RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 44, NUMBER 14 1 OCTOBER 1991-II

Temperature-dependent low-frequency conductivity in marginal-Fermi-liquid theory

E. J. Nicol and J. P. Carbotte
Physics Department, McMaster University, Hamilton, Ontario, Canada L8S 4M1
(Received 8 July 1991)

We have calculated the temperature-dependent optical conductivity within phenomenological marginal-Fermi-liquid theory. A "coherencelike" peak occurs from a drop in the quasiparticle damping in the superconducting state. Results are given for varying impurity scattering rate.

Low-frequency conductivity, such as measured by microwave and far-infrared experiments, has traditionally played an important role in the investigations of the superconducting state. This is due to the possibility of extracting, among other things, information about the energy gap [in conventional superconductors, such as Pb (Ref. 1)] and the quasiparticle damping rate.

Within BCS theory, the temperature-dependent low-frequency conductivity has type-II coherence factors as does the NMR relaxation rate and, hence, would normally exhibit a coherence peak just below T_c, where the superconducting-state conductivity is enhanced above the normal-state conductivity.[2] Recent work by Akis and Carbotte[3] and Marsiglio[4] has demonstrated that within the strong-coupling theory of superconductivity, the coherence factors can be suppressed; however, normal impurity scattering is shown to enhance the peak. In addition, Akis and Carbotte[5] and Allen and Rainer[6] have shown that a suppression of the coherence peak in the NMR relaxation rate also occurs with increased strong coupling and that the formulas for the NMR and the conductivity are essentially the same, such that the suppression of the peak in one quantity would be accompanied by the suppression of the peak in the other quantity. The observation of the absence of a coherence peak in the NMR experiments[7,8] on the high-T_c superconductors can be taken as evidence for strong-coupling effects,[5,6] anisotropy,[9] etc., but the essential point is that the coherence effects have been suppressed in some manner and, hence, are likewise suppressed in the conductivity.

In this paper, we present results of calculations of the low-frequency conductivity in the phenomenological marginal-Fermi-liquid theory of Varma *et al.* [10] and find a "coherence-type" peak that is not due to coherence effects but due to the quasiparticle scattering rate. Similar results have been obtained by Nuss *et al.* [11] who have also calculated the NMR relaxation rate in this model and find an absence of a peak in this latter quantity.[11,12]

Varma *et al.* [10] have proposed a phenomenological polarizability for charge and spin fluctuations that leads to a quasiparticle residue z_k that vanishes at the Fermi surface in a very weak manner (i.e., $\lim_{\omega\to 0}[z_k^{-1}\sim\ln(\omega/\omega_c)]$) and call this a marginal Fermi liquid. The essence of this polarizability is that it leads to a self-energy that yields the linear temperature and frequency scattering rate seen in so many of the normal-state properties of the high-T_c superconductors. Since their initial proposal, much work has been done to investigate the superconducting-state properties,[12–17] which have been based on an approximation to the polarizability given by Kuroda and Varma[13] to yield an effective electron-boson spectral density $\alpha^2F(\omega)$ of the form

$$\alpha^2F(\omega)\propto\frac{1}{\pi}\tanh\left(\frac{\omega}{2T}\right),\quad \omega\le\omega_c\,,$$
$$=0,\quad \omega>\omega_c\,, \tag{1}$$

where ω_c is a very-high-energy cutoff on the fluctuation spectrum. This spectral density enters into Eliashberg equations[14] which are formally similar to the standard form for phonons plus paramagnons.[18–20] The imaginary axis gap equations in Eliashberg theory are given as[14]

$$\tilde\Delta_n=\pi T\sum_m[\lambda^-(m-n)-\mu^*]\frac{\tilde\Delta_m}{(\tilde\omega_m^2+\tilde\Delta_m^2)^{1/2}} \tag{2}$$

and

$$\tilde\omega_n=\omega_n+\pi T\sum_m\lambda^+(m-n)\frac{\tilde\omega_m}{(\tilde\omega_m^2+\tilde\Delta_m^2)^{1/2}}\,, \tag{3}$$

where $\omega_n=(2n+1)\pi T$ with $n=0,\pm1,\pm2,\ldots$, μ^* is the Coulomb pseudopotential, $\tilde\Delta_n=\Delta(i\omega_n)Z(i\omega_n)$, $\tilde\omega_n=\omega_nZ(i\omega_n)$, and

$$\lambda^\pm(m-n)=(\lambda_\rho\pm\lambda_\sigma)\frac{1}{\pi}\int_0^{\omega_c}\frac{2\omega\tanh(\omega/2T)}{\omega^2+(\omega_m-\omega_n)^2}d\omega\,. \tag{4}$$

In the marginal Fermi liquid, charge fluctuations couple to the superconducting electrons through a dimensionless coupling λ_ρ, while coupling to spin is described by λ_σ.

