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Perturbation approach for the Kondo Hamiltonian

Guang-Ming Zhang
Pohl Institute of Solid State Physics, Tongji University, Shanghai 200092, People's Republic of China

Hong Chen and Xiang Wu
Center of Theoretical Physics, Chinese Center of Advanced Science and Technology (World Laboratory); International Center for Material Physics, Academia Sinica; and Phol Institute of Solid State Physics, Tongji University, Shanghai 200092, People's Republic of China
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The bosonized Kondo Hamiltonian is obtained by comparing the long-time limit of the reduced grand partition function for the impurity spin. We analyze this bosonized Kondo Hamiltonian at zero temperature and derive an effective Hamiltonian describing the effective interaction between the conduction electrons via an impurity-spin scattering. The infrared divergences encountered in the conventional perturbation theory are caused by this growth effective coupling at the low-energy limit. With the Bogoliubov transformation, we have developed a perturbation approach for this effective Hamiltonian and investigated the nontrivial ferromagnetic-antiferromagnetic crossover of the impurity spin. The critical condition derived here is in agreement with the renormalization-group numerical result. In particular, the ground-state wave function and its excitation spectrum of the conduction electrons are also obtained. Moreover, this effective Hamiltonian may be mapped onto a modified quantum sine-Gordon model. Paralleling the renormalization-group theory of the quantum sine-Gordon model, we straightforwardly reproduce previous results and derive the higher-order terms and a new universal correction in the flow equations.

I. INTRODUCTION

The Kondo problem is one of the prominent topics in both many-body physics and solid-state theory. In the past two decades, many authors attracted a great deal of attention to solving this problem characterized by the growth of an effective coupling at the low-energy limit.[1] The principal difficulty is that the relevant low-energy phenomena cannot be treated in the framework of the conventional perturbation theory. This implies that the genuine ground state of the system is radically different from that of the conventional perturbation theory.[1] The most successful approaches for the Kondo problem are the renormalization-group numerical analysis[2] and the Bethe-ansatz solution.[3] But these two methods are not perturbation theories and cannot tell us what the ground state of the system is. Thus, it is necessary to know whether further insight into the Kondo problem may be obtained from other quantum many-body theories.[4] It is well known that many phenomena that are difficult to understand in the Fermi language have simple explanations in the Bose language. The main purpose of this paper is to analyze the Kondo Hamiltonian in its bosonized form and try to develop a perturbation approach for this problem. In addition, we also demonstrate the effective Hamiltonian describing the conduction electrons to be mapped onto a modified quantum sine-Gordon model, and employ the renormalization-group theory of the quantum sine-Gordon model to investigate this effective Hamiltonian.

This paper is organized as follows. In Sec. II, the bosonized Kondo Hamiltonian is derived by comparing the long-time limit of the reduced grand partition function for the spin, and an effective Hamiltonian describing the effective interaction between the conduction electrons via an impurity spin scattering will be presented. In Sec. III, we will propose a perturbation approach for this effective Hamiltonian and investigate the nontrivial ferromagnetic-antiferromagnetic crossover of the impurity spin. In Sec. IV, we employ the renormalization-group theory of the quantum sine-Gordon model to treat the effective Hamiltonian of the conduction electrons. The conclusion will be given in Sec. V.

II. DERIVATIONS OF THE BOSONIZED KONDO HAMILTONIAN AND THE EFFECTIVE HAMILTONIAN OF THE CONDUCTION ELECTRONS

The simplest Kondo problem is concerned with a single magnetic impurity of spin $\frac{1}{2}$ which interacts with a band of conduction electrons via an exchange scattering potential. Since the exchange interaction is assumed to be pointlike, only s-wave scattering occurs. Expanding the plane-wave electron states $\mathbf{k}$ in spherical waves around the impurity site, we may characterize the relevant states simply by the magnitude $|\mathbf{k}|$ of the wave vector, which reduces the problem to an essentially onedimensional problem. For the long-time and low-temperature limits, the dominant excitations are those in the immediate vicinity of the Fermi surface and we may linearize the dispersion relation of the conduction electrons around the Fermi energy.[5] Thus, the usual Kondo Hamiltonian

