Vortex inertial mass for a discrete type-II superconductor

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The inertial mass per unit length of vortex in a discrete type-II superconductor is found. Using a Josephson-coupled-layer model, the expression for the inertial mass per unit length for a continuous type-II superconductor is continued from high temperatures (near the transition temperature T_c) to zero absolute temperature for the case that the vortex is oriented parallel to the layers. The possible application of the vortex inertial mass in studies of microwave and infrared response is discussed.

INTRODUCTION

The inertial mass of a vortex threading a type-II superconductor is of interest because of its importance in the description of either single or collective vortex dynamics (e.g., Refs. 1 and 2). For instance, using a Hamiltonian formalism, collective vortex oscillations can be studied and dispersion relations derived.[1] The relevance of retaining a vortex inertial mass in the equation of motion in the study of microwave response was observed some time ago[2] and the added importance for high-temperature superconductors has been pointed out.[3]

In this paper we are concerned with calculating the inertial mass of a vortex penetrating a type-II superconductor, taking into account atomic-level discreteness in the material. We expect the resulting expression for the inertial mass to be applicable to various layered superconductors, including the high-temperature copper oxides. The known high-T_c superconductors are generally accepted to have a perovskite crystal structure[4] and an associated layered structure and small coherence length (ξ_c) in the c direction. The anisotropic Ginzburg-Landau (GL) theory employing an effective mass tensor (e.g., Refs. 5 and 6) provides a useful tool for the phenomenology of these materials. However, the theory is strictly valid only over a very limited temperature range (near the transition temperature T_c). Further, the GL theory is inadequate for those situations where atomic-level discreteness of the superconductor comes into play. When calculating the inertial mass of a vortex for a discrete type-II superconductor, both of these considerations apply.

It is possible for the vortex inertial mass to arise from several mechanisms, including variation of the magnitude of the order parameter,[7] elastic deformation with an accompanying strain field,[8] and generation of an electric field from vortex motion. For a discrete superconductor with Josephson vortices present, as we consider, a major contribution is expected to be the latter electromagnetic one.

Using a Josephson-coupled-layer model,[9] the expression for the inertial mass per unit length for a continuous type-II superconductor found from anisotropic GL theory[10] is continued from high temperatures to zero absolute temperature for the case that the vortex is oriented parallel to the layers. We expect our result to be valid below a crossover temperature T^*, where $\xi_c(T)$ becomes of the order of the lattice constant c.[11–15] To our knowledge, this provides the first estimate of the vortex inertial mass for discrete three-dimensional (3D) superconductors. In particular, our result may apply to the classes of layered organic superconductors,[16–19] organically intercalated metals,[15] and high-T_c materials.[12–14] Our form of the inertial mass μ is compared to that of Lebwohl and Stephen[20] who treated the case of a vortex in a single Josephson junction. (In a well-known mechanical analogy, the mass of a fluxon in a single Josephson junction is related to the junction capacitance.[21])

We point out the great enhancement in μ for high-T_c materials, which arises from the small Josephson core size. This also leads us to discuss the possible application of the vortex inertial mass in studies of superconductor microwave and infrared response. An appropriate function for characterizing vortex dynamic response in these applications is the complex-valued dynamic mobility.[22] In this paper it is shown how the dynamic vortex mobility previously obtained to include both pinning and flux-creep effects can be extended to include a nonzero inertial mass. The mobility is written in terms of a general inertial mass per length so that other forms of the mass could be employed.

Using a model square lattice of superconducting grains weakly coupled by Josephson junctions, Eckern and Schmid have studied quantum vortex dynamics in granular superconducting films.[23] These authors have developed an effective action for a fluxon which uses an effective mass. A similar 2D model for granular superconducting systems has been used by Larkin *et al.*[24] to study quantum tunneling of vortices. Larkin *et al.* found an effective Lagrangian and action, incorporating an effective mass for vortices, and used it to examine collective low-frequency 2D vibrations in a network of Josephson junctions. A recent work[25] employed an effective mass for a vortex in a 2D superconducting ring to study voltage quantization.