The essential difference between the conventional Eliashberg theory and this model is that the spectral density is temperature dependent, and its frequency dependence is flat and featureless, extending to high frequencies. It is the same spectrum (except for coupling strength) for both the charge and spin fluctuations (unlike phonons and paramagnons where the two spectra can be different). The most interesting feature of this spectrum in the superconducting state is that the spectral density is gapped by $2\Delta(T)$ at the lower end because, to create fluctuations from the superconducting electrons themselves, a Cooper pair must be liberated from the condensate. Therefore, in the solutions of Eqs. (2) and (3), we must apply a lower limit of $2\Delta(T)$ to the integrals in Eqs. (4) and (5), which we take as a sharp cutoff. Solution of the equations then proceeds self-consistently in a procedure already outlined.[14]

To calculate the optical conductivity, we first solve for the imaginary axis gaps and frequencies and then use the analytic continuation of Marsiglio, Schossmann, and Carbotte[21] to obtain the real frequency complex gap function and renormalized energies, $\tilde{\Delta}(\varepsilon)$ and $\tilde{\varepsilon}(\varepsilon)=\varepsilon Z(\varepsilon)$, respectively, given by the self-consistent solution of

$$\tilde{\varepsilon}(\varepsilon)=\varepsilon+i\pi T\sum_{m=0}^{\infty}\frac{\tilde{\omega}_m}{(\tilde{\omega}_m^2-\tilde{\Delta}_m^2)^{1/2}}[\lambda^+(\varepsilon-i\omega_m)-\lambda^+(\varepsilon+i\omega_m)]+i\pi(\lambda_p+\lambda_\sigma)\int_{-\infty}^{+\infty}dz\frac{\tilde{\varepsilon}(\varepsilon-z)}{[\tilde{\varepsilon}^2(\varepsilon-z)+\tilde{\Delta}^2(\varepsilon-z)]^{1/2}}\times\alpha^2F(z)[N(z)+f(z-\varepsilon)] \tag{5}$$

and

$$\tilde{\Delta}(\varepsilon)=i\pi T\sum_{m=0}^{\infty}\frac{\tilde{\Delta}_m}{(\tilde{\omega}_m^2-\tilde{\Delta}_m^2)^{1/2}}[\lambda^-(\varepsilon-i\omega_m)+\lambda^-(\varepsilon+i\omega_m)-2\mu^*]+i\pi(\lambda_p-\lambda_\sigma)\int_{-\infty}^{+\infty}dz\frac{\tilde{\Delta}(\varepsilon-z)}{[\tilde{\varepsilon}^2(\varepsilon-z)-\tilde{\Delta}^2(\varepsilon-z)]^{1/2}}\times\alpha^2F(z)[N(z)+f(z-\varepsilon)]\,, \tag{6}$$

where

$$\lambda^{\pm}(\varepsilon)=-(\lambda_p\pm\lambda_\sigma)\int_{-\infty}^{+\infty}\frac{d\Omega\,\alpha^2F(\Omega)}{\varepsilon-\Omega+i0^+}\,, \tag{7}$$

with $\alpha^2F(\omega)$ defined in Eq. (1) for $2\Delta(T)<\omega<\omega_c$. In these equations, $N(z)$ is the boson occupation $N(z)=1/(e^{\beta z}-1)$ and $f(z)$ the fermion occupation $f(z)=1/(e^{\beta z}+1)$, with $\beta=1/k_BT$ and k_B the Boltzmann constant. We then use these in the formula for the conductivity derived by Lee, Rainer, and Zimmermann[22] which is given as

