Conductivity in one-dimensional highly correlated electron systems

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The dc conductivity for the t-J model is exactly obtained at $t=J$ and for the Hubbard model in one dimension. Our calculation is carried out for arbitrary band filling by use of the finite-size-scaling method based on the Bethe-ansatz solution under twisted boundary conditions. The effective transport mass defined as the inverse of the conductivity serves as a sensitive probe of the metal-insulator transition. The effective mass is extremely enhanced near half-filling both for the t-J model at $t=J$ and the repulsive Hubbard model, which is a direct evidence of the metal-insulator transition due to the electron correlation. We observe that the large antiferromagnetic coupling J in the t-J model increases the effective mass. On the other hand, the conductivity in the attractive Hubbard model exhibits quite different properties from the repulsive models.

I. INTRODUCTION

The metal-insulator transition is a fundamental phenomenon in which strong electron correlations play an essential role. In order to characterize the insulating state, Kohn pointed out many years ago that the low-frequency conductivity is directly related to the shift of the energy levels due to twisted boundary conditions.[1] We note that this relation holds for systems in any dimension. Recently Shastry and Sutherland have reexamined the idea by studying one-dimensional Bethe-solvable models.[2] An interesting issue is to see if (or how) the effective transport mass defined from the conductivity grows to infinity as the correlated system approaches the insulating phase.

In this paper, based on the idea described above, we present exact results on the conductivity of the one-dimensional (1D) correlated electron systems. We consider the t-J model at $t=J$ and the Hubbard model with repulsive as well as attractive Coulomb interaction. In 1D correlated systems without any randomness, the electrons are accelerated by the external electric field. The acceleration behavior is affected by the correlation effects as for the other static quantities. Here we note that the periodicity of the lattice system also plays an important role in controlling the conductivity. In fact, for the Galilean invariant system, the conductivity should not be renormalzied at all even if the interaction is strong. On the contrary, as we shall see, the conductivity of lattice electrons is strongly affected by the interaction and vanishes at the point of the metal-insulator transition (Mott-Hubbard transition). Thus we can observe the electron correlation effect quite clearly by looking at the conductivity or the transport mass.[2-5]

In the next section, we illustrate how to calculate the conductivity by studying finite-size effects in the ground-state energy under twisted boundary conditions. For this purpose, we take the t-J model with $t=J$ at which the model can be solved exactly by the Bethe ansatz.[6,7] The same procedure is straightforwardly applied to the repulsive and attractive Hubbard models. The properties of the conductivity, or the effective transport mass, of these models are studied in detail in Sec. III. We will encounter the nonrenormalization nature of the conductivity of the t-J model and the repulsive Hubbard model in the low-concentration limit. This is further examined in Appendix A. In the attractive Hubbard model the conductivity exhibits the logarithmic behavior near half-filling. Simple renormalization-group analysis to explain this is given in Appendix B.

II. TWISTED BOUNDARY CONDITIONS AND CONDUCTIVITY

Let us consider the ring system of length N under periodic boundary conditions. We introduce a uniform time-dependent electric field along the 1D chain to study the response. The external vector potential A couples to the system through the Peierls phase factor in the hopping term of the Hamiltonian. The ground-state energy E depends on the phase $\phi \propto A$. An important consequence of the linear-response theory is[1]

$$\lim_{\omega\to 0}\omega\,\mathrm{Im}\sigma(\omega)=\frac{1}{N}\frac{\partial^2 E}{\partial\phi^2}\ , \tag{2.1}$$

where $\sigma(\omega)$ is the conductivity at frequency ω. Perturbation theory gives

$$E(\phi)-E(0)=D_c\phi^2/N+O(N^{-2})\ , \tag{2.2}$$

where D_c is the stiffness constant. These imply that the dc part of the conductivity is expressed as[2]

$$\mathrm{Re}\sigma(\omega)=\frac{2\pi e^2}{\hbar}D_c\delta(\hbar\omega)\ . \tag{2.3}$$

Now it is crucial, though elementary, to notice that the

uniform electric field is gauged away from the Hamiltonian. The external field effect is then incorporated by twisting boundary conditions on the electronic wave function, $\Psi(x+N)=e^{i\phi}\Psi(x)$. Consequently the conductivity, or the stiffness, can be evaluated from the energy shift under twisted boundary conditions.

Along this line of thought let us calculate the conductivity of the 1D t-J model. The Hamiltonian is well known,[8]

$$\mathcal{H}=-t\sum_{i,\sigma}(c_{i\sigma}^{\dagger}c_{i+1\sigma}+c_{i+1\sigma}^{\dagger}c_{i\sigma}) +2J\sum_{i}(\mathbf{S}_i\cdot\mathbf{S}_{i+1}-\tfrac{1}{4}n_in_{i+1})-\mu\sum_{i}n_i\ , \tag{2.4}$$

where $c_{i\sigma}$ ($\sigma=\uparrow$ or $\downarrow$) is the spin-σ electron annihilation operator at the ith site, $\mathbf{S}_i=c_{i\sigma}^{\dagger}\mathbf{S}_{\sigma\sigma'}c_{i\sigma'}$ with the spin-$\frac{1}{2}$ matrix $\mathbf{S}$, the number operator $n_{i\sigma}=c_{i\sigma}^{\dagger}c_{i\sigma}$, $n_i=n_{i\uparrow}+n_{i\downarrow}$, and μ is the chemical potential. Here the spin coupling $J>0$ and the double occupancy of every site is strictly forbidden.

