PHYSICAL REVIEW B VOLUME 44, NUMBER 16 15 OCTOBER 1991-II

Parametric excitation of Alfvén and helicon waves in a magnetoactive compensated semiconductor by microwave radiation

A. A. Mamun and M. Salimullah
Department of Physics, Jahangirnagar University, Savar, Dhaka, Bangladesh
(Received 10 September 1990; revised manuscript received 26 December 1990)

A theoretical investigation is made on the parametric excitation of electromagnetic Alfvén and helicon waves by microwave radiation in a magnetoactive solid-state plasma, viz., compensated germanium. The Krook model has been used to solve the Boltzmann transport equation to find the nonlinear response of electrons and holes in the highly collisional semiconductor plasma. It is noted that the growth rate of excitation of the helicon wave is one order of magnitude higher than that for the Alfvén-wave excitation for the same set of plasma parameters. The excitation of the Alfvén and helicon waves for parallel propagation is higher than that for the transverse propagation of the incident microwave radiation.

I. INTRODUCTION

There have been extensive studies on the interactions of electromagnetic waves in solid-state plasmas for various optical properties and reliable diagnostics of metals, semimetals, and semiconductors.[1–8] One may evaluate a number of parameters from the dispersion relation of the propagating electromagnetic waves in a semiconductor. On the other hand, a number of collective modes may be excited in a magnetized semiconductor plasma due to the nonlinear interactions at relatively large amplitudes of the propagating waves. Semiconductor properties may depend largely on these excited modes.[9–16]

In the presence of external electromagnetic fields of moderate power density, the electrons and holes having very small effective masses may attain relatively large drift velocities in a semiconductor plasma. Consequently, the cyclotron and plasma frequencies of electrons and holes may be much higher than the angular frequency of the incident wave. Thus, in the presence of an external static magnetic field, two typical plasma modes, viz., the electromagnetic Alfvén and helicon waves, may be excited by an external microwave radiation in a sample of the semiconductor. To the best of our knowledge, there are no rigorous theoretical studies on the excitation of electromagnetic waves by external microwave sources.

Usually the electromagnetic microwaves at the Alfvén- and helicon-wave frequencies are generated in solid-state devices using the negative differential conductivity (NDC) properties of semiconductors. This is called microwave generation by the Gunn effect.[17] When the dc (bias) field lies in the NDC regime, the semiconductor specimen is unstable for the space-charge density perturbations, where growth rate falls in the microwave range of frequencies. This instability is the basic principle underlying present-day solid-state microwave sources.[18] The nonlinearities causing the NDC effect arise due to the intervalley scattering of electrons, i.e., field-dependent effective mass and effective electron-phonon collision frequency of electrons.[19] In the present paper we have studied the nonlinear excitation of microwave, viz., Alfvén and helicon waves, through parametric instabilities in semiconductor plasmas. We have considered two standard geometries, viz., transverse and parallel propagation of the pump wave with respect to the external magnetic field. The appropriate kinetic equation has been employed to obtain the nonlinear response of electrons and holes in the collision-dominated semiconductor plasma. The growth rate of the parametric excitation of the electromagnetic Alfvén and helicon waves for the parallel propagation of the pump wave is higher than that for the transverse propagation. We study the parametric interaction of an external electromagnetic microwave with a low-frequency electrostatic perturbation mode, e.g., an ion-acoustic mode and the excited electromagnetic Alfvén and helicon waves which propagate along the external magnetic field in a compensated semiconductor, viz., Ge. Since the semiconductor plasma is highly collisional and since the Larmor radii of electrons and holes may be comparable or even larger than the wavelengths of the waves involved—the microwave, Alfvén or helicon wave, and short-wavelength ion-acoustic modes—we employ the kinetic equation, viz., the Boltzmann transport equation, for the description of the nonlinear response of plasma electrons and holes in the magnetized semiconductor. For the usual plasma parameters in compensated germanium the growth rates of the parametric instabilities generating microwaves turn out to be in the range of Alfvén- and helicon-wave frequencies in the semiconductor.

In Sec. II we solve the Boltzmann transport equation to obtain the nonlinear density fluctuation associated with the low-frequency electrostatic mode in the magnetized semiconductor when the external microwave propagates along the static magnetic field in the right-hand circularly polarized mode. To simplify the complicated collisional term in the Boltzmann equation we employ the well-known Krook model[20] for this average effective col-

$$f_0^0 = n_0^0 (m_e/2\pi k_B T_e)^{3/2} \exp(-m_e v^2/2k_B T_e) , \quad (7)$$

where k_B is the Boltzmann constant and v is the random speed of electrons.

By using the Krook model solution the collision term in Eq. (5) can be written as[20]

$$\left[\frac{\partial f^t}{\partial t}\right]_{\text{collision}} = -\nu(f^t - f_0^0) , \quad (8)$$

where ν is the average effective electron-phonon collision frequency in the semiconductor plasma.

The linear response of electrons due to the incident wave is obtained by solving the Boltzmann transport equation, Eq. (5). To include the first-order effect of the external magnetic field in the distribution function of the incident wave, we take the distribution function in the zero-order approximation of the unmagnetized plasma as $-ef_0^0 \mathbf{E}_0\cdot\mathbf{v}/k_B T_e(\nu - i\omega_0 + i\mathbf{k}_0\cdot\mathbf{v})$. Thus, the linear distribution function corresponding to the incident wave, the generated Alfvén wave $(\omega_1, \mathbf{k}_1)$, and the perturbation mode $(\omega, \mathbf{k})$ including the effect of external magnetic field in the plasma can be written from Eq. (5) as

$$f_0^L = \frac{-ef_0^0}{k_B T_e(\nu - i\omega_0 + i\mathbf{k}_0\cdot\mathbf{v})}\left[\mathbf{E}_0\cdot\mathbf{v} + \frac{\mathbf{E}_0\cdot\mathbf{v}\times\boldsymbol{\omega}_c}{\nu - i\omega_0 + i\mathbf{k}_0\cdot\mathbf{v}}\right] , \quad (9)$$

$$f_1^L = \frac{-ef_0^0}{k_B T_e(\nu - i\omega_1 + i\mathbf{k}_1\cdot\mathbf{v})}\left[\mathbf{E}_1\cdot\mathbf{v} + \frac{\mathbf{E}_1\cdot\mathbf{v}\times\boldsymbol{\omega}_c}{\nu - i\omega_1 + i\mathbf{k}_1\cdot\mathbf{v}}\right] , \quad (10)$$

$$f^L = \frac{-ef_0^0}{k_B T_e(\nu - i\omega + i\mathbf{k}\cdot\mathbf{v})}\left[\mathbf{E}\cdot\mathbf{v} + \frac{\mathbf{E}\cdot\mathbf{v}\times\boldsymbol{\omega}_c}{\nu - i\omega + i\mathbf{k}\cdot\mathbf{v}}\right] . \quad (11)$$

