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Phase-dependent energy spectrum of quasiparticles in a superconducting superlattice

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The energy spectrum of the quasiparticle state in a superconducting superlattice is investigated based on the Kronig-Penney superlattice model. Andreev reflection which originates from the multilayered structure is completely taken into account. The energy spectrum of quasiparticles is strongly influenced by the lattice period and the pair-potential phase increment (ϕ) over one period. From the energy spectrum, the presence of the three states, i.e., the superconducting state, the gapless superconducting state, and the normal state is elucidated and the phase diagram is obtained in the parameter space of the lattice period and the phase difference. If the unit period is short, the reduction of the pair potential is caused by the proximity effect and the difference (ϕ) of the phase. In such a case, the ϕ dependence of the supercurrent is very different from the prediction of previous perturbational theories.

I. INTRODUCTION

Nowadays, because of the development of microfabrication techniques, various kinds of artificial atomic-scale superconducting system can be designed for device applications. Superconducting superlattices are interesting from this view point. However, it is often not known in detail as to how the superconducting properties are affected by the construction of a multilayered structure.

There exist many kinds of superconducting superlattice.[1,2] But superconducting superlattices realized so far have been mostly dirty due to the effect of atomic diffusion near the interface. Thus the interference of the wave function due to the multilayered structure cannot be expected to be fully operative. There are several theories of superconducting superlattices and layered materials.[3] Klemm, Luther, and Beasley[4] treated layered material with the use of tunneling model in the limit of weak interlayer coupling. On the other hand, Takahashi and Tachiki[5] investigated superconducting superlattices coupled by the proximity effect in the dirty limit. In order to treat the interference of the electron wave function due to the multilayered structure from the strong-interlayer-coupling regime to the weak-interlayer-coupling regime in a unified formalism, we have developed a theory of superconducting superlattice based on the Kronig-Penney model.[6] It is found that the critical temperature oscillates as a function of the length of the unit period a. The origin of the oscillation is ascribed to the oscillation of the density of states (DOS) with lattice constant a. The interference of the wave function due to the multilayered structure can be seen in the variation in the critical temperature.

However, the validity of the above theories[4-6] is restricted to a region near the critical temperature and the nature of multilayered structure in the superconducting system has not been fully investigated. In this paper, basic properties of a superconducting superlattice at temperatures below the critical temperature is discussed. In particular, we will discuss how the periodic distribution of the finite pair potential and its phase variation influences the energy spectrum of quasiparticles.

The quasiparticle state in a nonuniform superconducting system has been investigated since the discovery of superconductivity, and many important features have been clarified. For example, in the superconductor–normal-material junction in which the pair potential abruptly changes, Andreev reflection occurs.[7] The Tomash effect,[8] Rowell-McMillan oscillations,[9] and de Gennes–Saint James bound states[10] all result from interferences between the electronlike and hole-like quasiparticles.

A powerful technique for investigation of the electronic state of the surface, called STS (scanning tunneling spectroscopy), has been developed.[11] Recently STS has been applied in the case of superconductors, and the energy spectrum of quasiparticles have been obtained.[12,13] So at this time it is interesting to clarify the energy spectrum of quasiparticles in a superconducting superlattice. In this paper, we investigate the energy spectrum of quasiparticles using the Kronig-Penney superconducting superlattice model. This is a simple model which enables us to solve the quasiparticle state in a microscopically correct way. We consider the case with finite phase difference of the pair potential between the adjacent superconductors, i.e., the case in which a finite supercurrent flows perpendicularly to the layer. For thin superconducting layers, the superconducting proximity effect becomes significant, and the amplitude of the pair potential in the superconductor region is reduced. This value is also influenced by the phase difference ϕ of the pair potential between the adjacent superconductors. So we determine the pair potential $\Delta(x)$ self-consistently with the use of the Bogoliubov equation.[14]

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This paper is organized as follows. In Sec. II the method of self-consistent calculation of the pair potential in the multilayered system is expressed. In Sec. III, the energy spectrum of quasiparticles is discussed. From the energy spectrum, the presence of the three states, i.e., superconducting state, gapless superconducting state, and normal state, will be elucidated. How the supercurrent depends on the difference ϕ of the phase in the layered structure is also discussed.