The model can be solved by the Bethe ansatz for the special case of $t=J$.[6,7] We put $t=J=1$ for brevity and take the 1D lattice with even number of sites, N. Among the total of N_c electrons, the number of down-spin electrons is M. The Hamiltonian is diagonalized in two steps. First we seek for the wave function as a superposition of the plane waves characterized by the charge rapidity p_j ($j=1,2,\ldots,N_c$). The complete integrability is then ensured by the factorization of the scattering matrix (Yang-Baxter relation).[6,7] On imposing twisted boundary conditions with phase deviations $\phi_\uparrow$ ($\phi_\downarrow$) from periodic ones, for the up (down) -spin electrons, respectively, we reduce the problem to the ancillary one in the spin space. This problem can be solved by the generalized Bethe ansatz introducing the spin rapidity Λ_α ($\alpha=1,2,\ldots,M$). The resultant Bethe-Yang transcendental equations are given in terms of $k_j=\frac{1}{2}\cot(p_j/2)$ and Λ_α

$$\left[\frac{k_j+i/2}{k_j-i/2}\right]^N=\exp(i\phi_\uparrow)\prod_{\beta=1}^{M}\frac{k_j-\Lambda_\beta+i/2}{k_j-\Lambda_\beta-i/2}\ , \qquad j=1,2,\ldots,N_c$$

$$\prod_{j=1}^{N_c}\frac{\Lambda_\alpha-k_j+i/2}{\Lambda_\alpha-k_j-i/2}=-\exp(i\phi_\downarrow-i\phi_\uparrow)\prod_{\beta=1}^{M}\frac{\Lambda_\alpha-\Lambda_\beta+i}{\Lambda_\alpha-\Lambda_\beta-i}\ , \qquad \alpha=1,2,\ldots,M\ . \tag{2.5}$$

We note that the ground state is described by $N_c/2$ pairs of the charge complex solution $k_\alpha=\Lambda_\alpha\pm i/2$, which corresponds to the singlet ground state.[6]

The finite-size corrections to the ground-state energy due to twisting are now calculated, following the procedure explained in Ref. 4. In order to obtain the electric conductivity, it is sufficient to consider the twisting effects only on the charge sector. Thus we set $\phi_\uparrow=\phi_\downarrow=\phi$ hereafter. The logarithm of (2.5) for the ground-state solution with $k_\alpha=\Lambda_\alpha\pm i/2$ gives the expression

$$2N\tan^{-1}(\Lambda_\alpha)=2\pi J_\alpha+2\phi+2\sum_{\beta=1}^{M}\tan^{-1}(\Lambda_\alpha-\Lambda_\beta)\ , \qquad \alpha=1,2,\ldots,M \tag{2.6}$$

where $J_\alpha=(N_c+M+1)/2$ mod 1. The ground-state energy has the form

$$E=-2\sum_{j=1}^{N_c}\cos p_j=-2N_c+2\sum_{\alpha=1}^{N_c/2}\frac{1}{\Lambda_\alpha^2+1}\ . \tag{2.7}$$

We assume that the total electron number is even. Under periodic boundary conditions ($\phi_\uparrow=\phi_\downarrow=0$), the solutions k_j and Λ_α for the ground state are distributed symmetrically with respect to the origin, say $|\Lambda_\alpha|\geq Q$.[6] It is seen from (2.6) that the presence of the phase shift ϕ leads to a small asymmetry of the order of $1/N$ in the distribution of the solution Λ_α. Hence we have to compute the excitation energy originating from the asymmetric distribution.

In the thermodynamic limit, the distribution of Λ is obtained by taking the derivative of (2.6)

$$\rho(\Lambda)=\frac{1}{\pi}\frac{1}{\Lambda^2+1}-\int_q d\Lambda'\frac{1}{\pi}\frac{\rho(\Lambda')}{(\Lambda-\Lambda')^2+1} \tag{2.8}$$

in terms of which we express the energy density as

$$E/N=-2N_c/N+2\int_q d\Lambda\frac{\rho(\Lambda)}{\Lambda^2+1}\ . \tag{2.9}$$

Here we have introduced the abbreviation of the integral

$$\int_q=\int_{-\infty}^{Q^-}+\int_{Q^+}^{\infty}\ . \tag{2.10}$$

For subsequent calculation it is convenient to convert the above integral into $\int_{Q^-}^{Q^+}$. This can be performed straightforwardly by Fourier transform.[4] One finds

$$\rho(\Lambda)=R(\Lambda)+\int_{Q^-}^{Q^+}d\Lambda' R(\Lambda-\Lambda')\rho(\Lambda')\ , \tag{2.11}$$

with the kernel being

$$R(x)=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{\exp(-i\omega x)}{1+\exp|\omega|}\ . \tag{2.12}$$