In the presence of an external homogeneous magnetic field $\mathbf{B}_s$ the interaction of the generated sideband $(\omega_1, \mathbf{k}_1)$ with the incident pump wave $(\omega_0, \mathbf{k}_0)$ produces a force varying with the frequency ω. This force, known as the ponderomotive force, amplifies and drives the low-frequency perturbation mode $(\omega, \mathbf{k})$. Thus, a nonlinear effect occurs in the low-frequency perturbation mode. The nonlinear distribution function corresponding to the low-frequency perturbation mode $(\omega, \mathbf{k})$ can also be obtained from the Boltzmann transport equation of the form

$$\frac{\partial f^t}{\partial t} + \mathbf{v}\cdot\nabla f^t + \nu f^t = \frac{e}{2m_e}\left[\mathbf{E}_0\cdot\nabla_v f_1^L + \mathbf{E}_1\cdot\nabla_v f_0^L + \frac{\mathbf{v}\times\mathbf{B}_0}{c}\cdot\nabla_v f_1^L + \frac{\mathbf{v}\times\mathbf{B}_1}{c}\cdot\nabla_v f_0^L\right] . \quad (12)$$

In a semiconductor plasma the average effective electron-phonon collision frequency is very large compared to the frequencies of the microwave radiation, the generated Alfvén wave, and the perturbation. Thus, we can reasonably take the approximations $\nu > \omega_{0,1}, \mathbf{k}_{0,1}\cdot\mathbf{v}$ and $\nu > \omega, \mathbf{k}\cdot\mathbf{v}$. Under these approximations the nonlinear distribution function corresponding to the perturbation wave $(\omega, \mathbf{k})$ can finally be written as

$$f^{\text{NL}} = -\frac{e^2 f_0^0 E_{0y} E_{1y}}{2m_e k_B T_e \nu^2}\left[1 - \frac{ikv_z}{\nu}\right]\left\{2(1+\beta) - \frac{2m_e}{k_B T_e}\left[\left[\beta + \frac{i\omega_c}{2\nu}(1-\beta)\right]v_x^2 + \left[1 - \frac{i\omega_c}{\nu}(1-\beta)\right]v_y^2 - i(1-\beta)\left[1 - \frac{i\omega_c}{\nu}\right]v_x v_y\right] - (1+\beta)\left[\left[\frac{k_0}{\omega_0} + \frac{k_1}{\omega_1}\right] + \frac{i\omega_c}{\nu}\left[\frac{k_0}{\omega_0} - \frac{k_1}{\omega_1}\right]\right]v_z\right\} . \quad (13)$$

The mass of electrons in semiconductors is very small. As a result, the electron-cyclotron frequency ω_c becomes large compared to the electron-phonon collision frequency. Thus, we can take $\omega_c > \nu$. Taking this approximation and integrating Eq. (13), we obtain the nonlinear density perturbation for electrons associated with the low-frequency electrostatic mode as

$$n_e^{\text{NL}} \simeq \frac{n_0^0 e^2 E_{0y}\omega_c k}{2m_e^2 \nu^4}(1+\beta)\left[\frac{k_0}{\omega_0} - \frac{k_1}{\omega_1}\right]E_{1y} . \quad (14)$$

Following the same procedure the nonlinear density perturbation for holes is given by

$$n_h^{\text{NL}} \simeq -\frac{n_0^0 e^2 E_{0y}\omega_c k}{2m_e^2 \nu^4}(1+\beta)\left[\frac{k_0}{\omega_0} - \frac{k_1}{\omega_1}\right]\frac{m_e^3}{m_h^3}E_{1y} . \quad (15)$$

In the presence of the external magnetic field the interaction of the electromagnetic incident wave and the electrostatic perturbation mode gives rise to a nonlinear response of electrons at the frequency of the generated Alfvén wave, which is given by lision frequency in the semiconductor plasma. We obtain expressions for current densities corresponding to the excited Alfvén and helicon waves. Then we derive the nonlinear dispersion relation for the low-frequency electrostatic mode. In Sec. III this nonlinear dispersion relation is solved to obtain expressions for the growth rates of this three-wave parametric excitation process. The decay modes—the low-frequency electrostatic ion-acoustic mode and the electromagnetic Alfvén or helicon wave propagating along the external magnetic field—grow at the same growth rate at the expense of the energy delivered to the semiconductor by the incident microwave.

In a similar way, in Sec. IV we study the three-wave parametric excitation process for the situation when the incident microwave propagates in the extraordinary mode in the direction transverse to the direction of the external magnetic field. We obtain the nonlinear dispersion relation and the growth rates following the procedures of the preceding sections. The numerical appreciation of the results obtained in Secs. III and IV and a brief discussion is presented in Sec. V. Finally, a conclusion is drawn in Sec. VI.

II. KINETIC ANALYSIS FOR NONLINEAR RESPONSE OF ELECTRONS AND HOLES

We consider the propagation of a right-hand circularly polarized electromagnetic microwave radiation $(\omega_0,\mathbf{k}_0)$ of angular frequency ω_0 and propagation vector $\mathbf{k}_0$ in a semiconductor plasma immersed in an external static magnetic field $(\mathbf{B}_s\|\mathbf{k}_0\|\hat{\mathbf{z}})$. The electric and magnetic fields $\mathbf{E}_0$ and $\mathbf{B}_0$ of this incident electromagnetic wave (the homogeneous pump wave) are described by[21]

$$\mathbf{E}_0=\mathbf{E}_0'\exp[-i(\omega_0 t-k_0 z)]\ ,\qquad \mathbf{B}_0=c\mathbf{k}_0\times\mathbf{E}_0/\omega_0\ , \tag{1}$$

where

$$E_{0x}=-iE_{0y}\ ,\qquad k_0=\frac{\omega_0}{c}\left[\epsilon_L-\frac{\omega_p^2(1+\omega_c/\omega_0)}{\omega_0^2-\omega_c^2}\right]^{1/2} . \tag{2}$$

Here, ϵ_L is the lattice dielectric constant, $\omega_p=(4\pi e^2 n_0^0/m_e)^{1/2}$ is the electron plasma frequency, $\omega_c=eB_s/m_e c$ is the electron cyclotron frequency, $-e$ is the electronic charge, n_0^0 is the equilibrium electron or hole density in the semiconductor, m_e is the average effective mass of electrons in the semiconductor, and c is the speed of light in vacuum.