II. THE METHOD OF CALCULATION

The periodic square-well pair potential assumed in the three-dimensional Kronig-Penney model of a superconducting superlattice is shown in Fig. 1. The system is a periodic array of the material S and N, with respective thicknesses d_S and d_N, which have the same single-particle potential. Quasiparticles which travel in the x direction are affected by the spatial variation of the pair potential, while this system is uniform in the y and z directions. Under this assumption, the pair potential is taken to be independent of y and z. In our model, the phase of the pair potential $\Delta(x)$ is considered to be constant in each S region and obeys the following equations:

$$\Delta(x+a)=\Delta(x)\exp(i\phi)\ ,$$
$$\Delta(x)=\begin{cases}\Delta_0, & -d_S<x<0\\ 0, & 0<x<d_N\ .\end{cases} \tag{2.1}$$

In the above, Δ_0 is the amplitude of the pair potential in the S material. Since our system has translational invariance in the y and z directions, the Bogoliubov equation[14] is written as

$$H(x,k_\parallel)u_{\nu,k_\parallel}(x)+\Delta(x)v_{\nu,k_\parallel}(x)=\varepsilon_{\nu,k_\parallel}u_{\nu,k_\parallel}(x)\ ,$$
$$H(x,k_\parallel)v_{\nu,k_\parallel}(x)-\Delta^*(x)u_{\nu,k_\parallel}(x)=-\varepsilon_{\nu,k_\parallel}v_{\nu,k_\parallel}(x)\ . \tag{2.2}$$

In the above, $H(x,k_\parallel)$ is given as

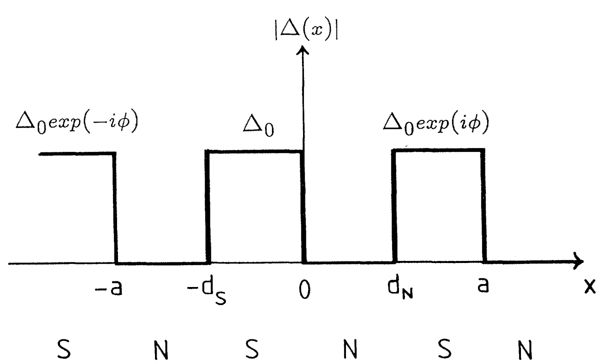

FIG. 1. The assumed spatial dependence of the pair potential in the Kronig-Penney superlattice model (S = superconductor, N = normal material).

$$H(x,k_\parallel)=-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+U(x)-\mu-\frac{\hbar^2k_\parallel^2}{2m}\ , \tag{2.3}$$

where μ is the chemical potential, $U(x)$ is the electron-ion potential, and $k_\parallel$ is the momentum of the quasiparticle parallel to the interface. The energy scale of $U(x)$ is a few eV, and the energy band of the normal state is determined by $U(x)$. On the other hand, the order of $\Delta(x)$ is a few meV. Consider the energy spectrum near the Fermi energy, namely, the excitation of quasiparticles with energy of order several meV. The qualitative features of this excitation do not depend on the shape of the energy band in the normal state. So we choose $U(x)=0$ for simplicity. The pair potential $\Delta(x)$ is related to $u_{\nu,k_\parallel}$ and $v_{\nu,k_\parallel}$ as follows:

$$\Delta(x)=TV(x)\sum_{\nu,k_\parallel}u_{\nu,k_\parallel}(x)v^*_{\nu,k_\parallel}(x)[1-2f(\varepsilon_{\nu,k_\parallel})]\ , \tag{2.4}$$

where the interelectron potential $V(x)$ is assumed to satisfy

$$V(x+a)=V(x)\ ,$$
$$V(x)=\begin{cases}V_0, & -d_S<x<0\\ 0, & 0<x<d_N\ .\end{cases} \tag{2.5}$$

Since Eq. (2.3) has translational invariance in the x direction, eigenfunctions $u_{\nu,k_\parallel}(x)$ and $v_{\nu,k_\parallel}(x)$ satisfy

$$u_{\nu,k_\parallel}(x+a)=u_{\nu,k_\parallel}(x)\exp(ika+i\phi/2)\ , \tag{2.6a}$$

$$v_{\nu,k_\parallel}(x+a)=v_{\nu,k_\parallel}(x)\exp(ika-i\phi/2)\ . \tag{2.6b}$$

The energy dispersions in this system are given as follows:

$$\cos(\bar{k}a)=\cos(\gamma d_S)\cos\left[\delta d_N-\frac{\phi}{2}\right]-\left[\frac{E}{\Omega}\right]\sin(\gamma d_S)\sin\left[\delta d_N-\frac{\phi}{2}\right]\ , \tag{2.7a}$$

$$\cos(\tilde{k}a)=\cos(\gamma d_S)\cos\left[\delta d_N+\frac{\phi}{2}\right]-\left[\frac{E}{\Omega}\right]\sin(\gamma d_S)\sin\left[\delta d_N+\frac{\phi}{2}\right]\ , \tag{2.7b}$$

where,

$$\bar{k}=k_x-k_{Fx},\quad \tilde{k}=k_x+k_{Fx}\ , \tag{2.8}$$

$$\gamma=\frac{\Omega}{\hbar v_{Fx}},\quad \delta=\frac{E}{\hbar v_{Fx}},\quad \Omega=\sqrt{E^2-\Delta_0^2}\ , \tag{2.9}$$

$$\frac{mv_{Fx}^2}{2}=\mu-\frac{\hbar^2k_\parallel^2}{2m},\quad \frac{\hbar k_{Fx}}{m}=v_{Fx}\ . \tag{2.10}$$

