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Dynamical screening of polar optical phonons in quantum wells

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We show that in polar semiconductors near a quantum well a mixed phonon-plasmon vibration can be localized. The dispersion of the vibration is investigated for the simple case where one can neglect the difference of the lattice properties within and outside the well. The magnetic-field dependence of the dispersion law is discussed. Under the same conditions, we have analyzed penetration of a longitudinal electric field of a polar optical vibration into a quantum well. It appears that dynamical screening decreases considerably the field within the well. We give physical reasons for the decrease and come to the conclusion that in most cases of interest the localized vibrations are more effective scatterers of the conduction electrons of the quantum well than the bulk optical phonons. This can explain the observed shift of the magnetophonon resonance in the quantum wells.

I. INTRODUCTION

The purpose of the present paper is to investigate theoretically dynamical screening of polar optical phonons by the electrons of a quantum well. Knowledge of the details of this screening is of basic interest for understanding many physical phenomena. Among these one may name transport phenomena both in the absence and especially in the presence of an external magnetic field, and various aspects of the hot-electron problem.

In this paper we shall investigate two effects brought about by the interaction of a two-dimensional electron gas (2DEG) with polar optical phonons. We shall show that the interaction between bulk optical phonons and 2D plasmons leads to the localization of coupled phonon-plasmon excitations in the plane of the quantum well. The frequency of such a coupled excitation differs from the limiting frequency of a longitudinal optical phonon Ω_l. A brief discussion of this effect has been given by the authors in Ref. 1.

Another effect of by no means less importance is the variation of the space-coordinate dependence of an optical vibration within the region of a quantum well. There is no alteration of the frequency of such a vibration due to interaction with the electrons, as the electrons are assumed to occupy only a small part of the whole volume. We shall see, however, that as a result of the interaction the scattering of the electrons by the bulk optical phonons can be substantially suppressed.

In the present paper we are going to discuss these two effects in detail. We shall exploit the simplest model that permits us to investigate these effects. We assume that the lattice properties of both components of the heterostructure are the same. It means that the phonons interact only with the electric field produced by the electrons of the quantum well. This means that we disregard another possible mechanism of localized phonon formation. By that we mean formation of localized phonons near a border between two different substances that have different dielectric and lattice properties, or phonon confinement between such borders.

II. POLARIZATION OF AN ELECTRON GAS IN A QUANTUM WELL

Working out a theory of dynamical screening of an optical vibration one should make allowances for the absence of homogeneity in the system. Let us consider a response of the electrons to an external perturbation whose potential has the form

$$\varphi(\mathbf{r})=\phi(z)e^{i(\mathbf{q}\cdot\mathbf{r}_\perp-\omega t)} . \tag{1}$$

Here the z axis is assumed to be perpendicular to the plane of the quantum well xy, whereas the wave vector $\mathbf{q}$ and vector $\mathbf{r}_\perp$ are within the plane of the quantum well.

The unperturbed states of an electron in a quantum well are characterized by the one-particle wave functions

$$\Psi_{i,\mathbf{k}}(\mathbf{r})=\frac{1}{\sqrt{S}}\psi_i(z)e^{i\mathbf{k}\cdot\mathbf{r}_\perp} . \tag{2}$$

Here $S=L_xL_y$ is the normalization area, and $\psi_i(z)$ are orthogonal and normalized wave functions describing electron motion across the well.

We confine ourselves to the first approximation in the electron-electron interaction. In this approximation the excess electron density due to a potential φ is given by

$$\delta\rho(\mathbf{r})=e^2\sum_{i,j}\langle j|\phi(z)|i\rangle\psi_i^*(z)\psi_j(z)\Pi_{ij}(\omega,q)e^{i(\mathbf{q}\mathbf{r}_\perp-\omega t)} . \tag{3}$$

Here $|i\rangle\equiv\psi_i(z)$, and $\Pi_{ij}(q,\omega)$ are elements of the 2DEG polarization operator matrix. In the absence of a magnetic field the matrix has the form

$$\Pi_{ij}(\omega,q)=\int \frac{2[n_i(\mathbf{k}-\mathbf{q})-n_j(\mathbf{k})]d^2k}{[E_j(\mathbf{k})-E_i(\mathbf{k}-\mathbf{q})-\hbar\omega-i\delta](2\pi)^2} , \quad (4)$$

where $n_i(\mathbf{k})$ and $E_i(\mathbf{k})$ are the occupation number and the energy in the state $|i\mathbf{k}\rangle$, respectively. The expression for $\Pi_{11}(q,\omega)$, the polarization operator of a 2DEG, for the case of Fermi statistics is well known (see, for example, Ref. 2).

Here we shall be interested in the case of Boltzmann statistics. In the region $q \ll q_T \equiv \sqrt{2mk_BT}/\hbar$ we have

$$\Pi_{ii}(q,\omega)=-\frac{n_s^{(i)}q^2}{m\omega^2} \quad \text{for } q \ll \omega/v_T , \quad (5a)$$

$$\Pi_{ii}(q,\omega)=\frac{n_s^{(i)}}{k_BT} \quad \text{for } q \gg \omega/v_T , \quad (5b)$$

where $n_s^{(i)}$ is the 2D electron concentration in the subband i. The full 2D electron concentration is in this case given by

$$n_s=\sum_i n_s^{(i)}=n_s^{(1)}\sum_i e^{-\Delta_{1i}/k_BT} \quad \text{where } \Delta_{ij}=E_j-E_i .$$

For off-diagonal elements $\Pi_{ij}(q,\omega)$ we have, for $i<j$, $k_BT \ll \Delta_{ij}$, $q \ll q_T$, and $q \ll |\Delta_{ij}-\hbar\omega|/v_T\hbar$,

$$\Pi_{ij}(q,\omega)=\frac{n_s^{(i)}}{\Delta_{ij}-\hbar\omega} , \quad (6a)$$

$$\Pi_{ji}(q,\omega)=\frac{n_s^{(i)}}{\Delta_{ij}+\hbar\omega} . \quad (6b)$$

One can see that for $k_BT \ll \Delta_{1j}$, $\Pi_{ij}(q,\omega)$ are exponentially small as compared to $\Pi_{1j}(q,\omega)$. Under this condition the induced charge density can be written as

$$\delta\rho(\mathbf{r})=e^2\sum_j \langle j|\phi(z)|1\rangle\psi_1^*(z)\psi_j(z)\tilde{\Pi}_{1j}(q,\omega)e^{i(\mathbf{q}\cdot\mathbf{r}_\perp-\omega t)} \quad (7)$$

where

$$\tilde{\Pi}_{11}(q,\omega)=\Pi_{11}(q,\omega), \quad \tilde{\Pi}_{1j}(q,\omega)=\Pi_{1j}(q,\omega)+\Pi_{j1}(q,\omega) .$$

In the presence of external magnetic field $\mathbf{H}\|z$ the electron wave function in the Landau gauge $\mathbf{A}=(Hy,0,0)$ is given by

$$\Psi_{i,N,k_x}(\mathbf{r})=\frac{1}{\sqrt{L_x}}\psi_i(z)\chi_N(y-y_0)e^{ik_xx} , \quad (8)$$

where $\chi_N(y)$ is the wave function of a harmonic oscillator with the quantum number N, $y_0=-l^2k_x$, $l=\sqrt{c\hbar/eH}$.