$$H_K = T + J\mathbf{S}\cdot\mathbf{s}(0) \tag{1}$$

can be changed into the following form:

$$H_K = T + \frac{J_\parallel}{4L}\sigma_z \sum_{k,k'} (c_{k,\uparrow}^\dagger c_{k',\uparrow} - c_{k,\downarrow}^\dagger c_{k',\downarrow}) + \frac{J_\perp}{2L}\sum_{k,k'} (\sigma_+ c_{k,\downarrow}^\dagger c_{k',\uparrow} + \sigma_- c_{k,\uparrow}^\dagger c_{k',\downarrow}), \tag{2}$$

where the conduction-electron kinetic energy (T) is measured relative to the Fermi energy. The impurity spin is $\mathbf{S}$ and its Pauli matrix vector is $\boldsymbol{\sigma}$, with $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$; whereas

$$\mathbf{s}(0) = \sum_{\sigma,\sigma} c_\sigma^\dagger \sigma_{\sigma,\sigma'} c_{\sigma'}/2$$

denotes the effective spin due to the conduction electrons at the impurity site $r=0$. The coupling constant $J_\parallel$ is positive for an antiferromagnetic coupling and negative for a ferromagnetic coupling, and the sign of $J_\perp$ is irrelevant.

Following the same approach of Anderson, Yuval, and Hamann,[5] we integrate out the electronic degrees of freedom and consider the resulting reduced path integral for the imaginary-time history of the impurity spin. Then the reduced grand-partition function for the spin in an expansion in the number of spin flips caused by the $J_\perp$ term is derived

$$\bar{z} = \sum_{n=0}^{\infty} \left[\frac{\rho J_\perp}{2}\right]^{2n} \int_0^{\beta n} \frac{d\tau_{2n}}{\tau_c} \cdots \int_0^{\tau_2-\tau_c} \frac{d\tau_1}{\tau_c} \exp\left[2(1-\rho J_\parallel)\sum_{i>j}(-1)^{i+j}\ln\left|\frac{\beta\hbar}{\pi\tau_c}\sin\left[\frac{\pi(\tau_i-\tau_j)}{\beta\hbar}\right]\right|\right], \tag{3}$$

where $\tau_c = \hbar/\varepsilon_F$ is a short-time cutoff. Since the coupling constants $J_\parallel$ and $J_\perp$ have dimensions of energy times length, the relevant dimensionless coupling parameters are $\rho J_\parallel$ and $\rho J_\perp$ with $\rho = (2\pi\hbar v_f)^{-1}$ as the single spin density of states at the Fermi surface. It is most interesting that this reduced grand-partition function is also obtained in the study of a spin-boson problem with Ohmic dissipation.[6-7] Thus, from the point view of the statistics thermodynamics, the Kondo Hamiltonian is exactly equivalent to an Ohmic spin-boson Hamiltonian; the bosonized Kondo Hamiltonian is given by[6]

$$H_K^B = (\varepsilon_F/2)(\rho J_\perp)\sigma_x + \tfrac{1}{2}(J_\parallel/2 - 1/\rho)\sigma_z \sum_k (|k|/\pi L)^{1/2}(a_k + a_{-k}^\dagger) + \sum_k \hbar v_f |k| a_k^\dagger a_k\ . \tag{4}$$

Here a_k and $a_k^\dagger$ are corresponding Bose operators, and a cutoff in energy around the Fermi energy ε_F cuts off the momenta k around k_c with $k_c^{-1} = \hbar v_f/\varepsilon_F$. Physically, $\hbar/\varepsilon_F$ may be thought of as the time for an electron to pass the local impurity spin. In fact, this correspondence between the two models had been derived from the bosonization procedures.[6,8] Although their methods are certainly not rigorous, they are relatively simple and it is easy to understand the relation between the physics of the two models.

Let us now concentrate on the bosonized Kondo Hamiltonian (4). Throughout the following calculations, the emphasis is on the properties of the conduction electrons rather than the impurity spin itself.[2] Suppose two conduction electrons, both with spin up, try to spin-flip scatter from a spin-down impurity. The first electron can spin-flip scatter, but the result is to leave the impurity with spin up. The second electron cannot spin-flip scatter because this would violate spin conservation. Thus, the impurity forces us to treat the conduction electrons as a many-electron system.

