PHYSICAL REVIEW B VOLUME 44, NUMBER 14 1 OCTOBER 1991-II

Superconducting phase transition in a two-dimensional Chern-Simons theory

J. Kapusta
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455

M. E. Carrington
Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455

B. Bayman, D. Seibert, and C. S. Song
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455
(Received 9 October 1990; revised manuscript received 14 May 1991)

We consider a two-dimensional Chern-Simons gauge theory where the gauge field couples to an internal degree of freedom of the fermion. For a fixed gauge coupling g there exists a finite window of Chern-Simons magnetic moment coupling g' for which a nonzero Chern-Simons magnetic field is generated at $T=0$. The system then is superconducting without Cooper pairing. There is a second-order thermodynamic phase transition at $T_c>0$ where the Chern-Simons magnetic field goes to zero and superconductivity terminates.

I. INTRODUCTION

Particles in three dimensions obey either Bose or Fermi statistics. In two dimensions, a continuous range of possibilities (anyons) exists between these two limits. They are said to obey fractional statistics.

Much of the recent interest in anyons is due to the possibility of their relevance to certain types of elementary excitations in condensed matter physics. They have been shown to play a role in the fractional quantum Hall effect.[1] Furthermore, Laughlin has suggested that the charge carriers in high-temperature superconductors may be anyons.[2] It has been demonstrated that particles which have fractional statistics, or anyons, can be described by means of an Abelian Chern-Simons field theory.[3,4] Both the Goldstone pole in the current-current correlation function and the Meissner effect at zero temperature have been derived.[5,6]

One difficulty with the previous discussions of anyon high-temperature superconductivity is that there is no evidence that these theories imply a superconducting phase transition at $T>0$. In this paper we propose a modification of the theory which *does* lead to a second-order transition to a superconducting state at a critical temperature $T_c>0$ when the parameters of the theory lie within a certain range.

This paper is organized as follows. In Sec. II we formulate our theory. The theory is analyzed in the mean-field approximation. By extremizing the thermodynamic potential, we show numerically that there is a second-order phase transition. We obtain a phase diagram for the critical temperature as a function of Chern-Simons magnetic moment coupling, and show that there exists a finite region of this coupling for which superconductivity exists. An analytic expansion of the thermodynamic potential verifies that the transition is second order, and reproduces the upper and lower critical values of the coupling. In Sec. III we study the Meissner effect at $T>0$. Near the critical temperature T_c the penetration depth λ behaves as $(T_c-T)^{-1/2}$, which indicates a second-order phase transition. In Sec. IV we summarize our results.

II. FORMULATION

The Chern-Simons Lagrangian studied in Ref. 5 is

$$\mathcal{L}=\psi^\dagger iD_0\psi-\frac{1}{2m}|D_k\psi|^2+\tfrac{1}{2}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda \ , \tag{1}$$

where $D_\mu=\partial_\mu-iga_\mu$ is the covariant derivative, a_μ is the Chern-Simons (sometimes called statistical) gauge field, and ψ is the fermion field. The fermion can be chosen to have spin-0 or spin-$\frac{1}{2}$, it makes little difference in this model. An operator identity following from (1) is

$$f_{12}\equiv\partial_1a_2-\partial_2a_1=-g\psi^\dagger\psi \ . \tag{2}$$

Thus, an average Chern-Simons magnetic field $b=\langle f_{12}\rangle$ is generated by a net particle density $n=\langle\psi^\dagger\psi\rangle$.

$$b=-gn \ . \tag{3}$$

In the mean-field approximation the fermions move in Landau orbitals with a cyclotron frequency $\omega_c=g|b|/m$. Usually, g is chosen so that $g^2/2\pi=1/N$, where N is an integer. Then an integral number of Landau levels are completely filled at $T=0$. Equation (3) is a self-consistency condition on b if one chooses as independent variables chemical potential μ and temperature T. A Meissner effect has been proven, both at $T=0$ and $T>0$.[5–7]

If this model is to be applied to real materials it has the failing that it has no phase transition and the Meissner effect persists to arbitrarily high temperatures. In our view this can be traced to Eqs. (2) and (3).

A minimal modification of (1) would be to have two types of fermions, called + and −, which couple to the Chern-Simons field with equal and opposite couplings $\pm g$. Then (3) would be replaced by

$$b=-g(n_+-n_-) . \tag{4}$$

If the − fermion had a lower energy than the + fermion with an energy difference proportional to b, then it could happen that $b\neq0$ at low temperatures but that $b=0$ at high temperatures due to thermal fluctuations. Let us therefore consider the Lagrangian

$$\mathcal{L}=\psi^\dagger iD_0\psi-\frac{1}{2m}|D_k\psi|^2+\tfrac{1}{2}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda -f_{12}\frac{1}{2m}\psi^\dagger(g'\sigma_3+g'')\psi , \tag{5}$$

where $D_\mu=\partial_\mu-iga_\mu\sigma_3$, ψ is a two-component spinor and σ_3 is the Pauli matrix. There are two types of magnetic coupling with coupling constants g' and g''. The σ_3 coupling is the important one, since it splits the energy levels of the $\pm$ fermions, and gives us the effect we want.