The polarization operator for a 2DEG is in this case given by

$$\Pi_{ij}^H(q,\omega)=\frac{1}{\pi l^2}\sum_{M=0}^{\infty}\sum_{N=0}^{\infty}F_{MN}(q)\frac{n_{i,N}-n_{j,M}}{\hbar\omega_c(M-N)+\Delta_{ij}-\hbar(\omega+i0)} . \quad (9)$$

Here $n_{i,N}$ is the occupancy of the Nth Landau level belonging to the ith subband:

$$F_{MN}(q)=\frac{M!}{N!}Q^{N-M}e^{-Q}[L_M^{N-M}(Q)]^2 \quad \text{for } N \geq M ,$$

$$F_{MN}(q)=F_{NM}(q) \quad \text{for } N<M .$$

$L_M^{N-M}(Q)$ are the generalized Laguerre polynomials, $Q=q^2l^2/2$.

For a nondegenerate electron gas our calculation gives

$$\Pi_{ij}^H(q,\omega)=\exp(-Q\coth\alpha)\sum_{N=\infty}^{\infty}\frac{n_s^{(i)}e^{-N\alpha}-n_s^{(j)}e^{N\alpha}}{\Delta_{ij}-\hbar(\omega-N\omega_c)} \times I_N\left[\frac{Q}{\sinh\alpha}\right] , \quad (10)$$

where $\alpha=\hbar\omega_c/2k_BT$, $\omega_c=eH/mc$. For $i=j$ we have

$$\Pi_{ii}^H(q,\omega)=\frac{4n_s^{(i)}}{\hbar}\exp(-Q\coth\alpha) \times\sum_{N=1}^{\infty}\frac{N\omega_c}{N^2\omega_c^2-\omega^2}\sinh(N\alpha)I_N\left[\frac{Q}{\sinh\alpha}\right] . \quad (11)$$

The case of degenerate electrons has been considered in Refs. 3 and 4. It is interesting to compare Eqs. (10) and (11) with the corresponding equations for a 3DEG (see Refs. 13 and 14):

$$\mathrm{Im}\Pi(q,\omega)=\sqrt{\pi}\frac{2n}{|q_z|v_T}\exp\left[-Q_\perp\coth\alpha-\frac{q_z^2}{2q_T^2}\right]\sinh\left[\frac{\hbar\omega}{2k_BT}\right]\sum_{N=-\infty}^{\infty}I_N\left[\frac{Q_\perp}{\sinh\alpha}\right]\exp\left[-\frac{(N\omega_c-\omega)^2}{q_z^2v_T^2}\right] , \quad (12a)$$

$$\mathrm{Re}\Pi(q,\omega)=\frac{n}{|q_z|v_T}\exp(-Q_\perp\coth\alpha)\sum_{N=-\infty}^{\infty}e^{N\alpha}I_N\left[\frac{Q_\perp}{\sinh\alpha}\right][W(x_+)+W(x_-)] . \quad (12b)$$

Here

$$Q_\perp=\frac{q_\perp^2 l^2}{2},\quad x_\pm=\frac{\hbar q_z^2/2m\pm\omega+N\omega_c}{|q_z|v_T},$$
$$W(x)=\exp(-x^2)\int_0^x\exp(u^2)du .$$

It is worthwhile to note that in the 2D case, in contrast to 3D one, there is no Landau damping of plasma oscillations. In other words, the polarization operator is real. This is due to the fact that the average electron velocity along any direction vanishes for a 2DEG. At the same time in the absence of magnetic field at $q\cong\omega/v_T$ the polarization operator has a considerable imaginary part.

Note also that in the presence of magnetic field for $ql\ll 1$

$$\Pi_{ii}^H(q,\omega)\simeq\frac{n_s^{(i)}q^2l^2\omega_c}{\hbar(\omega_c^2-\omega^2)}\propto q^2 ,$$

i.e., it is proportional to q^2 as in the absence of magnetic field. The quadratic dependence at small q is due to the term $I_1(Q/\sinh\alpha)$. The term with $I_0(Q/\sinh\alpha)$ vanishes as it enters along with $\sinh(N\alpha)$. For the same reason $\Pi_{ij}^H(q,\omega)$ [Eqs. (12)] are finite at $q=0$ as this time the term with $N=0$ is also finite.

III. COUPLED PHONON-MAGNETOPLASMON MODES: CONFINEMENT OF THE COUPLED MODES NEAR A QUANTUM WELL

A. Phonon-plasmon excitations at H=0

We wish to investigate principal characteristics of dynamical screening of polar optical phonons by a quasi-two-dimensional electron gas. For this purpose, we shall investigate the simplest model where the lattice properties of the two crystalline substances making the heterostructure are the same. In this way we completely disregard, in particular, phonon localization due to the different dielectric properties of the substances (see Refs. 5–8). We take into account the interaction of the electric field of an optical vibration with the plasma vibrations of the electrons of the quantum well. We shall be interested not only in the dispersion relation of the coupled modes, which has been studied in Refs. 9–11, but also in spatial distribution of the electric field.

If we disregard the region of very small values of $q\sim\omega/c$ (c being the light velocity), i.e., disregard retardation, then the potential of the electric field φ should satisfy the Poisson equation

$$\Delta\varphi=-4\pi\delta\rho+4\pi\,\mathrm{div}\mathbf{P} . \tag{13}$$

Here $\delta\rho$ is given by Eq. (3), and $\mathbf{P}$ is the lattice polarization. Neglecting the intrinsic space dispersion of the optical vibrations we can set $\mathbf{P}=[\epsilon(\omega)-1]\nabla\varphi/4\pi$ where $\epsilon(\omega)$ is given by

$$\epsilon(\omega)=\epsilon_\infty\frac{\omega^2-\Omega_l^2}{\omega^2-\Omega_t^2} .$$

Then one can present Eq. (13) in the form

$$\epsilon(\omega)\Delta\varphi=-4\pi\delta\rho . \tag{14}$$

To solve the equation we shall make one more simplification. Let us assume that $\hbar\omega$ is smaller than the energy gap between the two lowest subbands $\hbar\omega\ll\Delta_{21}$, and that the electrons have an appreciable concentration only in the lowest subband ($k_BT\ll\Delta_{21}$ for a nondegenerate case). This means that (as already indicated) we can use for $\delta\rho$ Eq. (7) instead of Eq. (3).

Let us assume further that the contribution of virtual transitions into higher subbands is negligibly small (in spite of the fact that the corresponding matrix elements of the polarization operator are relatively large, so that $|\tilde{\Pi}_{1j}|\gg|\tilde{\Pi}_{11}|$). In what follows we shall give reasons for this assumption. Then we are left with only one term in Eq. (7) instead of the whole sum over the subbands.