To derive the above dispersion relations we have used the relations

$$\frac{\gamma}{k_{Fx}}\ll 1,\quad \frac{\delta}{k_{Fx}}\ll 1\ , \tag{2.11}$$

where k_{Fx} is the x component of the Fermi wave number. It should be noted that in the case of $\phi=0$ we reproduce the results obtained by Gelder[15] and Gallagher.[16] To discuss the electronic property of this system, it is convenient to use the Matsubara Green's function which satisfies the following Gor'kov's equations:[17]

$$[i\omega_n - H(x,k_\parallel)]G(x,x';i\omega_n,k_\parallel) - \Delta(x)F(x,x';i\omega_n,k_\parallel) = \delta(x-x') \,, \tag{2.12}$$

$$[i\omega_n + H(x,k_\parallel)]F(x,x';i\omega_n,k_\parallel) - \Delta^*(x)G(x,x';i\omega_n,k_\parallel) = 0 \,.$$

The self-consistent relation to determine $\Delta(x)$ is given as follows:

$$\Delta(x) = -V(x)T\sum_{n,k_\parallel} F^*(x,x;i\omega_n,k_\parallel) \,. \tag{2.13}$$

The quantities $G(x,x';i\omega_n,k_\parallel)$ and $F(x,x';i\omega_n,k_\parallel)$ are expressed with use of $u_{\nu,k_\parallel}(x)$ and $v_{\nu,k_\parallel}(x)$ as follows:

$$G(x,x';i\omega_n,k_\parallel) = \sum_\nu \left[\frac{u_{\nu,k_\parallel}(x)u^*_{\nu,k_\parallel}(x')}{i\omega_n - \varepsilon_{\nu,k_\parallel}} + \frac{v^*_{\nu,k_\parallel}(x)v_{\nu,k_\parallel}(x')}{i\omega_n + \varepsilon_{\nu,k_\parallel}} \right] , \tag{2.14}$$

$$F(x,x';i\omega_n,k_\parallel) = \sum_\nu \left[\frac{v_{\nu,k_\parallel}(x)u^*_{\nu,k_\parallel}(x')}{i\omega_n - \varepsilon_{\nu,k_\parallel}} - \frac{u^*_{\nu,k_\parallel}(x)v_{\nu,k_\parallel}(x')}{i\omega_n + \varepsilon_{\nu,k_\parallel}} \right] .$$

Therefore, using Eq. (2.14), the following relations are obtained:

$$G(x+a,\, x'+a;i\omega_n,k_\parallel) = G(x,x';i\omega_n,k_\parallel) \,, \tag{2.15}$$

$$F(x+a,\, x'+a;i\omega_n,k_\parallel) = F(x,x';i\omega_n,k_\parallel)\exp(-i\phi) \,.$$

Extending the method introduced by McMillian,[18] Ishii,[19] and Furusaki[20] in the S-N-S junction to the present system, we obtain $G(x,x';i\omega_n,k_\parallel)$, $F(x,x';i\omega_n,k_\parallel)$ analytically. With use of these Green's functions, $\Delta(x)$ can be obtained by Eqs. (2.12). The obtained pair potential does not take a constant value in the superconductor region and deviates from the assumed value. It is difficult to take full account of the spatial dependence of the pair potential. Therefore we use the following iteration procedure. First, we calculate the averaged value of the pair potential Δ_1 in S region,

$$\Delta_1 = \frac{\int_{-d_S}^{0} \Delta(x)dx}{d_S} \,, \tag{2.16}$$

and require the self-consistency condition

$$\Delta_1 = \Delta_0 \,. \tag{2.17}$$

We replace Δ_0 in Eq. (2.1) for Δ_1, and the same procedure is repeated to obtain the new average of the pair potential in the S region, Δ_2. These processes are repeated many times until the difference between Δ_n and Δ_{n+1} becomes sufficiently small. Although this process includes approximation of averaging over the spatial change of the pair potential in S, the amplitude of the pair potential is obtained self-consistently. $\Delta_{\rm av}$ denotes the converged value of the averaged pair potential hereafter.

