^{1/2}, & \phi \lesssim \phi_c \\ [-\ln(\phi/\phi_0)]^{1/2}, & \phi \gtrsim \phi_c \ , \end{cases} \tag{30}$$

which applies to 2D samples and 3D slabs. Beyond ϕ_0, $\langle M \rangle = \chi_L H \mathcal{V}$, while the lower limit for the fluctuation is set by $\mu_B (k_F l)$.[30]

Combining Eqs. (2) and (29), we can express the maximal and the "typical" (single Fourier harmonic) average paramagnetic moments in terms of the rms fluctuation at $\phi \simeq \phi_0$ as

$$\langle M \rangle_{\max} \simeq \delta M g^{-1/2} \text{ and } \langle M \rangle_{\text{typ}} \simeq \delta M g^{-1} \ . \tag{31}$$

All these quantities are clearly of the same order of magnitude when $g \sim 1$. This circumstance becomes possible in a dirty sample or a narrow ring. It would also signal the breakdown of the diffusive approximation in a quantum coherent sample so that the condition $\Delta \lesssim T \lesssim E_c$, or $L \lesssim L_T \lesssim L g^{1/2}$, is violated. In this regard, it should be noted that the conductance of a 3D slab of thickness a is $(k_F a/\pi)$ larger than the conductance of its 2D counterpart. Therefore, in 3D, the above constraint is less restrictive for $a \gg l$ than in 2D.

The interaction-localization correction is given by the following expression:

$$\langle \Delta M \rangle \simeq \lambda \mu_B (k_F l)(\phi/\phi_0) \begin{cases} \ln(T\tau), & \phi \lesssim \phi_c \\ \ln(\tau_H^{-1}\tau), & \phi \gtrsim \phi_c \ . \end{cases} \tag{32}$$

We would like to stress that whereas the average paramagnetic moment peaks at $\phi \simeq \phi_c$, the interaction-localization correction peaks at $\phi \simeq \phi_0$. Therefore, it is, in principle, possible to separate these effects experimentally, provided that the temperature is sufficiently low. We also point out that in addition to the interaction correction considered in this paper, there is yet another important interaction mechanism which must be taken into account, namely, the interaction induced via spin-flip scattering off Kondo impurities.[32] It can be shown that due to the anomalous frequency dependence of this interaction, its contribution to the magnetic moment peaks at $\phi \simeq \phi_c$ and could be comparable in magnitude to the average moment, as well as to the interaction correction considered above. Furthermore, in principle, it could be of any sign although in the copper we expect it to be paramagnetic.

In terms of the future effort in the field, we believe that understanding of the current picture is imperative. For instance, one would like to know the typical values of circulating diamagnetic and Hall currents (in the presence of electric field) and their spatial correlations. Also, one would like to have the complete picture of the orbital magnetic response over the entire range of fields: at present, our understanding of the orbital magnetic response beyond the linear-response regime is clearly lacking. In particular, the evaluation of orbital response in a strong magnetic field is especially challenging. Depending on the field strength, two regimes should be investigated. The first regime is identified as such that the Landau orbit becomes shorter than the electron mean free path, $l \gg L_H$, leading to quasiballistic impurity scattering.[33] The second regime is identified as such that the cyclotron radius becomes shorter than the mean free path, $l \gg L_C$, marking the onset of the dHvA oscillations of the average magnetic moment. Among other issues, the "clean-limit" response and its connection to the underlying chaotic scattering and Dyson statistics awaits a thorough investigation. Finally, the problem of the diamagnetic sign of the AB response, which has been observed in recent experiments on narrow metallic rings,[9] remains unresolved.

Note added. Recently, spin-orbit effects have attracted much interest. We have shown [A. Yu Zyuzin and R. A.

Serota (unpublished)] that in addition to the usual fourfold reduction of the density-of-states fluctuations, coupling of spin and orbital degrees of freedom leads to orders-of-magnitude fluctuations of the local magnetization and, accordingly, the electron g factor. Preliminary estimates indicate that the fluctutation of the total sample moment due to this effect is $(k_F l)^{1/2}$ smaller than fluctuation due to orbital effects considered in this paper.

Note added in proof. After the acceptance of this paper for publication, the paper by J. M. van Ruitenbeck and D. A. van Leeuwen was published in Phys. Rev. Lett. **67**, 640 (1991) [see also J. M. van Ruitenbeck, Z. Phys. D **19**, 247 (1991)], where the zero-field response of a 2D rectangle was investigated in light of the compensation of the Langevin and van Vleck responses. Their simulations showed large fluctuations of the van Vleck component with the number of electrons and/or the ratio of the sides of the rectangle. Clearly, this is a result of Poisson level statistics and the correlations between the energies and the matrix elements of the angular momentum.

ACKNOWLEDGMENTS

We express our deep gratitude to B. Altshuler and H. Fukuyama, who participated in this work at various stages and without whom its completion would hardly be possible. We are grateful to B. Spivak, L. Glazman, and M. Raikh for numerous illuminating discussions and helpful suggestions. We acknowledge extremely useful correspondence with U. Eckern and V. Ambegaokar regarding the interaction-localization correction which has led us to the gauge-invariant formulation of the correction to the zero-mode Cooper-propagator eigenvalue. We are indebted to P. Suranyi, M. Ma, P. Esposito, and B. Goodman for sharing their insights on this problem as well. We wish to thank D. Reich for bringing Ref. 21 to our attention and R. Bhatt and M. Dyakonov for calling our attention to the "clean-limit" response. A significant portion of this work has been initiated at the Aspen Center for Physics, whose hospitality is gratefully acknowledged. Research at the University of Cincinnati is supported by the U.S. Army Grant No. DAAL03-89-K-0108.