The subject of dimensional crossover in a layered anisotropic superconductor was studied by Klemm, Luther, and Beasley.[11] At a crossover temperature, defined by $\xi_c(T^*)=\sqrt{2}s$, where s is the stacking periodicity, the upper critical field parallel to the layers $H_{c2\parallel}$ can show a strong upward curvature.[11] Possible experimental verification of this theory for $H_{c2\parallel}$ in intercalated layer compounds may be found in Ref. 15. The existence of a crossover temperature for high-T_c materials may be indicated by data from torque-magnetometry experiments on untwinned single crystals of $YBa_2Cu_3O_{7-\delta}$ (Y-1:2:3).[12] Current theory based on a 3D London treatment appears inadequate to explain these results at lower temperatures when the magnetic field lies close to the CuO planes.[12,26]

When a vortex is present in a layered, discrete superconductor, as the temperature is lowered, several related effects occur. For the sake of simplicity, unless stated otherwise in this paper, the vortex will be assumed to be oriented parallel to the superconducting layers of the specimen. As the temperature passes through T^*, the vortex core structure changes from an Abrikosov type to a Josephson type.[9] (The order parameter is no longer depressed to essentially zero on the vortex axis.) The dimensions of the vortex core change, which affects derived quantities such as the line energy, lower critical field, and viscous drag coefficient.[9,27] Specifically, if ξ_c is the smallest length entering in a GL expression, it is replaced with some other length, usually the repeat distance s between superconducting layers. One example of this is the lower critical field H_{c1} where the Ginzburg-Landau parameter κ is replaced by the ratio of penetration depth to the stacking periodicity.[27] As the vortex inertial mass at high temperatures is proportional to the upper critical field H_{c2}, we expect the product of coherence lengths, which is a measure of the vortex core area, to be replaced by an altered vortex core size.

BACKGROUND ON MODEL OF DISCRETE, LAYERED SUPERCONDUCTORS

Here we recall and slightly extend some of the results of Ref. 9 that we require in computing the vortex inertial mass in the next section. For description and application of the layer model with more detail than is provided here, the reader may consult Refs. 9 and 27. The model used is based on an infinite periodic stack of Josephson-coupled, parallel superconducting layers. The superconducting layers of thickness d_s alternate with insulating layers of thickness d_i, giving a periodicity length of $s=d_i+d_s$. The layers in the model are taken to be parallel to the xy (ab) plane, with the center of the insulating layers at $z=z_n=ns$, $n=0,\pm1,\pm2,\ldots$. The major results that we require include expressions for the gauge-invariant phase difference, $\Delta\gamma_n(y)$, of the superconducting wave function across the junction between successive superconducting layers when a single vortex is present in the central (or $n=0$) insulating layer and is aligned parallel to the x axis (a direction). In giving the expressions for $\Delta\gamma_n$, it is convenient to make use of the abbreviations

$$\begin{aligned} \tilde{y} &\equiv y/\lambda_c \ , \\ \tilde{z} &\equiv z/\lambda_b \ , \\ \tilde{u}_0 &\equiv s/2\lambda_b \ , \\ \tilde{R} &\equiv (\tilde{u}_0^2+\tilde{y}^2+\tilde{z}^2)^{1/2} \ , \end{aligned} \tag{1}$$

where the penetration depths λ_b and λ_c govern the exponential decay of the components of the supercurrent density along the b and c directions, respectively. In the Josephson weak-coupling limit, we have $\lambda_c^2=c\phi_0/8\pi^2 sJ_0$ to good approximation (ϕ_0 is the flux quantum and J_0 is the maximum Josephson current density). For the central junction (at $z=0$), an expression for the gauge-invariant phase difference well suited to our purposes is

$$\Delta\gamma_0(y)=\pi-2\tan^{-1}(\tilde{y}/\tilde{u}_0) \ , \quad |\tilde{y}|\ll 1 \ , \tag{2}$$

valid for small values of $\tilde{y}$. Similarly, for junctions with $n\neq 0$, a useful approximation for the gauge-invariant phase difference is

$$\Delta\gamma_n(y)\simeq\frac{2\tilde{u}_0\tilde{y}}{\tilde{R}^2} \ , \quad \tilde{R}\ll 1 \ , \tag{3}$$

valid for small $\tilde{R}$. Equations (2) and (3) result from the asymptotic forms of modified Bessel functions K_ν.[9]