We now assume a low-frequency perturbation $(\omega,\mathbf{k})$ which may be present in the semiconductor plasma due to an ion-acoustic mode or some other reasons. The oscillatory drift velocities of electrons and holes and the magnetic field of the pump wave $(\omega_0,\mathbf{k}_0)$ interact parametrically with the low-frequency perturbation $(\omega,\mathbf{k})$ and produce a high-frequency sideband $(\omega_1=\omega-\omega_0,\ \mathbf{k}_1=\mathbf{k}-\mathbf{k}_0)$. We choose the low-frequency perturbation to be purely electrostatic and is propagating along the direction of the magnetic field, that is, $\mathbf{E}=-\nabla\Phi$ and $\mathbf{k}\|\mathbf{k}_1\|\mathbf{B}_s\|\hat{\mathbf{z}}$.

In the presence of an external magnetic field the electromagnetic microwave whose frequency is less than both the electron plasma frequency and the electron cyclotron frequency, i.e., $\omega_0<\omega_p,\omega_c$, may propagate either as an Alfvén-wave mode or a helicon-wave mode in the semiconductor plasma. The microwave whose frequency is less than the electron plasma frequency and the electron cyclotron frequency, and which propagates along the direction of the external magnetic field with a velocity that depends only on the field strength $(\mathbf{B}_s)$ and the particle density (n_0^0), is known as the Alfvén wave. The Alfvén wave in this case is an elliptically polarized wave that propagates in the semiconductor plasma with the dispersion relation given by[22]

$$k_1=(|\omega_1|/V_A)(1+V_A^2/c^2)^{1/2}\ ,\qquad E_{1x}=i\beta E_{1y}\ . \tag{3}$$

Here, $V_A=B_s/(4\pi n_0^0 m_h)^{1/2}$ is the Alfvén speed in which m_h is the average effective mass of holes and β is the polarization constant.

The helicon wave is a large-amplitude low-frequency electromagnetic wave which may be propagated in a solid-state plasma under the conditions $\omega_1<\omega_c$ and $\omega_1\omega_c<\omega_p^2$. Similarly, if the generated wave is an elliptically polarized electromagnetic helicon wave, the condition $\omega_1<\omega_p,\omega_c$ must also be satisfied. The generated helicon wave propagates in the semiconductor plasma satisfying the following dispersion relation:[16,20,21]

$$k_1'=\frac{|\omega_1|}{c}\left[\epsilon_L+\frac{\omega_P^2}{|\omega_1|\omega_c}\right]^{1/2} . \tag{4}$$

The response of electrons of these coupled modes of the three-wave parametric process may be obtained by solving the Boltzmann transport equation[23]

$$\frac{\partial f^t}{\partial t}+\mathbf{v}\cdot\nabla f^t-\frac{e}{m_e}\left[\mathbf{E}+\frac{\mathbf{v}\times\mathbf{B}}{c}\right]\cdot\nabla_v f^t=\left[\frac{\partial f^t}{\partial t}\right]_{\text{collision}}, \tag{5}$$

where the superscript t refers to the total quantity involved. The total velocity distribution function f^t of the electrons in this three-wave interaction can be expanded as

$$f^t=f_0^0+f_0(\omega_0,\mathbf{k}_0)+f_1(\omega_1,\mathbf{k}_1)+f(\omega,\mathbf{k})\ , \tag{6}$$

where $f_{0,1}$ are the distribution functions corresponding to the incident wave $(\omega_0,\mathbf{k}_0)$ and generated sideband $(\omega_1,\mathbf{k}_1)$, $f(\omega,\mathbf{k})$ is the distribution function corresponding to the perturbation $(\omega,\mathbf{k})$, and f_0^0 is the equilibrium distribution function taken as a Maxwellian distribution function at temperature T_e:

$$f_1^{\rm NL}=\frac{e}{2m_e(\nu-i\omega_1+i\mathbf{k}_1\cdot\mathbf{v})}\left[\mathbf{E}_0^*\cdot\nabla_v f^L+\mathbf{E}\cdot\nabla_v f_0^{L*}+\frac{\mathbf{v}\times\mathbf{B}_0^*}{c}\cdot\nabla_v f^L\right], \tag{16}$$

where the asterisk denotes the complex conjugate of the quantity involved. Using $\nu>\omega_1$, $\mathbf{k}_1\cdot\mathbf{v}$, Eq. (16) reduces to

$$f_1^{\rm NL}\simeq\frac{ie^2f_0^0E_{0y}^*k\Phi}{2m_ek_BT_e\nu^2}(iv_x+v_y)\left\{\frac{k_0}{\omega_0}-\frac{m_ek}{2k_BT_e\nu}v_z^2-2\left[\frac{k_0k}{\omega_0\nu}+\frac{m_e}{k_BT_e}\left[1+\frac{i\omega_c}{2\nu}\right]\right]v_z\right\}. \tag{17}$$

Using the relation for the current density we obtain the nonlinear current density due to the motion of electrons for the generated Alfvén wave

$$\mathbf{J}_{1e}^{\rm NL}=-e\int\mathbf{v}f_1^{\rm NL}d\mathbf{v}\simeq\frac{n_0^0e^3E_{0y}^*k\Phi}{2m_e^2\nu^2}\left[\frac{k_0}{\omega_0}+\frac{k}{\nu}\right](\hat{\mathbf{x}}-i\hat{\mathbf{y}}). \tag{18}$$