*Permanent address: A. F. Ioffe Physical-Technical Institute, Leningrad 194021, U.S.S.R.

[1]P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).

[2]For a review see P. A. Lee, A. D. Stone, and H. Fukuyama, Phys. Rev. B **35**, 1039 (1987).

[3]B. L. Altshuler, A. G. Aronov, and A. Yu. Zyuzin, Zh. Eksp. Teor. Fiz. **84**, 1525 (1983) [Sov. Phys.—JETP **57**, 889 (1983)].

[4]B. L. Altshuler, D. E. Khmelnitskii, and B. Z. Spivak, Solid State Commun. **48**, 841 (1983).

[5]B. L. Altshuler and B. Spivak, Zh. Eksp. Teor. Fiz. **92**, 609 (1987) [Sov. Phys.—JETP **65**, 343 (1987)].

[6]B. L. Altshuler, A. G. Aronov, and B. Z. Spivak, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 101 (1981) [JETP Lett. **33**, 94 (1981)].

[7]B. Z. Spivak and D. E. Khmelnitskii, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 334 (1982) [JETP Lett. **35**, 412 (1982)].

[8]B. L. Altshuler and B. I. Shklovskii, Zh. Eksp. Teor. Fiz. **91**, 220 (1986) [Sov. Phys.—JETP **64**, 127 (1986)]. Level statistics of compound nuclei had been studied by F. J. Dyson and M. L. Metha, J. Math, Phys. **4**, 701 (1963). It was subsequently applied by Gor'kov and Eliashberg, Zh. Eksp. Teor. Fiz. **48**, 1407 (1965) [Sov. Phys.—JETP **21**, 940 (1965)] to the evaluation of the electromagnetic response of minute metallic particles. Altshuler and Shklovskii studied statistical description of electron levels in the context of mesoscopic fluctuations and evaluated the level-density correlation function.

[9]L. P. Levy, G. Dolan, J. Dunsmuir, and H. Bouchiat, Phys. Rev. Lett. **64**, 2074 (1990); V. Chandrasekhar, R. A. Webb, M. B. Ketchen, W. J. Gallagher, M. J. Brady, and A. Kleinsasser (unpublished); R. A. Webb (private communication).

[10]H. F. Cheung, E. K. Riedel, and Y. Gefen, Phys. Rev. Lett. **62**, 587 (1989).

[11]V. Ambegaokar and U. Eckern, Phys. Rev. Lett. **65**, 381 (1990).

[12]H. Bouchiat and G. Montambaux, J. Phys. (Paris) **50**, 2695 (1989).

[13]G. Montambaux, H. Bouchiat, D. Sigeti, and R. Friesner (unpublished).

[14]B. L. Altshuler, Y. Gefen, and Y. Imry, Phys. Rev. Lett. **66**, 88 (1991).

[15]A. Schmid, Phys. Rev. Lett. **66**, 80 (1991).

[16]O. D. Cheishvili, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 206 (1988) [JETP Lett. **48**, 225 (1988)]; H. Fukuyama, J. Phys. Soc. Jpn. Lett. **58**, 47 (1989); E. Akkermans and B. Shapiro, Europhys. Lett. **11**, 467 (1990); R. A. Serota and S. Oh, Phys. Rev. B **41**, 10 523 (1990); see also N. Dupuis and G. Montambaux (unpublished).

[17]R. A. Serota, S. Feng, C. Kane, and P. A. Lee, Phys. Rev. B **36**, 5031 (1987); R. A. Serota, *ibid.* **39**, 12 640 (1988); D. B. Mast and R. A. Serota, *ibid.* **41**, 7850 (1990); R. A. Serota, J. Yu, and Y. H. Kim, *ibid.* **42**, 9724 (1990).

[18]L. D. Landau and E. M. Lifshitz, *Statistical Physics, Part 1* (Pergamon, New York, 1980).

[19]U. Sivan and J. Imry, Phys. Rev. Lett. **61**, 1001 (1988).

[20]H. Fukuyama, Prog. Theor. Phys. **45** 704 (1971).

[21]K. Nakamura and H. Thomas, Phys. Rev. Lett. **61**, 247 (1988).

[22]See, for instance, S. V. Vonsovsky, *Magnetism* (Wiley, New York, 1974).

[23]M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983); H. F. Cheung, Y. Gefen, E K. Riedel, and W. H. Shih, Phys. Rev. B **37**, 6050 (1988).

[24]P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill, New York, 1953).

[25]N. Trivedi and D. Browne, Phys. Rev. B **38**, 9581 (1988).

[26]I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products* (Academic, New York, 1965).

[27]L. G. Aslamazov and A. I. Larkin, Zh. Eksp. Teor. Fiz. **67**, 647 (1974) [Sov. Phys.—JETP **40**, 321 (1975)].

[28]B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interactions in Disordered Systems,* edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985].

[29]B. L. Altshuler, D. E. Khmelnitskii, A. I. Larkin, and P. A. Lee, Phys. Rev. B **22**, 5142 (1980).

[30]B. L. Altshuler and A. G. Aronov, Pis'ma Zh. Eksp. Teor.

Fiz. **33**, 515 (1981) [JETP Lett. **33**, 499 (1981)].
[31]U. Eckern (unpublished), (private communication).
[32]A. Yu. Zyuzin and R. A. Serota, J. Appl. Phys. (to be published); A. G. Aronov and A. Yu. Zyuzin, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 369 (1984) [JETP Lett. **39**, 444 (1984)].
[33]V. M. Gasparyan and A. Yu. Zyuzin, Fiz. Tverd. Tela (Leningrad) **27**, 1662 (1985) [Sov. Phys.—Solid State **27**, 999 (1985)].