In this paper we also make use of the results of fluxoid quantization obtained in Ref. 9. An extension made here is deriving a modified 2D time-dependent sine-Gordon equation for the phase difference $\Delta\gamma_n(y,t)$. Subject to the assumption $\lambda_c^2\gg\lambda_b^2$, there results from fluxoid quantization the equation

$$b(y,z)=-\frac{\phi_0}{2\pi s}\frac{\partial}{\partial y}\Delta\gamma_n(y,t)+\frac{4\pi\lambda_b^2}{c}\frac{\partial}{\partial z}J_y(y,z) \ , \tag{4}$$

where b is the magnetic field of the vortex and J_y is the y component of the supercurrent in the layers. By combining the Josephson tunneling current relation with Ampere's law, including the displacement current, we obtain

$$\begin{aligned} J_z(y,z,t)&=J_0\sin\Delta\gamma_n(y,t) \\ &=-\frac{c}{4\pi}\frac{\partial b}{\partial y}(y,z)-\frac{\epsilon}{4\pi}\frac{\partial E_z(y,z,t)}{\partial t} \end{aligned} \tag{5}$$

when there is an applied electric field in the z direction, ϵ being the electrical permittivity of the insulating layers. The assumption is made here that the electric field is small enough so that, to first order, it induces a magnetic field negligible to b.

Now we use the relation between the electric field and the time variation of the phase difference,[21,28,29]

$$E_z(y,z,t)=\frac{\hbar}{2ed_i}\frac{\partial\Delta\gamma_n(y,t)}{\partial t} \ , \tag{6}$$

and Eqs. (4) and (5) to obtain a modified 2D sine-Gordon equation

$$\frac{\partial^2\Delta\gamma_n(y,t)}{\partial y^2}-\frac{1}{J_0}\frac{\lambda_b^2}{\lambda_c^2}\frac{\partial^2}{\partial y\partial z}J_y(y,z)-\frac{1}{\bar{c}^2}\frac{\partial^2\Delta\gamma_n(y,t)}{\partial t^2}=\frac{1}{\lambda_c^2}\sin\Delta\gamma_n(y,t) \ , \tag{7}$$

where the term with J_y is a modification of the usual (1+1)-dimensional (Lorentz invariant) sine-Gordon equation. From Eq. (7) we have thus found the speed of light $\bar{c}$ in the insulating layers of the stack model. We have

$\bar{c}^2 = c^2 d_i/\epsilon s$, with the geometrical factor d_i/s, which is usually of order unity. This geometrical factor is analogous to, but distinct from, that for a single Josephson junction (e.g., Refs. 20, 21, and 28) where the geometrical factor is the ratio of insulator thickness to magnetic thickness and is usually very small. We further have the plasma frequency in the stack, $\omega_p = \bar{c}/\lambda_c = c\sqrt{d_i/\epsilon s}/\lambda_c$. Alternatively, the plasma frequency $\omega_p = 1/\sqrt{L_0 C}$, where the junction inductance $L_0 = \hbar/2eAJ_0$ and capacitance $C = \epsilon A/4\pi d_i$ with area A. It is then possible to define a McCumber-Stewart parameter[29] for the stack $\beta_C = \omega_p RC$, where R is the normal resistance, which characterizes the effect of the junction capacitance. As seen below, the identification of the speed of light in the insulating layers of the stack model is useful in rewriting the expression for the vortex inertial mass.