Let us look for the potential φ in the form of Eq. (1). Then Eq. (13) will have the form

$$\epsilon(\omega)\left[\frac{d^2\phi}{dz^2}-q^2\phi\right]=4\pi e^2\langle 1|\phi|1\rangle\Pi_{11}(q,\omega)\psi_1^2(z) , \tag{15}$$

whereas the spatial distribution of the field will be given by

$$\phi(z)=\phi_0\int_{-\infty}^{+\infty}dz'\psi_1^2(z')e^{-q|z-z'|} . \tag{16}$$

For a rectangular potential well of the width a with infinitely high walls we have

$$\psi_1(z)=\sqrt{2/a}\cos\frac{\pi z}{a} . \tag{17}$$

In this case the space distribution of the potential given by Eq. (14) can be written as

$$\varphi=\phi_0 e^{i(\mathbf{q}\cdot\mathbf{r}_\perp-\omega t)}\times\begin{cases}\left[\exp\left[\frac{qa}{2}\right]\times\left[1+\frac{a^2q^2}{2\pi}\cos^2\frac{\pi}{a}z\right]-\cosh qz\right], & |z|\le\frac{a}{2}\\ \sinh\frac{qa}{2}\times\exp\left[q\left[\frac{a}{2}-|z|\right]\right], & |z|>\frac{a}{2} .\end{cases} \tag{18}$$

The coordinate dependence of E_x and E_z is plotted in Fig. 1. It is worthwhile to note that there is a phase shift $\pi/2$ between E_x and E_z. This means that outside the well, the electric-field vector performs a circular motion. The field is localized within the well and falls off exponentially over the length q outside it. This conclusion has been made in Ref. 12 for the model of electron density uniformly distributed within the well. We shall see that

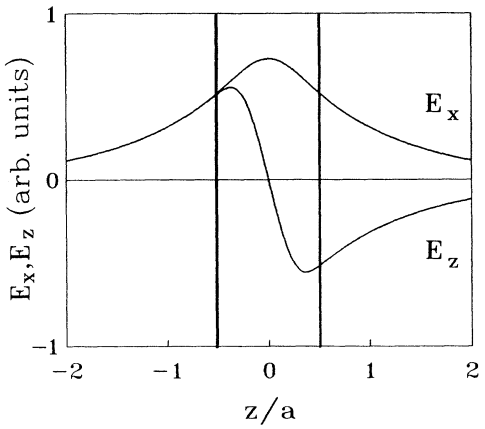

FIG. 1. Coordinate dependence of the amplitudes of x and z components of the electric field for the case of an infinitely deep rectangular potential well. The vertical lines indicate the boundaries of the well.

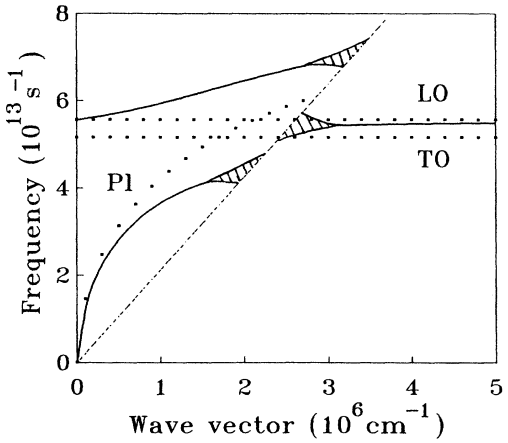

FIG. 2. Dispersion curves for the mixed modes in the absence of a magnetic field. The quantum well is assumed to be rectangular, with the infinitely high walls. The values of the parameters for GaAs correspond to $n_s=10^{12}$ cm^{-2}, $T=100$ K, $a=100$ Å. The regions of the strong damping are shaded. The dotted lines represent the spectrum of the plasma oscillations in the absence of the interaction, and the frequencies Ω_l and Ω_t (the horizontal dotted lines).

taking into account the true electron density distribution does not change this conclusion. Making use of the equation for $\delta\rho$ in the self-consistent-field approximation, we obtain the following dispersion relation:

$$\epsilon(\omega)+\frac{2\pi e^2}{q}\Pi_{11}(\omega,q)f(q)=0\ . \tag{19}$$

Here $f(q)$ is given by

$$f(q)=\int_{-\infty}^{+\infty}dz|\psi_1(z)|^2\int_{-\infty}^{+\infty}dz'|\psi_1(z')|^2e^{-q|z-z'|}\ , \tag{20}$$

while for $\Pi_{11}(q,\omega)$ we have Eq. (4).

In the limit $qa<1$ the dispersion relation does not differ in principle from that obtained in Ref. 9 for a plasmon-phonon excitation and in Ref. 10 for a magnetoplasmon-phonon excitation. To visualize it let us insert in Eq. (15) the explicit expressions for $\epsilon(\omega)$ and $\Pi_{11}(q,\omega)$. Then in the absence of external magnetic field we get for the frequencies of the coupled excitation

$$\omega_{\pm}^2=\frac{\Omega_l^2+\omega_p^2}{2}\pm\left[\left[\frac{\Omega_l^2+\omega_p^2}{2}\right]^2-\Omega_t^2\omega_p^2\right]^{1/2}\ , \tag{21}$$

where

$$\omega_p^2=\frac{2\pi e^2 n_s}{\epsilon_\infty m}qf(q)$$

or

$$\omega_-^2=\omega_p^2\epsilon_\infty/\epsilon_0,\quad \omega_+^2=\Omega_l^2+\omega_p^2(1-\epsilon_\infty/\epsilon_0)$$

$$\text{for } \omega_p\ll\Omega_l\ , \tag{22a}$$

$$\omega_-^2=\Omega_t^2,\quad \omega_+^2=\Omega_l^2-\Omega_t^2+\omega_p^2\quad \text{for } \omega_p\gg\Omega_l \tag{22b}$$

(cf. with Ref. 13 and see Fig. 2). One should remember that these equations are valid provided $q<q_T$ and $q<\omega/v_T$. If the last condition is violated, there is the Landau damping for the plasma vibrations which may result in a strong damping of the coupled modes. Furthermore, in the region $q\gg\omega/v_T$ there is only the phonon mode with the frequency given by

$$\omega^2=\frac{\Omega_l^2+\Omega_t^2 f(q)/qR_s}{1+f(q)/qR_s}\ ,$$

where

$$R_s=\frac{\epsilon_\infty k_B T}{2\pi e^2 n_s}.$$

B. Phonon-magnetoplasmon excitations

In the presence of magnetic field perpendicular to the layer, the spatial distribution of the electric field and the dispersion relation for the coupled vibrations are given by Eqs. (15), (17), and (19) with the only modification that in this case one should insert Eq. (11) for the polarization operator into Eq. (19). However, the physical situation changes significantly. Instead of a single branch of plasma vibrations there is a set of magnetoplasmon branches starting at $q=0$ from $\omega=N\omega_c$, where $N=1,2,\ldots$. In this case there is strong interaction between electron and phonon systems provided the optical-phonon frequency is almost equal to a multiple of the cyclotron frequency $\Omega_l\simeq N\omega_c$. These are the magnetophonon-resonance conditions (see Ref. 14). If $|N\omega_c-\Omega_l|\ll\omega_c$, i.e., near the Nth resonance, we have obtained the following dispersion relation for the coupled vibration:

$$\omega_{\pm}^2(q)=\frac{\Omega_l^2+\omega_{mp}^2}{2}\pm\left[\left[\frac{\Omega_l^2-\omega_{mp}^2}{2}\right]^2+(\Omega_l^2-\Omega_t^2)(\omega_{mp}^2-N^2\omega_c^2)\right]^{1/2}. \tag{23}$$

Here ω_{mp} is the frequency of the Nth magnetoplasma branch which is given by

$$\omega_{mp}^2=N^2\omega_c^2+\frac{2N\omega_p^2}{Q}\exp(-Q\,\mathrm{cotanh}\alpha)\times\sinh(N\alpha)I_N\left[\frac{Q}{\sinh\alpha}\right]. \tag{24}$$

Here, as above,

$$\omega_p^2=\frac{2\pi e^2 n_s}{\epsilon_\infty m}qf(q),\quad Q=\frac{q^2l^2}{2}.$$

The dispersion curves for the coupled excitations for various values of the magnetic field are given in Fig. 3 (cf. with Ref. 10 where such curves are given for Fermi statistics). It is interesting to note that if the cyclotron frequency is equal to the transverse-optical-phonon frequency Ω_t, one of the coupled branches appears to be dispersionless and has the frequency Ω_t [see Fig. 3(c)].