Similarly the energy density can be rewritten in a succinct form

$$e(\phi)\equiv E(\phi)/N=\int_{Q^-}^{Q^+}R(\Lambda)\varepsilon(\Lambda)d\Lambda\ , \tag{2.13}$$

where the charge excitation energy $\varepsilon(\Lambda)$ is defined by[4]

$$\varepsilon(\Lambda)=2\pi R(\Lambda)-(2+\mu)+\int_{Q^-}^{Q^+}d\Lambda' R(\Lambda-\Lambda')\varepsilon(\Lambda')\ . \tag{2.14}$$

The parameters Q^+ and Q^- corresponding to the Fermi points are determined so that $\varepsilon(\Lambda)$ should vanish at these points

$$\varepsilon(Q^+)=\varepsilon(Q^-)=0\ . \tag{2.15}$$

As we have mentioned, the twist shifts the interval form $[-Q,Q]$ to $[Q^-,Q^+]$, and hence the ground-state energy deviates from the periodic one. Now the energy density $e(\phi)$ is expended up to $(Q^{\pm}\mp Q)^2$. We readily

verify $\partial e(\phi)/\partial Q^{\pm}=0$ using (2.15). The second derivative of the energy takes the form

$$\frac{\partial^2 e(\phi)}{\partial (Q^{\pm})^2}=\pm R(Q^{\pm})\frac{\partial \varepsilon(\Lambda)}{\partial Q^{\pm}}\bigg|_{\Lambda=Q^{\pm}} + \int_{Q^-}^{Q^+} R(\Lambda)\frac{\partial^2 \varepsilon(\Lambda)}{\partial (Q^{\pm})^2}d\Lambda \ . \tag{2.16}$$

For $\partial^2\varepsilon(\Lambda)/\partial(Q^{\pm})^2$ we derive the integral equation from (2.14)

$$\frac{\partial^2 \varepsilon(\Lambda)}{\partial (Q^{\pm})^2}=\pm\frac{\partial \varepsilon(\Lambda)}{\partial Q^{\pm}}\bigg|_{\Lambda=Q^{\pm}} R(\Lambda-Q^{\pm}) + \int_{Q^-}^{Q^+} d\Lambda' R(\Lambda-\Lambda')\frac{\partial^2 \varepsilon(\Lambda')}{\partial (Q^{\pm})^2} \ , \tag{2.17}$$

where we have again used the condition (2.15). Furthermore, for the first derivative of $\varepsilon(\Lambda)$, there exists the relation

$$\frac{\partial \varepsilon(\Lambda)}{\partial Q^{\pm}}\bigg|_{\Lambda=Q^{\pm}}=-\frac{\partial \varepsilon(\Lambda)}{\partial \Lambda}\bigg|_{\Lambda=Q^{\pm}} \tag{2.18}$$

in accordance with (2.15).[4] Inserting the iteration solution of (2.17) into (2.16), and making use of (2.18), we obtain

$$e(\phi)-e(0)\simeq\pi[\rho(Q)]^2 v_c[(Q^+-Q)^2+(Q^-+Q)^2] \ , \tag{2.19}$$

where the velocity of the charge excitation is defined by[9]

$$v_c=\frac{\varepsilon'(Q)}{2\pi\rho(Q)} \ . \tag{2.20}$$

Here $\rho(Q)$ and $\varepsilon'(Q)=\partial\varepsilon(\Lambda)/\partial\Lambda|_{\Lambda=Q}$ are those obtained from (2.11) and (2.14) with $Q^{\pm}=\pm Q$. We next express the change of the parameters $Q^{\pm}$ in terms of ϕ. Replacing the summation in (2.6) by the integral and differentiating both sides with respect to $Q^{\pm}$, we obtain the expression for $\partial\phi/\partial Q^{\pm}$. The same steps described above are then followed to find

$$Q^+-Q=Q^-+Q\simeq\frac{\xi(Q)}{2\pi\rho(Q)}\frac{\phi}{N} \ , \tag{2.21}$$

where a function $\xi(\Lambda)$ defined by

$$\xi(\Lambda)=1+\int_{-Q}^{Q} d\Lambda' R(\Lambda-\Lambda')\xi(\Lambda') \tag{2.22}$$

is called a dressed charge function[10,11] since it corresponds to the effective charge of the elementary charge excitation. Inserting (2.21) into (2.19) yields the final expression for the energy up to ϕ^2,

$$E(\phi)-E(0)\simeq\frac{[\xi(Q)]^2 v_c\phi^2}{2\pi N}+O(N^{-2}) \ . \tag{2.23}$$

