Josephson-oscillator spectrum and the reentrant phase transition in granular superconductors

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The superconductive phase transition in a granular superconductor, modeled as a Josephson-junction array, is studied using a self-consistent mean-field theory. In the case when the Josephson oscillator was treated as a pendulum with discrete energy levels, no reentrant behavior was found. In the opposite extreme when it was treated as a quantum particle in a periodic field, having a band-energy spectrum, there exists a reentrance from the superconductive to the normal state as the temperature is further lowered.

I. INTRODUCTION

As was shown by Rosenblatt,[1] arrays of Josephson junctions become superconducting in two stages. At the bulk transition temperature T_0, the magnitude of the order parameter of each grain becomes nonzero. If the energy of Josephson coupling between grains E_J is much less than T_0, thermal fluctuations cause the phases φ of the order parameter on different grains to be uncorrelated until the temperature is lowered such that the condition $E_J \geq T$ is fulfilled.

As Abeles[2] has pointed out, when the charging energy E_c, i.e., the electrostatic energy of an excess Cooper pair localized on a fixed grain, is comparable to E_J, then zero-point fluctuations of φ may destruct the long-range superconductive order even at zero temperature.

Several authors[3-14] have theoretically investigated the effects of charging energy on the phase ordering transition. Some of them[3,5-7,11] found that this charging energy can lead to a reentrance of the normal state from the superconductive state as the temperature is further lowered. Several experimenters have claimed to observe a reentrant [16] or nearly reentrant[15,17-19] transition.

The aim of this Brief Report is to investigate how the reentrant superconductive transition in Josephson arrays is connected with the spectrum of the Josephson oscillator.

II. SINGLE JOSEPHSON OSCILLATOR

A Josephson junction is described by a Hamiltonian, obtained by Anderson:[20]

$$\hat{H}=(2e)^2\hat{n}^2/(2C)-E_J\cos\varphi \ . \tag{1}$$

Here C is the junction capacitance; $2e$ is the Cooper-pair charge; E_J is the Josephson coupling energy; $n=-i\partial/\partial\varphi$ is an excess Cooper-pair number operator conjugate to the phase difference φ across the junction.

As Likharev and Zorin[21] pointed out, Hamiltonian $\hat{H}$ is similar to that of two different physical systems: a pendulum (here φ is an angular coordinate), and a one-dimensional quantum particle moving along the φ axis. These two systems have different properties with respect to translations $\varphi\rightarrow\varphi+2\pi$.

For the pendulum the states before and after translation are indistinguishable and its wave function $\Psi(\varphi)$ should be 2π periodic. This leads to the discrete energy spectrum. The states of quantum particle before and after translation are distinguishable. The only condition is that $\Psi(\varphi)$ should be limited when φ increases *ad infinitum*. This leads to the band-energy spectrum.

As was shown by some authors,[21-23] when the junction is coupled to an environment (for example by normal shunt resistance) which permits a continuous change of the charge on the junction the Josephson oscillator can be treated having the band-energy spectrum.

Hamiltonian (1) may be rewritten in the form

$$\hat{H}=-E_c(\partial^2/\partial\varphi^2+g\cos\varphi) \ . \tag{2}$$

Here E_c denotes the Coulomb energy $(2e)^2/(2C)$ and $g=E_J/E_c$.

Let us consider the case of small g. In this case the "potential energy" $g\cos\varphi$ may be treated as a perturbation.

The nonperturbed Hamiltonian is

$$H_0=-E_c\partial^2/\partial\varphi^2 \ ,$$

with eigenfunctions

$$\Psi_n^0(\varphi)=\exp(in\varphi) \ ,$$

where the characteristic number n is real.

The thermodynamical average value $\langle\cos\varphi\rangle$ is given by an equation:

$$\langle\cos\varphi\rangle=\frac{\int_{-\infty}^{+\infty}\exp(-E_n/T)\langle\Psi_n|\cos\varphi|\Psi_n\rangle}{\int_{-\infty}^{+\infty}dn\,\exp(-E_n/T)} \ . \tag{3}$$

From the Hellman-Feynman theorem[24] is obtained[10]

$$\langle\Psi_n|\cos\varphi|\Psi_n\rangle=-(1/E_c)\partial E_n/\partial g \ . \tag{4}$$

Perturbation theory gives

$$E_n=E_cn^2-E_c\frac{1}{1-(2n)^2}g^2/2 \ . \tag{5}$$

From Eqs. (3)–(5) we obtain

$$\langle \cos\varphi \rangle = g \frac{\displaystyle ⨍_{-\infty}^{+\infty} dn \exp(-E_c n^2/T) \frac{1}{1-(2n)^2}}{\displaystyle \int_{-\infty}^{+\infty} dn \exp(-E_c n^2/T)} . \qquad (6)$$

Here $⨍$ denotes the main value of the integral.

