RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 44, NUMBER 12 15 SEPTEMBER 1991-II

Nuclear-spin relaxation and spin excitons in a two-dimensional electron gas

S. V. Iordanskii,* S. V. Meshkov,† and I. D. Vagner‡
Max-Planck-Institut für Festkörperforschung, Hochfeld Magnetlabor, 38042 Grenoble, France
(Received 21 February 1991)

The nuclear-spin relaxation in the vicinity of a two-dimensional electron channel is considered in terms of the creation and annihilation of spin excitons. This approach provides a qualitative picture of the competition between the inhomogeneity of the channel and the Coulomb repulsion of the electrons. The spin-flip processes are allowed energetically in the presence of the disorder. The *e-e* interaction suppresses the relaxation through the enhancement of the magnetic-field-induced spin splitting of the Landau level.

The spin exchange between nuclei of lattice atoms and conducting electrons due to the Korringa relaxation process is well studied in metals and semiconductors. Much more difficult is the observation of the Korringa relaxation in a two-dimensional (2D) electron gas in semiconductor heterostructures because of the relatively small number of electrons and nuclei interacting with them. Nevertheless, recently the nuclear-electron relaxation rate was successfully measured in high-quality heterojunctions under high magnetic field.[1,2] The nuclear-spin polarization was produced by means of dynamic polarization in electron-spin resonance (ESR) experiments. This polarization shifts the position of ESR due to the Overhauser effect, so measuring the rate of nuclear-spin relaxation becomes possible.[2] These experiments show unambiguously that the Korringa-like relaxation takes place due to exchange by the spin moment between lattice nuclear spins and electrons in a quantum 2D channel, as was previously proposed theoretically.[3]

As usual, the Korringa relaxation is an energy-consuming process due to the difference in magnetic moments of nuclear and electron spins involved. In a 3D metal the energy required for the relaxation can be provided by the kinetic energy of the electrons even in a high magnetic field because the longitudinal motion remains free and the energy spectrum of the electrons is continuous. The principal feature of the ideal 2D electron gas under high magnetic field is the completely discrete energy spectrum, so that the conservation of energy prevents the spin-flip transition in the ideal system and some sort of inhomogeneity is necessary for the nuclear-spin relaxation.[3]

However, the Coulomb interaction of electrons plays an important role because of its strong influence on the splitting of Landau levels (enhancement of the g factor by *e-e* interaction). An early attempt to take it into account[2] was based on a rather crude self-consistent scheme. The relaxation rate was taken as an overlap of two peaks of the one-electron density of states broadened by the random potential and splitted by the enhanced g factor. The correlation in electron energy before and after transition, which essentially suppresses the effect of both the random potential and the Coulomb interaction (see below), was ignored.

It is known that spin excitation for a fully occupied Landau level with lowest-energy spin polarization can be considered as a special quasiparticle (spin exciton) with conserved momentum $\hbar\mathbf{k}$ and energy $E(\mathbf{k})$.[4] In this Rapid Communication we shall use this concept to take into account both electronic interaction and random potential resulting from impurities in a natural way. In a combination of the optimal fluctuation treatment of the weak random potential, this approach gives us a possibility to avoid complicated and uncontrollable self-consistent procedures. As a result we obtain a rather clear qualitative picture of the influence of various physical factors.

We begin with the situation when the sublevel $\uparrow$ is completely occupied. The nuclei are dynamically polarized opposite to their equilibrium direction. In the absence of disorder and electron-electron (*e-e*) interaction the minimal energy of spin excitations is $\Delta=|g|\mu_B B$ (μ_B is the Bohr magneton, B is the magnetic field). The energy of the nuclear spin flip $\Delta_N=\mu_N B$ (μ_N is the nuclear magneton) is small compared with the cyclotron energy $\hbar\omega_c$ and Δ, therefore the relaxation is energetically forbidden in the ideal system.

