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Vortex-glass transition in three dimensions

J. D. Reger
Institut für Physik, Universität Mainz, D-6500 Mainz, Germany

T. A. Tokuyasu
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

A. P. Young*
Laboratoire de Physique Statistique, Ecole Normale Supérieure, Paris 75231 CEDEX 05, France

Matthew P. A. Fisher
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
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We investigate the possibility of a vortex-glass transition in a disordered type-II superconductor in a magnetic field in three dimensions by numerical studies of a simplified model. Monte Carlo simulations at finite temperature and domain-wall renormalization-group calculations at $T=0$ indicate that $d=3$ is just above the lower critical dimension d_l, though the possibility that $d_l=3$ cannot be definitely ruled out. A comparison is made with XY and Ising spin glasses. The (effective) correlation-length exponent ν and dynamical exponent z are in fairly good agreement with experiment.

Since fluctuation effects play a much more important role in high-temperature superconductors than in conventional superconducting materials, there has been a great deal of effort[1] to understand the behavior of type-II superconductors in a magnetic field, including the effects of disorder, when one goes beyond the mean-field picture of BCS or Ginzburg-Landau theories. One intriguing aspect which has emerged is the possibility of a vortex-glass phase[2,3] in which the off-diagonal long-range order of the pair condensate has a phase which is random in space but frozen in time, much like the order parameter in a spin glass.[4] This can arise because the Abrikosov flux lattice, which forms in pure samples, is destroyed by disorder in less than four dimensions[5] beyond a certain length scale, l_{dis}. Disorder also destroys orientational order in the flux lattice[6] if one neglects orientational couplings between it and the crystal lattice. The phase of the condensate does not then form a regular periodic pattern on scales larger than l_{dis}, but, according to the vortex-glass hypothesis, the system undergoes a transition into a spin-glass-like state in which the phase is frozen in time. At the transition, the vortex-glass correlation length ξ diverges. A number of experiments[7] have found evidence for such a transition in the I-V characteristics of Y-B-Cu-O samples. Only if there is a vortex-glass phase does the resistance really vanish[1] for $H>H_{c_1}$. Otherwise, the resistance is, in principle, finite because clusters of vortices on scale ξ can move by thermal activation over barriers, a process known as "flux creep."[8] These effects are observable[9] in high-T_c compounds, since they have much larger fluctuations than conventional materials. The purpose of this paper is to investigate whether a vortex-glass phase occurs at finite temperature in a disordered type-II superconductor in a field greater than H_{c_1} by numerical studies of a simplified model system.

The model that we study, known as the "gauge glass," has the following Hamiltonian:

$$\mathcal{H}=-\sum_{\langle i,j\rangle}\cos(\phi_i-\phi_j-A_{ij})\,. \tag{1}$$

The phase ϕ_i is defined on each site of a regular lattice, which we take here to be simple cubic with $N=L^3$ sites and periodic boundary conditions. The sum is over all nearest-neighbor pairs on the lattice. The effects of the magnetic field and disorder are represented by quenched vector potentials A_{ij} which we take to be independent random variables with a uniform distribution between 0 and 2π. This model seems to be the simplest model with the correct ingredients of randomness, frustration, and order parameter symmetry. It does, however, ignore screening, and therefore corresponds to an extreme type-II limit in which $\kappa=\lambda/\xi\to\infty$, where λ is the penetration length. Since $\kappa\gg1$ in the high-T_c superconductors, this limit is not unreasonable. It is unclear, however, how much inclusion of screening via a fluctuating gauge field would modify the behavior of Eq. (1).

If the A_{ij} are restricted to the values 0 and π, the model becomes the XY spin glass, for which the lower critical dimension is believed[10] to be 4. However, earlier work[11,12] has shown that the gauge glass is in a different universality class from the XY spin glass, presumably because it does not have the "reflection" symmetry, $\phi_i\to-\phi_i\ \forall i$.[11]

It is useful[13] to look at the following dimensionless ratio of moments of the probability distribution for q, the gauge glass order parameter:

$$g\equiv2-\frac{[\langle|q|^4\rangle_T]_{\text{av}}}{[\langle|q|^2\rangle_T]_{\text{av}}^2}\,. \tag{2}$$

This has the property in the thermodynamic limit that it goes to zero above T_c and tends to unity in the ordered phase. We define the thermal average over a single sample by $\langle\cdots\rangle_T$ and the configurational average over samples by $[\cdots]_{\rm av}$. Because it is dimensionless, g has the simple finite-size scaling form,[13]

$$g(L,T) \approx \tilde{g}[L^{1/\nu}(T-T_c)] , \tag{3}$$

where T_c is the transition temperature and ν the correlation length exponent. This shows that curves of g against T for different sizes should intersect at T_c and splay out again at lower temperatures. Huse and Seung[11] report data for g for different sizes which come together at a finite temperature, $T \simeq 0.6$, but then do not splay out again at lower temperatures. This is very similar to the results for the three-dimensional (3D) Ising spin glass,[13] and, by itself, does not indicate clearly whether an ordered phase exists.