In applying the usual canonical transformation,

$$U_1 = \exp\left[\sigma_z(\rho J_\parallel/2 - 1)\sum_k (\pi/|k|L)^{1/2}(a_k - a_{-k}^\dagger)\right], \tag{5}$$

to the Hamiltonian (4), we have

$$\begin{aligned}\bar{H}_K^B &= U_1^\dagger H_K^B U_1 \\ &= \sum_k \hbar v_f |k| a_k^\dagger a_k + (\varepsilon_F/2)(\rho J_\perp)\sigma_x \cosh\left[(\rho J_\parallel/2 - 1)\sum_k (4\pi/|k|L)^{1/2}(a_k - a_{-k}^\dagger)\right] \\ &\quad - (\varepsilon_F/2)(\rho J_\perp) i\sigma_y \sinh\left[(\rho J_\parallel/2 - 1)\sum_k (4\pi/|k|L)^{1/2}(a_k - a_{-k}^\dagger)\right],\end{aligned} \tag{6}$$

where we have omitted an irrelevant constant. The transformed Hamiltonian includes two kinds of interaction terms. The first is the diagonal interaction term containing σ_x, which represents the diagonal interaction between the conduction electrons and the ground state ($\sigma_x = -1$) or the excited state ($\sigma_x = +1$) of the impurity spin. The second is the nondiagonal interaction term containing σ_y, which determines the transition between the ground state and excited state of the impurity spin. This case is analogous to that of a small-polaron problem.[9] Thus, the nondiagnal interaction term will be important only at finite temperature and we can neglect it at zero temperature.[9] Then we obtain an effective Hamiltonian for the conduction electrons:

$$H_K^{\rm eff}=\sum_k \hbar v_f|k|a_k^\dagger a_k-(\varepsilon_F/2)(\rho J_\perp)\cosh\left[(\rho J_\parallel/2-1)\sum_k(4\pi/|k|L)^{1/2}(a_k-a_{-k}^\dagger)\right] . \tag{7}$$

From the above Hamiltonian (7), we can see that there exists an effective interaction between conduction electrons via the impurity spin scattering. It is difficult to find an exact ground state of such a model and we have to look for approximations. To the zero of

$$[(\rho J_\parallel/2-1)(4\pi/|k|L)^{1/2}],$$

the ground state of this effective Hamiltonian is a vacuum state, which just corresponds to that of the conventional perturbation theory.[5] But, we cannot make such an expansion because the quantity

$$[(\rho J_\parallel/2-1)(4\pi/|k|L)^{1/2}]$$

will increase from

$$[(\rho J_\parallel/2-1)(4\pi/k_cL)^{1/2}]$$

to infinity as k goes from k_c to zero. This implies that there still exists a strong effective interaction between the conduction electrons at the low-energy limit even though the coupling constant $|\rho J|$ is very small. Moreover, it is this growing interaction that is responsible for the infrared divergences of the Kondo problem at zero temperature.[1]

III. PERTURBATION APPROACH FOR THE EFFECTIVE HAMILTONIAN OF THE CONDUCTION ELECTRONS

In order to make an appropriate perturbation expansion, we employ the following Bogoliubov transformation to rescale the normal-mode coordinates and momenta of the conduction electron[10,11]

$$U_2=\exp\left[-\sum_k\gamma_k(a_k^\dagger a_{-k}^\dagger-a_ka_{-k})\right] \tag{8}$$

with

$$b_k=U_2^\dagger a_kU_2=a_k\cosh 2\gamma_k-a_{-k}^\dagger\sinh 2\gamma_k, \tag{9a}$$

$$b_k^\dagger=U_2^\dagger a_k^\dagger U_2=a_k^\dagger\cosh 2\gamma_k-a_{-k}\sinh 2\gamma_k, \tag{9b}$$

$$(b_k-b_{-k}^\dagger)=U_2^\dagger(a_k-a_{-k}^\dagger)U_2 =(a_k-a_{-k}^\dagger)\exp(2\gamma_k) . \tag{9c}$$