C. The limits of applicability

Now we should justify the assumption that one may neglect the high-subband contribution. To do this we

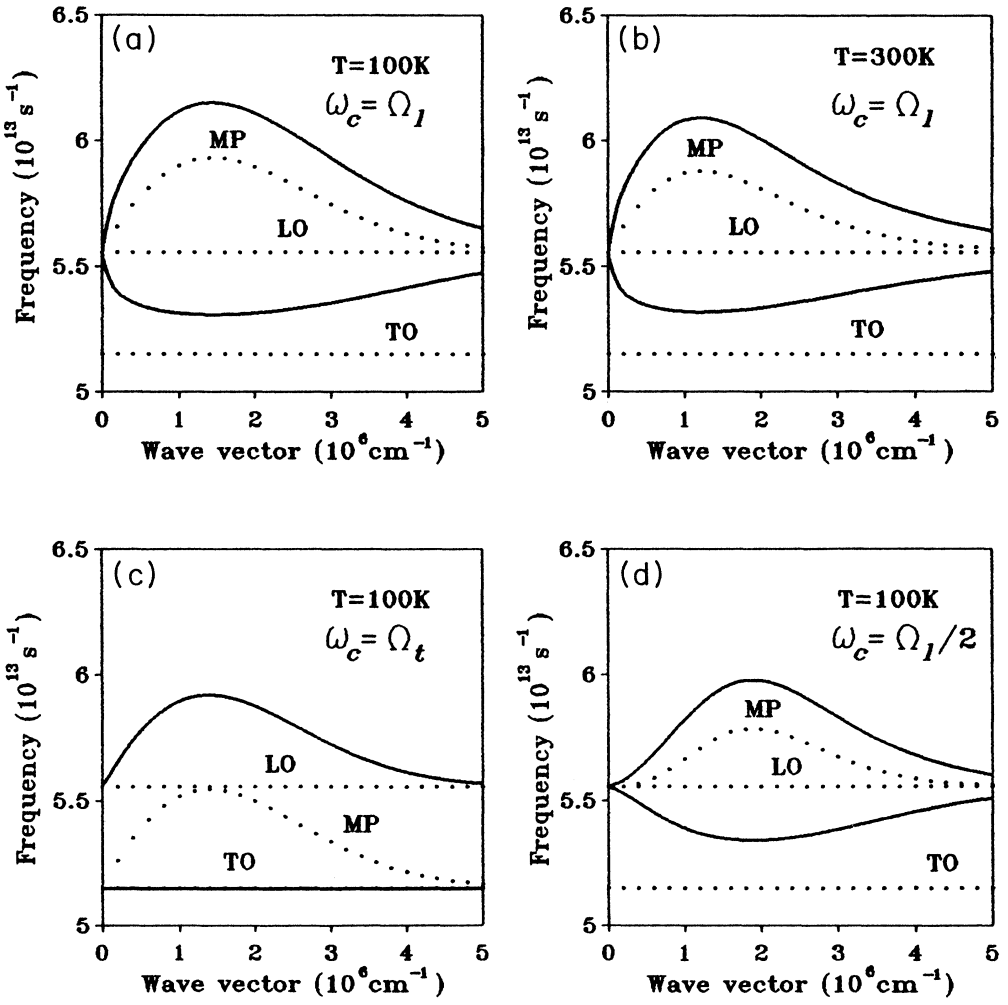

FIG. 3. Dispersion curves for the mixed modes in magnetic field. The dotted lines represent the spectrum of magnetoplasmal oscillations in the absence of the interaction, and the frequencies of the bulk longitudinal and transverse optical phonons, Ω_l and Ω_t. The values of the parameters for GaAs correspond to $n_s=2\times10^{11}$ cm^{-2}, $a=100$ Å. (a) $T=100$ K, $\omega_c=\Omega_t$; (b) $T=300$ K, $\omega_c=\Omega_l$; (c) $T=100$ K, $\omega_c=\Omega_t$, the lower branch coincides with Ω_t; (d) $T=100$ K, $\omega_c=\Omega_l/2$, which corresponds to the second maximum of the magnetophonon resonance.

write the full Eq. (14) where ρ is given by Eq. (7),

$$\epsilon(\omega)\left[\frac{d^2\phi}{dz^2}-q^2\phi\right]$$

$$=4\pi e^2\sum_j \langle j|\phi(z)|1\rangle\psi_1(z)\psi_j(z)\tilde{\Pi}_{1j}(q,\omega) . \quad (25)$$

This equation can be transformed to the form

$$\phi(z)=\frac{2\pi e^2}{q\epsilon(\omega)}\sum_j \langle j|\phi(z)|1\rangle\tilde{\Pi}_{1j}(q,\omega) \times\int_{-\infty}^{+\infty}dz'\psi_1(z')\psi_j(z')e^{-q|z-z'|} . \quad (26)$$

As $\phi(z)$ enters the right-hand side under the sign of the summation it is difficult to obtain the explicit coordinate dependence of $\phi(z)$. However, it is possible to get order-of-magnitude estimates of the contribution of the terms with $j\neq 1$ using as the zeroth approximation $\phi(z)$ [Eq. (16)] calculated without regard to these terms.

The term due to subband j is proportional to

$$\tilde{\Pi}_{1j}(q,\omega)\zeta_j(z)\int_{-\infty}^{\infty}\zeta_j(u)\psi_1^2(u)du , \quad (27)$$

where

$$\zeta_j(z)=\int_{-\infty}^{\infty}\psi_1(z')\psi_j(z')e^{-q|z-z'|}dz' .$$

One sees immediately that for spatially symmetric quantum wells only the subbands with odd values of j give contributions. For a rectangular potential well with infinitely high walls the contribution from the nearest subband of the same parity is as small as

$$\frac{\hbar^2\omega^2}{\Delta_{31}^2-\hbar^2\omega^2}$$

for $H=0$. For nonvanishing magnetic field H this contribution is as small as

$$\frac{\hbar(\omega_c^2-\omega^2)}{\omega_c(\Delta_{31}-\hbar\omega)}\frac{a^2}{4\pi^2 l^2}$$

(this estimate is valid for $N=1$, i.e., $|\omega_c-\omega|\ll\omega_c$). Note that even if it is necessary to take into account the next-subbands' contributions the general conclusion concerning the localization of the phonon-plasmon mode may still remain valid: it may be sufficient that $\psi_1(z)$ be well localized within the well to arrive at such a conclusion.

Now we turn to the discussion of the limits of applicability of the theory. It is natural to expect that in the limit where the electron concentration vanishes the coupled vibration should be delocalized. However, the derivation given above does not seem to demand a limitation on the electron concentration. The limitation will appear if we take into account the dispersion of the optical vibrations. To do this let us introduce into the equations of motion for the lattice the terms that are the second derivatives of the displacement by coordinates. For simplicity, let us consider these terms as isotropic. We have the following set of equations for the vector $\mathbf{u}$ proportional to the relative displacement of the sublattices and for the lattice polarization $\mathbf{P}$ (see Ref. 7):

$$\ddot{\mathbf{u}}=-\Omega_t^2\mathbf{u}+\sqrt{(\epsilon_0-\epsilon_\infty)/4\pi}\,\Omega_t\mathbf{E}-\beta_a^2\nabla(\nabla\cdot\mathbf{u})-\beta_b^2\Delta\mathbf{u} , \quad (28a)$$

$$\mathbf{P}=\sqrt{(\epsilon_0-\epsilon_\infty)/4\pi}\,\Omega_t\mathbf{u}+[(\epsilon_0-\epsilon_\infty)/4\pi]\mathbf{E} . \quad (28b)$$

The dispersion relation for the longitudinal optical phonon appears to be $\omega^2(q)=\Omega_l^2-\beta^2q^2$, where $\beta^2=\beta_a^2+\beta_b^2$. (For GaAs according to Ref. 7, $\beta\simeq 4.7\times10^5$ cm/s, i.e., it is of the order of the sound velocity.) Making use of Eqs. (13), (28a), (28b), and the approximations introduced above for the electrons, instead of (15), we end up with the equation

$$\epsilon_\infty\frac{\Omega_l^2-\omega^2-\beta^2(k^2+q^2)}{\Omega_t^2-\omega^2-\beta^2(k^2+q^2)}(k^2+q^2)\phi_k =-4\pi e^2\langle 1|\phi|1\rangle\Pi_{11}(q,\omega)\int_{-\infty}^{+\infty}e^{ikz}\psi_1^2(z)dz . \quad (29)$$

Here ϕ_k is the Fourier component of $\phi(z)$.

