Inelastic scattering in a doped polar semiconductor at finite temperature

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We calculate, using the Born approximation, the *finite*-temperature scattering rate for hot electrons injected into n-type doped GaAs for various doping densities and injected-electron energies. We treat the base region as a three-dimensional coupled electron-phonon system, so that the Coulomb and Fröhlich interactions with the injected electron are put on an equal footing. For moderate to high doping densities ($n \gtrsim 8\times10^{17}$ cm^{-3}) and injection energies of $E-E_F \gtrsim 100$ meV, our results indicate that (1) the increase in the scattering rate from $T=0$ K to $T=300$ K is typically less than a factor of 2, and (2) the scattering rates decrease with increasing doping density.

Recently there has been considerable interest in fabricating hot-electron transistors, in which electrons are injected from an emitter into a thin, doped, base region and collected at a collector on the other side of the base.[1-4] Typically, the base region is made of a high-mobility semiconductor such as GaAs. Because the base regions of these transistors are extremely thin, the electrons can be transferred from the emitter to the collector in a very short time. This fast response time makes this device an attractive candidate for a high-speed transistor of the future.

For this device to be viable as a practical transistor, the fraction of electrons α that traverse the base region ballistically (i.e., without scattering) must be close to unity. If the thickness of the GaAs layer is d, and the mean free path of the injected electron is l, then $\alpha \sim e^{-d/l}$. For the hot-electron transistor to be viable, one must have $l \gg d$, which can be achieved either by making l large or d small (or a combination of both).

Unfortunately, the base region must be doped to a fairly high density and must be of a minimum width (approximately 500 Å) for technological reasons (e.g., so that electrical contacts to the base can be made). The minimum width of the case sets a lower bound on d, while the presence of conduction electrons contributed by dopants in the base typically increases electron-electron scattering, which decreases l. With these restrictions, it is important to have a quantitative determination of the electron mean free path to determine if a hot-electron transistor is in fact technologically feasible.

Recently, the inelastic-scattering rates of such a doped base region in GaAs for zero temperature have been calculated.[5,6] However, for the hot-electron transistor to be a viable transistor in practical situations, its characteristics at *finite* temperature must be known. The typical energy scales for doped GaAs are given by the Fermi energy ($\sim$30–3000 K in the doping density range 10^{16}–10^{19} cm^{-3}), the longitudinal-optic phonon energy ($\sim$400 K), and the plasmon energy ($\sim$50–1600 K in the doping density range from 10^{16}–10^{19} cm^{-3}) so that a temperature variation of $T=0$–300 K may be quantitatively significant for electronic scattering rates (unlike in bulk metals, where $E_f, \omega_p \sim 10^4$ K). Rorison and Herbert[7] calculated the finite-temperature inelastic-scattering rates of electrons in n-type GaAs, using the plasmon-pole approximation for the dielectric function and assuming a Maxwell-Boltzmann distribution for the electrons. In contrast, the calculation reported here uses the exact finite-temperature dielectric and electron distribution functions (within the random-phase approximation) to obtain the finite-temperature inelastic-scattering rates.

As in Ref. 5 we assume the following. (1) The electrons are in a parabolic band corresponding to the Γ valley in GaAs, so that $E(\mathbf{k})=k^2/2m^*$, where m^* is 0.067 times the bare electron mass ($\hbar=1$ throughout this paper). (2) The electrons interact with each other via the direct Coulomb interaction and with bulk longitudinal-optic phonons of GaAs via the long-range Fröhlich interaction. The electrons and longitudinal-optic phonons in the base combine to form a coupled electron-phonon system with which the injected electrons interact. (3) The system is effectively three dimensional.[8] We calculate the scattering within the Born approximation,[9] in which the probability that an electron transfers a momentum $\mathbf{q}$ and energy ω to the coupled electron-phonon system is given by

$$P(\mathbf{q},\omega)=\frac{8\pi e^2}{q^2}\left[-\mathrm{Im}\frac{1}{\epsilon_t(\mathbf{q},\omega)}\right]\frac{1}{1-e^{-\beta\omega}}\ , \tag{1}$$