At this point we do not specify the origin of the two fermion types. It could be spin: in this case (5) is to be viewed as an "Ising" approximation since spin SU(2) is reduced to U(1). If this symmetry breakdown is spontaneous, then magnons should be added to (5). However, these magnons would not affect the mean-field analysis presented below. One could consider elevating the Chern-Simons gauge field to a non-Abelian field $\mathbf{a}_\mu$, in which case the extension of (5) would have a local unbroken SU(2) symmetry when $g''=0$. Of course, coupling to an internal fermion degree of freedom other than spin is also a possibility. In any event, we focus here on (5) as our model and analyze its thermodynamic properties.

We allow the fermions to move independently in the mean fields $a_0=\text{const}$, $b=\text{const}$. These fields are generated by the fermions and so will be T and μ dependent. The fermions move in Landau orbitals with energies

$$E_{i\sigma}=\left[i+\frac{1}{2}+\frac{1}{2}\frac{g''}{g}+\frac{1}{2}\frac{g'}{g}\sigma\right]\omega_c$$

and effective chemical potentials $\mu_\sigma=\mu+ga_0\sigma$, where $\sigma=\pm1$ and $i=0,1,2,\ldots$. The thermodynamic potential is

$$\Omega=\frac{-g|b|T}{2\pi}\sum_{i\sigma}[\tfrac{1}{2}\beta E_{i\sigma}+\ln(1+e^{-\beta(E_{i\sigma}-\mu_\sigma)})]-a_0b . \tag{6}$$

The shift in the zero-point energy (Casimir effect) due to $b\neq0$ is computed by standard methods to be

$$\Omega_{\text{zero}}=\frac{(3g'^2+3g''^2-g^2)b^2}{48\pi m} . \tag{7}$$

Ω is an extremum with respect to the condensates a_0 and b at fixed μ and T. The condition $\partial\Omega/\partial a_0=0$ just reproduces (4)

$$b=\frac{-g^2|b|}{2\pi}\sum_{i\sigma}\sigma\rho_{i\sigma} , \tag{8a}$$

where

$$\rho_{i\sigma}=\frac{1}{e^{\beta(E_{i\sigma}-\mu_\sigma)}+1} . \tag{8b}$$

This can now be thought of as determining $a_0=a_0(b,\mu,T)$. This solution is substituted back into Eqs. (6) and (7) and Ω is minimized with respect to b. In general, this must be done numerically.

As an example we take $m=10m_e$ (electron mass) and $N=10$ $(g^2/2\pi=0.1)$. We also take $g'/g=9$; this perhaps can be motivated by saying that, although the dynamic mass of the fermion (the mass that enters in the kinetic energy) is enhanced by many-body effects, the mass that enters in the expression for the magnetic moment is not modified,[8] hence, we might expect $g'/g\simeq m/m_e$. Since the energy splitting between + and − states does not depend on g'', it is an inessential parameter and we set it to zero. Finally, we take $\mu=0.01$ eV. The result of a numerical calculation is displayed in Fig. 1. The Chern-Simons magnetic field $b(T)$ goes smoothly to zero at a critical temperature $T_c=125$ K, as is characteristic of a second-order phase transition. Since μ is fixed in this calculation the number density $n=n_++n_-$ varies slowly with T: $n(0\text{ K})=0.7\times10^{-2}$ Å^{-2} and $n(T_c)=1.6\times10^{-2}$ Å^{-2}.

By varying g' we obtain the phase diagram shown in Fig. 2 from which we can make the following observations. The critical temperature is a sensitive function of g'. Our model does not predict the critical temperature, but reproduces an experimentally typical value for physically reasonable choices of the parameters. There are upper and lower critical values for superconductivity to occur. From Fig. 2 we have approximately $g'_{\min}=4.6$ and $g'_{\max}=27$. We can interpret this as follows. From (4), it is only when we have unequal number densities for

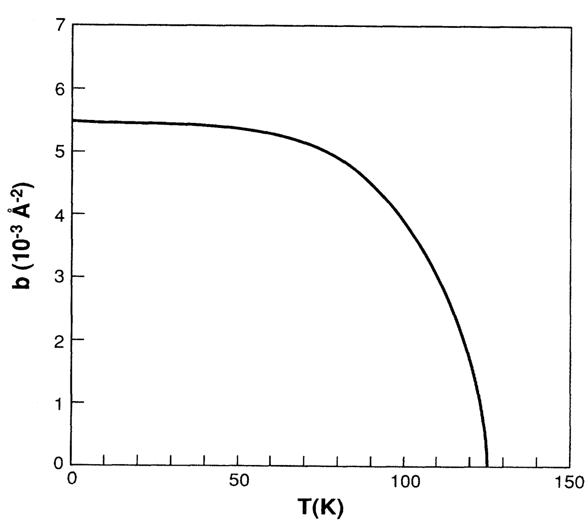

FIG. 1. Plot of the mean Chern-Simons magnetic field b as a function of temperature T. This curve is obtained by numerical solution of Eqs. (6)-(8), using the parameters specified in the text.