The "contact" interaction of 2D electrons with the nuclear spin positioned at the point $\mathbf{R}$ can be written using the local Fermi ψ operators and spin-flip operators $S^{\pm}$, for nucleus

$$H_{e\text{-}n}=S^{+}\psi_{\uparrow}(\mathbf{R})\psi_{\downarrow}^{\dagger}(\mathbf{R})+\text{H.c.} \tag{1}$$

Because the position of the nucleus is arbitrary, we also put $\mathbf{R}=\mathbf{0}$. Obviously, in rewriting ψ through the creation and annihilation operators for stationary states we can include only the states of the given Landau level. In the Landau gauge $A_y=ec^{-1}Bx$ these states are classified by the wave number k_y quantized due to the finite size of the sample by $2\pi/L_y$. In the resulting expression for the interaction, the operators a_{k_y}, $a_{k_y}^{\dagger}$ can be combined into second-quantization operators of the spin exciton

$$A_{\mathbf{k}}^{\dagger}=\frac{2\pi l^2}{L_x L_y}\sum_p e^{ik_x p}a_{\uparrow,p-k_y/2}a_{\downarrow,p+k_y/2}^{\dagger}=(A_{\mathbf{k}})^{\dagger}, \tag{2}$$

where $l=(c\hbar/eB)^{1/2}$ is the magnetic length. The interaction (1) takes the form

$$H_{e\text{-}n}\propto\frac{1}{\pi^{1/2}L_x L_y}\sum_{\mathbf{k}}L_n(l^2k^2/2)\times\exp(-l^2k^2/4)[S^{+}A_{\mathbf{k}}+S^{-}A_{\mathbf{k}}], \tag{3}$$

containing the Laguerre polynomial L_n corresponding to the considered Landau level. The summation is assumed over the 2D spin-exciton wave vector $\mathbf{k}$ with components quantized by $2\pi/L_x$ and $2\pi/L_y$, respectively. The spin-exciton representation of the e-n interaction (3) allows us to write the golden rule formula for the nuclear depolarization rate through the imaginary part of the spin-exciton Green function

$$T_1^{-1} \propto \sum_k \int \mathrm{Im} G_{\mathbf{k}}(E)\delta(E-\Delta_N)L_n(l^2k^2/2)e^{-l^2k^2/2}dE\,. \tag{4}$$

Thus the depolarization rate is proportional to the density of states of spin excitons at the energy Δ_N. The formula (3) is written for an arbitrary Landau level, but for the sake of simplicity we will consider below only the lowest Landau level $n=0$.

The dispersion law $E(\mathbf{k})$ of the spin exciton has a gap Δ at $\mathbf{k}=\mathbf{0}$. At finite $\mathbf{k}$ the Coulomb interaction leads to the monotonic growth of $E(\mathbf{k})$ with k. The characteristic scale of this interaction is the average energy per particle $E_C=e^2/\epsilon l$ (ϵ is dielectric constant), and $E(\mathbf{k})$ can be written in the form

$$E(\mathbf{k})=\Delta+E_C E_0(k)\,, \tag{5}$$

$$E_0(k)=\frac{1}{2\pi}\int (1-e^{il\mathbf{k}\cdot\boldsymbol{\xi}})e^{-(\xi^2/2)}\xi^{-1}d^2\xi\,.$$

(The integration gives the standard form[4]

$$E_0=(\pi/2)^{1/2}[1-\exp(-l^2k^2/4)I_0(l^2k^2/4)]$$

with the modified Bessel function I_0.) In line with the above discussion, the spin-exciton density of states is zero at low energy Δ_N and the depolarization is possible only in the presence of some additional factors. Here we consider the influence of the random potential $U(\mathbf{r})$ coming from the inhomogeneities of the 2D channel (impurities, etc.), which also can be written in the spin-exciton representation

$$H_{\mathrm{imp}}=\frac{1}{L_xL_y}\sum_{\mathbf{k},\mathbf{q}}U(\mathbf{q})\{\exp(il^2[\mathbf{kq}]/2) - \exp(-il^2[\mathbf{kq}]/2)\}A^{\dagger}_{\mathbf{k}+\mathbf{q}}A_{\mathbf{k}}\,, \tag{6}$$

where $U(\mathbf{q})\equiv\int U(\mathbf{r})e^{-i\mathbf{q}\cdot\mathbf{r}}d^2r$ is the Fourier component of random impurity potential for electrons, [ab] designates a z component of the vector product.