Consequently we obtain the charge stiffness D_c in the conductivity

$$D_c=\frac{1}{2\pi}[\xi(Q)]^2 v_c \ . \tag{2.24}$$

This expression can be recast into other forms. First we note that the critical exponent α_c for the $4k_F$ oscillating part of the charge correlation function (k_F being the Fermi momentum) is given by $2[\xi(Q)]^2$.[9,12,13] We thus obtain the relation between the conductivity and the correlation exponent[3,5]

$$D_c=\frac{1}{4\pi}\alpha_c v_c \ . \tag{2.25}$$

It is not surprising to have the $4k_F$ exponent α_c since the energy shifts under consideration are closely related to those occurring for the charge-density excitation with $4k_F$ momentum transfer.[5,12,13] It is also possible to rewrite (2.25) by the bulk quantities only. It was shown in Refs. [5, 12, and 13] that α_c is expressed in terms of the compressibility χ_c and the specific-heat coefficient γ_c coming from the charge fluctuation part, $\alpha_c=2\pi^2\chi_c/(3\gamma_c)$, where the Boltzmann constant $k_B=1$. Consequently

$$D_c=\frac{\pi^2}{18}\frac{\chi_c}{\gamma_c^2} \ . \tag{2.26}$$

In a similar way, we calculate the finite-size corrections in the Hubbard model using the Bethe-ansatz solution. As a result we arrive at the same expression for the conductivity although each quantity (such as χ_c) takes different values from the t-J model (see Sec. III). To conclude this section, we emphasize that expressions (2.24)–(2.26) for the conductivity are not specific to the t-J model and the Hubbard model, but are valid generally for the 1D correlated electron systems, which is one of the universal aspects of Luttinger liquids introduced by Haldane.[10]

III. CONDUCTIVITY AND EFFECTIVE TRANSPORT MASS

We discuss the electron correlation effects on the electric conductivity. In order to see the correlation effects clearly, we introduce the effective transport mass[2] m^* defined by the relation

$$\mathrm{Re}\sigma(\omega)=\left[\frac{m^*}{m}\right]^{-1}\frac{4e^2}{\pi\hbar\chi_0}\delta(\hbar\omega) \ , \tag{3.1}$$

where χ_0 is the compressibility of the noninteracting system and $\nu=N_c/(2N)$ is the electron concentration ($\upsilon=\frac{1}{2}$ for half-filling). Notice that the mass enhancement factor m^*/m is unity irrespective of the interaction strength if the system is Galilean invariant. On the other hand, for a system undergoing the metal-insulator transition, the mass is expected to be greatly enhanced close to the insulating phase and eventually diverge at the transition point. Hence the effective transport mass will serve as a sensitive probe of the metal-insulator transition.

A. t-J model

We begin with the discussion of the t-J model. In Fig. 1, we show the result for the charge stiffness D_c obtained from (2.26). For comparison the static quantities χ_c and γ_c are also shown in Fig. 2.

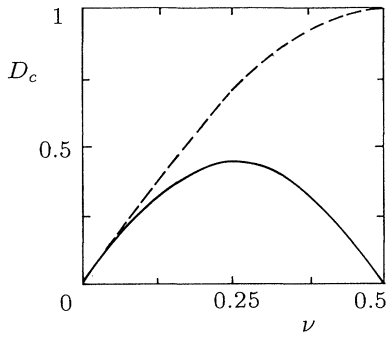

FIG. 1. Normalized charge stiffness D_c for the t-J model at $t=J$. The result for the noninteracting case is plotted (dashed line) for comparison.

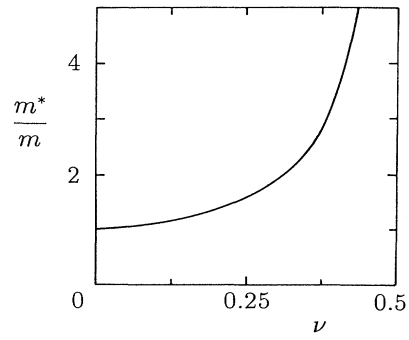

FIG. 3. Effective transport mass m^*/m for the t-J model at $t=J$.

In the low-density limit the conductivity vanishes linearly as the electron concentration decreases. This is simply due to the decrease of the carrier density just as for the noninteracting case. On the other hand, as $\nu \to \frac{1}{2}$, the conductivity decreases and finally vanishes at half-filling in contrast with noninteracting model. This reflects the fact that the system undergoes the metal-insulator transition due to strong electron correlation. We notice that the curve drawn in Fig. 1 is not symmetric about the $\nu=\frac{1}{4}$ point, though it looks almost symmetric [compare (3.4) and(3.5)].

The electron correlation effect on the conductivity can be seen more clearly if we plot the enhancement factor of the transport mass. See Fig. 3. We immediately observe that the mass is not renormalized ($m^*/m \simeq 1$) in the low-concentration limit. We refer to Appendix A for the analytic derivation of this result. Physically it means that in such a low-concentration limit, the system becomes effectively Galilean invariant and any strong interaction cannot modify the transport mass. This holds valid also for the repulsive Hubbard model but not for the attractive model, as will be studied in Sec. III B.

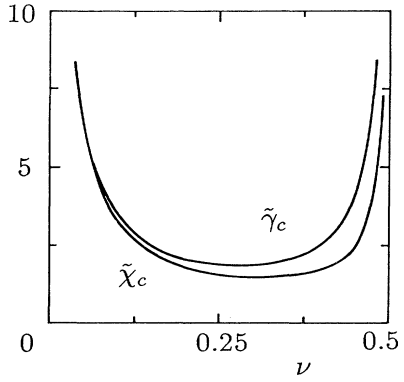

FIG. 2. Compressibility $\tilde{\chi}_c$ and specific-heat coefficient $\tilde{\gamma}_c$ for the t-J model at $t=J$. We plot the quantities normalized so that $\tilde{\chi}_c=\tilde{\gamma}_c=1$ at $\nu=\frac{1}{2}$ for the noninteracting case.