We obtain

$$H_K^{\rm eff}=\sum_k\hbar v_f|k|[b_k^\dagger b_k\cosh 4\gamma_k+(b_k^\dagger b_{-k}^\dagger+b_kb_{-k})(\sinh 4\gamma_k)/2+(\sinh 2\gamma_k)^2] -(\varepsilon_F/2)(\rho J_\perp)\cosh\left[(\rho J_\parallel/2-1)\sum_k(4\pi/|k|L)^{1/2}\exp(-2\gamma_k)(b_k-b_{-k}^\dagger)\right] . \tag{10}$$

Then we may expand the hyperbolic function with respect to

$$[(\rho J_\parallel/2-1)(4\pi/|k|L)^{1/2}\exp(-2\gamma_k)].$$

By normal ordering boson operators and up to

$$[(\rho J_\parallel/2-1)(4\pi/|k|L)^{1/2}\exp(-2\gamma_k)]^2,$$

we have

$$\begin{aligned}H_K^{\rm eff}=&\sum_k[\hbar v_f|k|\cosh 4\gamma_k+(\rho J_\parallel/2-1)^2(4\pi/|k|L)(\varepsilon_f/2)(\rho J_\perp)K\exp(-4\gamma_k)]b_k^\dagger b_k\\&+\sum_k[\hbar v_f|k|\sinh 4\gamma_k-(\rho J_\parallel/2-1)^2(4\pi/|k|L)(\varepsilon_F/2)(\rho J_\perp)K\exp(-4\gamma_k)]\\&\times(b_k^\dagger b_{-k}^\dagger+b_kb_{-k})/2+\sum_k\hbar v_F|k|\sinh 2\gamma_k)^2-(\varepsilon_F/2)(\rho J_\perp)K,\end{aligned} \tag{11}$$

where we have omitted the interactions between different momentum modes and introduced a renormalized parameter of the impurity spin-flip frequency, which is defined by the following overlapping integral:

$$\begin{aligned}K&=\langle{\rm vac}|U_2[U_1^\dagger(\sigma_z=-1)][U_1(\sigma_z=+1)]U_2^\dagger|{\rm vac}\rangle\\&=\exp\left[-(\rho J_\parallel/2-1)^2\sum_k(2\pi/|k|L)\exp(-4\gamma_k)\right] .\end{aligned} \tag{12}$$

The parameters γ_k are chosen to diagonalize H_K^{eff} in (11), and it gives

$$H_K^{\text{eff}} = \sum_k [\hbar v_f |k| \exp(4\gamma_k) b_k^\dagger b_k + \hbar v_f |k| (\sinh 2\gamma_k)^2] - (\varepsilon_F/2)(\rho J_\perp) K , \tag{13}$$

$$\gamma_k = \tfrac{1}{8} \ln \left[1 + 4(\rho J_\parallel/2-1)^2 (2\pi/L)(\varepsilon_F/2)(\rho J_\perp) \left[\frac{K}{\hbar v_f k} \right] \right] . \tag{14}$$

Thus, the ground-state wave function of the Hamiltonian (7) is a pairing quasiparticle state $U_2^\dagger |\text{vac}\rangle$, which will be discussed in detail at the end of this section. It should be pointed out that, if we calculate the ground-state energy of the system

$$\begin{aligned} E_g &= \langle \text{vac} | U_2 H_K^{\text{eff}} U_2^\dagger | \text{vac} \rangle \\ &= (\varepsilon_F/2)(\rho J_\perp) \exp[-(\rho J_\parallel/2-1)^2 \sum_k (2\pi/|k|L) \exp(-4\gamma_k)] + \hbar v_f \sum_k |k| \sinh^2 2\gamma_k \end{aligned} \tag{15}$$

and minimize it with respect to γ_k, we can also get the same condition for γ_k as Eq. (14). This means that the above ground state of the system makes the ground-state energy of the system minimize automatically. It can easily be found that the

$$[(\rho J_\parallel/2-1)(4\pi/|k|L)^{1/2} \exp(-2\gamma_k)]$$

in condition (14) is convergent for the zero-momentum mode, and the infrared divergences appearing in the conventional perturbation theory are eliminated due to the exponential factor. Since the higher-order terms in the expansion of Eq. (10) is much smaller than the second-order terms, our approach for this effective Hamiltonian is reliable.