At low temperatures ($x = E_c/T \gg 1$) Eq. (6) has an asymptotic:

$$\langle \cos\varphi \rangle = g(1+2T/E_c) . \qquad (7)$$

Equation (6) may be transformed to the form suitable for numerical calculations (see Appendix A):

$$\langle \cos\varphi \rangle = g \frac{\displaystyle \int_0^x dy \exp(y/4) \frac{1}{4\sqrt{y}}}{\exp(x/4)/\sqrt{x}} . \qquad (8)$$

At high temperatures ($x \ll 1$) the asymptotic of (8) is

$$\langle \cos\varphi \rangle = gx/2 .$$

Remember that $g = E_J/E_c$ and $x = E_c/T$. So the last equation takes the form

$$\langle \cos\varphi \rangle = E_J/(2T) . \qquad (9)$$

The last equation may be obtained in a more simple way using classical statistics (see Appendix B).

The inverse inductance of the Josephson junction is given by equation:

$$L^{-1} = (2e/\hbar)^2 E_J \langle \cos\varphi \rangle , \qquad (10)$$

as Mirhashem and Ferrell[25] have shown. So using Eqs. (7) and (9) we see that temperature dependence of inverse inductance is nonmonotonous (in the case $g \ll 1$).

III. JOSEPHSON ARRAYS

The generalization of Hamiltonian (1) for Josephson arrays, obtained by Efetov,[6] is

$$\hat{H} = \sum_{ik} P_{ik}(2e)^2 \hat{n}_i \hat{n}_k - \sum_{ik} E_{ik} \cos(\varphi_i - \varphi_k) . \qquad (11)$$

Here matrix elements P_{ik} are the Coulomb interactions and E_{ik} are the Josephson energies between the i and k grains; $\hat{n}_k = -i\partial/\partial\varphi_k$ is the excess Cooper-pair number operator, conjugate to the phase φ_k of the k grain.

The mean-field Hamiltonian in Hartree approximation should be obtained by replacing all operators, except two conjugate operators corresponding to chosen grain, by their average values:

$$\hat{H}_{mf} = P(2e)^2 \hat{n}^2 - z\mu E_J \cos\varphi . \qquad (12)$$

Here z is the lattice coordination number; $P = P_{ii}$ is the diagonal element of the potential matrix; μ is the average around array value of $\cos\varphi$; the average values of $\hat{n}$ is zero because of electroneutrality of a sample, and this is why the mean-field Hamiltonian depends on diagonal elements of potential matrix only.

The Hamiltonian (12) is similar to that of an isolated Josephson oscillator (2) with

$$E_c = P(2e)^2 \text{ and } g = z\mu E_J/E_c . \qquad (13)$$

Near the phase transition point $\mu \ll 1$ and condition $g \ll 1$ fulfills.

For theory to be self-consistent the average value $\langle \cos\varphi \rangle$ obtained using the mean-field Hamiltonian should be equal to μ.

It may be shown that the next term in Eq. (7) which is proportional to g^3 is negative. For zero temperature it was obtained by Ferrell and Mirhashem.[10] At low temperatures it has small correction, which cannot change its sign. The same result may be simply obtained for high temperatures using Eq. (B1). So we will consider the term g^3 to be negative for all temperatures.

So it is clear the μ will be nonzero if

$$\partial \langle \cos\varphi \rangle / \partial g \geq E_c/(zE_J) . \qquad (14)$$

At low temperatures ($T \ll E_c$) using Eqs. (7) and (14) we obtain that long-range order exists ($\mu \neq 0$) if

$$zE_J/E_c \geq 1 - 2T/E_c . \qquad (15)$$

At high temperatures ($T \gg E_c$) we easily obtain from Eqs. (9) and (14) that long-range superconductive order exists if

$$zE_J/E_c \geq 2T/E_c . \qquad (16)$$

The results for intermediate temperatures calculated numerically using Eqs. (8) and (14) are plotted in Fig. 1.

The occurrence of superconductivity goes together with occurrence of nonzero inverse inductance which is proportional[26] to μ^2.