From Eq. (5) we see that the enhancement of Zeeman energy due to e-e interaction will appear only for nonvanishing spin-exciton momentum $\mathbf{k}\neq\mathbf{0}$, and cannot be described by a uniform enhanced g factor. This follows from the fact that the electron and hole orbital wave functions are identical and spin independent in the absence of the spin-orbit interaction. The same effect takes place for the interaction of the spin exciton with a random impurity potential: for $\mathbf{k}=\mathbf{0}$ an electron with spin up and a hole with spin down, being at the same place, interact with opposite signs with the random field. This yields an exact cancellation at $\mathbf{k}=\mathbf{0}$, according to Eq. (6).

Equations (5) and (6) now are to be considered as a kinetic and potential energy of the one-particle Hamiltonian of the spin exciton in a random field, so that an appropriate Schrödinger equation for the spin-exciton wave function Ψ, can be directly written in a trivial way. Since the random potential is assumed to be weak, a rather large fluctuation is required to produce the potential well deep enough to compensate the kinetic energy of the spin exciton and obtain the level with the energy Δ_N. In this situation the standard way to describe the density of states is to use the concept of the optimal fluctuation (see, e.g., Ref. 5). In this approach the random potential is assumed to be Gaussian and is described completely by its correlator $Q(\mathbf{r}-\mathbf{r}')\equiv\langle U(\mathbf{r})U(\mathbf{r}')\rangle$ with Fourier harmonics $Q_{\mathbf{q}}\equiv\int Q(\mathbf{r})e^{-i\mathbf{q}\cdot\mathbf{r}}d^2r$. The averaging of the Green function over the random potential leads to the exponential

$$\langle \mathrm{Im}G(E,\mathbf{k})\rangle \propto \Psi^2(\mathbf{k})\exp(-S_E\{\Psi\})\,, \tag{7}$$

which has in the exponent a functional of the normalized wave function of the spin exciton $\Psi(\mathbf{k})$,

$$S_E\{\Psi\}=\frac{(E-\bar{E}\{\Psi\})^2}{2(L_xL_y)^{-1}\sum_{\mathbf{q}}Q_{\mathbf{q}}W^2_{\mathbf{q}}\{\Psi\}}\,,$$

where

$$\bar{E}\{\Psi\}\equiv\sum_{\mathbf{k}}E(k)\Psi^2(\mathbf{k})\,,$$

$$W_{\mathbf{q}}\{\Psi\}\equiv 2\sum_{\mathbf{k}}\sin(l^2[\mathbf{kq}]/2)\Psi(\mathbf{k})\Psi(\mathbf{k}+\mathbf{q})\,.$$

The function $\Psi(\mathbf{k})$ should be chosen to maximize the depolarization rate (1) with the Green function (7).

We consider here the simplest case of the white-noise random potential $Q_{\mathbf{q}}\equiv Q=\mathrm{const}$ and use the variational trial function

$$\Psi(\mathbf{k})=A\exp[-(l\mathbf{k}-\mathbf{K})^2/4D^2]\,, \tag{8}$$

normalized by a coefficient A to a unit sum $\sum\Psi^2(\mathbf{k})$. Under this ansatz, the evaluation of the functional $S_E\{\Psi\}$ is easy and leads to