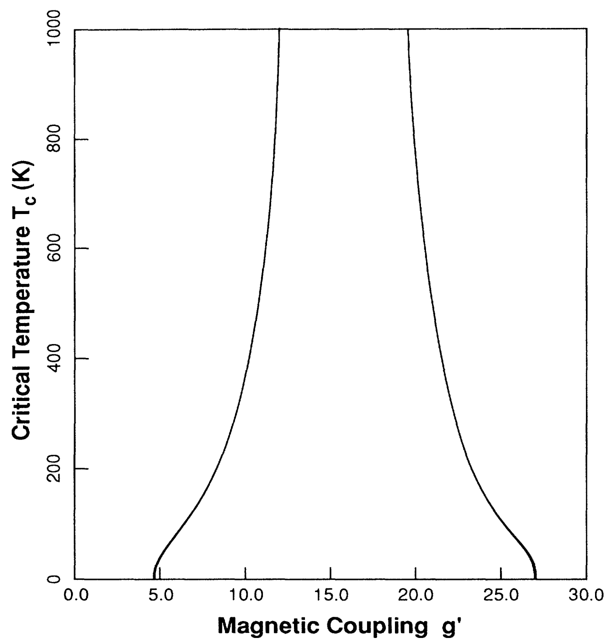

FIG. 2. Plot of the critical temperature as a function of magnetic coupling for fixed $g^2/2\pi=0.1$.

the two species of fermions that we have nonzero magnetic field. To have a significant difference in the number densities, we must have a large enough asymmetry in the energies of the two species and thus the magnetic coupling must be greater than a certain lower critical value. Conversely, if the magnetic coupling is too large, the zero-point contributions (7) dominate in the expression for the thermodynamic potential (6), and the only solution of the extremization condition occurs for $b=0$. Finally, we note that there is a singularity at $\frac{1}{2}g'/g=N$. This singularity is presumably an unphysical artifact of the mean-field approximation. Numerically we find that the condensates a_0 and b approximately satisfy $ga_0=-2.47\omega_c$ for $\frac{1}{2}g'/g<N$ and $ga_0=2.47\omega_c$ for $\frac{1}{2}g'/g>N$.

We have looked at the sensitivity of the critical temperature as a function of chemical potential. With $N=10$, and $g'/g=9$, chemical potentials of $\mu=0.005$, 0.01, and 0.02 eV give critical temperatures of 62, 125, and 348 K, respectively. Thus, the critical temperature is also a fairly sensitive function of chemical potential. We will comment further on the significance of this phase diagram in Sec. IV

We have also done an analytic calculation that verifies that the phase transition is second order. For small values of b the sum involved in (6) can be approximated by the Euler-MacLaurin formula. To order b^4 we find, after eliminating a_0,

$$\Omega(b)-\Omega(0)=-\tfrac{1}{2}c_2\omega_c^2+\tfrac{1}{4}c_4\omega_c^4 , \tag{9}$$

where

$$\begin{aligned} c_2&=m\left[\frac{g'}{g^3}-\frac{(3g'^2-g^2)}{24\pi g^2}-\frac{u}{12\pi}-\frac{\pi}{g^4}\frac{1}{u}\right] , \\ c_4&=\frac{m}{96\pi T^2}(1-u)(2u-1)u \\ &\quad\times\left[\left[\frac{2\pi}{g^2}\right]^4\frac{1}{u^4}-2\left[\frac{2\pi}{g^2}\right]^2\frac{1}{u^2}+\frac{7}{240}\right] , \\ u&=\frac{1}{1+e^{-\beta\mu}} . \end{aligned} \tag{10}$$

It is apparent from (9) that the phase transition is second order. The possibility that the phase transition is weaker than the second-order transition predicted by mean-field theory cannot be ruled out. Some Abelian two-dimensional continuum gauge theories have been shown to possess a Kosterlitz-Thouless transition.[9] The original Chern-Simons theory (1) has been formulated on the lattice and it has been argued to also undergo a Kosterlitz-Thouless transition.[10] So far, vortex solutions have not been found in the continuum theories (1) or (5). Therefore, the softening of the transition due to the Kosterlitz-Thouless mechanism in these theories remains an open problem.