In the vicinity of half-filling, on the other hand, the effective mass of the electron becomes heavier and heavier, indicating the metal-insulator transition. The critical behavior in this region is now clarified. First we find that the compressibility near half-filling is given as[3]

$$\chi_c \simeq -\frac{2}{\pi}\frac{[R(0)]^2}{R''(0)}(1-2\nu)^{-1} , \tag{3.2}$$

where $R(0)=(1/\pi)\ln 2$, $R''(0)=-(3/2\pi)\zeta(3)$ with ζ being the Riemann zeta function. The dressed charge becomes

$$\xi(Q) \simeq 1+(1-2\nu) . \tag{3.3}$$

Substituting these expressions into (2.24), we obtain the effective transport mass near half-filling

$$\frac{m^*}{m} \simeq \frac{16(\ln 2)^2}{3\pi\zeta(3)}(1-2\nu)^{-1} \simeq 0.679(1-2\nu)^{-1} . \tag{3.4}$$

This behavior should be compared with the $U \to \infty$ Hubbard model near half-filling,

$$\frac{m^*}{m} \simeq \frac{2}{\pi}(1-2\nu)^{-1} \simeq 0.637(1-2\nu)^{-1} , \tag{3.5}$$

which will be derived from (3.10). We thus observe that the mass of the t-J model ($t=J$) is about 7% enhanced compared with the infinite-U Hubbard model near half-filling. Based on these results, we can say that the large antiferromagnetic interaction in the t-J model has a tendency to enhance the effective transport mass. It is understood noting that the large antiferromagnetic coupling favors singlet paris of the nearest-neighbor electrons, which restricts the free motion of each electron. This situation of favoring singlet pair seems to be similar to that of the attractive Hubbard model. In the attractive case, however, the pairing effect modifies the effective mass rather drastically (see the next section).

B. Hubbard model

We consider the one-dimensional Hubbard chain described by the Hamiltonian

$$\mathcal{H} = -t \sum_{\langle ij \rangle, \sigma} a_{i\sigma}^{\dagger} a_{j\sigma} + U \sum_i a_{i\uparrow}^{\dagger} a_{i\uparrow} a_{i\downarrow}^{\dagger} a_{i\downarrow} , \tag{3.6}$$

where the electrons are assumed to transfer between the nearest-neighbor sites. Both repulsive and attractive cases are discussed using the Bethe-ansatz solution.[14]

1. Repulsive case

The calculation goes in parallel with the t-J model. We note here that the twisting effect on the Hubbard model was examined by Frahm and Korepin,[13] and the conductivity was discussed by the present authors[3] following the method of Shastry and Sutherland.[2] Independently Schulz obtained the same result earlier using the bosonization method with the aid of the Bethe-ansatz solution.[5]

The conductivity is given by (2.24), where we should use the following charge function:

$$\xi(k)=1+\int_{-Q}^{Q} dk' \cos k' \, R(\sin k-\sin k')\xi(k') \tag{3.7}$$

and the velocity $v_c=\varepsilon'(Q)/[2\pi\rho(Q)]$ with

$$\rho(k)=\frac{1}{2\pi}+\cos k\int_{-Q}^{Q} dk' \, R(\sin k-\sin k')\rho(k') , \tag{3.8}$$

$$\varepsilon(k)=-2t\cos k-\mu + \int_{-Q}^{Q} dk' \cos k' \, R(\sin k-\sin k')\varepsilon(k') , \tag{3.9}$$

where Q is determined by $\varepsilon(\pm Q)=0$. In Figs. 4 and 5, we plot the charge stiffness and the corresponding mass enhancement factor as a function of the electron concentration. As in the t-J model, the effective mass for low concentration is not renormalized (see Appendix A). When the concentration is increased toward half-filling, the effective mass is considerably enhanced due to the metal-insulator transition.[3,5]

Near half-filling the mass enhancement for the Hubbard model is explicitly obtained as[3]

$$\frac{m^*}{m}\simeq\frac{A^2}{B}(1/2-\nu)^{-1} ,$$

$$A=\int_1^{\infty} dz\,(z^2-1)^{-1/2}\mathrm{csch}(2\pi tz/U) , \tag{3.10}$$

$$B=\frac{\pi^3 t}{4U}\int_1^{\infty} dz\,(z^2-1)^{1/2}\frac{[2\coth^2(2\pi tz/U)-1]}{\sinh(2\pi tz/U)} .$$

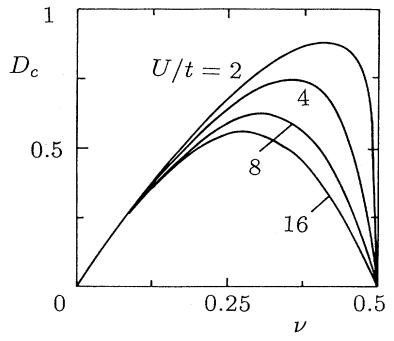

FIG. 4. Normalized charge stiffness [$D_c=\sin(\pi\nu)$ for $U=0$] for the repulsive Hubbard model. See also results by Schulz (Ref. 5).