$$\bar{E}\{\Psi\}=\Delta+E_C\bar{E}_0(K,D)\,,$$

$$\bar{E}_0(K,D)=\left(\frac{\pi}{2}\right)^{1/2}\left[1-(1+D^2)^{-1/2}\exp\left(-\frac{K^2}{4(1+D^2)}\right)I_0\left(\frac{K^2}{4(1+D^2)}\right)\right]\,,$$

$$W_{\mathbf{q}}=2\sin(l^2[\mathbf{Kq}]/2)\exp(-l^2q^2/4D^2)\,,$$

$$S_E\{\Psi\}=S(E,K,D)=\frac{[E-\Delta+E_C\bar{E}_0(K,D)]^2}{(2/\pi)l^{-2}QD^2[1-\exp(-D^2K^2/2)]}\,.$$

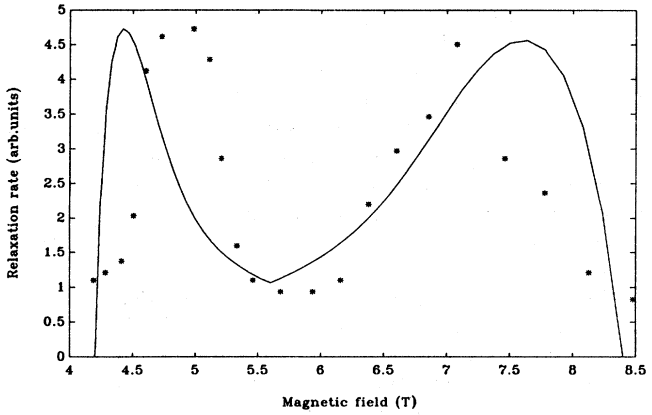

FIG. 1. The normalized magnetic-field dependence of the nuclear relaxation rate given by Eq. (10) fitted to the experimental data (Ref. 2) by the value of the amplitude of white-noise potential $Q=300\ \mathrm{meV^2nm^2}$.

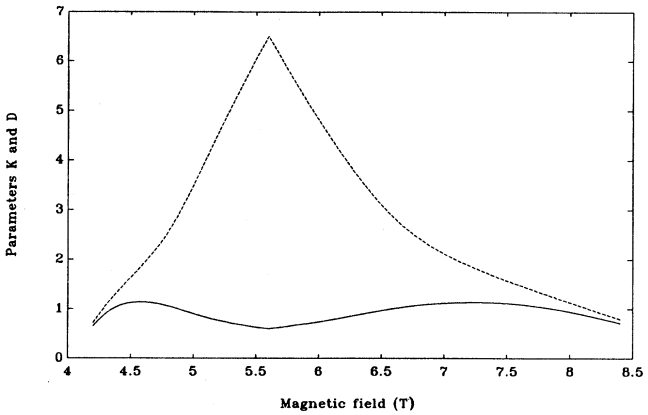

FIG. 2. The magnetic-field dependence of the variational parameters K (solid line) and D (dashed line) in the variational ansatz (8) corresponding to Fig. 1.

The expression for the whole depolarization rate to be maximized is

$$T_1^{-1} \propto \frac{1}{1+D^2}\exp\left[-\frac{K^2}{2(1+D^2)}-S(\Delta_N,K,D)\right]. \quad (9)$$

In this paper we do not touch the problem of the rigorous evaluation of the preexponential coefficient, but the coefficient in (9) is kept because it gives the qualitatively correct behavior.

Up to this moment we have discussed the case of completely filled sublevel (↑) and empty sublevel (↓), for which the concept of the spin exciton has the most clear and exact sense. In the case of an arbitrary filling of the Landau levels the most important modification takes place in the Coulomb interaction of electrons. Namely, in the same way as the enhanced electron g factor, the parameter E_C must be corrected by the factor $(\nu_\uparrow-\nu_\downarrow)$ containing the filling factors of participating spin sublevels.[6] Another change is the factor $\nu_\uparrow(1-\nu_\downarrow)$ in the preexponential coefficient in Eq. (9) proportional to the number of occupied states on the initial sublevel and the number of empty states on the final one. The resulting expression for the relaxation rate is