III. ENERGY SPECTRUM OF QUASIPARTICLES IN A SUPERCONDUCTING SUPERLATTICE

In this section, based on the obtained pair potential, we discuss the energy spectrum of quasiparticles. In Figs. 2 and 3, the local DOS (LDOS)

$$\rho(x,E) = -\frac{1}{\pi}\lim_{\delta\to 0}\sum_{k_\parallel} \mathrm{Im}[G(x,x;E+i\delta,k_\parallel)] \tag{3.1}$$

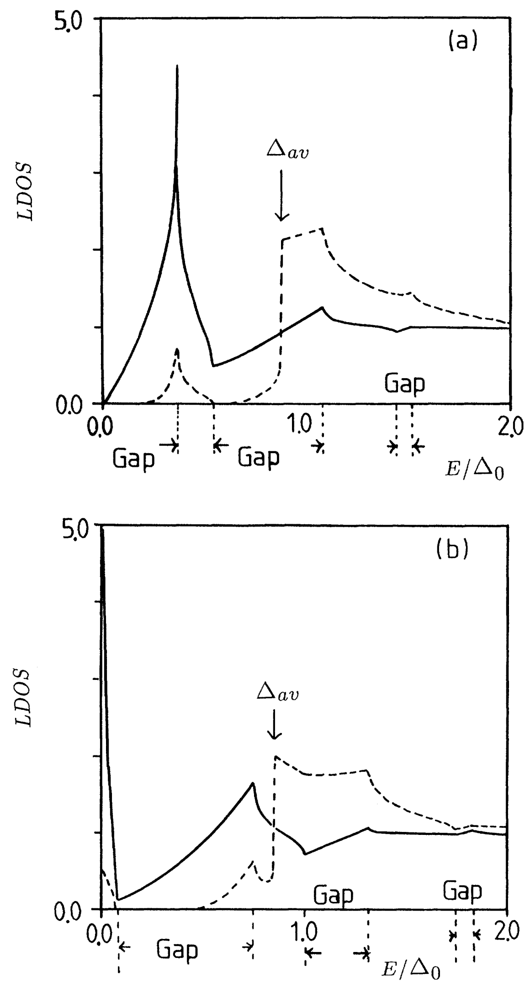

FIG. 2. Typical LDOS in the normal region for $d_S=d_N=5000$ Å at $(T/T_{C0}=0.1)$. The energy is normalized by Δ_0 which is the value of the pair potential of bulk S material. T_{C0} is the critical temperature of bulk S material. (a) $\phi=0.0$ and (b) $\phi=\pi$.

is plotted with different d_S and d_N at $T = 0.1T_{C0}$ in the N region (full line) and in the S region (dashed line). The quantity E is the energy of a quasiparticle measured from the Fermi level in units of Δ_0, which is the value of the pair potential of the bulk S material. The quantity T_{C0} is the critical temperature of the bulk S material. The LDOS $\rho(x,E)$ is a periodic function of ϕ with period 2π. The quantity ϕ is chosen as (a) $\phi = 0.0$ and (b) $\phi = \pi$, respectively. In the N region, the interference between the electronlike and the holelike quasiparticle does not occur since the pair potential vanishes. For this reason, $\rho(x,E)$ becomes constant in N. On the other hand, in the S region where the pair potential takes finite value, $\rho(x,E)$ depends on x. Both in Figs. 2 and 3, x is chosen as $x = -d_S/2$ as a representative value. In the k_x direction, quasiparticles form energy bands due to the periodic distribution of the pair potential, which generates multiple Andreev reflections at the S-N interfaces. To see the electronic structure of this system more clearly, the position of the energy gaps in the one-dimensional dispersion in the k_x direction are indicated in Figs. 2 and 3 for the states with $k_\parallel = 0$.