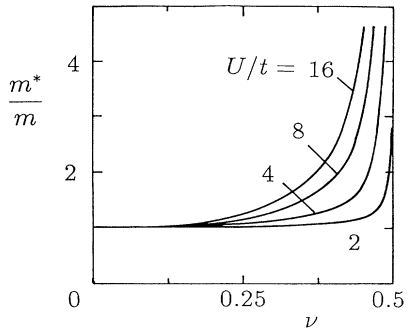

FIG. 5. Effective transport mass m^*/m for the repulsive Hubbard model.

For small U, m^*/m shows the essential singularity property which can be expressed in terms of the Mott-Hubbard gap E_g in the universal form

$$\frac{m^*}{m}=\frac{2E_g}{\pi t}(1-2\nu)^{-1} , \tag{3.11}$$

$$E_g=8(tU/\pi^2)^{1/2}\exp(-2\pi t/U) . \tag{3.12}$$

This indicates that the mass enhancement is directly related to the formation of the Mott-Hubbard gap associated with the metal-insulator transition. In the strong-coupling limit it follows the spinless fermion behavior $m^*/m\simeq(2/\pi)(1-2\nu)^{-1}$ as given in (3.5).

2. Attractive case

We next turn to the attractive case and see that the behavior of the conductivity is quite different from the repulsive case. The analysis helps to get an intuitive interpretation of the mass enhancement for the repulsive case. The conductivity is again given by (2.24) to which we substitute the dressed charge $\xi(\Lambda)$ and velocity $v_c=\varepsilon'(Q)/[2\pi\rho(Q)]$ obtained from

$$\xi(\Lambda)=1+\int_{-Q}^{Q} d\Lambda' K(\Lambda-\Lambda')\xi(\Lambda') , \tag{3.13}$$

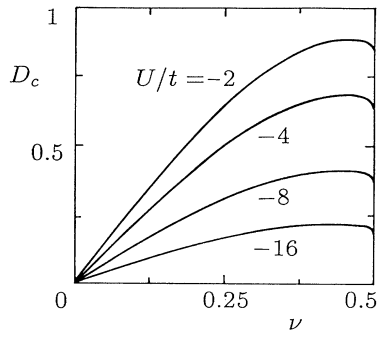

FIG. 6. Normalized charge stiffness [$D_c=\sin(\pi\nu)$ for $U=0$] for the attractive Hubbard model. There exist the logarithmic behaviors near half-filling.

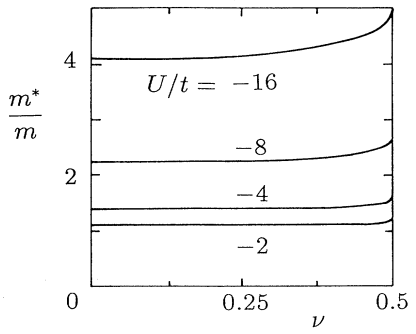

FIG. 7. Effective transport mass m^*/m for the attractive Hubbard model.

$$\rho(\Lambda)=\frac{1}{\pi}\mathrm{Re}\{1-[\Lambda+iU/(4t)]^2\}^{-1/2} + \int_{-Q}^{Q} d\Lambda' K(\Lambda-\Lambda')\rho(\Lambda') , \tag{3.14}$$

$$\varepsilon(\Lambda)=-4t\,\mathrm{Re}\{1-[\Lambda+iU/(4t)]^2\}^{1/2}-2\mu + \int_{-Q}^{Q} d\Lambda' K(\Lambda-\Lambda')\varepsilon(\Lambda') , \tag{3.15}$$

where the integral kernel reads $K(\Lambda)=[U/(2\pi t)][\Lambda^2+U^2/(4t^2)]^{-1}$.

In Figs. 6 and 7, we depict the conductivity and the effective transport mass as a function of the electron concentration. The stronger the attractive interaction is, the larger the effective mass becomes over the whole range of the electron concentration. It is noteworthy that at the low-density limit, the effective mass is enhanced by the interaction strength, which is quite distinct from the repulsive case. The enhancement is due to the fact that the singlet electron pairs are developed by the attractive force, which restricts the free motion of electrons considerably. The effective mass in the strongly attractive region is analytically shown to be proportional to $|U|/t$, since the effective electron hopping around the sites is possible only by breaking the electron pair whose binding energy is of the order of $|U|$. The formation of the singlet pairs of the electrons is also understood from the behavior of the pair correlation function.[15] We present in Fig. 8 the correlation exponent β for the singlet pairing correlation function $P(r)\sim r^{-\beta}$ at large distance r.

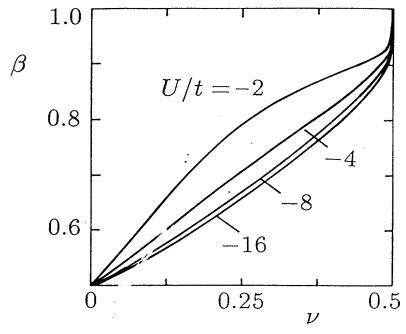

FIG. 8. Critical exponent β of the singlet-pairing correlation function for the attractive Hubbard model.