where $\beta=(k_B T)^{-1}$ is the inverse of the lattice temperature and ϵ_t is the total dielectric function of the coupled electron-phonon system. The total scattering rate of an electron with momentum $\mathbf{k}$, $\gamma(E(\mathbf{k}))$, is given by an integral of $P(\mathbf{q},\omega)$ over all $\mathbf{q}$ and ω which are consistent with energy-momentum conservation and the Pauli exclusion principle:

$$\gamma(E(\mathbf{k}))=\int\frac{d\mathbf{q}}{(2\pi)^3}\int d\omega\, P(\mathbf{q},\omega)\delta\left[\omega-\frac{\mathbf{k}\cdot\mathbf{q}}{m}-\frac{q^2}{2m}\right]\times[1-f_{eq}(\mathbf{k}-\mathbf{q})]\ , \tag{2}$$

where $f_{eq}(\mathbf{k})$ is the finite-temperature Fermi-Dirac distribution. The $\gamma(E(\mathbf{k}))$ obtained in this manner is equivalent to the scattering rate given by the many-body theory in which the electron self-energy is calculated to leading order in the total effective (i.e., screened) dynamical interaction, as was calculated in Ref. 5 for the zero-temperature case.

The $\epsilon_t(\mathbf{q},\omega)$, which determines the scattering probability through Eq. (1), is obtained by assuming that the polarizability of the coupled electron-phonon system is given by the sum of the polarizabilities of the electron and the phonons. This assumption implies that the total dielectric function is given by[10]

$$\epsilon_t=\epsilon_e+\epsilon_{\rm ph}-\epsilon_\infty \ , \tag{3}$$

where ϵ_e ($\epsilon_{\rm ph}$) is the bare electron (bare phonon) part of the dielectric function, and ϵ_∞ is the optical (high-frequency) dielectric constant at the material. Here, the optical dielectric constant simply renormalizes the Coulomb interaction by a factor of ϵ_∞^{-1} because the relevant frequencies in this problem are much lower than optical frequencies.

The standard dispersionless form for the phonon dielectric in a polar material is[10,11] $\epsilon_{\rm ph}(\omega)=\epsilon_\infty(\omega^2-\omega_{\rm LO}^2)/(\omega^2-\omega_{\rm TO}^2)$ where $\omega_{\rm LO}$ ($\omega_{\rm TO}$) are the bare longitudinal (transverse) polar optic-phonon frequencies. The electron dielectric function is given by $\epsilon_e=\epsilon_\infty-V_q\Pi_0(\mathbf{q},\omega)$, where $V_q=4\pi e^2/q^2$ is the Fourier transform of the Coulomb interaction and Π_0 is the finite-temperature random-phase approximation (Lindhard) form for electron polarizability[11]

$$\Pi_0(\mathbf{q},\omega)=2\int\frac{d\mathbf{k}}{(2\pi)^3}\times\frac{f_{eq}(\mathbf{k}+\mathbf{q}/2)-f_{eq}(\mathbf{k}-\mathbf{q}/2)}{E(\mathbf{k}+\mathbf{q}/2)-E(\mathbf{k}-\mathbf{q}/2)-\omega-i0^+} \ . \tag{4}$$

With these forms for $\epsilon_{\rm ph}$ and ϵ_e, the total dielectric function is

$$\epsilon_t(\mathbf{q},\omega)=\epsilon_\infty\frac{\omega^2-\omega_{\rm LO}^2}{\omega^2-\omega_{\rm TO}^2}-V_q\Pi_0(\mathbf{q},\omega) \ . \tag{5}$$

The dielectric function of this form can be rigorously derived from a many-body formalism in which the bare Coulomb and phonon interaction lines are screened by the electron gas within the random-phase approximation.[11]

For finite temperatures, $\Pi_0(\mathbf{q},\omega)$ was numerically evaluated using a formula given by Maldague.[12] This gave us $P(\mathbf{q},\omega)$, the scattering probability, which we then numerically integrated, using Eq. (2), to obtain $\gamma(E(\mathbf{k}))$.