Similarly, the nonlinear current density due to the motion of holes for the generated Alfvén wave is given by

$$\mathbf{J}_{1h}^{\rm NL}\simeq-\frac{n_0^0e^3E_{0y}^*k\Phi}{2m_h^2\nu^2}\left[\frac{k_0}{\omega_0}+\frac{k}{\nu}\right](\hat{\mathbf{x}}-i\hat{\mathbf{y}}). \tag{19}$$

Substituting the expression for the nonlinear current density in the wave equation for the generated electromagnetic Alfvén wave we obtain[24]

$$\overleftrightarrow{\mathbf{D}}_1\cdot\mathbf{E}_1=\frac{4\pi i\omega_1}{c^2}(\mathbf{J}_{1e}^{\rm NL}+\mathbf{J}_{1h}^{\rm NL}), \tag{20}$$

where $\overleftrightarrow{\mathbf{D}}_1$ is the dispersion tensor and is given by

$$\overleftrightarrow{\mathbf{D}}_1=k_1^2\overleftrightarrow{\mathbf{I}}-\mathbf{k}_1\mathbf{k}_1-(\omega_1^2/c^2)\overleftrightarrow{\epsilon}_1, \tag{21}$$

$\overleftrightarrow{\mathbf{I}}$ is the unit tensor of rank 2, and the linear dielectric tensor for the generated Alfvén wave is given by[22]

$$\overleftrightarrow{\epsilon}_1=\begin{bmatrix}\epsilon_L+\frac{\omega_p^2}{\omega_c^2}\left[1+\frac{m_h}{m_e}\right] & \frac{2i\omega_p^2}{\omega_c\omega_1} & 0\\ -\frac{2i\omega_p^2}{\omega_c\omega_1} & \epsilon_L+\frac{\omega_p^2}{\omega_c^2}\left[1+\frac{m_h}{m_e}\right] & 0\\ 0 & 0 & \epsilon_L-\frac{\omega_p^2}{\omega_1^2}\left[1+\frac{m_e}{m_h}\right]\end{bmatrix}. \tag{22}$$

Substituting the nonlinear density perturbations of electrons and holes, Eqs. (18) and (19), into the Poisson equation we obtain

$$\epsilon\Phi=-(4\pi e/k^2)(n_e^{\rm NL}-n_h^{\rm NL}). \tag{23}$$

Equations (20) and (23) represent two coupled equations, which are to be decoupled and solved to obtain the nonlinear dispersion relation for the low-frequency electrostatic mode $(\omega,\mathbf{k})$.

Using Eqs. (14), (15), (18), and (20) the nonlinear density perturbations for electrons and holes, $n_e^{\rm NL}$ and $n_h^{\rm NL}$, associated with the low-frequency electrostatic mode are finally given by

$$n_e^{\rm NL}\simeq-\frac{n_0^0e^3|E_{0y}|^2\omega_p^2\omega_c\omega_1^3k^2\epsilon_{1zz}}{4m_e^3\nu^6c^4}\times\frac{\Phi}{|\overleftrightarrow{\mathbf{D}}_1|}(1+\beta)\left[1-\frac{m_e^2}{m_h^2}\right]X, \tag{24}$$

$$n_h^{\rm NL}\simeq\frac{n_0^0e^3|E_{0y}|^2\omega_p^2\omega_c\omega_1^3k^2\epsilon_{1zz}}{4m_e^3\nu^6c^4}\times\frac{\Phi}{|\overleftrightarrow{\mathbf{D}}_1|}\frac{m_e^3}{m_h^3}(1+\beta)\left[1-\frac{m_e^2}{m_h^2}\right]X, \tag{25}$$

where

$$|\overleftrightarrow{D}_1| = -\frac{\omega_1^2}{c^2}\epsilon_{1zz}\left[\left[k_1^2 - \frac{\omega_1^2}{c^2}\epsilon_{1zz}\right]^2 - \frac{4\omega_p^4\omega_1^2}{\omega_c^2 c^4}\right],$$

$$\epsilon_{1xx} = \epsilon_L + \frac{\omega_p^2}{\omega_c^2}\left[1 + \frac{m_h}{m_e}\right],$$

$$\epsilon_{1zz} = \epsilon_L - \frac{\omega_p^2}{\omega_1^2}\left[1 + \frac{m_e}{m_h}\right], \tag{26}$$

$$X = \left[\frac{k_0}{\omega_0} + \frac{k}{\nu}\right]\left[\frac{k_0}{\omega_0} - \frac{k_1}{\omega_1}\right]\left[k_1^2 - \frac{\omega_1^2}{c^2}\left[\epsilon_{1xx} + \frac{2\omega_p^2}{\omega_c\omega_1}\right]\right].$$

Now eliminating Φ from the coupled equations (20) and (23) we obtain the following nonlinear dispersion relation for the low-frequency electrostatic mode:

$$\epsilon = \mu_1 / |\overleftrightarrow{D}_1| , \tag{27}$$

where the coupling coefficient μ_1 is given by

$$\mu_1 \simeq \frac{|V_{0y}/C_s|^2\omega_p^4\omega_c^3 C_s^2\epsilon_{1zz}}{4\nu^6 c^4}(1+\beta) \times \left[1 - \frac{m_e^2}{m_h^2}\right]\left[1 + \frac{m_e^3}{m_h^3}\right] X ,$$

$$|V_{0y}| \simeq eE_{0y}/m_e\omega_c , \tag{28}$$

and C_s is the ion acoustic speed in the semiconductor plasma.

III. GROWTH RATES OF THE INSTABILITY

In the absence of the pump wave $(\omega_0, \mathbf{k}_0)$, Eq. (27) would result in $\epsilon(\omega,\mathbf{k}) = 0$ and $|\overleftrightarrow{D}_1|(\omega_1,\mathbf{k}_1) = 0$ which represent the linear dispersion relations of the low-frequency perturbation $(\omega,\mathbf{k})$ and the generated Alfvén wave $(\omega_1,\mathbf{k}_1)$, respectively. However, in the presence of the pump wave the coupling coefficient μ_1 takes some finite value $(\mu_1 \neq 0)$ and the angular frequency ω of the low-frequency mode becomes complex. When the resonance conditions $\omega_1 = \omega - \omega_0$, $\mathbf{k}_1 = \mathbf{k} - \mathbf{k}_0$ are satisfied, we can expand ϵ and $|\overleftrightarrow{D}_1|$ as follows:[24]

$$\omega = \omega_r + i\gamma ,$$

$$\epsilon = i(\gamma + \gamma_L)(\partial\epsilon_r/\partial\omega) , \tag{29}$$

$$|\overleftrightarrow{D}_1| = i(\gamma + \gamma_{L1})(\partial|\overleftrightarrow{D}_1|_r/\partial\omega_1) ,$$

where the subscript r denotes the real part of the quantity involved, and γ_L and γ_{L1} are the linear damping rates of the low-frequency electrostatic mode and the electromagnetic Alfvén wave, respectively.