This expansion also reproduces the upper and lower critical values of g' which were obtained numerically (see Fig. 2). In order that a nonzero b is generated, c_2 must be positive. At $T=0$ this means that only if g' lies in the window

$$2-\left[2-\frac{1}{3N^2}\right]^{1/2}<\frac{gg'}{2\pi}<2+\left[2-\frac{1}{3N^2}\right]^{1/2} \tag{11}$$

will a Chern-Simons magnetic field be generated. With $N=10$, (11) gives $g'_{\rm min}=4.65$ and $g'_{\rm max}=27$. Finally, for g' in this window, T_c can be computed from the condition $c_2(T_c)=0$. For the numerical example above, $u_c=0.7172$, $\beta_c\mu=0.9307$ which gives $T_c=125$ K.

III. FINITE-TEMPERATURE MEISSNER EFFECT

There is a simple argument, due to Chen *et al.*,[11] that the Chern-Simons model described by (1) exhibits the Meissner effect at $T=0$ as long as $2\pi/g^2=N=$integer. It is based on the observation that, with these values of the coupling, there is complete band filling up to some Fermi energy. This leads to an energy gap. Imposition of an external magnetic field costs an amount of energy proportional to $|B|$. Therefore, magnetic fields will tend to be excluded. In our case, the self-consistency condition (8) evaluated at $T=0$ is $2\pi/g^2=N_--N_+$, where N_+ and N_- are the number of filled levels of $\sigma=+1$ and $\sigma=-1$, respectively. If we choose $2\pi/g^2=N$, then one finds numerically that $N_+=0$ and $N_-=N$ for $T=0$. Hence, we have completely filled levels for the $-$ particle, and the elementary argument of Chern *et al.* can be applied. At $T>0$ the analysis is more involved. That is the topic of this section.

We have carried out finite-temperature calculations analogous to the zero-temperature calculations of Ref. 5

which showed that the model (1) exhibits a Meissner effect. We couple the fermions to the electromagnetic vector potential A_μ with the covariant derivative $D_\mu=\partial_\mu-ieA_\mu-iga_\mu\sigma_3$, and include the Zeeman term $-(e/2m)B\psi^\dagger\sigma_3\psi$ in the Lagrangian.[12] The material occupies the half-plane $x_2\geq 0$. A real, weak, static, magnetic field B is applied in a direction perpendicular to the plane. We work in the Landau gauge. Inside the material the vector potentials are $A_0(x_2)$, $A_1(x_2)$, $A_2=A_3=0$, and the condensates develop nonzero fluctuations such that

$$a_0(x_2)=a_0+a_0^{(1)}(x_2)\ ,$$

$$a_1(x_2)=-b^{(0)}x_2+a_1^{(1)}(x_2)\ ,$$

$$a_2=0\ ,$$

where the superscript denotes the shift in the Chern-Simons vector potential caused by the imposition of the real magnetic field.

We proceed in the following way. First we calculate the electromagnetic and Chern-Simons currents to first order in perturbation theory by writing the Hamiltonian as $H=H_0+V$, where V is first order in the small quantities $A_0(x_2)$, $A_1(x_2)$, $a_0^{(1)}(x_2)$, and $a_1^{(1)}(x_2)$. Then we substitute these results into the equations of motion from (5) to obtain four equations for the four unknown variables $e^2=f^{20}$, $E^2=F^{20}$, $b=f^{12}$, $B=F^{12}$. Note that $e^1=E^1=j_2=J_2=0$. Finally, we solve these equations by implementing a local-density approximation as in Ref. 5; that is, we assume that the perturbing fields vary over a distance much larger than the average Bohr-Landau radius.

The Hamiltonian, to first order, is

$$H=H_0+V\ , \tag{12}$$

$$H_0=-\frac{1}{2m}[(\partial_1-igbx_2\sigma_3)^2+\partial_2^2]-ga_0\sigma_3+\frac{b^{(0)}}{2m}(g'\sigma_3+g'')\ , \tag{13}$$

$$V=-\frac{1}{m}(ga_1^{(1)}\sigma_3+eA_1)(gbx_2\sigma_3+i\partial_1)+\frac{1}{2m}[(g'\sigma_3+g'')b^{(1)}+eB\sigma_3]-(ga_0^{(1)}\sigma_3+eA_0)\ . \tag{14}$$

The zeroth-order solutions satisfy

$$H_0\psi_{nk\sigma}^{(0)}=\epsilon_{n\sigma}\psi_{nk\sigma}^{(0)}\ , \tag{15}$$

$$\epsilon_{n\sigma}=\left[n+\frac{1}{2}+\frac{1}{2}\frac{g''}{g}+\frac{1}{2}\frac{g'}{g}\sigma\right]\omega_c\ ,\quad \omega_c=\frac{g|b|}{m}\ . \tag{16}$$