As we have assumed $\lambda_b^2/\lambda_c^2 \ll 1$, for those cases where $|\partial^2(J_y/J_0)/\partial y \partial z| \le \lambda_c^2$, the J_y term in Eq. (7) may be neglected. (This is probably a valid approximation outside of the vortex core region.) However, in the core region, specifically in the central layer, the behavior of the phase difference is truly 2D.[9] Equation (7) is a nonlinear wave equation for the gauge-invariant phase difference which does not contain any dissipative terms. For the calculation of the vortex inertial mass below, we assume that such terms can be ignored to leading order. In this regard, we mention that perturbed versions of the (1+1)-dimensional sine-Gordon equation have been studied.[30] These treatments have usually focused on soliton dynamics in the presence of weak perturbations.

CALCULATION OF VORTEX INERTIAL MASS

We consider the vortex to move at constant velocity v in the y direction (see Fig. 1). This is similar to the situation where the vortex (moving parallel to the layers) is acted upon by a Lorentz force due to an applied current density in the z direction. It will be seen that the calculation of the inertial mass is similar to that for the energy dissipation (e.g., Refs. 9 and 29). The vortex inertial mass is obtained by equating the vortex kinetic energy to the electric field energy produced by the vortex motion. This yields the expression for the inertial mass per unit length

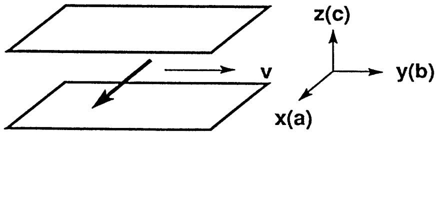

FIG. 1. Geometry for the calculation of the vortex inertial mass in a Josephson-coupled-layer model. The vortex (bold arrow), in the central insulating layer, is aligned parallel to the a axis and moves with constant velocity **v** in the b direction.

$$\mu = \frac{\epsilon}{4\pi v^2} \int E^2(\mathbf{x}) d^2x \quad (v \ll \bar{c}) . \tag{8}$$

If we were to include relativistic effects, we would need to use the expression[31] $K = (\gamma - 1)\mu c^2$ for the kinetic energy where the factor $\gamma = 1/\sqrt{1 - v^2/\bar{c}^2}$. By using Eq. (6) to express the electric field in terms of the gauge-invariant phase difference, integrating over the y direction, and summing over all junctions in the model (in the z direction), we have

$$\mu = \frac{\epsilon s}{4\pi} \left[\frac{\phi_0}{2\pi c} \right]^2 \frac{1}{d_i^2} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\frac{\partial \Delta\gamma_n(y)}{\partial y} \right]^2 dy , \tag{9}$$

where we took $\Delta\gamma_n(y,t) = \Delta\gamma_n(y - vt)$, which should be valid for $v \ll \bar{c}$ [cf. Eq. (7)]. Using Eq. (2), it is found that the central junction provides the dominant contribution to the inertial mass. Using Eq. (3), all the other layers in the model simply provide a small correction to μ. For this correction from all layers $j \neq 0$, we introduce the infinite sum

$$\Sigma_1 \equiv \sum_{n=1}^{\infty} \frac{1}{(1+4n^2)^{3/2}} \simeq 0.113\,08 . \tag{10}$$

Then the vortex inertial mass per unit length is

$$\mu = \frac{\epsilon s}{8\pi^2} \frac{\phi_0^2}{c^2 d_i^2} \frac{1}{\tilde{u}_0 \lambda_c} (1 + \Sigma_1) . \tag{11}$$