Using Eqs. (29), the dispersion relation, Eq. (27), gives the following relation for the growth rate of the three-wave parametric instability:

$$(\gamma + \gamma_L)(\gamma + \gamma_{L1}) \simeq \gamma_0^2 = -\mu_1 \Big/ \left[\frac{\partial\epsilon_r}{\partial\omega}\right]\left[\frac{\partial|\overleftrightarrow{D}_1|_r}{\partial\omega_1}\right], \tag{30}$$

where γ_0 is the growth rate in the absence of the linear damping of the decay waves.

Under the approximations, $\nu > \omega, \mathbf{k}\cdot\mathbf{v}$ and integrating the distribution function corresponding to the perturbation wave $(\omega,\mathbf{k})$, we obtain the linear electron and hole density perturbation n_e^L and n_h^L given by

$$n_e^L = (\chi_e k^2/4\pi e)\Phi ,$$
$$n_h^L = (\chi_h k^2/4\pi e)\Phi , \tag{31}$$

where the electron and hole susceptibilities of the semiconductor plasma, χ_e and χ_h, are given by

$$\chi_e = \frac{\omega_p^2}{\nu^2}\left[1 - \frac{\omega^2}{\nu^2} + \frac{2i\omega}{\nu}\right],$$
$$\chi_h = -\frac{\omega_p^2}{\nu^2}\left[1 - \frac{\omega^2}{\nu^2} + \frac{2i\omega}{\nu}\right]\left[\frac{m_e}{m_h}\right]. \tag{32}$$

Thus, the dielectric function[5–9] $\epsilon = \epsilon_L + \chi_e + \chi_h$ can be expressed as

$$\epsilon = \epsilon_L + \frac{\omega_p^2}{\nu^2}\left[1 - \frac{\omega^2}{\nu^2} + \frac{2i\omega}{\nu}\right]\left[1 - \frac{m_e}{m_h}\right]. \tag{33}$$

Therefore, with the help of Eqs. (26), (28), (30), and (33) the growth rate of the instability γ_0 in the absence of linear damping of the decay waves is given by

$$\gamma_0 \simeq \left[-\frac{|V_{0y}/C_s|^2\omega_p^2\omega_c^3\omega_1^2 C_s^2\epsilon_{1zz}}{16\nu^2\omega c^2}(1+\beta) \times \left[1 + \frac{m_e}{m_h}\right]\left[1 + \frac{m_e^3}{m_h^3}\right]\frac{X}{P+Q}\right]^{1/2}, \tag{34}$$

where

$$P = \frac{2\omega_p^2}{c^2}\left[\frac{2\omega_p^4}{\omega_c^2 c^2} + \epsilon_{1xx}\left[k_1^2 - \frac{\omega_1^2}{c^2}\right]\right]\left[1 + \frac{m_e}{m_h}\right],$$

$$Q = \epsilon_L\left\{k^4 - \frac{\omega_1^2}{c^2}\left[\frac{8\omega_p^4}{\omega_c^2 c^2} + \epsilon_{1xx}\left[4k_1^2 - 3\frac{\omega_1^2}{c^2}\epsilon_{1xx}\right]\right]\right\}, \tag{35}$$

and X is given by Eqs. (26). The linear damping rates γ_L and γ_{L1} are given by[24]

$$\gamma_L = \epsilon_i \Big/ \left[\frac{\partial\epsilon_r}{\partial\omega}\right] \simeq \nu ,$$

$$\gamma_{L1} = |\overleftrightarrow{D}_1|_i \Big/ \left[\frac{\partial|\overleftrightarrow{D}_1|_r}{\partial\omega_1}\right] = 0 , \tag{36}$$

where the subscript i denotes the imaginary part of the quantity involved. The overall growth rate γ of the instability is given by

$$\gamma = [(\gamma_L^2 + 4\gamma_0^2)^{1/2} - \gamma_L]/2 . \tag{37}$$

To obtain the growth rates γ_0, γ, and γ_L of the three-wave parametric instability for the helicon wave, the previous procedures are all applicable. We have to replace only the propagation constant $\mathbf{k}_1$ of the Alfvén wave by the propagation constant $\mathbf{k}_1'$ of the helicon wave, in Eq. (4). Thus following the same procedures the growth rates γ_0, γ, and γ_L can be obtained by replacing $\mathbf{k}_1$ by $\mathbf{k}_1'$.

IV. EXCITATION FOR TRANSVERSE PROPAGATION

In this section we consider the propagation of the incident electromagnetic microwave radiation $(\omega_0,\mathbf{k}_0)$ in a direction transverse to the direction of the static magnetic field in the semiconductor plasma $(\mathbf{k}_0\|\hat{\mathbf{x}},\mathbf{B}_s\|\hat{\mathbf{z}})$

$$\mathbf{E}_0=\mathbf{E}_0'\exp[-i(\omega_0 t-k_0 x)]\ , \tag{38}$$

where

$$k_0=\frac{\omega_0}{c}\left[\epsilon_L-\frac{\omega_p^2}{\omega_0^2}\frac{\omega_0^2-\omega_p^2}{\omega_0^2-\omega_p^2-\omega_c^2}\right]^{1/2},$$
$$E_{0x}=-i\beta_0 E_{0y}\ , \tag{39}$$
$$\beta_0=\frac{\omega_c}{\omega_0}\frac{\omega_p^2}{\omega_0^2-\omega_p^2-\omega_c^2}\ .$$

Due to the nonlinear interaction of this incident wave $(\omega_0,\mathbf{k}_0)$ with a low-frequency electrostatic mode $(\omega,\mathbf{k})$ in the semiconductor plasma, the Alfvén or helicon wave $(\omega_1,\mathbf{k}_1)$ will be excited with parametric conditions $\omega_1=\omega-\omega_0$, $\mathbf{k}_1=\mathbf{k}-\mathbf{k}_0$ satisfied. The excited Alfvén or helicon wave propagates in the z direction and the low-frequency mode is considered to be propagating in the xz plane.