$$\sigma(\omega,T)=\frac{N(0)e^2v_F^2}{4\pi^2\omega}\int_{-\infty}^{+\infty}d\varepsilon\left\{\tanh\left(\frac{\varepsilon}{2k_BT}\right)M(\varepsilon,\omega)[g(\varepsilon)g(\varepsilon+\omega)+h(\varepsilon)h(\varepsilon+\omega)+\pi^2]-\tanh\left(\frac{\varepsilon+\omega}{2k_BT}\right)M^*(\varepsilon,\omega)[g^*(\varepsilon)g^*(\varepsilon+\omega)+h^*(\varepsilon)h^*(\varepsilon+\omega)+\pi^2]+\left[\tanh\left(\frac{\varepsilon+\omega}{2k_BT}\right)-\tanh\left(\frac{\varepsilon}{2k_BT}\right)\right]L(\varepsilon,\omega)[g^*(\varepsilon)g(\varepsilon+\omega)+h^*(\varepsilon)h(\varepsilon+\omega)+\pi^2]\right\}, \tag{8}$$

where

$$g(\varepsilon)=\frac{-\pi\tilde{\varepsilon}(\varepsilon)}{[\tilde{\Delta}^2(\varepsilon)-\tilde{\varepsilon}^2(\varepsilon)]^{1/2}}\,, \tag{9}$$

$$h(\varepsilon)=\frac{-\pi\tilde{\Delta}(\varepsilon)}{[\tilde{\Delta}^2(\varepsilon)-\tilde{\varepsilon}^2(\varepsilon)]^{1/2}}\,, \tag{10}$$

$$M(\varepsilon,\omega)=\left([\tilde{\Delta}^2(\varepsilon+\omega)-\tilde{\varepsilon}^2(\varepsilon+\omega)]^{1/2}+[\tilde{\Delta}^2(\varepsilon)-\tilde{\varepsilon}^2(\varepsilon)]^{1/2}+\frac{1}{\tau}\right)^{-1}\,, \tag{11}$$

and

$$L(\varepsilon,\omega)=\left([\tilde{\Delta}^2(\varepsilon+\omega)-\tilde{\varepsilon}^2(\varepsilon+\omega)]^{1/2}+[\tilde{\Delta}^{*2}(\varepsilon)-\tilde{\varepsilon}^{*2}(\varepsilon)]^{1/2}+\frac{1}{\tau}\right)^{-1}\,. \tag{12}$$

In Eq. (8) $N(0)$ is the single spin electronic density of states at the Fermi surface, e is the electron charge, and v_F is the Fermi velocity. The impurity scattering time τ enters Eqs. (11) and (12). We quote values in terms of t^+ where $t^+=1/2\pi\tau$. The normal-state conductivity is obtained by setting the gaps to zero in the above procedure.

In Fig. 1, we show the frequency-dependent conductivity for several reduced temperatures $t=T/T_c$: $t=0.1$ (——), $t=0.6$ (– – –), $t=0.7$ (— - —), $t=0.8$ (---),

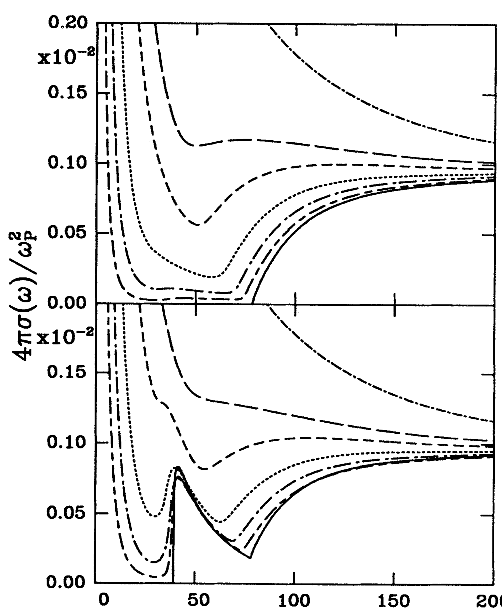

FIG. 1. The real part of the frequency-dependent conductivity $\sigma(\omega)$ vs frequency ω for several temperatures below T_c. In terms of reduced temperatures $t=T/T_c$ the curves are $t=0.1$ (——), $t=0.6$ (– – –), $t=0.7$ (— - —), $t=0.8$ (---), $t=0.9$ (---), $t=0.95$ (———), and $t=1.1$ (—-—-). The upper frame corresponds to the clean limit (i.e., $t^+=0$) and the lower frame corresponds to $t^+=1$ meV. The zero-temperature gap Δ_0 is 19.6 meV.