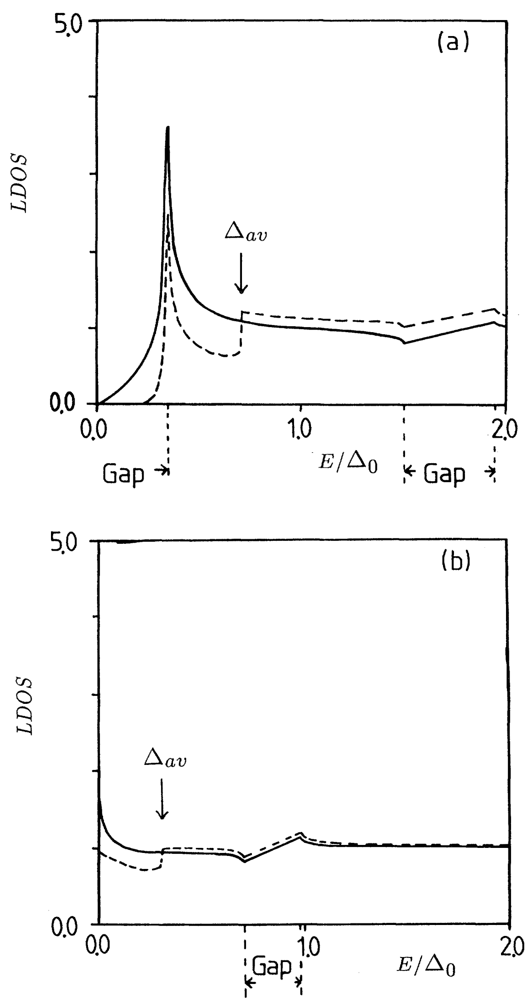

FIG. 3. Plots similar to Fig. 2 with $d_S = d_N = 2000$ Å, (a) $\phi = 0.0$ and (b) $\phi = \pi$.

First, let us make some comments on Fig. 2, in which the quantities d_S and d_N are chosen as $d_S = d_N = 5000$ Å$=2.3\xi$, where the coherence length ξ in the bulk S material is given as

$$\xi = \hbar v_F / \Delta_0 \ . \tag{3.2}$$

Due to the proximity effect, and the destruction of the pair potential by finite ϕ, Δ_{av} becomes smaller than Δ_0. The ratio Δ_{av}/Δ_0 becomes 0.88 and 0.85 for cases (a) and (b), respectively. In both cases, the LDOS in the S region takes smaller value than that in the N region when $E < \Delta_{av}$ is satisfied. However in the case of $E > \Delta_{av}$, the LDOS in the S region becomes larger than that in the N region. It should be remarked that in the case of $E < \Delta_{av}$, the quasiparticle localizes in the N region. When E becomes larger than Δ_{av}, the amplitude of quasiparticle is larger in the S region than that in the N region. With further increases in E, the distribution of quasiparticles becomes uniform in both the S and N regions. The dashed line shows discontinuous jump at Δ_{av} in each case. The jump of the LDOS at $E = \Delta_{av}$ arises from the following fact. In the S region, a quasiparticle is sensitive to the presence of the averaged pair potential Δ_{av}. Therefore the LDOS of the bulk S material diverges when $E = \Delta_{av}$ is satisfied. However, this singular behavior of the LDOS is weakened by the proximity effect due to the adjacent N materials, and only a discontinuous jump in the LDOS remains.

Since the LDOS is an even function of the energy E, the width of the first energy gap in the one-dimensional dispersion centers around $E = 0.0$, and is found to be nearly $0.7\Delta_0$ from Fig. 2(a). In this region the LDOS increases with increasing E. In the lowest-energy band (from $0.38\Delta_0$ to $0.54\Delta_0$), the LDOS decreases with increasing E. The second energy gap is situated at the region from $0.54\Delta_0$ to $1.08\Delta_0$ and the width of this gap is smaller than that of the first gap. With the increase of E, the width of the energy gap decreases, and finally vanishes. In such a case, the LDOS becomes 1.0, which is the value of the normal state.

In the case of Fig. 2(b) ($\phi = \pi$), the energy $E = 0.0$ does not lie in the gap, but locates at the one-dimensional dispersion in the k_x direction. The energy dependence of the LDOS is drastically changed from that in case (a). The LDOS takes peak at $E = 0.0$. It should be remarked, that although Δ_{av} is finite, the energy gap does not exist at $E = 0.0$. This is a gapless state of the superconductivity. However there are also some features similar to (a). When the LDOS increases with increasing E, E is in the energy gap in the k_x direction, and when the LDOS decreases with increasing E, E is in the energy band in the k_x direction. Also as E increases, the LDOS becomes unity as in (a).

In Fig. 3 the quantities d_S and d_N are chosen as $d_S = d_N = 2000$ Å$=0.92\xi(k_\parallel = 0)$, while the other parameters are the same as in Fig. 2. As compared to Figs. 2(a) and 2(b), the qualitative features of the LDOS are similar. However, since the length of the unit period a is smaller in the case of Fig. 3 than in the case of Fig. 2, the proximity effect becomes more significant. For this reason, Δ_{av} becomes smaller as compared to Fig. 2, namely; $\Delta_{av}=0.7\Delta_0$ and $\Delta_{av}=0.3\Delta_0$ for (a) ($\phi=0.0$) and (b) ($\phi=\pi$), respectively. Since the length of the unit period becomes smaller, the band width of the one-dimensional dispersion becomes larger. So, as seen from Figs. 3(a) and 3(b), the width between the nth peak and the $(n+1)$th peaks is broader than that of in the case of Figs. 2(a) and 2(b).