Transforming Eq. (29) we get

$$\phi(z)=\frac{4\pi e^2}{\epsilon(\omega)}\langle 1|\phi(z)|1\rangle\Pi_{11}(q,\omega) \times\int_{-\infty}^{+\infty}dz'\psi_1^2(z')\int_{-\infty}^{+\infty}\frac{dk}{2\pi}\frac{e^{ik(z'-z)}}{k^2+q^2} \times\frac{\Omega_t^2-\omega^2-\beta^2(k^2+q^2)}{\Omega_l^2-\omega^2-\beta^2(k^2+q^2)} . \quad (30)$$

The integrand in the integral over k in Eq. (30) has four poles, $k_1=\pm iq$, $k_2=\pm i[q^2+(\omega^2-\Omega_l^2)/\beta^2]^{1/2}$. Calculating the residues one should take into account only the poles corresponding to the values of k that are modulus much smaller than π/a_0 (a_0 being the lattice constant), which corresponds to the limits of applicability of expansion (28a). This condition actually concerns the second pair of poles $\pm k_2$, which gives the following limitation on the frequency ω:

$$\Omega_l^2-\beta^2q^2-\beta^2\pi^2/a_0^2<\omega^2<\Omega_l^2-\beta^2q^2+\beta^2\pi^2/a_0^2 . \quad (31)$$

In other words, these inequalities determine an interval of frequency variation where the spatial variation of $\phi(z)$ deviates from that given by Eq. (16) and, at the same time, can be described by our theory. However, even in this region one can discard the second pole term in Eq. (30) provided $|k_2|\gg k_1|$, and we are still left with Eq. (16) obtained above (for more details see Appendix B).

In regard to the electron concentration these considerations give the limitation

$$n>n_c , \quad (32)$$

where for $H=0$

$$n_c=\epsilon_c\frac{m\beta^2}{e^2a} \quad (33)$$

(here $\epsilon_c^{-1}=\epsilon_\infty^{-1}-\epsilon_0^{-1}$). In a magnetic field for $N=1$, i.e., provided the first Landau subband is involved, we have

$$n_c=\epsilon_c\frac{m\beta^2}{e^2a}\frac{\Omega_l^2-\omega_c^2}{\Omega_l^2} \tag{34}$$

(for a more general case, see again Appendix B).

The condition given by Eqs. (32) and (33) practically coincides with the estimate of our paper[1] obtained by qualitative considerations. For a typical quantum-well width 100 Å, the estimate gives, for $H=0$, $10^{10}-10^{11}$ cm^{-2}.

It is worthwhile to note that, comparing Eqs. (33) and (34), the minimum electron concentration sufficient to bring about localization of the excitations may be much lower in the vicinity of the magnetophonon resonance than far from the resonance.

IV. PENETRATION OF A BULK OPTICAL VIBRATION INTO A QUANTUM WELL

To consider this problem one should take into account the spatial dispersion of the dielectric susceptibility from the very beginning. Physically this is due to the fact that an energy transfer by a bulk vibration is determined exclusively by the dispersion.

The electrostatic potential of an optical vibration should satisfy Eq. (30) considered above. One should take into account, however, that in contrast to the localized excitations a bulk excitation has a z component of the wave vector. To stress this point, let us replace q by $q_\perp$ in Eq. (30) and take into account that the bulk phonon dispersion is given by

$$\omega^2(q)=\Omega_l^2-\beta^2(q_\perp^2+q_z^2)\ . \tag{35}$$

We wish to emphasize that this relation neglects small corrections proportional to the ratio of the quantum-well width to the linear dimension of the whole specimen. In other words, its validity presupposes a smallness of the relative volume occupied by a quantum well (or by quantum wells if there are several such wells unconnected to each other). Then one can write Eq. (30) in the form

$$\phi(z)=\frac{4\pi e^2}{\epsilon(\omega)}\langle 1|\phi(z)|1\rangle\Pi_{11}(q,\omega)\int_{-\infty}^{+\infty}dz'\psi_1^2(z')\int_{-\infty}^{+\infty}\frac{dk}{2\pi}\frac{e^{ik(z'-z)}[\Omega_t^2-\omega^2-\beta^2(k^2+q^2)]}{\beta^2(k^2+q_\perp^2)(q_z^2-k^2)}\ . \tag{36}$$

The optical vibration frequency is assumed to be quite near Ω_l, i.e.,

$$\beta^2q^2\ll\Omega_l^2-\Omega_t^2\ ,$$

where $q^2=q_\perp^2+q_z^2$. Therefore one can discard the term with β in the numerator of the integrand and at the same time retain it in the denominator because there it represents the poles of the integrand.

The integration gives (see Appendix C)

$$\phi(z)=-\frac{2\pi e^2}{\epsilon_\infty}\langle 1|\phi(z)|1\rangle\Pi_{11}(q,\omega)\frac{\Omega_l^2-\Omega_t^2}{\beta^2q^2}\times\int_{-\infty}^{+\infty}dz'\psi_1^2(z')\left[\frac{1}{q_\perp}e^{-q_\perp|z-z'|}+\frac{1}{q_z}e^{-iq_z|z-z'|}\right]\ . \tag{37}$$

We have obtained a particular solution of the Poisson equation (14). One should add to this a general solution of the homogeneous equation

$$\phi_0(z)=Ae^{-iq_zz}+Be^{iq_zz}+Ce^{-q_\perp z}+De^{q_\perp z}\ . \tag{38}$$

Obviously, one should set $C=D=0$ because otherwise the last two terms cannot be bounded in the whole interval of z variation. We wish to underline that, in the problem thus formulated, there are no other boundary conditions. The general equation for the potential of a bulk optical vibration is

$$\phi(z)=Ae^{-iq_zz}+Be^{iq_zz}-\frac{2\pi e^2\Omega_l^2}{\epsilon_c\beta^2q^2}\langle 1|\phi(z)|1\rangle\Pi_{11}(q,\omega)\times\int_{-\infty}^{+\infty}dz'\psi_1^2(z')\left[\frac{1}{q_\perp}e^{-q_\perp|z-z'|}+\frac{1}{q_z}e^{-iq_z|z-z'|}\right]\ . \tag{39}$$

We are interested in an optical vibration falling from the space region $z<0$. It means that at $z\to+\infty$ we are left only with a solution e^{-iq_zz}. The minus sign in the exponent means that the group velocity of the optical vibration is antiparallel to its wave vector as the dispersion is negative. In other words, the energy is transferred in the direction $-\mathbf{q}$. Therefore we shall set $B=0$.