In Fig. 1 we show the scattering rate and the corresponding inelastic mean free path of electrons as a function of their injected electron density and of temperature, for a density of $n=8\times10^{17}$ cm^{-3}. As the temperature increases, the scattering rate typically rises because of the increase in the number of thermally excited quasiparticles. The presence of these thermally excited quasiparticles enhances the scattering rates by opening up another channel for the scattering of injected electrons; i.e., scattering by absorption of a quasiparticle. However, the scattering rate at fixed k does not always necessarily increase monotonically as a function of temperature because the scattering rate $P(\mathbf{q},\omega)$ also changes with temperature, since $\Pi_0(\mathbf{q},\omega)$ varies with temperature through

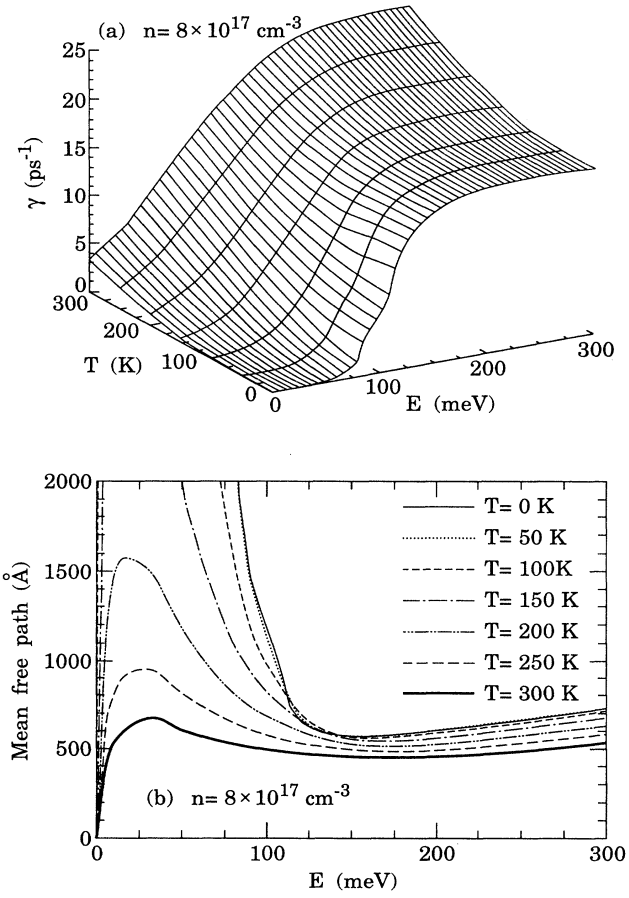

FIG. 1. (a) Scattering rates and (b) mean free path of injected electrons in n-type GaAs doped at $n=8\times10^{17}$ cm^{-3} as a function of injection energies and temperature. For this density, the Fermi energy and plasma frequency are $E_F=47$ meV ($\sim$550 K) and $\omega_p=38$ meV ($\sim$440 K), respectively. The scattering rates are calculated from the Born approximation, using a dielectric function which describes the doped GaAs as a coupled electron-phonon system. The mean free paths are given by $l(E)=v(E)/\gamma(E)$, where $v(E)=(2E/m^*)^{1/2}$ is the electron velocity. As the temperature increases, the scattering rate typically increases because electrons absorb thermally excited quasiparticles. The cusps in the scattering rate that are present at $T=0$ are smeared out as the temperature increases because the quasiparticle peaks of the coupled electron-phonon system are thermally broadened by Landau damping. For $E\gtrsim150$ meV, the scattering rate increases by approximately 30% when the temperature is changed from 0 to 300 K.

the change in $f_{eq}(\mathbf{k})$. In Fig. 2 we show the scattering rate as a function of temperature for a lower ($n=4\times10^{16}$ cm^{-3}) and for a higher ($n=1\times10^{19}$ cm^{-3}) doping density.