It is interesting to clarify how the supercurrent depends on the phase ϕ. So we calculate the supercurrent I with use of the obtained Green's function $G(x,x';k_{\parallel},i\omega_n)$ as follows:

$$I=\frac{e\hbar}{2im}\sum_{k_{\parallel}}\lim_{x'\to x}\left[\frac{\partial}{\partial x'}-\frac{\partial}{\partial x}\right]k_B T\sum_n \mathrm{Tr}\{G(x,x';k_{\parallel},i\omega_n)\}\ ,$$

$$=\sum_{k_{\parallel}}\frac{2iek_BT}{\hbar}\sum_n\left[\sum_{n'=0}^{n'=\infty}A_{n'}(\bar{\delta}d_n,\bar{\gamma}d_S;\omega_n,\Delta_{av})\exp(2n'\bar{\delta}d_n)\sin(n'\phi)\right]\ . \tag{3.3}$$

In the above, $\bar{\delta}$, $\bar{\gamma}$, and ω_n are defined by

$$\bar{\delta}=|\omega_n|/\hbar v_{Fx},\quad \bar{\gamma}=\sqrt{\omega_n^2+\Delta_{av}^2}/\hbar v_{Fx},\quad \omega_n=2\pi k_B T(n+\tfrac{1}{2})\ , \tag{3.4}$$

and R denotes $(\pi\hbar/e^2)$.[13] It should be remarked that the supercurrent can be written with a series of $\sin(n'\phi)$ and the n'th term corresponds to the process of the quasiparticle reflected at the S-N interface $2n'$ times. In the case when d_S is infinite, our theory reproduces the previous theory of S-N-S junction by Ishii.[19] In such a case, when d_N is sufficiently large, the quasiparticle which reflects many times at the S-N interface does not contribute to the supercurrent and only the term proportional to $\sin(\phi)$ remains in Eq. (3.3). This feature reproduces the previous results by Josephson,[21] Ambegaokar and Baratoff.[22]

In the three-dimensional system, there are many kinds of quasiparticle which have different wave numbers. We have divided RI by $(L_{\parallel}k_F)^2/4\pi$, which is the number of quasiparticles in the y,z directions. In Fig. 4, RI is plotted as a function of the phase difference ϕ for various d_N with $d_S=5000$ Å$=2.3\xi$ at $T/T_{C0}=0.1$. In the case of $\phi=0$ and $\phi=\pi$, the supercurrent becomes zero for all cases of d_N. This property can be seen from Eq. (3.4). The quantity RI becomes maximum at a certain intermediate value of ϕ. This ϕ is called ϕ_C hereafter. The qualitative features of Fig. 4 can be summarized as follows. With the decrease of d_N, RI becomes larger. When the unit period d_N is sufficiently large (a) ($a=6.9\xi$), the supercurrent is roughly proportional to $\sin(\phi)$, and ϕ_C is 0.5π. As the period d_N decreases (b) ($a=4.6\xi$), ϕ_C deviates from 0.5π. In cases (a) and (b), d_S and d_N is sufficiently large, RI is roughly proportional to $\sin(\phi)$ as discussed above. However in the case of (c) and (d), where the quasiparticle makes Andreev reflections many times, RI cannot be expressed by $\sin(\phi)$.

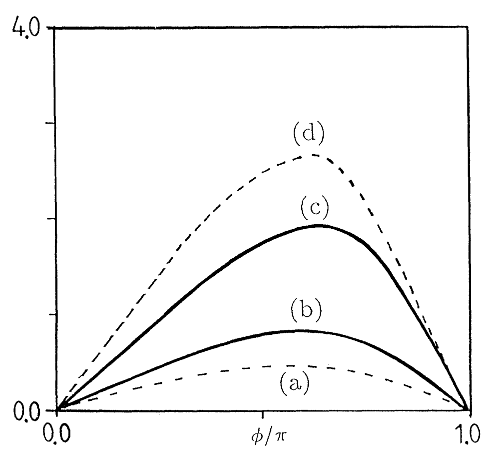

FIG. 4. The supercurrent I through the junction is plotted as the function of ϕ for various d_N, with fixed d_S ($d_S=5000$ Å$=2.3\xi$) at ($T/T_{C0}=0.1$). T_{C0} is the critical temperature of bulk S material and R_n is the resistance in the normal state. (a) $d_N=7500$ Å$=3.45\xi$, (b) $d_N=5000$ Å$=2.3\xi$, (c) $d_N=2000$ Å$=0.92\xi$, and (d) $d_N=1000$ Å$=0.46\xi$.