$$T_1^{-1} \propto \min_{K,D}\left[\frac{\nu_\uparrow(1-\nu_\downarrow)}{1+D^2}\exp\left(-\frac{K^2}{2(1+D^2)}-\frac{[\Delta+(\nu_\uparrow-\nu_\downarrow)E_C\bar{E}_0(K,D)-\Delta_N]^2}{(2/\pi)l^{-2}QD^2[1-\exp(-D^2K^2/2)]}\right)\right]. \quad (10)$$

The example of the magnetic-field dependence of the depolarization rate obtained from Eq. (10) is plotted in Fig. 1 together with experimental data[2] with parameters corresponding to the GaAs heterostructure used in this experiment. The amplitude of the white-noise potential was used as a fitting parameter (the chosen value is $Q=300\ \mathrm{meV^2nm^2}$). The interval of magnetic field corresponds to the range of filling factor ν from 2 to 4. Although our formulas are written for the lowest Landau level $0<\nu<2$, the qualitative agreement between the theory and experiment is evident. The drop of the curve between two peaks is explained by Coulomb enlargement of the spin-exciton kinetic energy. This effect is most strong in the region close to the odd filling factor $\nu\simeq 3$ ($\nu_\uparrow\simeq 1$, $\nu_\downarrow\simeq 0$) and causes the exponentially small probability of the fluctuation required for the spin-flip process.

The corresponding magnetic-field dependence of the variational parameters K and D is plotted in Fig. 2. The most important finding here is the rather high value of the parameter D (in the scale of inverse magnetic length l^{-1}). This means that the wave function of the spin exciton and the optimal fluctuation of the random potential are usually well localized in the coordinate space.

One can note the finite value of T_1^{-1} at the ends of the presented interval of the magnetic field corresponding to even filling $\nu=2$ and $\nu=4$. The theoretical curve drops to zero at these points because we consider the zero-temperature case and do not include the contribution of the neighboring Landau levels. The generalization of the approach proposed here to an arbitrary number of Landau levels and a finite temperature would improve the agreement with the experiment.

Note added in proof. We would like to thank D. Antoniou and A. H. MacDonald for sending us a copy of their work on a perturbation-theory analysis of nuclear-spin relaxation and spin-wave collective modes in a disordered two-dimensional electron gas.

Two of us (S.I. and S.M.) are grateful to P. Wyder for the hospitality and support during their stay in the Hochfeld Magnetlabor of the Max-Planck-Institut in Grenoble. This research was partly supported by the German-Israel Foundation (GIF) for Scientific Research and Development, Grant No. G-112-279.7/88.

*Permanent address: L. D. Landau Institute for Theoretical Physics, Kosygina 2, Moscow, U.S.S.R.

†Permanent address: Institute of Solid State Physics, 142432 Chernogolovka, Moscow district, U.S.S.R.

‡Present address: Department of Physics & Astronomy, The Johns Hopkins University, Baltimore, MD 21218.

[1]M. Dobers, K.v. Klitzing, G. Weiman, and K. Ploog, Phys. Rev. Lett. **61**, 1650 (1988).

[2]A. Berg, M. Dobers, R. R. Gerhardts, and K.v. Klitzing, Phys. Rev. Lett. **64**, 2563 (1990).

[3]I. D. Vagner and T. Maniv, Phys. Rev. Lett. **61**, 1400 (1988); T. Maniv and I. D. Vagner, Surf. Sci. **229**, 134 (1990).

[4]Yu. A. Bychkov, S. V. Iordanskii, and G. M. Eliashberg, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 152 (1981) [JETP Lett. **33**, 143 (1981)].

[5]B. I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors* (Springer-Verlag, Berlin, 1984), Chap. 12.

[6]C. Kallin and B. I. Halperin, Phys. Rev. B **30**, 5655 (1984).