Inserting Eq. (14) into Eq. (12), we get

$$-\ln K = 2(\rho J_\parallel/2-1)^2 \int_0^1 \frac{dx}{x} \left[1 + \left[\frac{K}{B} \right] x^{-2} \right]^{-1/2} \tag{16}$$

with

$$B = \tfrac{1}{2} (k_c/2\pi/L)(\rho J_\parallel/2-1)^{-2} (\rho J_\perp)^{-1} . \tag{17}$$

It is important that, from the self-consistent Eq. (16), the renormalized impurity spin-flip frequency can be calculated, and we can use it to discuss the nontrivial ferromagnetic-antiferromagnetic crossover of the impurity spin. For instance, the magnetic impurity appears as a finite moment as K vanishes. Finishing the integral in Eq. (16), we obtain

$$-\ln K = \frac{\alpha}{1-\alpha} \ln[B^{1/2} + (B+K)^{1/2}]^2 , \tag{18}$$

where we have defined another dimensionless coupling coefficient; that is,

$$-\sqrt{\alpha} = (\rho J_\parallel/2-1) . \tag{19}$$

Considering the usual weak-coupling limit $|\rho J| \ll 1$, for $(\rho J_\parallel) > 0$, we always have a nonzero solution of K, and the impurity spin has been completely screened by its conduction-electron spins and the whole system is nonmagnetic. While, for $(\rho J_\parallel) < 0$, we only have a zero solution of K, which implies the impurity spin is free and the system appears as a finite moment. Thus, our derived condition for the nontrivial ferromagnetic-antiferromagnetic crossover impurity spin is $(\rho J_\parallel)_c = 0$, in agreement with the renormalization-group numerical analysis.[2]

In order to give further investigations, it is crucially important to discuss the ground-state properties of the effective Hamiltonian (7). First, its ground-state wave function is

$$|G\rangle = \prod_k \exp[\gamma_k (a_k^\dagger a_{-k}^\dagger - a_k a_{-k})] |\text{vac}\rangle . \tag{20}$$

Employing the following useful identity:[12]

$$\exp[\gamma_k (a_k^\dagger a_{-k}^\dagger - a_k a_{-k})] = \exp[(\tanh\gamma_k) a_k^\dagger a_{-k}^\dagger] \exp\{[\ln(\cosh\gamma_k)](1 + a_k^\dagger a_k + a_{-k}^\dagger a_{-k})\} \times \exp[-(\tanh\gamma_k) a_k a_{-k}] , \tag{21}$$

we then obtain

$$|G\rangle = \prod_k (\cosh\gamma_k) \exp[(\tanh\gamma_k) a_k^\dagger a_{-k}^\dagger] |\text{vac}\rangle . \tag{22}$$

This shows that the ground state of the conduction electrons in the Kondo problem is a pairing quasiparticle state, which is analogous to the BCS superconducting state. To our knowledge, it is the first time such a new ground state was found. According to Eq. (14), we can see that this ground state will return to that of the conventional perturbation theory when the renormalized impurity spin-flip factor K approaches zero. It is also proved that the Kondo problem with antiferromagnetic coupling cannot be treated in the framework of the conventional perturbation theory because the ground state of the system is radically different from that of the conventional perturbation theory. When the conduction electrons are in the above pairing state, (22), the ground-state energy of the system is

$$E_g = -(\varepsilon_F/2)(\rho J_\perp)K + (\varepsilon_F/2)(k_c L/2\pi)[(1+K/B)^{1/2}-1] \sim -(\varepsilon_F/2)(\rho J_\perp)K[1-(\rho J_\parallel/2-1)^2] . \quad (23)$$

From the ground-state energy, the conduction electrons of this system will change from the pairing bound state to the free state when the coupling parameter $(\rho J_\parallel)$ or the renormalized factor K is close to zero. This just corresponds to the above nontrivial ferromagnetic-antiferromagnetic crossover of the impurity spin. In addition, we also obtain the excitation spectrum of the conduction electrons

$$\omega_k = |k|[1+(K/B)(k_c/k)^2]^{1/2} . \quad (24)$$

There exists a gap when the momentum approaches zero, and it will disappear as the vanishing of the renormalized factor K, which is also related to the fact that the pairing quasipartical state returns to the ground state of the conventional perturbation theory. This excitation spectrum of the conduction electrons is similar to that of the BCS superconducting case.