We impose a periodic boundary condition in the x_1 direction, $\psi(t,x_1,x_2)=\psi(t,x_1+L,x_2)$. Taking the thermodynamic limit $L\to\infty$, and defining the magnetic length l by $l^{-2}=|gb^{(0)}|$, we have

$$\psi_{nk\sigma}^{(0)}=\frac{1}{(lL)^{1/2}}e^{-ikx_1}u_n\left[\sigma\frac{x_2}{l}+kl\right]v_\sigma\ ,$$

$$u_n(z)=\left[\frac{1}{2^n n!\sqrt{\pi}}\right]^{1/2}e^{-z^2/2}H_n(z)\ , \tag{17}$$

$$k=\frac{2\pi}{L}p\ ,\quad p\in\mathbb{Z}$$

$$\sigma_3v_\sigma=\sigma v_\sigma\ .$$

The first-order corrections to the wave function are given by

$$\psi_{nk\sigma}^{(1)}(x_1,x_2)=\sum_{k',\sigma'}\sum_{n'\neq n}\frac{\psi_{n'k'\sigma'}^{(0)}(x_1,x_2)}{\epsilon_{n\sigma}-\epsilon_{n'\sigma'}}\left[\int dy_1dy_2\psi_{n'k'\sigma'}^{(0)*}(y_1,y_2)V(y_1,y_2)\psi_{nk\sigma}^{(0)}(y_1,y_2)\right] \tag{18}$$

$$=-\sum_{k',\sigma'}\sum_{n'\neq n}\frac{\delta_{\sigma\sigma'}\delta_{kk'}}{m}\int_{-\infty}^{\infty}\frac{dy}{l}u_{n'}(\sigma y)u_n(\sigma y)C(y,k,\sigma)\ , \tag{19}$$

$$C(y,k,\sigma)=[ga_1^{(1)}(y)\sigma+eA_1(y)][gb^{(0)}y\sigma+k]+m[ga_0^{(1)}(y)\sigma+eA_1(y)]-\tfrac{1}{2}[g'\sigma+g'')b^{(1)}(y)+eB(y)\sigma]\ . \tag{20}$$

Now we can calculate the currents to first order. For example, to first order, the zeroth component of the electromagnetic current is given by

$$\langle J_0(x)\rangle=e\langle\hat{\psi}^\dagger(\mathbf{x},t)\hat{\psi}(\mathbf{x},t)\rangle\ , \tag{21}$$

$$\hat{\psi}(\mathbf{x},t)=\sum_{n,k,\sigma}a_{nk\sigma}(\psi_{nk\sigma}^{(0)}+\psi_{nk\sigma}^{(1)})e^{-i\epsilon_{nk\sigma}t}\ , \tag{22}$$

$$\langle a_{n'k'\sigma'}^\dagger a_{nk\sigma}\rangle=\delta_{nn'}\delta_{kk'}\delta_{\sigma\sigma'}\rho_{n\sigma}\ . \tag{23}$$

Substituting in, we obtain

$$\frac{1}{e}\langle J_0(x_2)\rangle=\frac{g|b|}{2\pi}\sum_{i,\sigma}\rho_{i\sigma}+K[C(y,k,\sigma);x_2]\ , \tag{24}$$

where the linear functional is

$$K[h(y,k,\sigma);x_2]=\frac{1}{\pi}\sum_{n,\sigma}\rho_{i\sigma}\sum_{i'\neq i}\frac{1}{i'-i}\int_{-\infty}^{\infty}d\left[\frac{y}{l}\right]\int_{-\infty}^{\infty}d(kl)h(y,k,\sigma)u_{i'}\left[\sigma\frac{y}{l}+kl\right]$$
$$\times u_i\left[\sigma\frac{y}{l}+kl\right]u_{i'}\left[\sigma\frac{x_2}{l}+kl\right]u_i\left[\sigma\frac{x_2}{l}+kl\right]. \tag{25}$$

Recall that $u_i(x)$ are the normalized, one-dimensional harmonic-oscillator wave functions. The integrand contributes significantly only in the vicinity of $y=x_2$ because of the Gaussian falloff of the Hermite polynomials. We evaluate the integral by expanding the function $C(y,k,\sigma)$ in a Taylor series about x_2. Using the orthogonality relations of the Hermite polymonials, we obtain

$$\frac{1}{e}J_0(x_2)\simeq\frac{gb}{2\pi}\sum_{n,\sigma}\rho_{n\sigma}\left[1+l^2(gb^{(1)}+e\sigma B)+l^4m\,\partial_2(g\sigma f_{02}+eE_2)\right.$$
$$\left.+\frac{l^4}{2}(g'\sigma+g'')\partial_2^2b^{(1)}+\frac{l^4}{2}e\sigma\partial_2^2B+\frac{3}{4}\sigma l^4(2n+1)\partial_2^2(g\sigma b^{(1)}+eB)\right]. \tag{26}$$