Now, for the Ising spin glass, it was also possible to perform domain-wall renormalization-group studies at $T=0$,[14] in which the difference between the ground states with, say, periodic and antiperiodic boundary conditions is calculated for a large number of samples. This quantity fluctuates between samples, with a distribution whose mean is zero for a spin glass and a width, measured, e.g., by the expectation value of the absolute value, which varies with the size of the system as

$$\Delta E \sim L^{\theta} . \tag{4}$$

If θ is positive then one argues that there is an order phase and a finite-temperature transition, while a negative value of θ indicates a transition only at $T=0$. The marginal case, $\theta=0$, corresponds to the system being at its lower critical dimension. For the 3D Ising spin glass, $\theta \simeq 0.20$,[14] indicating an ordered state.

It is possible that ΔE turns out to be a more sensitive test of ordering than g for the 3D Ising spin glass because the former *diverges* with L in the low-temperature phase while the latter is *bounded* by unity. Hence the data for g has only a restricted range in which it can splay out below T_c, so, if θ is fairly small, corrections to finite-size scaling may make the data for g look marginal, i.e., independent of size, for the range of sizes that can be studied.

The purpose of this paper is, therefore, as follows: (i) to compute ΔE for the 3D gauge glass, and (ii) to perform finite-temperature Monte Carlo simulations in which some new quantities are calculated which have the desired properties that they are (a) independent of size at T_c and (b) diverge with L at lower temperatures.

We begin with our results for ΔE, shown in Fig. 1. Apart from the point for $L=2$, which is a bit low, as was also found[12] for the analogous calculations in $d=2$, the results are independent of size, within the error bars. This indicates that $\theta \simeq 0$ and hence that the lower critical dimension is at or close to 3. At least for these sizes, the 3D gauge glass seems to be much more marginal than the Ising spin glass, for which $\theta \simeq 0.20$.[14] Our results disagree with those of Cieplak, Banavar, and Khurana[15] who find $\theta \simeq 0.3$, though very recently Gingras[16] finds results quite similar to those in Fig. 1.

We now proceed to the Monte Carlo simulations. At finite temperature, the domain-wall energy ΔE is replaced by the free energy ΔF, which is expected[17] to vary as L^{θ} *everywhere* in the low-temperature phase and to be independent of L at T_c. Now ΔF should scale in the same

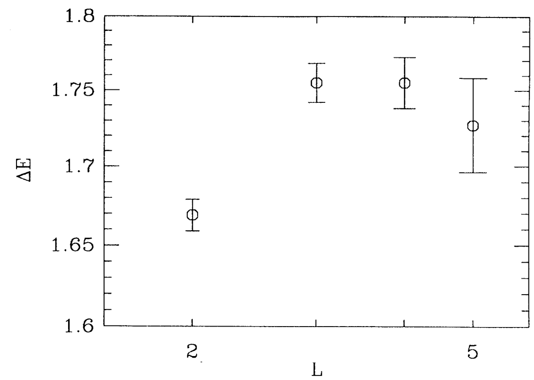

FIG. 1. A double logarithmic plot of the domain-wall energy ΔE, defined by $\Delta E = [|E_{\rm per} - E_{\rm aper}|]_{\rm av}$, where $E_{\rm per}$ is the total ground-state energy with periodic boundary conditions and $E_{\rm aper}$ is the energy with antiperiodic boundary conditions, at $T=0$ for different sizes. The number of samples used varied between 15000 for $L=2$ and 1900 for $L=5$. The error bar represents one standard deviation. The slope of the data is the exponent θ characterizing the low-temperature phase. Apart from the point for $L=2$, which lies a little low, the energy is roughly independent of size, $\theta \simeq 0$, i.e., the system is at, or close to, its lower critical dimension.