Following the same procedures described in Sec. III the nonlinear density perturbations for electrons and holes associated with the low-frequency electrostatic mode are given by

$$n_{eT}^{\rm NL}\simeq\frac{n_0^0 e^2 E_{0y}E_{1y}\omega_c X_T}{2m_e^2\nu^4}\ ,$$
$$n_{hT}^{\rm NL}\simeq-\frac{n_0^0 e^2 E_{0y}E_{1y}\omega_c}{2m_h^2\nu^4}\frac{m_e}{m_h}X_T\ , \tag{40}$$

where the subscript T refers to the corresponding quantity for the transverse propagation and X_T is given by

$$X_T=\frac{k_0k_x}{\omega_0}\beta-\frac{k_1k_z}{\omega_1}(\beta_0+\beta)\ . \tag{41}$$

The nonlinear current densities in the generated Alfvén wave due to the motion of electrons and holes are given by

$$\mathbf{J}_{1eT}^{\rm NL}\simeq\frac{in_0^0 e^3E_{0y}^*k_0k_x\Phi}{2m_e^2\nu^2\omega_0}\left[\frac{\omega_c}{\nu}\hat{\mathbf{x}}-\hat{\mathbf{y}}\right],$$
$$\mathbf{J}_{1hT}^{\rm NL}\simeq\frac{in_0^0 e^3E_{0y}^*k_0k_x\Phi}{2m_h^2\nu^2\omega_0}\left[\frac{\omega_c}{\nu}\left[\frac{m_e}{m_h}\right]\hat{\mathbf{x}}+\hat{\mathbf{y}}\right]. \tag{42}$$

Decoupling and solving the coupled equations

$$\overleftrightarrow{\mathbf{D}}_1\cdot\mathbf{E}_1=\frac{4\pi i\omega_1}{c^2}(\mathbf{J}_{1eT}^{\rm NL}+\mathbf{J}_{1hT}^{\rm NL})\ ,$$
$$\epsilon\Phi=-\frac{4\pi e}{k^2}(n_{eT}^{\rm NL}-n_{hT}^{\rm NL})\ , \tag{43}$$

we obtain the following dispersion relation for the low-frequency electrostatic mode:

$$\epsilon_T=\mu_{1T}/|\overleftrightarrow{\mathbf{D}}_1|\ , \tag{44}$$

where the coupling coefficient μ_{1T} for the transverse incident wave is given by

$$\mu_{1T}\simeq-\frac{|V_{0y}/C_s|^2\omega_p^4\omega_c^3\omega_1^3k_0k_xC_s^2\epsilon_{1zz}}{4\nu^6\omega_0c^4k^2}\times\left[1-\frac{m_e^2}{m_h^2}\right]\left[1+\frac{m_e^3}{m_h^3}\right]X_TY_T\ , \tag{45}$$

where

$$Y_T=k_1^2-\frac{\omega_1^2}{c^2}\epsilon_{1xx}\ . \tag{46}$$

Following the preceding sections the growth rate in the absence of linear damping of the decay waves and the linear damping rates γ_{LT} and γ_{L1T} of the decay waves, are given by

$$\gamma_{0T}\simeq\left[\frac{|V_{0y}/C_s|^2\omega_p^2\omega_c^3\omega_1^2k_0k_xC_s^2\epsilon_{1zz}}{16\nu^2\omega_0\omega c^2k^2}\times\left[1+\frac{m_e}{m_h}\right]\left[1+\frac{m_e^3}{m_h^3}\right]\frac{X_TY_T}{P_T+Q_T}\right]^{1/2},$$
$$|\gamma_{LT}|\simeq\nu,\quad \gamma_{L1T}=0\ , \tag{47}$$

where

$$P_T=\frac{2\omega_p^2}{c^2}\left[\frac{2\omega_p^4}{\omega_c^2c^2}+\epsilon_{1xx}\left[k_1^2-\frac{\omega_1^2}{c^2}\right]\right],$$
$$Q_T=\epsilon_L\left\{k_1^4-\frac{\omega_1^2}{c^2}\left[\frac{8\omega_p^4}{\omega_c^2c^2}+\epsilon_{1xx}\left[4k_1^2-\frac{3\omega_1^2}{c^2}\epsilon_{1xx}\right]\right]\right\}. \tag{48}$$

X_T and Y_T are given in Eqs. (41) and (46), respectively. Therefore, the overall growth rate γ_T of the instability for transverse propagation is given by

$$\gamma_T=[(\gamma_{LT}^2+4\gamma_{0T}^2)^{1/2}-\gamma_{LT}]/2\ . \tag{49}$$

To obtain the results for the growth rates γ_{0T}, γ_{LT}, and γ_T for the helicon wave we have to replace only the propagation constant $\mathbf{k}_1$ of the Alfvén wave by that of the helicon wave $\mathbf{k}_1'$ in all final expressions involving $\mathbf{k}_1$. The expressions without involving $\mathbf{k}_1$ in the case of the helicon wave are exactly similar to those in the case of the Alfvén wave.

V. NUMERICAL RESULTS AND DISCUSSIONS

To obtain some numerical appreciation of the results of the present theory, we have made a calculation of the