In the same way we calculate

$$j^0=g\psi^\dagger\sigma_3\psi\ , \tag{27}$$

$$J^i=\left[\psi^\dagger\frac{e\Pi^i}{2m}\psi+\text{H.c.}\right]+\frac{e}{2m}\epsilon^{ij}\partial^j(\psi^\dagger\sigma_3\psi)\ , \tag{28}$$

$$j^i=\left[\psi^\dagger\frac{g\sigma_3\overline{\Pi}^i}{2m}\psi+\text{H.c.}\right]+\frac{1}{2m}\epsilon^{ij}\partial^j[\psi^\dagger(g'\sigma_3+g'')\psi]\ , \tag{29}$$

$$\Pi^i=-i\partial^i+eA^i\ , \tag{30}$$

$$\overline{\Pi}^i=-i\partial^i+ga^i\ .$$

The results are

$$\frac{1}{g}\langle j_0(x_2)\rangle=\frac{g|b|}{2\pi}\sum_{i,\sigma}\sigma\rho_{i\sigma}+K[\sigma C(y,k,\sigma);x_2]$$
$$\simeq\frac{gb}{2\pi}\sum_{n,\sigma}\sigma\rho_{n\sigma}\left[1+l^2(gb^{(1)}+e\sigma B)+l^4m\,\partial_2(g\sigma f_{02}+eE_2)\right.$$
$$\left.+\frac{l^4}{2}(g'\sigma+g'')\partial_2^2b^{(1)}+\frac{l^4}{2}e\sigma\partial_2^2B+\frac{3}{4}\sigma l^4(2n+1)\partial_2^2(g\sigma b^{(1)}+eB)\right], \tag{31}$$

$$\frac{1}{e}\langle J^1(x_2)\rangle=-\frac{1}{m}\frac{g|b|}{2\pi}\sum_{i,\sigma}\rho_{i\sigma}(eA_1+g\sigma a_1^{(1)})+K\left[\left[\omega_c x_2\sigma+\frac{k}{m}\right]C(y,k,\sigma);x_2\right]-\frac{1}{2mg}\partial_2\langle j_0(x_2)\rangle$$
$$\simeq\frac{gb}{2\pi}\sum_{n,\sigma}\rho_{n\sigma}\left[-\sigma l^2(g\sigma f_{02}+eE_2)-\frac{\sigma l^2}{2m}\partial_2[(g'\sigma+g'')b^{(1)}+e\sigma B]\right.$$
$$\left.-\frac{l^2}{m}(2n+1)\partial_2(g\sigma b^{(1)}+eB)-\frac{3}{4}(2n+1)\sigma l^4\partial_2^2(g\sigma f_{02}+eE_2)\right]-\frac{1}{2mg}\partial_2\langle j_0(x_2)\rangle\ , \tag{32}$$

$$\frac{1}{g}\langle j^1(x_2)\rangle = -\frac{1}{m}\frac{g|b|}{2\pi}\sum_{i,\sigma}\sigma\rho_{i\sigma}\left[eA_1+g\sigma a_1^{(1)}\right]+K\left[\sigma\left[\omega_c x_2\sigma+\frac{k}{m}\right]C(y,k,\sigma);x_2\right]$$
$$-\frac{g'}{2mg^2}\partial_2\langle j_0(x_2)\rangle-\frac{g''}{2mge}\partial_2\langle J_0(x_2)\rangle$$
$$\simeq\frac{gb}{2\pi}\sum_{i,\sigma}\rho_{n\sigma}\left[-l^2(g\sigma f_{02}+eE_2)-\frac{l^2}{2m}\partial_2[(g'\sigma+g'')b^{(1)}+e\sigma B]\right.$$
$$\left.-\frac{l^2\sigma}{m}(2n+1)\partial_2(g\sigma b^{(1)}+eB)-\frac{3}{4}l^4(2n+l)\partial_2^2(g\sigma f_{02}+eE_2)\right]$$
$$-\frac{g'}{2mg^2}\partial_2\langle j_0(x_2)\rangle-\frac{g''}{2mge}\partial_2\langle J_0(x_2)\rangle\ . \tag{33}$$

We note that calculating the wave functions in first-order perturbation theory[5] is completely equivalent to using standard linear response theory.[13]

Substituting these results into the equations of motion from (5),

$$j^\mu=-\tfrac{1}{2}\epsilon^{\mu\nu\lambda}f_{\nu\lambda}\ , \tag{34}$$

$$\partial_\nu F^{\mu\nu}=J^\mu-en_0\delta^{\mu 0}\ , \tag{35}$$

where $-en_0$ is the background neutralizing charge density, we obtain a set of four coupled, linear, differential equations. We have analyzed this set of equations using the local-density approximation as given above. We assume the fields vary as $e^{-x_2/\lambda}$, where $\lambda \gg l$. We define

$$n_+=\frac{gb}{2\pi}\sum_n\rho_{n,1}\ , \tag{36}$$

$$n_-=\frac{gb}{2\pi}\sum_n\rho_{n,-1}\ , \tag{37}$$

$$\nu_+=\frac{gb}{2\pi}\sum_n(n+1/2)\rho_{n,1}\ , \tag{38}$$

$$\nu_-=\frac{gb}{2\pi}\sum_n(n+1/2)\rho_{n,-1}\ . \tag{39}$$

In the limit that $b\to 0$, we find that

$$n_0=n_++n_-\sim\text{const}\ ,$$
$$\bar{n}=n_+-n_-\sim b\ ,$$
$$\nu_0=\nu_++\nu_-\sim 1/b\ ,$$
$$\bar{\nu}=\nu_+-\nu_-\sim\text{const}\ .$$