$t=0.9$ (---), $t=0.95$ (— — —), and $t=1.1$ (-—-—). The marginal-Fermi-liquid parameters we have used are $\omega_c=200$ meV, $g=(\lambda_\rho-\lambda_\sigma)/(\lambda_\rho+\lambda_\sigma)=0.8$, and $T_c=100$ K, with μ^* taken to be zero. This results in $2\Delta_0/k_BT_c=4.55$, $\Delta_0=19.6$ meV, and $\lambda\sim1.3$, where λ is taken from $Z_1(\omega=0)=1+\lambda$. In the top frame the conductivity is shown in the clean limit (i.e., $t^+=0$). Notice, at zero temperature, no absorption occurs until $\omega=4\Delta_0$. This corresponds to $2\Delta_0$ of energy to create an electron-hole pair plus another $2\Delta_0$ of energy to reach the first frequency in the fluctuation spectrum (the creation of another pair), the requirement for conserving both energy and momentum. The $4\Delta_0$ result has been obtained previously by ourselves and other authors.[12,14,23] In the bottom frame of Fig. 1, $t^+=1.0$ meV. While a dip remains at $4\Delta_0$ at low temperatures, impurity-assisted absorption occurs at $2\Delta_0$. Notice that the $4\Delta_0$ feature exhibits the temperature dependence of the gap (taken as BCS here[15]) while the dip before the impurity peak fills in with the low-frequency absorption due to quasiparticle scattering.

In Fig. 2, we plot the low-frequency conductivity. The top frame shows the temperature-dependent conductivity for several frequencies: $\omega=1$ meV (——), $\omega=2$ meV (-—-), $\omega=5$ meV (— — —), and $\omega=10$ meV (---). The striking feature is a peak in the conductivity which is not suppressed until rather high frequencies. These curves are for $t^+=0$ (the clean limit). In the lower frame, we exhibit the conductivity as a function of impurity content, at a lower frequency of $\omega=0.05$ meV, for $t^+=0$ meV (——), $t^+=0.1$ meV (---), $t^+=0.2$ meV (---), $t^+=1$ (— — —), and $t^+=100$ meV (-—-—). The last curve is essentially the dirty limit and, thus, for the microwave frequency range, the peak is never suppressed in the near-to-clean limit (the limit in which the high-T_c superconductors are thought to be). Note that the behavior here is completely opposite to what occurs in the conventional strong-coupling case, where increased normal impurity scattering enhances the peak.[3]

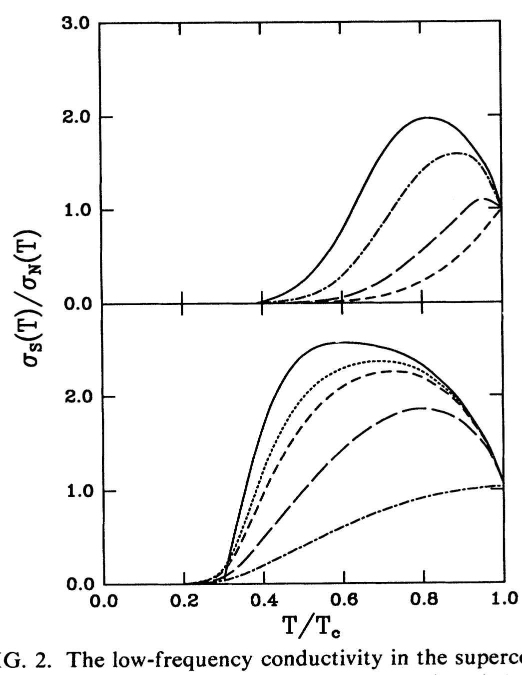

FIG. 2. The low-frequency conductivity in the superconducting state normalized by the normal state as a function of temperature. In the upper frame, curves are for the clean limit ($t^+=0$) and are drawn for several frequencies: $\omega=1$ meV (——), $\omega=2$ meV (-—-), $\omega=5$ meV (— — —), and $\omega=10$ meV (---). The lower frame shows the effect of normal impurity scattering suppressing the peak. These curves are for $\omega=0.05$ meV and $t^+=0$ meV (——), $t^+=0.1$ meV (---), $t^+=0.2$ meV (---), $t^+=1$ (— — —), and $t^+=100$ meV (-—-—).