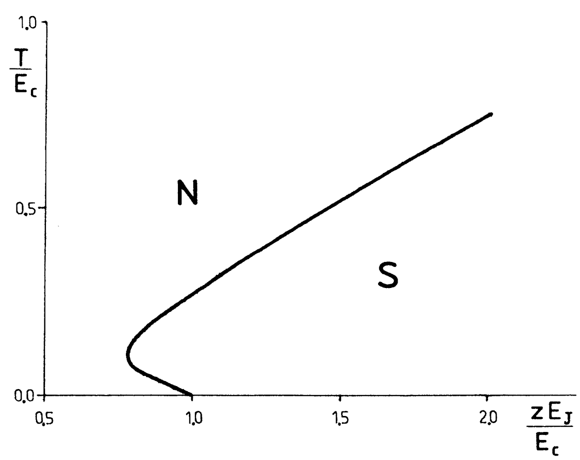

FIG. 1. The phase diagram of a granular superconductor, calculated using Eqs. (8) and (14). In the area marked N, the mean value $\mu = \langle \cos\varphi \rangle = 0$. In area S, $\mu \neq 0$ and the long-range superconductive order exists.

IV. DISCUSSION

If we will treat a Josephson oscillator as having discrete spectrum then integrals over n in Eq. (6) should be changed to sums over integer numbers:

$$\langle \cos\varphi \rangle = g \frac{\sum_{n=-\infty}^{+\infty} \exp(-E_c n^2/T) \frac{1}{1-(2n)^2}}{\sum_{m=-\infty}^{+\infty} \exp(-E_c/n^2/T)} ;$$

all terms except the one corresponding to $n=0$ (lowest energy level) are negative. So $\langle \cos\varphi \rangle$ decreases when temperature increases, and no reentrant behavior will occur.

On the other hand, Efetov,[6] Fazekas,[5] and Fishman and Stroud[11] obtained reentrant transition using a discrete spectrum. They used mean-field approximation only for the Josephson part of Hamiltonian (11) while they treated the charging term exactly. The reentrance in these treatments is connected with thermal excitations screening the Coulomb interactions and reducing the Coulomb energy. But in the most recent papers[8,14] some of these authors reported that if both parts of the Hamiltonian are treated on equal footing, reentrance disappears.

Simanek,[3,7] who predicted the reentrant transition, used 4π-periodic eigenfunctions in his calculations. His results can be rederived using perturbation theory with degeneracy. So it is not necessary to use a continuous spectrum to find this effect, but 2π periodicity should be violated.

Phase diagrams presented in Refs. 3, 5–7, and 11 are different from that obtained by me. The phase curve between the superconductive and normal state obtained in those references is perpendicular to abscissa (zE_J/E_c in Fig. 1), while that obtained by me has a finite slope given by Eq. (15).

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APPENDIX A

To calculate the integral

$$I(x) = \int_{-\infty}^{+\infty} dn \exp(-xn^2) \frac{1}{1-(2n)^2} ,$$

we rewrite $1/[1-(2n)^2]$ as $1+(2n)^2/[1-(2n^2]$ and rewrite $I(x)$ as a sum of two integrals:

$$I(x) = \int_{-\infty}^{+\infty} dn \exp(-xn^2) + \int_{-\infty}^{+\infty} dn \exp(-xn^2) \frac{4n^2}{1-(2n)^2} ;$$

the first integral is equal to $\sqrt{\pi/x}$ and the second is equal to $-4\partial I(x)/\partial x$. So we have a linear differential equation:

$$4\partial I(x)/\partial x + I(x) = \sqrt{\pi/x} .$$

Taking in attention condition $I(0)=0$, we obtain

$$I(x) = \exp(-x/4) \int_0^x dy \exp(y/4) \tfrac{1}{4} \sqrt{\pi/y} .$$

APPENDIX B

In the case $T \gg E_c$, φ and n are classical variables and the mean value $\langle \cos\varphi \rangle$ may be calculated via classical statistics:

$$\langle \cos\varphi \rangle = \frac{\int_{-\infty}^{+\infty} dn \int_0^{2\pi} d\varphi \exp\left[\frac{-1}{T}(E_c n^2 - E_J \cos\varphi\right] \cos\varphi}{\int_{-\infty}^{+\infty} dn \int_0^{2\pi} d\varphi \exp\left[-\frac{1}{T}(E_c n^2 - E_J \cos\varphi)\right]} = \frac{\int_0^{2\pi} d\varphi \exp\left[\frac{1}{T} E_J \cos\varphi\right] \cos\varphi}{\int_0^{2\pi} d\varphi \exp\left[\frac{1}{T} E_J \cos\varphi\right]} . \quad \text{(B1)}$$

In the case $E_J \ll T$ we easily obtain Eq. (9).

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