Finally we remark that the logarithmic behavior in the mass enhancement factor exists in the vicinity of half-filling, which is directly related to the existence of the marginally irrelevant operator. This point is discussed in Appendix B following the renormalization-group method.

IV. SUMMARY

In this paper we have discussed the conductivity of the 1D correlated electron systems based on the finite-size-scaling analysis under twisted boundary conditions. The extremely large enhancement of the transport mass has been observed near half-filling for both the t-J model and the repulsive Hubbard model, which is a hallmark of the metal-insulator transition.

We have shown another example of the mass enhancement in the attractive Hubbard model, in which the enhancement results from the formation of the electron pairs with antiparallel spins. In view of the pairing mechanism we understand that the mass enhancement near the metal-insulator transition in the t-J model and the repulsive model is caused by the formation of the *electron-hole* pair which also restricts the free motion of electrons. The Mott insulator can then be regarded as an extreme case in which the pairing of electron and hole severely restricts the electron motion so as to make the transport mass infinitely heavy.

The present discussion of the t-J model has been restricted to the integrable point $t=J$. In this special case we have seen that the large antiferromagnetic coupling J gives rise to the pairing effect between antiparallel spin electrons on the nearest-neighbor sites, which in turn makes the mass enhanced more than the repulsive Hubbard model. What will then happen when the strength of J is increased further? Intuitively the transport mass is expected to increase gradually and then diverge at the transition point to the phase separation. In fact, according to recent numerical calculations by Ogata *et al.*,[16] the compressibility becomes extremely large on the boundary of the phase transition. Based on formulas (2.24)–(2.26), we naturally confirm the above picture for the conductivity.

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APPENDIX A

We prove here that the mass is not renormalized in the low-density limit of the repulsive Hubbard model. The technique described here is applicable to many interacting 1D systems. For the t-J model, however, we need to invoke another technique, which will be mentioned in the end of this appendix.

In the low-density limit ($Q\to 0$), basic equations (3.7)–(3.9) for the repulsive Hubbard model are reduced to

$$\xi(k)=1+\int_{-Q}^{Q}dk'R(k-k')\xi(k') , \tag{A1}$$

$$\varepsilon(k)=-2t(1-k^2/2)-\mu+\int_{-Q}^{Q}dk'R(k-k')\varepsilon(k') , \tag{A2}$$

$$\rho(k)=\frac{1}{2\pi}+\int_{-Q}^{Q}dk'R(k-k')\rho(k') . \tag{A3}$$

We first express $\varepsilon'(Q)$ in terms of the dressed charge function $\xi(k)$. To do so, we consider the integral equation for the second derivative $\varepsilon''(k)$. From (3.9), one obtains

$$\begin{aligned}\varepsilon''(k)&=2t+\int_{-Q}^{Q}dk'R'(k-k')\varepsilon'(k')\\&=2t-[R(k-Q)+R(k+Q)]\varepsilon'(Q)\\&\quad+\int_{-Q}^{Q}dk'R'(k-k')\varepsilon''(k') ,\end{aligned} \tag{A4}$$

where we have used the condition $\varepsilon(\pm Q)=0$ in the first line. Since the equation is linear we may decompose ε'' as $\varepsilon''(k)=\varepsilon_1(k)+\varepsilon_2(k)$. $\varepsilon_1(k)$ [$\varepsilon_2(k)$] satisfies the equation with the driving term given by the first (second) term in (A4). It is then obvious

$$\varepsilon_1(k)=2t\xi(k) . \tag{A5}$$

The formal solution of ε_2 obtained by iteration yields the relation

$$\int_{-Q}^{Q}\varepsilon_2(k)dk=2[1-\xi(Q)]\varepsilon'(Q) . \tag{A6}$$

From (A4)–(A6) we get

$$\begin{aligned}\varepsilon'(Q)&=\int_{0}^{Q}\varepsilon''(k)dk\\&=\tfrac{1}{2}\int_{-Q}^{Q}2t\xi(k)dk+[1-\xi(Q)]\varepsilon'(Q) ,\end{aligned} \tag{A7}$$

from which we verify the useful identity

$$\varepsilon'(Q)=\frac{t}{\xi(Q)}\int_{-Q}^{Q}\xi(k)dk . \tag{A8}$$

Noting that $\xi(k)=2\pi\rho(k)$, we finally obtain the mass enhancement factor for (2.24),

$$m^*/m=1 \tag{A9}$$

in the low-concentration limit.

The above technique can be applied straightforwardly to the electron gas system and the bose gas system (with δ-function-type interaction), because the basic equations are given by essentially the same formula as (A1)–(A3). In these continuum models, it is thus shown that the transport mass is not renormalized for arbitrary electron (boson) concentrations.

A different method is required to discuss the case of the t-J model, since the low-concentration limit corresponds to $Q\to\infty$ limit. Employing the Wiener-Hopf method, which was discussed in detail in Ref. 6, we find the asymptotic relation for the electron concentration as $Q\to\infty$ (i.e., $\mu\to-2t$),

$$\nu\simeq\frac{1}{\pi}(\mu+2t)^{1/2} . \tag{A10}$$

The compressibility is evaluated

$$\chi_c\simeq\frac{1}{\pi^2 t}\nu^{-1} . \tag{A11}$$

This result together with $[\xi(Q)]^2\to 2$ (for $Q\to\infty$) leads to the desired result $m^*/m=1$ in the low-concentration regime of the t-J model.