Then, to lowest order in b, we must solve the four equations

$$0=E_2(-eml^4n_0/\lambda)+f_{02}(-gml^4\bar{\nu}/\lambda)$$
$$+b^{(1)}(l^2gn_0+3gl^4\nu_0/2\lambda^2)+B(el^2\bar{n}+3el^4\bar{\nu}/2\lambda^2)\ , \tag{40}$$

$$0=E_2(-eml^4\bar{n}/\lambda)+f_{02}(-gml^4n_0/\lambda)$$
$$+b^{(1)}(l^4g'n_0/2\lambda^2+3gl^4\bar{\nu}/2\lambda^2)$$
$$+B(el^2n_0+3el^4\nu_0/2\lambda^2)\ , \tag{41}$$

$$0=E_2(-l^2e\bar{n}-3el^4\bar{\nu}/2\lambda^2)+f_{02}(-l^2gn_0-3gl^4\nu_0/2\lambda^2)$$
$$+b^{(1)}\left[\frac{1}{2m^2}\right](g'l^2mn_0/\lambda+4gml^2\bar{\nu}/\lambda)$$
$$+B\left[\frac{1}{m^2}\right](2eml^2\nu_0/\lambda)\ , \tag{42}$$

$$0=E_2(-el^2n_0-3el^4\nu_0/2\lambda^2)+f_{02}(-3l^4g\bar{\nu}/2\lambda^2)$$
$$+b^{(1)}\left[\frac{1}{m^2}\right](2gml^2\nu_0/\lambda)+B\left[\frac{1}{m_2}\right](2eml^2\bar{\nu}/\lambda)\ . \tag{43}$$

It is straightforward to solve these equations for λ. It turns out that λ is complex:

$$\lambda=\lambda_R+i\lambda_I=3\left[\frac{\nu_0}{2n_0}\right]^{1/2}l(1\pm i\sqrt{2})\ . \tag{44}$$

We emphasize that this is valid only in the limit $T\to T_c$ from below. Physically, it is clear that the penetration depth should diverge in this limit. As $T\to T_c$, $n_0\to$const, but ν_0 and l^2 both diverge as $b^{-1}\sim(T_c-T)^{-1/2}$. Hence, $\lambda\sim(T_c-T)^{-1/2}$. In particular, the magnetic field behaves as

$$B(x_2)=B(0)e^{-x_2/\lambda_R}\cos\left[\frac{1}{\sqrt{2}}\frac{x_2}{\lambda_R}\right]\ .$$

There is a Meissner effect below T_c but not above. Superconductivity terminates at T_c. In the zero-temperature limit, it is easily demonstrated that the fields are exponentially screened with a penetration depth given by the London expression $\lambda(T=0)=\sqrt{m/e^2n_0}$.

IV. CONCLUSIONS

We have obtained a phase diagram for a superconducting phase transition that appears qualitatively similar to the one proposed by Chakraverty[14] which led Bednorz and Müller[15] to their discovery of high-temperature superconductivity in a material with copper-oxide planes that contain symmetric Cu^{3+} ions and Jahn-Teller-distorted Cu^{2+} ions.

It was recently pointed out by Heinz and Tscheuschner[16] that the electron-phonon interaction which produces the Jahn-Teller effect is mathematically similar to our own interaction term. These authors suggest that fictitious magnetism or anyon statistics is created from quantum-mechanical nonlinear acoustics in $(2+1)$ dimensions. Essentially, an electron imbedded in its own surrounding lattice distortion, and interacting with the vibration modes, is an anyon seeing an effective magnetic flux. Two electron bands give rise to two anyon types, or flavors.

It may also be possible to interpret our model in the following way. The Chern-Simons Lagrangian describes a system of particles with charge, with an extra term that gives these particles fictitious flux. These particles acquire fractional statistics through what is essentially the Aharanov-Bohm effect. It should be equivalent to study a system of particles with nonzero magnetic moment, with an extra term which gives these particles fictitious charge, since such particles would develop fractional statistics through the Aharanov-Casher effect.[17] Such a formulation is of interest to us because a nonrelativistic particle with magnetic moment and no charge in an electromagnetic field is equivalent to a particle with an additional flavor degree of freedom, minimally coupled to an SU(2)-valued gauge potential.[18,19]