The remaining terms in Eq. (39) have a simple physical meaning. The term Ae^{-iq_zz} describes the ingoing optical vibration while the integral term describes the response of the electron system. In order to find its relative value, i.e., to obtain a relation between A and $\langle 1|\phi(z)|1\rangle$, one should formulate the condition of self-consistency. It reads

$$A\langle 1|e^{-iq_z z}|1\rangle=\langle 1|\phi(z)|1\rangle \times\left[1+\frac{2\pi e^2\Omega_l^2}{\epsilon_c\beta^2q^2}\Pi_{11}(q,\omega) \times\left[\frac{f(q_\perp)}{q_\perp}+i\frac{f(iq_z)}{q_z}\right]\right]. \quad (40)$$

We wish to emphasize once again an important difference in treatment of the localized excitations and the bulk ones. In the first case one needs the condition of self-consistency to obtain the dispersion relation. In the second case the dispersion relation is considered as given, and one needs the self-consistency to find the constant A describing the response of the electron system to the bulk excitation. Such an excitation does not exist in the first case for the dielectric susceptibility $\epsilon(\omega)$ does not vanish at a frequency of the localized excitation. We feel that these physical considerations might be of a more general nature than the model of two components of heterostructure with the same lattice properties we are treating here.

Finally, for the potential of the bulk optical vibration, with regard to the screening, we have

$$\phi(z)=\phi_0\left[e^{-iq_z z}-\gamma\int_{-\infty}^{+\infty}dz'\psi_1^2(z') \times\left[\frac{1}{q_\perp}e^{-q_\perp|z-z'|}+\frac{1}{q_z}e^{-iq_z|z-z'|}\right]\right], \quad (41)$$

where

$$\gamma=\langle 1|e^{-iq_z z}|1\rangle\left[\frac{\epsilon_c\beta^2q^2}{2\pi e^2\Omega_l^2\Pi_{11}(q,\omega)}+\frac{f(q_\perp)}{q_\perp}+i\frac{f(iq_z)}{q_z}\right]^{-1}.$$

These relations are, however, rather complicated. It is easier to understand the physical meaning of the equations for the transmission and reflection coefficients of an optical vibration, T and R, that are given below. They can be obtained directly from Eq. (41). Let us assume that the electron concentration is so large that is satisfies the inequality (32). Then one may neglect the first term in large parentheses in the preceding equation for γ and we have

$$T=1-\frac{iq_\perp|\langle 1|e^{-iq_z z}|1\rangle|^2}{q_z f(q_\perp)+iq_\perp f(iq_z)}, \quad (42)$$

$$R=\frac{iq_\perp|\langle 1|e^{-iq_z z}|1\rangle|^2}{q_2 f(q_\perp)+iq_\perp f(iq_z)}. \quad (43)$$

For the long-wavelength phonons ($q_z a\ll 1, q_\perp a\ll 1$) the form factors in the denominators and the matrix elements in the numerators equal 1 and these equations take an especially simple form

$$|T|=\frac{q_z}{q}, \quad (44a)$$

$$|R|=\frac{q_\perp}{q}. \quad (44b)$$

It is also interesting to discuss, though briefly, the opposite limiting case of a layer filled with a classical electron gas. This corresponds to a large number of subbands of spatial quantization filled with the electrons. It appears that if the parameter β^2q^2/ω^2 is small enough for the reflection and transmission coefficients of the long-wavelength phonons, we get the same formulas (44). The amplitude of electric field within the layer appears to be smaller than outside, in proportion to this small parameter. This result can be visualized as follows.

Relatively small space-dispersion correction to the optical-phonon frequency means that the ion vibrations in the adjacent cells are almost independent. Therefore, as the screening shifts the eigenfrequency considerably so that it differs substantially from the frequency of a bulk phonon, the energy transfer from cell to cell also goes down considerably. The small parameter proportional to β^2q^2/ω^2 is a measure of how much the ions screened by the electrons are out of resonance.

The limiting expressions (44) themselves can also be understood easily. The electrons move freely within the xy plane under the action of an ac electric field. As a result, the corresponding field component is screened and does not take part in the excitation of the optical vibration behind the other side of the barrier.

At the same time in the limit $q_z a\ll 1$ the electrons do not move freely in the z direction and therefore cannot screen the z component of the field. As a result, the transmission coefficient appears to be proportional to q_z/q.

V. THE ELECTRON-PHONON INTERACTION

Scattering of a 3D electron by an optical phonon is described by the Fröhlich matrix element

$$|\langle\mathbf{k}'|\hat{H}_{e\text{-ph}}|\mathbf{k}\rangle|^2=C_q^2\delta_{\mathbf{k}',\mathbf{k}\pm\mathbf{q}}. \quad (45)$$

Here

$$C_q^2=\frac{2\pi\hbar e^2\Omega_l}{Vq^2\epsilon_c},$$

while V is the normalization volume.

To treat transport phenomena in quasi-2D systems a model is usually considered where the electrons are scattered by the bulk optical phonons. The motion of the latter is assumed to be unhindered by the existence of the well (see, for instance, Refs. 15 and 16). [We do not discuss here the papers where the boundaries between the components of a heterostructure were taken into account and the scattering by interface and film modes was considered (see, for example, Ref. 8)]. Then the scattering probability within the first subband would be given by

$$\sum_{q_z}|\langle\mathbf{k}'|\hat{H}_{e\text{-ph}}|\mathbf{k}\rangle|^2=\tilde{C}_{q_\perp}^2 f(q_\perp)\delta_{\mathbf{k}',\mathbf{k}\pm\mathbf{q}_\perp}, \quad (46)$$

where

$$\tilde{C}^{\,2}_{q_\perp}=\frac{\pi\hbar e^2\Omega_l}{Sq^2\epsilon_c} .$$

Here, as indicated above, S is the normalization area. The form factors $f(q_\perp)$ allow for a finite width of a quantum well. For a true 2D case $f(q_\perp)\equiv 1$.

These equations, however, by no means take account of the effect of screening discussed above. If one takes it into account the results are as follows.

The probability of electron scattering from a localized (magneto)plasma mode is given by the following matrix element squared (here and below ω stands for the frequency of the oscillations in question):

$$|\langle \mathbf{k}'|\hat{H}_{e-\text{loc ph}}|\mathbf{k}\rangle|^2 = \frac{\tilde{C}^{\,2}_q f(q)}{\left[1+\frac{2\pi e^2}{\epsilon_\infty q}\Pi_{11}(\omega,q)f(q)\right]^2}\delta_{\mathbf{k}',\mathbf{k}\pm\mathbf{q}} . \quad (47)$$

Comparison of this equation with the former one reveals an extra factor that is quite similar to the corresponding substitution taking into account the screening for the 3D case (see Refs. 13 and 14)

$$C_q \rightarrow \frac{C_q}{1+\frac{4\pi e^2}{\epsilon_\infty \hbar q^2}\Pi(\omega,q)} .$$

In our case taking into account the screening of a bulk optical vibration by the electrons of a quantum well results in the following substitution under the sign of summation in Eq. (46):

$$C_q \rightarrow \frac{C_q}{1+\frac{2\pi e^2\Omega_l^2}{\epsilon_c\beta^2 q^2}\Pi_{11}(q,\omega)\left[\frac{f(q_\perp)}{q_\perp}+i\frac{f(iq_z)}{q_z}\right]} . \quad (48)$$

One can easily derive this from Eq. (40) as the renormalization of the interaction is determined by the ratio

$$\frac{\langle 1|\phi(z)|1\rangle}{A\langle 1|e^{-iq_z z}|1\rangle} .$$

The ratio β^2q^2/Ω_l^2 is of the order $(qa_0)^2$, where a_0 is the lattice constant. For typical values of q the inverse parameter is very large. Therefore in the majority of typical situations the interaction with the bulk optical phonons is substantially suppressed.