In Fig. 5, plots similar to Fig. 4 are made for various values of the superlattice period $a=d_S+d_N$ with $d_S=d_N$ at $T/T_{C0}=0.1$. When the unit period a is sufficiently large (a) ($a=6.9\xi$), ϕ_C nearly equals 0.6π, and the supercurrent is roughly proportional to $\sin(\phi)$. Comparing Figs. 5(a) and 5(b), we can see that the quantity RI increases with the decrease of a in this region. These features are similar to the case of Figs. 4(a) and 4(b). As a decreases further [(c) ($a=1.84\xi$)], ϕ_C becomes smaller than that in cases (a) and (b). In such a case, the interference of the quasiparticle due to the periodic structure becomes significant and ϕ_C becomes smaller than 0.5π. Furthermore, the coupling between the adjacent superconductors becomes significant, and $\Delta(x)$ is much reduced when ϕ exceeds ϕ_C. When a decreases further to 2000 Å [Fig. 5(d)], ϕ_C becomes 0.20, and $\Delta(x)$ vanishes when ϕ is larger than 0.25. In this case the ϕ dependence of the supercurrent is very different from the previous theories.[21,22] To summarize the above features we get the following conclusions. When the period a becomes short, the width of the normal region decreases and this is favorable to obtain larger value of R_nI. But there is a disadvantage for superconductivity in the case of short a;

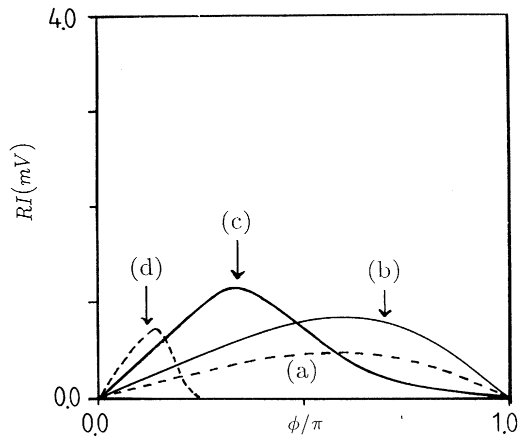

FIG. 5. Plots similar to Fig. 4 with $d_S=d_N(a=2d_S)$, (a) $a=15\,000$ Å$=6.9\xi$, (b) $a=10\,000$ Å$=4.6\xi$, (c) $a=4000$ Å$=1.84\xi$, and (d) $a=2000$ Å$=0.92\xi$.

the amplitude of the pair potential in the superconductor region decreases due to the proximity effect. Further, the coupling between the adjacent superconductors becomes significant, and the increase of ϕ makes superconducting state unstable. So, in order to obtain larger values of R_nI, the length of the unit period should be chosen properly.

Finally, we show the phase diagram (Fig. 6) at $T/T_{C0}=0.1$ as a function of the unit period a and the phase difference ϕ. There are three phases; the superconducting state (S), the normal state (N), and the gapless superconducting state (GS).

First, we discuss the boundary between the GS state and the S state. The LDOS at $E=0$ takes finite value for the GS state, while it is zero for the S state. Thus, we investigate the LDOS in the N region at $E=0.0$ which is given as

$$\rho_N=\lim_{E\to 0}\rho(x\in N,E)$$
$$=N(0)\lim_{E\to 0,\delta\to 0}\int_1^\infty \frac{dy}{y^2}\,\mathrm{Re}[Z(y,E+i\delta)]\ . \quad (3.5)$$

In the above, the quantity $Z(y,0)$ can be expressed as

$$Z(y,0)=\begin{cases}0, & g(y)>0\ ,\\ \mathrm{Re}\left[\dfrac{C}{D}\right]^{1/2}, & g(y)<0\ ,\end{cases} \quad (3.6)$$

with $y=k_F/k_{Fx}$, and

$$C\equiv[1+\exp(-yd_S/\xi)]^2[1-\exp(i\phi)]^2\ ,$$
$$D\equiv 4\exp(i\phi)[1-\exp(-yd_S/\xi)]^2 g(y)\ , \quad (3.7)$$
$$g(y)\equiv 1-\coth^2(yd_S/\xi)\sin^2(\phi/2)\ .$$

As seen from the above equations, $g(y)$ is a monotonously increasing function of y. For this reason, if $Z(y,0)$ is zero at $y=1$, then it will be zero for any $y>1$. On the other hand, when $Z(y,0)$ takes finite value for $y=1$, ρ_N does not vanish. Namely, when $g(y=1)<0$ is satisfied, our system is in the GS state. Therefore, we can determine the boundary between S and GS by the sign of $g(y=1)$. On the other hand, the boundary between the GS and the N state is determined by the following linearized Gor'kov's equation:

$$\Delta(x)=V(x)T\int_{-\infty}^{\infty}\sum_n\sum_{k_\parallel}K(x,x';\omega_n,k_\parallel)\Delta(x')dx'\ , \quad (3.8)$$
$$K(x,x';\omega_n,k_\parallel)\equiv G(x,x';i\omega_n,k_\parallel)G(x,x';-i\omega_n,k_\parallel)\ .$$

We have solved Eq. (3.8) numerically as an eigenvalue problem.