We note the following salient features. (1) As the temperature increases, the cusps in the scattering rates as a function of energy become rounded. This rounding occurs because the collective modes become increasingly broadened due to Landau damping as $f_{eq}(\mathbf{k})$ is thermally smeared. Therefore, the sharp emission and absorption thresholds in the scattering rates that occur at low temperatures, when the collective modes are undamped, become less sharp and distinct as the collective modes broaden with increasing temperature. (2) Typically as the temperature is increased from 0 to 300 K, the scattering rate at the typical energies at which the electrons are injected ($\gtrsim 100$ meV above the Fermi energy) increases by a factor of $\lesssim 2$ for the moderate to high densities ($n\gtrsim 8\times10^{17}$ cm^{-3}). At these densities, since E_F, $\omega_p > 300$ K, and thermal effects at 300 K do not alter the scattering substantially. (3) The scattering rate at the high doping density of $n=10^{19}$ cm^{-3} is significantly smaller (~ 5 ps^{-1}) than at the lower doping densities of $n=4\times10^{16}$ and 8×10^{17} cm^{-3} (~ 10–25 ps^{-1}). This decrease in $\gamma(E(\mathbf{k}))$ with increasing density might seem surprising at first, since at higher densities there are more quasiparticles available to scatter the electron. However, there are two other competing effects that play an important role: screening and degeneracy. At higher densities, the electron gas can screen more effectively, reducing the strength of the electron-electron and electron-phonon interactions. Also, as more states are filled by the electron gas, the phase space available to which an electron can be scattered decreases because of the Pauli exclusion principle. At lower energies, these two effects dominate, resulting in a decrease in γ with increasing density. A decreasing scattering rate with increasing dopant density has also been theoretically predicted in p-type bases[13] (i.e., in the presence of holes) and in high electric-field situations.[14] Our results indicate that this reduction in scattering rate persists even to room temperatures. Therefore, by increasing the dopant density considerably, one might be able to decrease the scattering rate and increase the transfer ratio α.

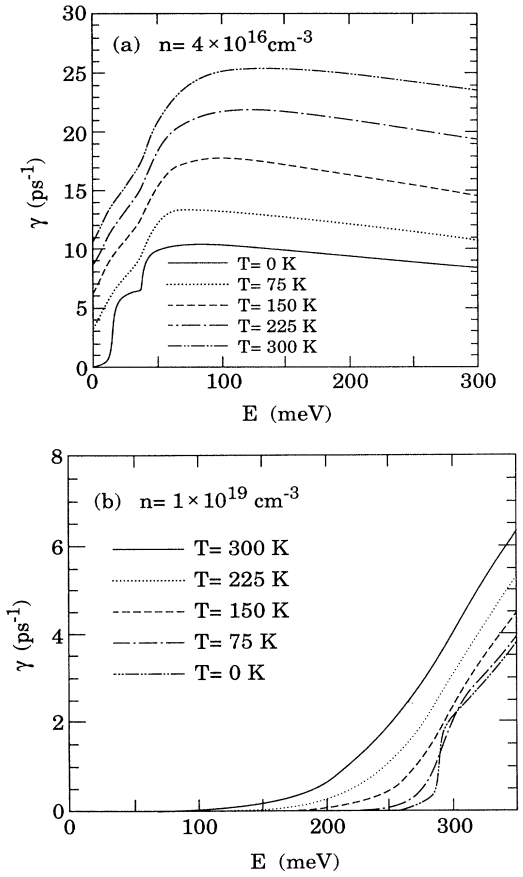

FIG. 2. Scattering rates γ of a coupled electron-phonon system for electron densities of (a) 4×10^{16} cm^{-3} and (b) 1×10^{19} cm^{-3}, as a function of energy, for various temperatures. For $n=4\times10^{16}$ cm^{-3}, $E_F=6$ meV (~ 70 K) and $\omega_p=8.5$ meV (~ 100 K), and for $n=1\times10^{19}$ cm^{-3}, $E_F=250$ meV (~ 2900 K) and $\omega_p=135$ meV (~ 1600 K). The scattering rates for $E\lesssim 300$ meV in the $n=1\times10^{19}$ cm^{-3} case is much smaller than that for the $n=4\times10^{16}$ and 8×10^{18} cm^{-3} cases. The γ decreases when the electron density is increased in spite of the increase in the number of quasiparticles with which the injected electron can scatter, because at high electron density (1) screening becomes more effective, weakening the electron-phonon and electron-electron interactions and (2) the number of states to which the injected electrons can be scattered is reduced because these states are already occupied by other electrons and hence are excluded by the Pauli principle.