This is due to the fact that the limiting phonon frequencies within and outside the well differ very much. As a result, a long-wavelength optical vibration (i.e., a vibration with $qa_0 \ll 1$) cannot penetrate into the well because there it is out of the resonance.

For a more general model where the heights of the potential wells are finite, some electron density should penetrate outside the walls and there is some interaction there. However, it should still be much smaller than in the theories disregarding the discussed effect.

VI. DISCUSSION AND CONCLUDING REMARKS

We have investigated localized (magneto)plasmon-phonon excitations and screening of the bulk optical vibrations within the quantum well using the same method.

We have discussed the model where the lattice properties of the components of the heterostructure do not differ too much. We have shown that under this assumption, provided the electron concentration exceeds some critical value given by Eq. (33) or (34), the scattering of electrons by bulk optical phonons is greatly suppressed. The main contribution to the scattering is given by the coupled excitations localized near the well.

To our opinion, the best way to observe these effects is to do this in an external magnetic field. There are several reasons for this. First, the restriction on the critical electron concentration is easier to meet in the presence of a magnetic field than in the absence of the field. Second, there is no Landau damping of the plasma oscillations in this case. Third, the spectrum of the coupled excitations is modified in such a way that they can be observed more easily.

In a number of experiments a shift of the magnetophonon resonance maxima has been observed in $Al_xGa_{1-x}As/GaAs$ quantum wells. This shift amounts to the apparent reduction of the optical-phonon frequency as compared to Ω_l (see Ref. 17). We believe that this shift can be due to the scattering of the electrons by the localized phonon-magnetoplasmon excitations. Although the components of the heterostructure have, in this case, different lattice properties, some qualitative conclusions of our theory should remain valid even for this case. The dispersion curves for the coupled vibrations show (see Fig. 3) that one of the branches has frequencies that are rather near those observed in the magnetophonon resonance experiments. The dispersion curve variation with the temperature is rather weak in the temperature interval 50–300 K [cf. Figs. 3(a) and 3(b)]. This also seems to be in agreement with the experimental data.

There is another class of systems where such effects could be observed. These are δ-doped semiconductors. Usually there are several levels of spatial quantization that are occupied in these systems. However, this should not modify qualitatively our predictions. Therefore we consider the experimental investigation of these systems to be very interesting.

On comparison of Eqs. (46) and (47) for the probabilities of the electron-phonon transition due to interaction with the bulk and localized phonons, an impression can be formed that they do not differ too much. Then there should not be much difference between the results of the theories taking into account these two types of scattering, although the theories themselves would look quite different.

We wish to emphasize that this should not be so in all cases where there is a substantial deviation of the phonon distribution function from the equilibrium. Here we mean both the drag of the electrons by the phonons and various "hot-electron" and "hot-phonon" phenomena. These phenomena are essentially determined by the mechanisms of the phonon relaxation which, in their

turn, depend on whether the phonon gas is two or three dimensional.

There is a possibility of the application of various optical methods, for instance, light scattering, for the investigation of the localized vibrations.

We have already mentioned that the minimum electron concentration that can bring about localization of the excitations may be lower near the magnetophonon resonance rather than far away from it. In this way one can control, by variation of the magnetic field, the Raman scattering of light by the localized vibrations.

Finally we wish to mention that one can also expect that confined optical phonons can be formed in the vicinity of other electron structures, such as 1D quantum wires and 0D quantum dots as various, more complicated, 2D structures in the absence and especially in the presence of external magnetic field.

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APPENDIX A

Here we calculate the polarization operator in the presence of magnetic field. To calculate polarization operator for a nondegenerate electron gas let us start with Eq. (9), which can be presented in the form

$$\Pi_{ij}^{H}(q,\omega)=G_i(x)+G_j(-x)\ , \tag{A1}$$

where

$$G_i(x)=\frac{1}{\pi l^2}\sum_{M=0}^{\infty}\sum_{N=0}^{\infty}F_{MN}(q)\frac{n_{i,N}}{\hbar\omega_c(M-N)+x}\ , \tag{A2}$$

$x=\Delta_{ij}-\hbar\omega$. Taking into account the explicit expression for $F_{MN}(q)$ [see the text after Eq. (9)] we get

$$G_i(x)=\frac{1}{\pi l^2}\sum_{M=0}^{\infty}\left[\sum_{N=0}^{M}\frac{n_{i,N}}{\hbar\omega_c(M-N)+x}\frac{N!}{M!}\times Q^{M-N}e^{-Q}[L_N^{M-N}(Q)]^2+\sum_{N=M+1}^{\infty}\frac{n_{i,N}}{\hbar\omega_c(M-N)+x}\frac{M!}{N!}\times Q^{N-M}e^{-Q}[L_M^{N-M}(Q)]^2\right]. \tag{A3}$$

Now we change the order of the summations in the first term in the large parentheses

$$\sum_{M=0}^{\infty}\sum_{N=0}^{M}\rightarrow\sum_{N=0}^{\infty}\sum_{M=N}^{\infty}\ ;$$

then we change the summation variable according to $K=|N-M|$ and take into account that for the Boltzmann statistics

$$n_{i,N}=\pi l^2 e^{-N\alpha}(1-e^{-\alpha})n^{(i)}\ ,$$

where $\alpha=\hbar\omega_c/2k_BT$. Then we can rewrite Eq. (A3) as

$$G_i(x)=(1-e^{-\alpha})n^{(i)}e^{-Q}\times\left[\sum_{K=0}^{\infty}\frac{e^{-K\alpha}Q^K}{x-K\hbar\omega_c}\sum_{M=0}^{\infty}\frac{M!}{(M+K)!}e^{-M\alpha}[L_M^K(Q)]^2+\sum_{K=1}^{\infty}\frac{Q^K}{x-K\hbar\omega_c}\sum_{N=0}^{\infty}\frac{N!}{(N+K)!}\times e^{-N\alpha}[L_N^K(Q)]^2\right]. \tag{A4}$$

We can calculate the sum over M and N in the first and second terms, respectively, using the sum rule

$$(1-t)^{-1}\exp\left[-\frac{2zt}{1-t}\right]z^{-\alpha}t^{-\alpha/2}I_\alpha\left[-\frac{2z\sqrt{t}}{1-t}\right]=\sum_{N=0}^{\infty}\frac{N!}{\Gamma(N+\alpha+1)}[L_N^\alpha(z)]^2t^N\ , \tag{A5}$$

where $\alpha>-1$, $|t|<1$ (see Ref. 18). Applying it to Eq. (A4) we end up with the following equation:

$$G_i(x)=n_s^{(i)}\exp(-Q\ \mathrm{cotanh}\alpha)\times\sum_{K=-\infty}^{+\infty}\frac{e^{-K\alpha}}{x-K\hbar\omega_c}I_K\left[\frac{Q}{\sinh\alpha}\right]. \tag{A6}$$

Substituting it into Eq. (A1) we get Eq. (10) for the polarization operator.