way if we apply a twist through an *arbitrary* angle Θ as it does for the case of $\Theta=\pi$ considered so far. It is therefore useful to calculate derivatives of the free energy with respect to Θ, so, for single sample, we define a current I and a stiffness Y by

$$I \equiv \frac{\partial F}{\partial \Theta} = \frac{1}{L}\sum_i \langle \sin\Delta_i \rangle_T , \tag{5}$$

$$Y \equiv \frac{\partial^2 F}{\partial \Theta^2} = \frac{1}{L^2}\left(\sum_i \langle \cos\Delta_i \rangle_T - \frac{1}{T}\sum_{i,j} [\langle \sin\Delta_i \sin\Delta_j \rangle_T - \langle \sin\Delta_i \rangle_T \langle \sin\Delta_j \rangle_T] \right) , \tag{6}$$

where $\Delta_i = \phi_i - \phi_{i+x} - A_{i,i+x}$, F is the total free energy, and $i+x$ refers to the nearest-neighbor site in the x direction from i. Note that both I and Y are gauge invariant so they are still useful even if one includes fluctuating gauge fields. By constrast, it is difficult to give a sensible gauge invariant definition of the order parameter q.

Below T_c, the free energy varies with twist angle on a scale L^{θ} so one might expect that Y would also vary in this way. However, for a large finite system we expect instead that the value of Y is, with high probability, due simply to spin waves, which gives a positive contribution scaling as L^{d-2}. In addition, as the twist angle is changed, the system will suddenly jump from one local minima to another by moving vortices, at which point the slope changes abruptly, so there is a small probability that the curvature is large and negative. Hence, we expect[18] that, in the thermodynamic limit, essentially all the weight will be in the peak at positive Y. Since Y is L^{d-2} times the superfluid density ρ_s, which, in turn, is related to the conductivity σ through $\sigma \sim \rho_s / i\omega$, this implies that the dc conductivity is infinite, as mentioned above. Nonetheless,

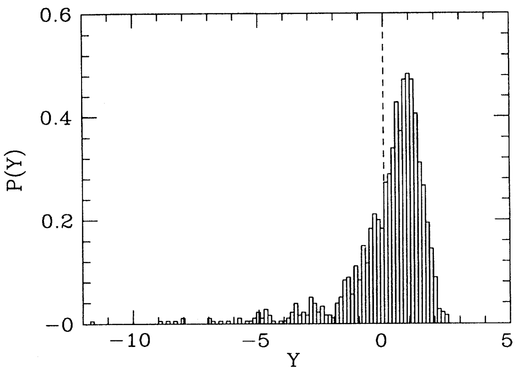

FIG. 2. A histogram of the estimates for the thermal expectation value of the stiffness, Y, defined in Eq. (6), for the different samples (1200 altogether) for $T=0.4$, $L=8$. This data is actually a convolution of the true distribution of the stiffness among samples with a "smearing function" due to the imperfect averaging of each sample by the Monte Carlo simulation. Notice that the distribution is *not* symmetric but has a peak at positive stiffness and a tail down to large negative values (which ensures that the mean is zero).

the tail for negative Y ensures that $[Y]_{av}=0$, which follows because $[F(\Theta)]_{av}$ is independent of Θ, since different choices for Θ correspond to different realizations of the A_{ij} with the same probability. It is also likely that the stiffness is self-averaging, i.e., the peak at positive Y becomes infinitely narrow in the thermodynamic limit. We find that the distribution of values of Y among different samples is indeed very asymmetric, as expected from this discussion; see Fig. 2. The distribution of I must, however, be symmetric. It is possible that, like the rms stiffness, the rms current, $\Delta I=[I^2]_{av}^{1/2}$, also diverges with an exponent larger than θ, although this is not necessary and it is perhaps more natural to assume that it is equal to θ. However, the precise value of the exponent is not critical for our analysis because it cannot be *less* than θ, so ΔI must diverge in the ordered phase. The finite-size scaling form for ΔI is expected to be the same as for g, given in Eq. (3).

Tests to ensure equilibration were carried out as described elsewhere.[13] Our results for ΔI are presented in Fig. 3. The data come together at $T \simeq 0.50$, slightly lower than the corresponding temperature for g.[11] At lower temperatures most of the results are size independent but at the lowest temperature, $T=0.4$, the data for the largest size, $L=8$, exceeds that for the smallest sizes by three standard deviations. This indicates that there is an ordered phase and that θ is slightly positive. Although we found no sign of a similar upturn in the calculations of ΔE shown in Fig. 1, the sizes there were smaller and it is possible that the somewhat larger sizes used in the Monte Carlo simulations are necessary to see ordering.