IV. THE EFFECTIVE HAMILTONIAN FOR CONDUCTION ELECTRONS AS A QUANTUM SINE-GORDON MODEL

The physics behind the effective Hamiltonian (7) appeared to be transparent if we introduced the following boson-field operators:

$$\Pi(x) = \sum_k (|k|/2L)^{1/2}[a_k \exp(ikx) + a_k^\dagger \exp(-ikx)] , \quad (25)$$

$$\Phi(x) = i\sum_k (2|k|L)^{-1/2}[a_k \exp(ikx) - a_k^\dagger \exp(-ikx)], \quad (26)$$

which obey the canonical commutation relation

$$[\Phi(x), \Pi(x')] = i\delta(x-x') . \quad (27)$$

Thus, the effective Hamiltonian (7) becomes

$$H_K^{\text{eff}} = \int dx\, \{[\Pi^2 + (\nabla\Phi)^2]/2 - (\varepsilon_F/2)(\rho J_\perp)\delta(x) \times \cos[(8\pi)^{1/2}(1-\rho J_\parallel/2)\Phi]\} . \quad (28)$$

This is a modified quantum sine-Gordon model in one spatial dimension.[13,14] Therefore, the effective Hamiltonian describing the effective interaction between the conduction electrons may be exactly mapped onto a more fundamental quantum model.

The behavior of the quantum sine-Gordon model is well known: a phase transition exists since the coupling constant is close to a critical value (here the $\rho J_\parallel$ approaches zero), and the renormalization-group theory has been successfully applied to it.[14] Since this theory is based on the analysis of the divergent behaviors of the vertex functions (which are confined to the point $x=0$), the δ function in the interaction term of Eq. (27) does not cause any difficulty in our calculations if we parallel this approach to analyze the nontrivial ferromagnetic-antiferromagnetic crossover of the impurity spin in the Kondo problem. First, we introduce an infrared cutoff factor (m) in order to circumvent the infrared divergences in the conduction-electron overlapping integral on the renormalized impurity spin-flip frequency K. In fact, we only consider the dependence of the high-energy cutoff ε_F and use it to set the scale of energies in the problem. Second, we perform a double-power-series expansion in the bare spin-flip frequency $(\rho J_\perp)/2$ and the dimensionless coupling constant $(\rho J_\parallel)$ because spontaneous symmetry breaking occurs, leading to a localized regime signaled by a vanishing of K at $(\rho J_\parallel)=0$, as $\varepsilon_F \to \infty$. Such an expansion suffices to remove all divergences order by order. Third, by computations of the vertex functions up to the third order $(\rho J_\perp)^3$, the renormalization-group flow equations can be obtained

$$\bar{\bar{Z}}_{(\rho J_\perp)} = (\rho J_\perp)(\rho J_\parallel) + a(\rho J_\perp)^3, \quad (29)$$

$$\bar{\bar{Z}}_{(\rho J_\parallel)} = 2(\rho J_\perp)^2 + b(\rho J_\perp)^2(\rho J_\parallel) \quad (30)$$

with $2a+b$ a universal number, where the detail calculations may be referred to the paper of Amit, Goldschmidt, and Grinstein.[14] The first terms on the right-hand side of Eqs. (29) and (30) correspond to those derived by the previous poorman scaling formalism,[5] which is in agreement with that of the Bethe-ansatz solution.[3] The second terms on the right-hand side of Eqs. (29) and (30) are the next-to-leading-order corrections which would be difficult to obtain in the previous scaling scheme.[5] These new terms do not change qualitatively in the flow equations. It is important that the combination of the coefficients of the higher-order terms introduces a new univeral quantity, which gives a universal correction to the vanishing of the renormalized impurity spin-flip frequency on the critical line as the scale of the energy is varied. More work is required to elucidate this result.