These surprising results are not due to coherence effects, which have been suppressed by strong coupling, but, rather, they are due to the quasiparticle damping rate $\Gamma(\omega,T)$ which is shown in Fig. 3.[17] This quantity is obtained from the imaginary part of the self-energy and is given in terms of the gap function $\Delta(\omega)=\Delta_1(\omega)+i\Delta_2(\omega)$ and the renormalization function $Z(\omega)=Z_1(\omega)+iZ_2(\omega)$ as[24]

$$\Gamma(\omega,T)=\frac{Z_2(\omega^2-\Delta_1^2)-\Delta_1\Delta_2Z_1}{\omega}. \tag{13}$$

Unlike the NMR relaxation rate, the conductivity samples this scattering rate and it is this difference that can produce a peak in the conductivity but not in the NMR. The damping rate depends on the availability of fluctuations for the quasiparticle to emit or absorb when making transitions between states. However, in the marginal-Fermi-liquid model, such fluctuations are absent for low frequencies and temperatures not near T_c, due to the gap of $2\Delta(T)$ in the fluctuation spectrum. It is only very close to T_c, where $2\Delta(T)$ is small, that the scattering processes can occur. Behavior, as shown in Fig. 3, recently has been observed experimentally.[25,26]

To aid in visualizing how the peak in the conductivity occurs and how it decreases with increasing impurity scattering, we use a very naive and simplistic model. We take the simple form for the Drude conductivity at zero frequency (as $\omega=0.05$ meV is quite small relative to Δ_0):

$$\sigma=\frac{ne^2\tau}{m}, \tag{14}$$

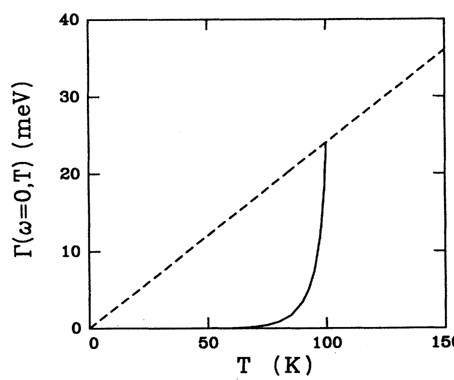

FIG. 3. The zero-frequency quasiparticle damping rate as a function of temperature. The dashed curve corresponds to the normal state and the solid curve to the superconducting state. There is a sudden drop in the damping rate just below T_c in the superconducting state, due to a gap opening up in the fluctuation spectrum.

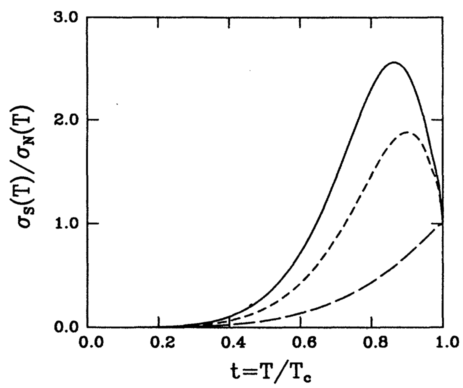

FIG. 4. A simulation of the low-frequency conductivity using a simple two-fluid model as described in the text. These curves are for illustrative purposes only and are not to be taken as quantitatively correct. Curves are drawn for $\omega=0$ and $t^+=1.0$ meV (——), $t^+=2.0$ meV (---), and $t^+=100.0$ meV (— — —).

and make this temperature dependent by replacing the density of electrons n by the two-fluid model temperature dependence[2] for normal electrons $n_n^S(T)$ (taking the quasiparticles to be a fluid of normal electrons in the superconducting state) and we also use the temperature dependence of τ given in Fig. 3. Hence,

$$\frac{\sigma_S(T)}{\sigma_N(T)}=\frac{n_n^S(T)[\Gamma_N(T)+\pi t^+]}{\Gamma_S(T)+\pi t^+}, \tag{15}$$

where we have included the effects of impurity scattering with $\pi t^+=1/(2\tau)$. In Fig. 4, we show the results of this simple picture and we see the same qualitative shape and behavior as we find in the full numerical solutions. The curves are drawn for $t^+=1.0$ meV (——), $t^+=2.0$ meV (---), and $t^+=100.0$ meV (— — —). It is the drop in the scattering rate that is causing the peak to appear in the conductivity. Of course, the position and shape of the peak depends on the details of how the gap forms in the fluctuation spectrum, for which there is no microscopic theory. The curves in Fig. 4 are not quantitatively correct and are for illustrative purposes only.

In conclusion, we have calculated the low-frequency conductivity within marginal-Fermi-liquid theory. Due to the drop in the quasiparticle damping rate, a peak occurs in the microwave conductivity, whereas no such peak occurs in the NMR relaxation rate in this model.[11,12] With increasing impurity scattering the peak will be reduced. Recent experiments have observed such a peak[11,26,27] and this model provides a possible explanation for the occurrence of a peak in the conductivity with an absence of the same in the NMR relaxation rate.