Equation (11) can be rewritten in several forms. One particularly illuminating form is to recognize that $\tilde{u}_0 \lambda_c = (s/2)\lambda_c/\lambda_b = y_{\max}$ is the dimension of the Josephson core along the y axis.[9] [This is the distance from the vortex axis along the y direction at which the gauge-invariant phase difference $\Delta\gamma_0(y)$ across the central junction becomes equal to $\pi/2$ and the Josephson current density reaches its maximum value of J_0.[9]] By also employing the expression for the speed of light in the insulating layers, $\bar{c}$, we have for the inertial mass per unit length

$$\mu = \frac{\phi_0^2 (1 + \Sigma_1)}{8\pi^2 \bar{c}^2 d_i y_{\max}} \tag{12}$$

for a vortex aligned and moving parallel to the layers. Now the appearance of the Josephson vortex core area can be seen, for the core dimension in the z direction is of order $s/2$ which, in turn, is roughly of the size of d_i. We next compare the result (12) with that from anisotropic GL theory and the single Josephson junction case of Lebwohl and Stephen, respectively.

In Ref. 10 the dipolar electric field induced by the motion of a single vortex moving at constant velocity in a continuous anisotropic superconductor was investigated. We showed that the standard dipolar field case can be extended by the inclusion of a mass anisotropy parameter β. Furthermore, by computing the electric field energy per unit length of vortex, we found a vortex inertial mass tensor per unit length. For comparison purposes in this paper, we recall that component of the (diagonal) inertial mass tensor corresponding to a vortex along the x axis moving in the y direction:[10]

$$\mu_y = \frac{\phi_0 H_{c2\parallel x}}{16\pi c^2}\left[3+\frac{1}{\beta}\right], \quad (13)$$

where $\beta = m_z/m_y$, $H_{c2\parallel x} = \phi_0/2\pi\xi_y\xi_z$, and m_y and m_z are the GL effective masses.[5,6] The result (13) should be very analogous to (12) because, although the (Abrikosov) vortex was assumed to have a normal core, (13) was derived by neglecting any variation in the order parameter as a result of the vortex motion. The result (13) for a continuous superconductor should be valid at low fields ($B \ll B_{c2}$), high temperatures ($|T-T_c| \ll T_c$), and when the GL parameter is large, $\kappa \gg 1$. We recall that, in the anisotropic model,[6,10] the vortex core is taken to be elliptical in shape with semimajor axis ξ_y and semiminor axis ξ_z (the GL coherence lengths). For a typical anisotropic high-T_c superconductor [e.g., Y-1:2:3 (Ref. 13)], the term $1/\beta$ can be ignored in Eq. (13). Then the vortex inertial mass for a continuous superconductor is of the order of

$$\mu_y \simeq \frac{3\phi_0^2}{32\pi^2 c^2 \xi_y \xi_z}. \quad (14)$$

The forms (12) and (14) exhibit the vortex inertial mass at low and high temperatures, respectively. As anticipated, as the temperature falls below the crossover temperature T^*, the vortex core area is replaced in the GL expression. Specifically, aside from numerical factors of order unity, ξ_z is replaced by d_i, ξ_y by $y_{\max}$, and c by the speed of light in the insulating barriers $\bar{c}$.

Using a relativistic formulation, Lebwohl and Stephen[20] (LS) found an effective mass per unit length of vortex line. They considered a vortex in a single Josephson junction, assuming isotropic superconductors with thicknesses much larger than the London penetration depth λ. Using the speed of light in the single insulating barrier, $\bar{c} = c\sqrt{d_i/\epsilon d}$, their result[20] can be written as

$$\mu_{\rm LS} = \frac{1}{2\pi^3}\frac{\phi_0^2}{\bar{c}^2 d\lambda_J}, \quad (15)$$