These ideas lead us to consider the Lagrangian of a particle with charge e coupled to a field a_μ, and a magnetic moment λ coupled to a field strength $F_{\mu\nu}$,

$$\mathcal{L}=\bar{\psi}\left[i\gamma_\mu(\partial_\mu-iea_\mu)-m-\frac{\lambda}{2}\sigma_{\mu\nu}F^{\mu\nu}\right]\psi\ . \tag{45}$$

Using a covariant generalization of the transformation of Ref. 17,

$$A^{\mu(3)}=\tfrac{1}{2}\epsilon^{\mu3\nu\lambda}F_{\nu\lambda}\ , \tag{46}$$

this Lagrangian can be rewritten in terms of a U(1) field a_μ and an SU(2) field A^k_μ. In $(2+1)$ dimensions, (45) and (46) become

$$\mathcal{L}=\bar{\psi}[i\gamma_\mu(\partial_\mu-iea_\mu)-m+i\lambda\gamma_\mu\gamma_\nu\epsilon^{\mu\nu\lambda}a^{(3)}_\lambda]\psi\ . \tag{47}$$

The magnetic-moment interaction is equivalent to a chiral-type interaction with a non-Abelian field. In the nonrelativistic limit this becomes

$$\mathcal{L}_{\rm NR}=\phi^\dagger iD_0\phi-\frac{1}{2m}|D_i\phi|^2-\frac{\lambda}{2m}B\phi^\dagger\phi-\frac{e}{2m}\phi^\dagger\sigma_3 b\phi\ , \tag{48}$$

where ϕ is a two-component Pauli spinor and $D_\mu=\partial_\mu-iea_\mu-i\lambda\sigma^3A^{(3)}_\mu$. A comparison of (48) and (5) seems to indicate that the flavor degree of freedom which we have introduced is related to a chiral degree of freedom. A Chern-Simons term does not appear at this level, but may appear at higher order in the expansion. These ideas will be explored in future work.

In summary, we have shown that there is a class of Chern-Simons theories in two dimensions which undergo superconducting phase transitions at finite temperature. An essential feature of these theories is that the Chern-Simons gauge field couples to an internal degree of freedom of the fermion. Superconductivity occurs only for a finite window of magnetic moment coupling g'. The phase transition appears to be second order. Further progress along these lines depends on either a demonstration or refutation of this class of theories in real materials.

ACKNOWLEDGMENTS

We are grateful to J. Hetrick, L. McLerran, and A. Srivastava for helpful discussions. This work was partially supported by the DOE under Grant No. DE-FG02-87ER40382.

[1]V. Kalmeyer and R. B. Laughlin, Phys. Rev. Lett. **59**, 2095 (1987).

[2]R. B. Laughlin, Phys. Rev. Lett. **60**, 2677 (1988).

[3]D. P. Arovas, J. R. Schrieffer, F. Wilczek, and A. Zee, Nucl. Phys. **B251**, 117 (1985).

[4]A. Goldhaber, R. MacKenzie, and F. Wilczek, Mod. Phys. Lett. A **4**, 21 (1989).

[5]Y. Hosotani and S. Chakravarty, Phys. Rev. B **42**, 342 (1990); J. E. Hetrick, Y. Hosotani, and B.-H. Lee, Ann. Phys. (N.Y.) **209**, 151 (1991).

[6]A. L. Fetter, C. B. Hanna, and R. B Laughlin, Phys. Rev. B **39**, 9679 (1989).

[7]S. Randjbar-Daemi, A. Salam, and J. Strathdee, Nucl. Phys. **B340**, 403 (1990).

[8]H. Shiba, Prog. Theor. Phys. **54**, 967 (1975); C. M. Varma, in *Trends in Theoretical Physics,* edited by P. J. Ellis and Y. C. Tang (Addison-Wesley, Reading, MA, 1990) Vol. 1.

[9]A. Kovner and B. Rosenstein, Phys. Rev. B **42**, 4748 (1990).

[10]Y. Kitazawa and H. Murayama, Nucl. Phys. B **338**, 777 (1990); Phys. Rev. B **41**, 11 101 (1990).

[11]Yi-Hong Chen, Frank Wilczek, Edward Witten, and Bertrand I. Halperin, J. Mod. Phys. B **3**, 1001 (1989).

[12]This assumes a spin interpretation of the two fermion types. There is also an ambiguity of whether to use $e/2m$ or $e/2m_e$. It depends on the exact nature of the fermion. Fortunately, this term is unimportant for the Meissner effect.

[13]A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).

[14]B. K. Chakraverty, J. Phys. Lett. **40**, L99 (1979).

[15]J. Bednorz and K. A. Müller, Z. Phys. B **64**, 189 (1986).

[16]A. Heinz and R. D. Tscheuschner, Phys. Rev. B **43**, 5601 (1991).

[17]Y. Aharanov and A. Casher, Phys. Rev. Lett. **53**, 319 (1984).

[18]J. Anandan, Phys. Lett. A **138**, 347 (1989).

[19]A. Goldhaber, Phys. Rev. Lett. **62**, 482 (1989).