We would like to thank Dr. P. B. Littlewood for discussions and for sending us both his and Dr. Nuss's results prior to publication. We also thank Dr. D. B. Tanner for illuminating discussions which led us to the simple two-fluid calculation. This work was partly supported by the Natural Sciences and Engineering Research Council of Canada (NSERC) and by the Canadian Institute for Advanced Research (CIAR).

[1] L. H. Palmer and M. Tinkham, Phys. Rev. **165**, 588 (1968).
[2] J. R. Schrieffer, *Theory of Superconductivity* (Addison-Wesley, New York, 1964).
[3] R. Akis and J. P. Carbotte, Solid State Commun. (to be published).
[4] F. Marsiglio, Phys. Rev. B **44**, 5373 (1991).
[5] R. Akis and J. P. Carbotte, Solid State Commun. **78**, 393 (1991).
[6] P. B. Allen and D. Rainer, Nature (London) **34**, 396 (1991).
[7] S. E. Barrett, D. J. Durand, C. H. Pennington, C. P. Slichter, T. A. Freidmann, J. P. Rice, and D. M. Ginsberg, Phys. Rev. B **41**, 6283 (1990).
[8] M. Takigawa, P. C. Hammel, R. H. Heffner, and Z. Fisk, Phys. Rev. B **39**, 7371 (1989).
[9] B. W. Statt, Phys. Rev. B **42**, 6805 (1990).
[10] C. M. Varma, P. B. Littlewood, S. Schmitt-Rink, E. Abrahams, and A. Ruckenstein, Phys. Rev. Lett. **63**, 1996 (1989); **64**, 497(E) (1990).
[11] M. C. Nuss, P. M. Mankiewich, M. L. O'Malley, E. H. Westerwick, and P. B. Littlewood, Phys. Rev. Lett. **66**, 3305 (1991).
[12] P. B. Littlewood and C. M. Varma, J. Appl. Phys. **69**, 4979 (1991).
[13] Y. Kuroda and C. M. Varma, Phys. Rev. B **42**, 8619 (1990).
[14] E. J. Nicol, J. P. Carbotte, and T. Timusk, Phys. Rev. B **43**, 473 (1991); Solid State Commun. **76**, 937 (1990).
[15] E. J. Nicol and J. P. Carbotte, Phys. Rev. B **43**, 1158 (1991).
[16] E. J. Nicol and J. P. Carbotte, Solid State Commun. **78**, 55 (1991).
[17] E. J. Nicol, J. P. Carbotte, and T. Timusk (unpublished).
[18] J. M. Daams, B. Mitrović, and J. P. Carbotte, Phys. Rev. Lett. **46**, 65 (1981).
[19] R. Baquero, J. M. Daams, and J. P. Carbotte, J. Low Temp. Phys. **42**, 585 (1981).
[20] H. G. Zarate and J. P. Carbotte, J. Low Temp. Phys. **57**, 291 (1984).
[21] F. Marsiglio, M. Schossmann, and J. P. Carbotte, Phys. Rev. B **37**, 4965 (1988).
[22] W. Lee, D. Rainer, and W. Zimmermann, Physica C **159**, 535 (1989).
[23] J. Orenstein, S. Schmitt-Rink, and A. Ruckenstein, in *Electronic Properties of High-T_c Superconductors and Related Compounds*, edited by H. Kuzmany, M. Mehring, and J. Fink (Springer, Berlin, 1990), p. 254.
[24] S. B. Kaplan, C. C. Chi, D. N. Langenberg, J. J. Chang, S. Jafarey, and D. J. Scalapino, Phys. Rev. B **14**, 4854 (1976).
[25] D. B. Tanner, D. B. Romero, K. Kamarás, G. L. Carr, L. Forro, D. Mandrus, L. Mihaly, and G. P. Williams, in *Proceedings of the Miami Workshop on Electronic Structure and Mechanisms for High Temperature Superconductivity*, edited by G. Vezzoli *et al.* (Plenum, New York, in press).
[26] D. A. Bonn, P. Dosanjh, R. Liang, and W. N. Hardy (unpublished).
[27] K. Holczer, L. Forro, L. Mihály, and G. Grüner, Phys. Rev. Lett. **67**, 152 (1991).