In the case of sufficiently large a ($a>0.7\xi$), a transition from S to GS occurs with the increase of ϕ. The N phase does not appear. When a becomes smaller ($0.2\xi<a<0.7\xi$), the transition from GS to N occurs. There are three phases: S, GS and N with the increase of ϕ from 0 to π. In the case of very short a, the region of GS and S phase decreases. As seen from the phase diagram, the ground state of the superconducting superlattice strongly depends on ϕ.

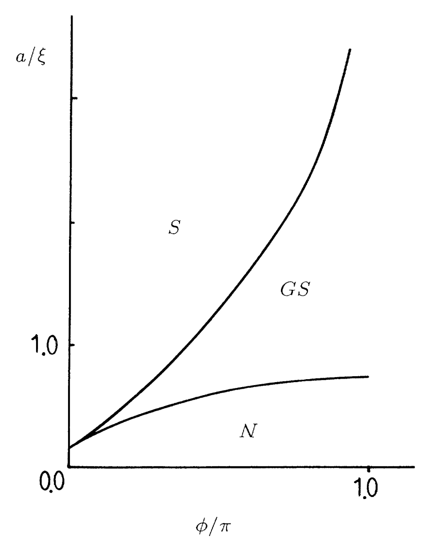

FIG. 6. The phase diagram is expressed as the function of a and ϕ.

IV. CONCLUSIONS

In this paper we have clarified basic properties of the superconducting superlattice below the critical temperature. With use of Gor'kov's Green's function we have determined the pair potential $\Delta(x)$ self-consistently, when the phase difference of the pair potential between the adjacent superconductors exists. In our treatment, although the spatial dependence of the pair potential has been averaged out, and thus the pair potential has not been obtained fully self-consistently, the essential point of the energy spectrum of the superconducting superlattice is clarified. In our previous one-dimensional theory, we have clarified that the quasiparticle consists of the energy band which originates from the Andreev reflection.[23] The feature of this band structure also appears in the LDOS in the three-dimensional system as seen from Figs. 2 and 3.

There appear three states, i.e., the superconducting state, the gapless superconducting state, and the normal state depending on the value of parameters of the superlattice period a and the phase difference ϕ. As ϕ increases from 0, the superconducting state becomes unstable in general. In the case of larger a, the superconducting state changes into the gapless superconducting state with the increase of ϕ. When a is reduced, $\Delta(x)$ vanishes for larger ϕ and the gapless superconducting state changes into the normal state. It should be remarked that the LDOS of the quasiparticle is influenced by ϕ as well as by the lattice constant of the unit period. So it is interesting to observe the LDOS by STS when the supercurrent is flowing perpendicularly to the layer.

We have also clarified the ϕ dependence of the supercurrent in the superconducting superlattice. In general, the supercurrent is expressed by an infinite series of $\sin(n\phi)$, where n is a positive integer. In the previous work in the weak-link limit by Ishii[19] and Kulik and Omelyanchuk,[24] the existence of the $\sin(n\phi)$ $(n>1)$ terms are predicted. Since these theories treated the infinite S system, the amplitude of the pair potential in the S region is insensitive to the difference of the phase ϕ and the length of the normal region d_N. In Ishii's theory,[19] only the bound state appears in the energy gap region. We have developed a theory of supercurrent which is valid for finite d_S as an extension of Ishii's theory, and which the band structure due to the Andreev reflection is considered. As seen from our theory, when the length of the unit period becomes shorter, the coupling between the adjacent superconductors becomes significant. In such a case, the destruction of the pair potential due to the proximity effect becomes significant, and the superconducting state is unstable for finite ϕ. When this effect becomes significant, the pair potential vanishes and the ϕ dependence of the supercurrent becomes very different from that of the previous theories.[19–22,24]

There are many limitations in our paper, which should be considered as future problems. In the present formalism, the spatial dependence of the pair potential is not obtained completely except near the critical temperature. Below the critical temperature, only the averaged value is obtained self-consistently. For a more quantitative discussion, the spatial dependence of the pair potential should be determined completely at any temperature. Furthermore, the quantity $U(x)$ is assumed to be zero. It is desired that $U(x)$ should be chosen as more realistic function. We also did not consider the effect of the magnetic field on the supercurrent. The obtained ϕ dependence of the supercurrent should be observed by the experiment under the magnetic field. It is also interesting to clarify how the supercurrent is influenced by the microwave. The interference of the quasiparticle in the multilayered structure and the energy spectrum of the quasiparticle may be observed by the Shapiro step, and by STS, respectively.

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