APPENDIX B

To understand in which way the additional pair of poles in Eq. (29) would change the resulting equations, we start with the simpler case $H=0$. If the electron concentration is sufficiently low so that $\omega_p\ll\Omega_i$, then in accordance with Eq. (21) there are plasmalike and phononlike branches with the frequencies

$$\omega_-^2=\omega_p^2\epsilon_\infty/\epsilon_0$$

and

$$\omega_+^2=\Omega_l^2+\omega_p^2(1-\epsilon_\infty/\epsilon_0)\ .$$

The branch ω_- is of no interest for us as it represents mainly oscillations of the electron density and at $\pi\beta/a_0\ll\Omega_l$ it obviously does not satisfy inequality (31). This means that we need not take into account the poles k_2 doing the contour integration in Eq. (30), and are left with the old result Eq. (16). It reflects the obvious fact

that the plasmalike oscillation is localized near the quantum well.

The situation with the phononlike branch is more intriguing. Calculating the integral in Eq. (30) one should take into account also the pole

$$k_2 = i\sqrt{q^2+(\omega^2-\Omega_l^2)/\beta^2} = \frac{i}{\beta}\sqrt{\beta^2 q^2+\omega_p^2\epsilon_\infty/\epsilon_c}$$

provided, of course, that the electron concentration is not so large that condition (31) is violated. We have

$$\int_{-\infty}^{+\infty}\frac{dk}{2\pi}\frac{e^{ik(z'-z)}}{k^2+q^2}\frac{\Omega_t^2-\omega^2-\beta^2(k^2+q^2)}{\Omega_l^2-\omega^2-\beta^2(k^2+q^2)} = \frac{\pi}{\omega^2-\Omega_l^2}\left[\frac{\omega^2-\Omega_t^2}{q}e^{-q|z-z'|}+\frac{\omega^2-\Omega_l^2}{q'}e^{-q'|z-z'|}\right], \tag{B1}$$

where

$$q' = |k_2| = \frac{1}{\beta}\sqrt{\beta^2 q^2+\omega_p^2\epsilon_\infty/\epsilon_c}\ .$$

One can see that if $q' \gg q$, i.e., $\omega_p^2\epsilon_\infty/\epsilon_c \gg \beta^2 q^2$ one can discard the second term in the large parentheses in Eq. (B1), which gives again the result (16). If, however, the inequality is reversed, the second term contributes almost as much as the first. Then the substitution of Eq. (B1) into Eq. (30), making use of self-consistency and a subsequent expansion in small parameter

$$\frac{\epsilon_\infty\omega_p^2}{\epsilon_c\beta^2 q^2}\ ,$$

gives the dispersion relation

$$1+\frac{\Omega_l\omega_p^2}{\omega^2\beta^2 q^2}=0\ . \tag{B2}$$

This has no solution describing propagating wave at all.

This means that if

$$\omega_p^2\epsilon_\infty/\epsilon_c \ll \beta^2 q^2\ ,$$

the localized vibration (15) having dispersion relation (21) cannot be considered as a first approximation and the assumption that the phononlike mode is localized is wrong. Substituting the explicit expression for ω_p into the inequality we get, for $q \approx 1/a$, the estimate (33).

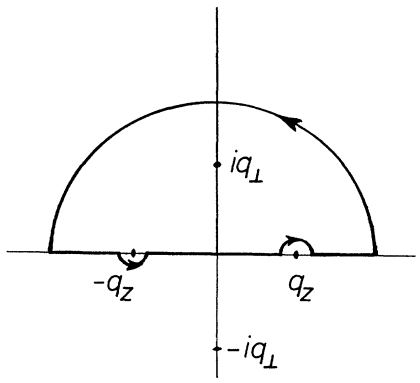

FIG. 4. Contour of integration in the complex plane of variable k.

If a magnetic field perpendicular to the layer is present it makes the analysis of Eq. (30) more complicated. The point is that in this case, according to Eq. (22), the frequency of the phononlike mode can be, depending on the magnitude of the magnetic field, either higher or lower than Ω_l. As a result, the pair of poles k_2 can appear either on the real or on the imaginary axis. One should take this into account calculating the contour integral. Omitting details, we shall give here only the result. The excitation is localized near a quantum well if the following condition is met:

$$\frac{\Omega_l^2(\omega_{mp}^2-N^2\omega_c^2)}{\Omega_l^2-N^2\omega_c^2}\frac{\epsilon_\infty}{\epsilon_c} \gg \beta^2 q^2\ . \tag{B3}$$

For $N=1$ this gives estimate (34).

APPENDIX C

To calculate the integral over k one should displace the poles $k=\pm q_z$ from the real axis. One can do this if one takes into account that function $1/\epsilon(\omega)$, being a so-called generalized susceptibility, has no singularities in the upper half plane of the complex variable ω (for a proof see, for instance, Ref. 19). It means that one should replace $(q_z^2-k^2)$ in the denominator of Eq. (36) by $(q_z^2-k^2-i\omega\delta)$ where $\delta>0$, $\delta\to+0$. Then for positive values of ω the poles are displaced as is shown in Fig. 4. The value of the integral is given by the residue at $k=-q_z$ and we obtain the result given in the text.

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[1]V. Gurevich and K. Shtengel, J. Phys. Condens. Matter **2**, 6323 (1990).
[2]T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).
[3]N. J. M. Horing and M. M. Yildiz, Ann. Phys. (N.Y.) **97**, 316 (1976).
[4]M. L. Glasser, Phys. Rev. B **22**, 472 (1980).
[5]F. Bechstedt and R. Enderlein, Phys. Status Solidi B **131**, 53 (1985).
[6]Klein, IEEE J. Quantum Electron. **QE-22**, 1760 (1986).
[7]M. Babiker, J. Phys. C **19**, 683 (1986).
[8]N. Mori and T. Ando, Phys. Rev. B **39**, 10 192 (1989).
[9]Xiaoguang Wu, F. M. Peeters, and J. T. Devreese, Phys. Rev. B **32**, 6982 (1985).
[10]H. C. A. Oji and A. H. MacDonald, Phys. Rev. B **34**, 1371 (1986).
[11]Xiaoguang Wu, Phys. Rev. B **38**, 4212 (1988).
[12]H. Sato and Y. Hori, Phys. Rev. B **39**, 10 292 (1989).
[13]Yu. A. Firsov and V. L. Gurevich, Zh. Eksp. Teor. Fiz. **41**, 512 (1961) [Sov. Phys.—JETP **14**, 367 (1962)].

[14]Yu. A. Firsov, V. L. Gurevich, R. V. Parfeniev, and I. M. Tsidilkovskii, in *Landau Level Spectroscopy,* edited by G. Landwehr and E. I. Rashba (Elsevier Science, New York, 1990).
[15]P. Price, Phys. Rev. B **30**, 2234 (1981).
[16]B. Ridley, J. Phys. C **15**, 5899 (1982).
[17]M. A. Brummel, R. J. Nicholas, M. A. Hopkins, J. J. Harris, and C. T. Foxon, Phys. Rev. Lett. **58**, 77 (1987); M. A. Brummell, D. R. Leadley, R. J. Nicholas, J. J. Harris, and C. T. Foxon, Surf. Sci. **196**, 451 (1988); R. J. Nicholas, M. A. Hopkins, M. A. Brummell, and D. R. Leadley, in *Interfaces, Quantum Wells and Superlattices,* Vol. 179 of *NATO Advanced Study Institute Series B: Physics,* edited by C. R. Leavens and R. Taylor (Plenum, New York, 1988), p. 243.
[18]E. Hill, Proc. Natl. Acad. Sci. **12** (1926).
[19]L. D. Landau and E. M. Lifsits, *Statistical Physics, Part I* (Pergamon, New York, 1980).