APPENDIX B

We discuss the logarithmic behavior of the conductivity of the attractive Hubbard model near half-filling. It is well known that under the canonical transformation $a_{i\uparrow}=d_{i\uparrow}$, and $a_{i\downarrow}=(-1)^i d_{i\downarrow}$, the Coulomb interaction changes its sign and the role of the chemical potential μ and the external field H is interchanged.[17] Thus the attractive Hubbard model $H=0$ away from half-filling is transformed into the repulsive model with nonzero external field at half-filling. This means that the compressibility near half-filling in the attractive case is replaced by the spin susceptibility of the repulsive model at half-filling. Since the latter model belongs to the same universality class as the spin-$\frac{1}{2}$ Heisenberg chain, the problem is reduced to evaluating the susceptibility of the quantum spin chain. Logarithmic corrections to the susceptibility have already been studied by means of the Wiener-Hopf method when the exact Bethe solution is available.[6,18–20] In the following, we give a simple renormalization-group argument to derive the result.

We consider the spin-s Hamiltonian and assume that it is attracted to the multicritical point whose critical behavior is described by the level-k ($=2s$) SU(2) Kac-Moody algebra.[21] In the spectrum of this theory there exists a marginally irrelevant operator with scaling dimension 2, which is usually responsible for logarithmic terms.

The ground-state energy E depends on an external field H as

$$E(H)-E(0)\simeq\frac{LH^2k}{4\pi v_s}\quad\text{for } L\to\infty ,\ H\to 0 \tag{B1}$$

where L is the system size and v_s denotes the spin-wave velocity. Notice that H has mass dimension 1. As first shown by Affleck[22] the zero-field susceptibility is thus given by

$$\chi_s=-\frac{\partial^2 E}{\partial H^2}=\frac{Lk}{2\pi v_s} . \tag{B2}$$

For finite H we expect the scaling form

$$E(H)-E(0)=-\frac{LH^2}{v_s}\Phi(g(H)) , \tag{B3}$$

where Φ is a universal function and $g(H)$ is a renormalized scaling variable corresponding to the marginal operator $\hat{\phi}(x)$ (Ref. 23)

$$g(H)=\frac{g}{1+\pi bg\ln(1/H)} . \tag{B4}$$

Here b is a universal constant fixed by a three-point function

$$\langle\hat{\phi}(x_1)\hat{\phi}(x_2)\hat{\phi}(x_3)\rangle=\frac{-b}{|x_1-x_2|^2\,|x_1-x_3|^2\,|x_2-x_3|^2}\ . \tag{B5}$$

Hence, for $\ln(1/H)\gg 1/(bg)$, we get

$$E(H)-E(0)\simeq-\frac{LH^2}{v_s}\left[\frac{k}{4\pi}+\frac{\Phi'(0)}{\pi b}\frac{1}{\ln(1/H)}\right], \tag{B6}$$

where $\Phi(0)=k/(4\pi)$ from (B1). This explains the universal logarithmic correction to χ_s as $H\to 0$.[18]

We next determine the coefficient $\Phi'(0)/b$. The correction to the fixed point Hamiltonian due to the marginal operator is

$$\delta\mathcal{H}=g\int dx\,\hat{\phi}(x)\ , \tag{B7}$$

where $\hat{\phi}(x)=-2\pi^2 b\mathbf{J}_L\cdot\mathbf{J}_R(x)$ and $\mathbf{J}_{L(R)}$ is the left (right) -moving SU(2) current.[24] At the first order of perturbation we have

$$E(H)-E(0)\simeq-\frac{LH^2k}{v_s}+v_s g\int dx\,\langle\hat{\phi}(x)\rangle_H\ . \tag{B8}$$

The magnetization $ML=-\partial E/\partial H$ is represented as $M=\langle(S_L^z+S_R^z)\rangle_H/L$, where $S_L^z=\int dx\,J_L^z(x)$. From this we may evaluate $\langle\mathbf{J}_L\cdot\mathbf{J}_R(x)\rangle=M^2/4$. Substituting this into (B8) and comparing with (B3) for $g\ll 1$ we obtain $\Phi'(0)=bk^2/8$. We thus find the susceptibility for a small external field

$$\chi_s=\frac{Lk}{2\pi v_s}\left[1+\frac{k}{2}\frac{1}{\ln(1/H)}\right], \tag{B9}$$

which agrees with the known results for $k=1$.[19,20] Finally the logarithmic correction to the compressibility in the attractive Hubbard model near half-filling is given as

$$\chi_c(\nu)=\chi_c(\nu=\tfrac{1}{2})\left[1-\frac{1}{2}\frac{1}{\ln(1-2\nu)}\right], \tag{B10}$$

thereby we obtain the mass enhancement factor

$$\frac{m^*}{m}=\frac{m^*}{m}(\nu=\tfrac{1}{2})\left[1+\frac{1}{2}\frac{1}{\ln(1-2\nu)}\right]. \tag{B11}$$

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