where $d = 2\lambda + d_i$ is the magnetic thickness and $\lambda_J = \sqrt{c\phi_0/8\pi^2 dJ_0}$ is the Josephson penetration depth. In terms of the lower critical field of a Josephson junction, $H_{c1J} = 2\phi_0/\pi^2\lambda_J d$ (Ref. 21, p. 109), (Ref. 32), the Lebwohl and Stephen effective vortex mass per length becomes $\mu_{\rm LS} = \phi_0 H_{c1J}/4\pi\bar{c}^2$. Because the product $d_i y_{\max}$ is much smaller than the product $d\lambda_J$, or alternatively, because $H_{c1J} \ll H_{c2}$, the LS result is much smaller than the inertial mass for a vortex in a Josephson-coupled stack. It is also smaller than the inertial mass obtained by Suhl by means of time-dependent GL theory[7] for an isotropic superconductor. (In Ref. 10 the result for an anisotropic superconductor was compared with Suhl's result.) This situation for the inertial masses is parallel to that for the viscous drag coefficients. As pointed out by Lebwohl and Stephen, the drag coefficient for a single Josephson junction is proportional to H_{c1J}. However, for a continuous superconductor, the drag coefficient is proportional to the upper critical field.[6]

To estimate the size of the inertial mass for high-T_c materials, we replace Eq. (12) by the approximation

$$\mu \simeq \frac{\phi_0^2}{4\pi^2}\frac{\lambda_b}{\lambda_c}\frac{1}{\bar{c}^2}\frac{1}{d_i s}. \quad (16)$$

Considering Y-1:2:3 as an example,[13] we take $\lambda_b/\lambda_c = 1/5.5$ and $s = 12$ Å. Assuming that $d_i = s/2$ and $\bar{c}^2 \approx c^2/100$, we have $\mu \approx 3\times 10^6\ m_e$/cm for temperatures below T^*. In making this estimate we ignored a possible frequency dependence in the dielectric constant ϵ for the insulating layers.[33] The corresponding vortex relaxation time is $\tau = \mu/\eta$, where η is the viscous drag coefficient.[9] We have that $\tau = c^2 s/\bar{c}^2 4\pi d_i \sigma_n$, where σ_n is the normal-state conductivity, which compares with $1/\pi\sigma_n$ for the case of an Abrikosov vortex in a continuous superconductor.[10]

VORTEX MOBILITY

The above estimate suggests that inertial effects may be significant in the vortex dynamics of high-temperature superconductors. An appropriate function for characterizing vortex dynamic response is the complex-valued dynamic mobility.[22] The dynamic mobility is useful in describing many applications, including high-frequency phenomena such as microwave[2,3,22,34] or infrared[35–37] absorption or reflection. The mobility has the added attraction that mechanisms in the vortex dynamics, e.g., thermal activation, can be included in a systematic manner. In particular, we show here how the dynamic vortex mobility obtained in Ref. 22 to include both pinning and flux-creep effects can be extended to include a nonzero inertial mass.

As an illustration, an often used vortex equation of motion is

$$\mu\ddot{u}(\mathbf{x},t) + \eta\dot{u}(\mathbf{x},t) + \kappa_p u(\mathbf{x},t) = f_d(\mathbf{x},t), \quad (17)$$

where u is the vortex displacement from its equilibrium pinning potential well, η is the viscous drag coefficient (e.g., Ref. 27), κ_p is the restoring force constant of the pinning potential, and f_d is the driving force, often the Lorentz force from a local current density. The corresponding mobility in linear response at angular frequency ω is given by

$$\tilde{\mu}(\omega) = \mu^{-1}(-i\omega + \eta/\mu + i\kappa_p/\mu\omega)^{-1}.$$

To go further, the dynamic mobility can be computed from its definition as a velocity correlation function which is a Fourier transform.[38,39] In a manner analogous to our treatment of the rf surface impedance,[22] we employ the result of Ref. 38 for a particle undergoing Brownian motion in a periodic potential. The effect of thermal agitation of the particle is described by the addition of a Langevin force on the right-hand side of Eq. (17), which is assumed to be Gaussian white noise with zero mean.[38,40] The form of the dynamic mobility that we use is based on Schneider's continued-fraction expansion of the Laplace transform of the velocity time correlation function.[41] A truncated continued-fraction expansion of the dynamic mobility has proven successful provided that independent information is built in for the dc mobility $\tilde{\mu}(0,T)$.[40,41]