Collapsing the data for ΔI onto a scaling plot (see Fig. 4), we find $T_c=0.45\pm0.05$, $\nu=1.3\pm0.4$. It is gratifying that this value of ν is in reasonably good agreement with experimental estimates,[19] though we should caution that the effects of systematic errors (due to corrections to finite size scaling) have not been incorporated in the estimate of the error bar and that both the experimental and theoretical values may be only effective exponents, valid in a re-

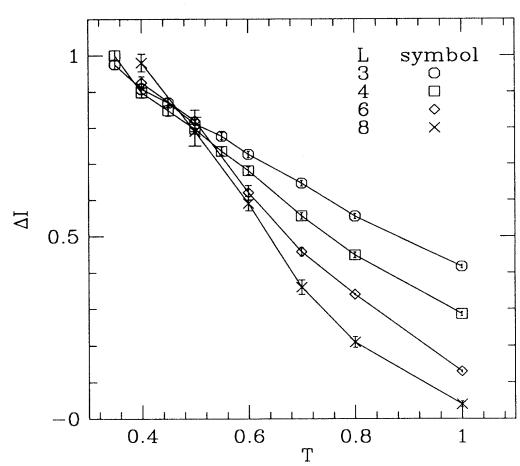

FIG. 3. The rms current, $\Delta I=[I^2]_{av}^{1/2}$, determined by Monte Carlo simulations for different sizes and temperatures. The curves for different sizes are expected to come together at T_c and, if there is order in the low-temperature state, to splay out again at lower temperatures. The data does indeed become independent of size for $T \simeq 0.50$. At lower temperatures, most of the data is independent of size, but for $T=0.4$ the data for $L=8$ is significantly greater than that of the lower sizes. More precisely, the values for $T=0.4$ are 0.980 ± 0.024 ($L=8$, 1200 samples), 0.925 ± 0.017 ($L=6$, 2000 samples), 0.898 ± 0.015 ($L=4$, 2000 samples), 0.908 ± 0.020 ($L=3$, 2000 samples). In general, the data for $L=8$ used 1000 samples and that for smaller sizes used 2000 or more samples. The error bars are obtained from the variance of the estimates from different samples and represent one standard deviation.

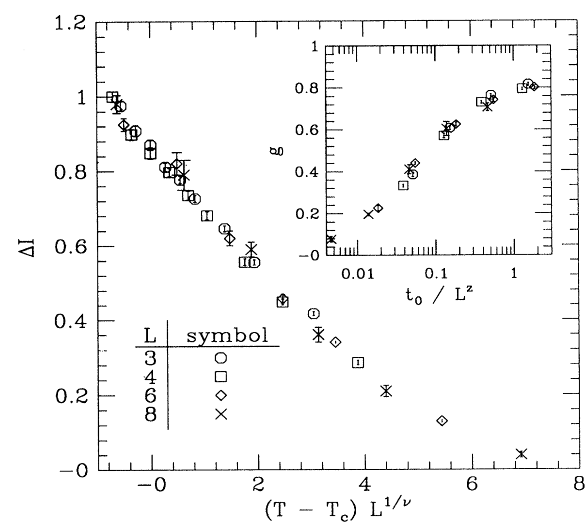

FIG. 4. The same data as in Fig. 3 but in a finite-size scaling plot similar to that for g in Eq. (3), with $T_c=0.45$ and $\nu=1.2$. The inset shows a scaling plot of data for g at $T=T_c=0.45$, during the *approach* to equilibrium in which t_0 sweeps are used for equilibration followed by $2t_0$ sweeps for averaging. One expects (Ref. 20) $g \approx \bar{g}(t_0/L^z)$, where z is the dynamical exponent, and here we took $z=4.8$.

stricted range of temperature and size, since it is unclear if the asymptotic critical region has been reached. We have also estimated the dynamical exponent z by determining the time to reach equilibrium at $T=T_c$, which varies with system size as L^z.[20] Our Monte Carlo dynamics neglects coupling of the order parameter to charge fluctuations, which is probably correct here since long-range forces make the relevant propagating mode plasmonlike and gapped,[21] even though charge is conserved. An appropriate scaling plot is shown in the inset to Fig. 4 with $z=4.8$. We estimate $z=4.7\pm0.7$, again in good agreement with experiment.[19]

To conclude, our results indicate that the gauge glass in 3D is just above its lower critical dimension, though it is also possible that the lower critical dimension is *precisely* 3.

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*Permanent address: Physics Department, University of California Santa Cruz, Santa Cruz, CA 95064.

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[19]For example, Olsson *et al.* in Ref. 7 find $\nu=1.1\pm0.4$, whereas somewhat larger values $\nu\simeq1.7$ and $\nu=2\pm1$, respectively, are found by Koch *et al.* and Gammel, Schneemener, and Bishop. For the dynamical exponent z the experimental results are $z=5.2\pm0.6$ (Olsson *et al.*), $z\simeq4.8$ (Koch *et al.*), and $z=4.3\pm1.5$ (Gammel, Schneemener, and Bishop).

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