It may be worthwhile to point out that for the doping densities considered in this paper, the effective r_s parameter ($r_s=[3/(4\pi n a^*)]^{1/3}$, where $a^*=\epsilon_\infty/(e^2m^*)$ is the effective Bohr radius) for GaAs is quite small ($r_s<1$) and the random-phase approximation should be quite valid for treating the electron-electron interaction.[11] Since the Fröhlich electron-phonon coupling is quite weak in GaAs (Ref. 15) (the Fröhlich coupling constant is around 0.07), we expect our leading-order calculation to be quite good for the electron-phonon interaction as well. Thus, from a theoretical viewpoint, the leading-order Born-approximation calculation as carried out in this paper is quite justified for obtaining the inelastic-scattering rate in doped GaAs.

In conclusion, using the Born approximation, we have calculated the finite-temperature inelastic-scattering rate of hot electrons in doped GaAs by including both electron-electron and electron–optic-phonon interactions on an equal footing. Our results indicate that, for typical injection energies and base doping densities, increasing the temperature from $T=0$ to 300 K increases the scattering rate by a factor of $\lesssim 2$. Finally, at moderate to high doping densities, the scattering rates for typical injection energies decrease with an increase in the doping density.

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[1]M. Heiblum, M. I. Nathan, D. C. Thomas, and C. M. Knoedler, Phys. Rev. Lett. **55**, 2200 (1985); M. Heiblum, M. V. Fishetti, W. P. Dumke, D. J. Frank, I. M. Anderson, C. M. Knoedler, and L. Osterling, *ibid.* **58**, 816 (1987); M. Heiblum, D. Galbi, and M. Weckwerth, *ibid.* **62**, 1057 (1989).

[2]A. F. J. Levi, J. R. Hayes, P. M. Platzman, and W. Wiegmann, Phys. Rev. Lett. **55**, 2071 (1985).

[3]S. Muto *et al.*, Electron. Lett. **21**, 555 (1985); N. Yokoyama *et al.*, Jpn. J. Appl. Phys. **24**, L853 (1985).

[4]A. P. Long *et al.*, Semicond. Sci. Technol. **1**, 63 (1986); J. R. Hayes *et al.*, Phys. Rev. Lett. **54**, 1570 (1985); M. A. Hollis *et al.*, IEEE Electron Devices Lett. **EDL-4**, 440 (1987).

[5]R. Jalabert and S. Das Sarma, Phys. Rev. B **41**, 3651 (1990).

[6]P. H. Beton, A. P. Long, and M. J. Kelly, Solid-State Electron. **31**, 637 (1988).

[7]J. M. Rorison and D. C. Herbert, J. Phys. C **19**, 6357 (1986).

[8]The justification for treating the base region as a three-dimensional system is that for widths of greater than 300 Å, many (> 10) subbands are occupied by the electrons forming the Fermi sea, as shown in Ref. 5.

[9]See, e.g., David Pines and Phillips Nozières, *The Theory of Quantum Liquids* (Benjamin, New York, 1966; reprinted by Addison-Wesley, Reading, MA, 1989), Vol. 1, Chap. 2.

[10]See, e.g., N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Saunders College, Philadelphia, 1976), pp. 515–518.

[11]See, e.g., Gerald D. Mahan, *Many Particle Physics* (Plenum, New York, 1981), Chap. 6.

[12]P. F. Maldague, Surf. Sci. **73**, 296 (1978); S. Das Sarma, Phys. Rev. B **33**, 5401 (1986).

[13]A. F. J. Levi, Electron. Lett. **24**, 1273 (1988).

[14]N. S. Wingreen, C. J. Stanton, and J. W. Wilkins, Phys. Rev. Lett. **57**, 1084 (1986); M. Combescot and R. Combescot, *ibid.* **59**, 375 (1987); N. S. Wingreen, C. J. Stanton, and J. W. Wilkins, *ibid.* **59**, 376 (1987).

[15]See, e.g., B. K. Ridley, *Quantum Processes in Semiconductors,* 2nd ed. (Oxford University Press, Oxford, 1988), pp. 62 and 63.