We have to point out here that the above renormalization-group approach did not directly consider, but circumvented the infrared divergences encountered in the quantum sine-Gordon model. The infrared cutoff factor introduced actually breaks the symmetry of the model Hamiltonian. As we have seen above, it is these infrared divergences that contribute to the great changes of the renormalized impurity spin-flip factor K when the dimensionless coupling constant $\rho J_\parallel$ approaches zero. For this reason, we develop the above perturbation treatment of this effective Hamiltonian without the infrared cutoff factor. It can also provide other properties of the Kondo problem.

V. CONCLUSION

Based on the analysis of the bosonized Kondo Hamiltonian at zero temperature, an effective Hamiltonian describes the effective interaction between the conduction electrons via an impurity spin scattering. We find that the infrared divergences encountered in the conventional perturbation theory are caused by this growing effective coupling at the low-energy limit. With the Bogoliubov transformation, we have developed a new perturbation approach for the Kondo problem and investigated the nontrivial ferromagnetic-antiferromagnetic crossover of the impurity spin. The critical condition obtained here is in agreement with the renormalization-group numerical analysis. In particular, the ground-state wave function of the conduction electrons is found to be a pairing quasiparticle state, which is analogous to the BCS superconducting state. Its excitation spectrum will open a gap near the zero-momentum mode. In addition, the effective Hamiltonian for the conduction electrons may also be mapped onto a modified quantum sine-Gordon model. Paralleling the renormalization-group theory of the sine-Gordon equation, we straightforwardly reproduce previous results, and derive the higher-order terms and a universal correction in the flow equations.

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[1]A. M. Tsvelick and P. B. Wiegman, Adv. Phys. **32**, 453 (1983), and references therein.
[2]K. G. Wilson, Rev. Mod. Phys. **47**, 773 (1975).
[3]P. B. Wiegmann, Pis'ma Zh. Eksp. Teov. Fiz. **31**, 421 (1980) [JETP Lett. **31**, 393 (1980)]; J. Phys. C **14**, 1463 (1981); N. Andrei, Phys. Rev. Lett. **45**, 379 (1980).
[4]A. Berkovich and J. H. Loweinstein, Phys. Rev. B **32**, 7263 (1985); S. Barnes, *ibid.* **33**, 3209 (1986); S. D'Elia, *ibid.* **41**, 11536 (1990); N. Kawakam and A. Okiji, *ibid.* **42**, 2383 (1990).
[5]P. W. Anderson, G. Yuval, and D. R. Hamann, Phys. Rev. B **1**, 4464 (1970); P. W. Anderson and G. Yuval, J. Phys. C **4**, 607 (1971).
[6]M. Blume, V. J. Emery, and A. Luther, Phys. Rev. Lett. **25**, 450 (1970).
[7]S. Chakravarty, Phys. Rev. Lett. **49**, 681 (1982); A. J. Bray and M. A. Moore, *ibid.* **49**, 1545 (1982).
[8]K. D. Schotte, Z. Phys. **230**, 99 (1970).
[9]G. D. Mahan, *Many Particle Physics* (Plenum, New York, 1980).
[10]J. P. Blaizot and G. Ripka, *Quantum Theory of Finite System* (MIT, Cambridge, MA, 1986).
[11]Yu-Mei Zhang, Hong Chen, and Xiang Wu, J. Phys.: Condens. Matter **2**, 3119 (1990); Guang-Ming Zhang, Hong Chen, and Xiang Wu, Phys. Rev. B **41**, 11 600 (1990).
[12]N. N. Bogoliubov and D. V. Shirkov, *Quantum Field* (Wiley, New York, 1986).
[13]S. Coleman, Phys. Rev. D **11**, 2088 (1975); P. B. Wiegman, J. Phys. C **11**, 1583 (1978).
[14]D. J. Amit, Y. Y. Goldschmidt, and G. Grinstein, J. Phys. A **13**, 585 (1980).