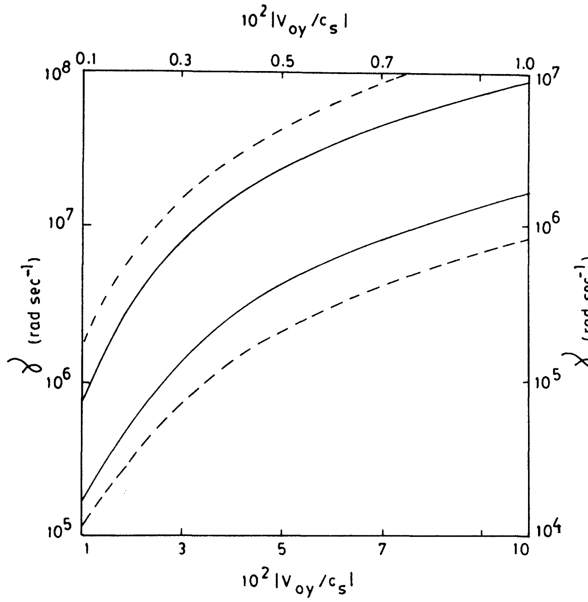

FIG. 1. The variation of the growth rates (γ) as a function of $|V_{0y}/C_s|$ for the following parameters in compensated Ge: $\epsilon_L=16$, $m_e=0.1m$ (where m is the free-electron mass), $m_h=3m_e$, $\nu=5\times10^{11}$ rad sec^{-1}, $\omega_0=10^{10}$ rad sec^{-1}, $C_s=10^6$ cm sec^{-1}, $n_0^0=10^{16}$ cm^{-3}, and $B_s=10$ kG for circular propagation and $B_s=50$ kG for transverse propagation. The left-hand side, lower scales, and dashed curves represent the growth rates for parallel propagation, whereas the right-hand side, upper scales, and solid curves represent the growth rates for the transverse propagation. The lower two curves are for the Alfvén wave and the upper two curves are for the helicon wave.

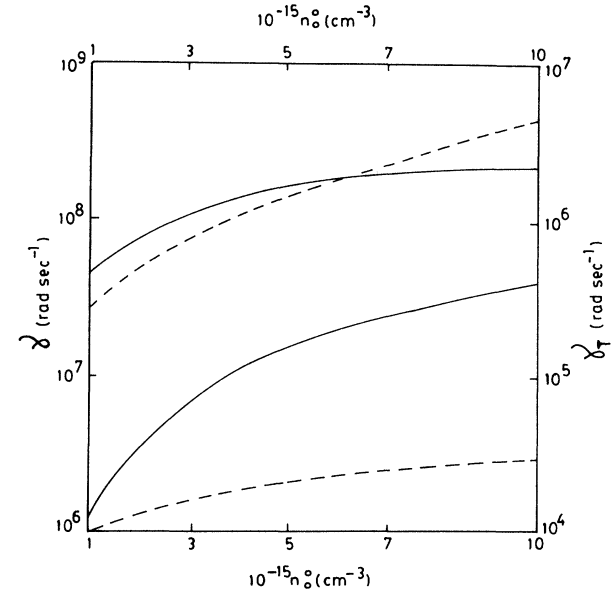

FIG. 2. The variation of the growth rates (γ) with n_0^0 for $|V_{0y}/C_s|=0.01$ for parallel propagation and $|V_{0y}/C_s|=0.5$ for transverse propagation. The other parameters and specifications are the same as in Fig. 1.

growth rates of the parametric excitation of Alfvén and helicon waves by the microwave radiation propagating in the direction parallel and perpendicular to the direction of the external static magnetic field $\mathbf{B}_s$ for the following plasma parameters in compensated Ge:

$$\epsilon_L=16,\quad m_e=0.1m\ ,$$

$$m_h=3m_e,\quad \nu=5\times10^{11}\ \mathrm{rad\,sec^{-1}}\ ,$$

$$\omega_0=10^{10}\ \mathrm{rad\,sec^{-1}}\ ,$$

$$C_s=10^6\ \mathrm{cm\,sec^{-1}}\ ,$$

$$|V_{0y}/C_s|=0.01-1.0\ ,$$

$$n_0^0=10^{15}-10^{16}\ \mathrm{cm^{-3}}\ ,$$

$$B_s=5-100\ \mathrm{kG}\ .$$

(Here m is the free-electron mass.) The results of the numerical calculations are shown in the form of curves in Figs. 1–3.

Figure 1 shows the variation of the growth rates as a function of the normalized and directed pump-induced drift velocity of electrons, $|V_{0y}/C_s|$ for different parameters of interest, for excitation of Alfvén and helicon waves in both circularly polarized and extraordinary modes of propagation of the incident microwave. The growth rate increases linearly with $|V_{0y}/C_s|$ for excitation of both Alfvén and helicon waves in both the modes of propagation of the incident microwave. The growth rates of the instability for the excitation of Alfvén and helicon waves in transverse propagation are relatively small in comparison with those for parallel propagation under the same set of plasma parameters. The growth rate for helicon-wave excitation is approximately one order higher than those for the Alfvén-wave excitation for both modes of propagation of the incident microwave. The behavior of all the curves is almost similar.

Figure 2 shows the variation of the growth rates (γ) with the equilibrium particle density n_0^0 for different plasma parameters of interest for excitation of Alfvén and helicon waves in parallel and transverse propagation of the incident wave. For parallel propagation of the incident microwave the growth rate of the instability for helicon-wave excitation increases faster than that for Alfvén-wave excitation, whereas for the transverse propagation of the incident wave the growth rate of the instability for Alfvén-wave excitation increases faster than that for the helicon-wave excitation. The growth rates for the excitation of helicon and Alfvén waves in the case of transverse propagation are relatively small compared with that in the case of parallel propagation for the same set of plasma parameters.

Figure 3 describes the variation of the growth rates for the excitation of Alfvén and helicon waves as a function of the external magnetic field B_s for different plasma parameters of interest for parallel and transverse propagation of the incident microwave. The growth rate of the instabilities for the excitation of Alfvén and helicon waves in the circularly polarized mode of propagation increases more rapidly than that for the extraordinary

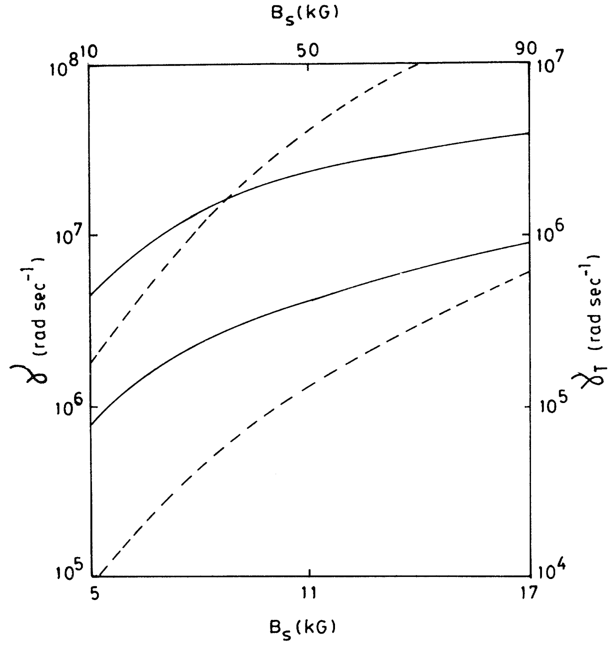

FIG. 3. The variation of the growth rates (γ) with B_s for $|V_{0y}/C_s|=0.01$ for parallel propagation and $|V_{0y}/C_s|=0.5$ for transverse propagation. The other parameters and specifications are the same as mentioned in Fig. 1.

mode of propagation of the incident wave. The growth rates in the extraordinary mode of propagation are relatively small compared to that for parallel propagation. The other characteristics of the curves in Fig. 3 are almost similar to those in Figs. 1 or 2.