The truncated continued fraction form of $\tilde{\mu}$ that we use is

$$\tilde{\mu}(\omega,T)=\mu^{-1}\left[-i\omega+\eta/\mu+\frac{\alpha\kappa_p/\mu}{-i\omega+g_0(T)}\right]^{-1}, \quad (18)$$

where $\alpha(\nu)$ is the ratio $I_1(\nu)/I_0(\nu)$, I_p is a modified Bessel function of the first kind of order p, the temperature-dependent argument $\nu=U(T)/2k_BT$, and U is the barrier height of the periodic potential. In Eq. (18), at nonzero temperatures, due to anharmonicity of the potential well, the squared frequency κ_p/μ is replaced by an effective squared frequency $\alpha\kappa_p/\mu$.[38] Our requirement on the dc limit of the mobility gives the condition $g_0(T)=\alpha\kappa_p/\{[\tilde{\mu}(0,T)]^{-1}-\eta\}$. This condition specifies the dynamic mobility (18), which includes inertial, viscous damping, pinning, and thermal activation effects. Of course, the mobility is written in terms of a general inertial mass per length so that forms of the mass other than Eq. (12) can be used. With Eq. (18), the effective dynamic resistivity due to vortex motion is given by[22] $\tilde{\rho}_v(\omega,T)=B\phi_0\tilde{\mu}(\omega,T)$, where B is the magnetic induction.

Analytic results for $\tilde{\mu}(0,T)$ exist in certain limiting cases. The limit of extreme damping was considered in Ref. 22. Here we consider the opposite case of no damping, $\eta\rightarrow 0$, suitable only when inertial effects dominate the dynamics. In this limit, the function g_0 which specifies the dynamic mobility is given by[38]

$$g_0(T)=\frac{a\sqrt{l}\,\kappa_p}{2}(2\pi\mu k_BT)^{-1/2}\frac{I_1(\nu)[I_0(\nu)+I_1(\nu)]}{I_0^3(\nu)}e^{\nu}, \quad (19)$$

where a, the period of the pinning potential, is related to the height by $a=\pi\sqrt{2U/l\kappa_p}$ and l is the length of vortex.

SUMMARY

By employing a Josephson-coupled-layer model, we calculated the inertial mass per unit length of vortex in a discrete type-II superconductor. The resulting expression, Eq. (12), includes the effect of superconductor discreteness and may be applicable to layered organics, intercalated metals, and high-temperature superconductors. The result (12) is expected to hold at low magnetic fields and at temperatures below a crossover temperature T^*, where the coherence length ξ_c becomes of the order of the lattice constant. Combining this result with Eq. (13) from anisotropic GL theory,[10] the expression for the vortex inertial mass has been continued from high temperatures (near the transition temperature T_c) to zero absolute temperature for the case that the vortex is oriented parallel to the layers. As anticipated, once the temperature falls below T^*, the vortex core area is replaced in the GL expression. In addition, the speed of light in Eq. (13) is replaced with the speed of light in the insulating layers, a further reflection of the change in vortex structure as the temperature decreases.

A rough estimate suggests that inertial effects may be significant in the vortex dynamics of high-temperature superconductors. Accordingly, there may be a need to include the vortex inertial mass in descriptions of microwave and infrared response. A convenient function for characterizing vortex dynamic response in such applications is the complex-valued dynamic mobility.[22] We showed how the dynamic vortex mobility obtained in Ref. 22 to include pinning, flux flow, and flux-creep effects can be extended to include a nonzero inertial mass.

In this paper we considered a 3D vortex with Josephson core aligned and moving parallel to the superconducting layers. One may wonder about obtaining an expression for the inertial mass for a discrete superconductor when the vortex is at an angle to these layers. An appropriate starting point for such a study may be provided by Ref. 42, which describes 2D pancake vortices in an infinite stack of Josephson-decoupled superconducting thin films. If Josephson coupling can be included in this model, a generalization of our result to a vortex inertial mass tensor may be possible.

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