VI. CONCLUSION

An electromagnetic right-hand circularly polarized microwave propagating in a compensated magnetoactive semiconductor is unstable and excites electromagnetic Alfvén and helicon waves efficiently. The growth rate of the instability for the three-wave excitation process increases with increasing the normalized and directed pump-induced drift velocity of electrons, $|V_{0y}/C_s|$, the equilibrium electron or hole density n_0^0, and the external magnetic field B_s in both the modes of propagation of the incident microwave radiation.

The parametric excitation of the Alfvén and helicon waves by microwave radiation propagating along the external magnetic field is higher than that for the propagation in a direction transverse to the direction of the external magnetic field.

The parametric excitation of the helicon wave is one order higher than that of the Alfvén wave in both the modes of propagation of the incident microwave radiation for the same set of plasma parameters. For typical parameters in compensated Ge:

$$\epsilon_L=16,\quad \nu=5\times10^{11}\ \text{rad sec}^{-1}\ ,$$

$$\omega_0=10^{10}\ \text{rad sec}^{-1},\quad n_0^0=10^{16}\ \text{cm}^{-3}\ ,$$

$$B_s=10\ \text{kG},\quad |V_{0y}/C_s|=0.01\ ,$$

the growth rates of the Alfvén and helicon waves for the parallel propagation turn out to be quite large ($\sim10^7$ rad sec^{-1} for the Alfvén wave and $\sim10^8$ rad sec^{-1} for the helicon wave) and those for the transverse propagation turn out to be quite small ($\sim10^2$ rad sec^{-1} for the Alfvén wave and $\sim10^3$ rad sec^{-1} for the helicon wave). The characteristics of all the curves are almost similar, i.e., the growth rate increases with $|V_{0y}/C_s|$, n_0^0, and B_s in nearly the same manner.

Thus, we notice that for the usual plasma parameters in compensated germanium the growth rates of the parametric instabilities exciting microwaves turn out to be in the range of Alfvén- and helicon-wave frequencies. From this study it may be suggested that an experiment may be designed in the laboratory to investigate the details of the excitation of the Alfvén and helicon waves through parametric instability and the results could be compared to those in the conventional methods in the bulk semiconductor. The details of the various aspects of the parametric instabilities might also be verified in the semiconductor plasma where the plasma parameters could be varied over a wide range of values without much difficulty.

[1]A. Hasegawa, J. Phys. Soc. Jpn. **20**, 1072 (1965).
[2]W. Beer, in *Semiconductors and Semimetals,* edited by R. K. Willardson (Academic, New York, 1966), Vol. 1, p. 417.
[3]S. J. Buchsbaum and P. M. Platzman, Phys. Rev. **154**, 395 (1967).
[4]J. R. Houck and R. Bowers, Phys. Rev. **166**, 397 (1968).
[5]M. C. Steele and B. Vural, *Wave Interactions in Solid State Plasmas, Advanced Physics Monograph Series* (McGraw-Hill, New York, 1969), p. 89.
[6]H. Hartnagel, *Semiconductor Plasma Instabilities* (Heinemann, London, 1969).
[7]S. Guha and S. Ghosh, J. Phys. Chem. Solids **40**, 775 (1979).
[8]Juras Pozhela, *Plasma and Current Instabilities in Semiconductors* (Pergamon, New York, 1981).
[9]S. Guha, P. K. Sen, and S. Ghosh, Phys. Status Solidi B **91**, K135 (1979).
[10]R. R. Sharma and V. K. Tripathi, Phys. Rev. B **20**, 748 (1979); M. Salimullah, F. Majid, and V. K. Tripathi, *ibid.* **23**, 869 (1981).
[11]S. Ghosh and V. K. Agrawal, Phys. Status Solidi B **112**, 119 (1982).
[12]Y. K. Pozhela, R. B. Totutis, and Z. K. Yankanshas, Radiophys. Quantum Electron. **27**, 552 (1984).
[13]S. Ghosh and S. Dixit, Phys. Status Solidi B **131**, 225 (1985).
[14]M. Longtin and B. U. O. Senerup, J. Geophys. Rev. **91**, 681 (1986).
[15]L. Stenflo and P. K. Shukla, Phys. Rev. B **39**, 12 941 (1989).
[16]M. Salimullah, S. M. Khurshed Alam, and M. N. Alam, Phys. Rev. B **35**, 3771 (1989).
[17]J. B. Gunn, Solid State Commun. **1**, 88 (1963).

[18] S. M. Sze, *Physics of Semiconductor Devices* (Wiley, New York, 1981), p. 637.
[19] M. Salimullah and V. K. Tripathi, Appl. Phys. **22**, 89 (1980); M. Salimullah, Indian J. Phys. A **62**, 255 (1988).
[20] D. R. Nicholson, *Introduction to Plasma Theory* (Wiley, New York, 1983).
[21] M. Salimullah, J. Appl. Phys. **54**, 2781 (1983).
[22] M. S. Sodha, A. K. Ghatak, and V. K. Tripathi, Prog. Opt. **13**, 179 (1976).
[23] N. A. Krall and A. W. Trivelpiece, *Principle of Plasma Physics* (McGraw-Hill, Kugakusha, 1973).
[24] C. S. Liu and P. K. Kaw, in *Advances in Plasma Physics*, edited by A. Simon and W. Thompsom (Wiley, New York, 1976), Vol. VI, p. 83.