Dipole-exchange modes in multilayers with out-of-plane anisotropies

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The collective modes of a multilayer constructed from alternating ultrathin films of ferromagnetic and nonmagnetic materials are calculated for cases in which the magnetic films have out-of-plane anisotropies. The collective surface and bulk modes of the system are predicted to strongly interact for certain values of the perpendicular anisotropy. A surprising result is the existence of a collective surface mode even when the nonmagnetic layer thicknesses are greater than the magnetic-layer thicknesses. Finally, the effects of small interlayer exchange interactions are examined and shown to result in a large broadening of the collective bulk spin-wave band.

I. INTRODUCTION

The availability of high-quality magnetic thin films and multilayers has led to a new generation of experimental studies in thin-film magnetism. Old questions concerning such topics as surface and interface anisotropies and surface roughness, for example, are now being examined in magnetic systems composed of one or more thin magnetic layers which are designed and manufactured with precision on the atomic scale. One area of research, which is of particular interest to magneto-optical storage technologies, is the appearance of strong interfacially induced perpendicular anisotropies.[1]

Strong perpendicular anisotropies have been observed for single thin films of Fe on Cu, Co on Pd, and Co on Au.[2–5] Multilayers and superlattices have also been constructed with perpendicular anisotropies that are also often large enough to create an out-of-plane easy axis.[6] Hysteresis measurements on superlattices of Co/Ag and Co/Pd, for example, show that the existence of a perpendicular easy axis is very sensitive to both the magnetic- and the nonmagnetic-film thicknesses.[7,8]

The physical origin of these observed perpendicular anisotropies is still an open problem. Several calculations have shown that the reduced magnetic symmetry of ultrathin films can lead to the creation of an out-of-plane easy axis.[9,10] Roughness and elastic stresses at the interfaces are also believed to contribute significantly to out-of-plane anisotropies.[10,11] The challenge to the experimentalist is to find ways to differentiate between these various contributions.

An effective method to study surface and interface anisotropies is to investigate the dynamics of the spin system. Brillouin light scattering is an extremely powerful tool for probing the long-wavelength excitations on ferromagnetic systems since it can be used to detect propagating excitations that are very sensitive to surface and interface anisotropy and mean exchange fields. Surface and interface anisotropies are measured indirectly with this technique by measuring shifts in the frequencies of long-wavelength spin waves as function of, for example, externally applied fields, propagation direction, and wavelength. The sensitivity is such that in-plane and out-of-plane anisotropies of different orders can be distinguished with this technique.[12]

Magnetic surface waves on ultrathin magnetic films and combinations of ultrathin films are especially interesting since effects due to exchange interactions are observable even for long-wavelength excitations.[13] These spin-wave modes are often called "dipole-exchange" modes in order to distinguish them from the dipolar Damon-Eschbach modes and short-wavelength exchange modes. Dipole-exchange modes exist for wavelengths in the $10^{5}-10^{-6}$-cm range and the surface modes, which have large stray magnetic fields outside the material, are readily studied with Brillouin light-scattering techniques.[14]

Comparatively little work has been done on examining how out-of-plane anisotropies affect the frequencies of dipole-exchange spin waves in magnetic systems composed of ultrathin magnetic films. This is despite the fact that multilayers are particularly interesting systems for experimental study since the presence of several interfaces effectively amplifies surface effects. The purpose of the present paper is to extend previous calculations to multilayers where it is possible to vary the magnitude of the out-of-plane component of the magnetization by applying an in-plane magnetic field.

In a previous work, the surface mode was shown to be sensitive to both exchange interactions and out-of-plane surface anisotropies for single ultrathin films.[15] The essential features are summarized in the following: The magnetization is assumed to be uniform across the film so that the equilibrium orientation of the magnetization is determined by the applied static field, demagnetizing fields, and anisotropy fields. This assumption is made only in the sense that long-wavelength spin-wave modes are sensitive only to magnetic properties averaged over several lattice sites. As in ferromagnetic resonance studies,[16] the assumption of uniformity should also be applicable to the surface mode on ultrathin films since the magnetic fields associated with this mode change very little from lattice site to lattice site. By the same reasoning, surface anisotropies are treated as effective bulk anisotropies which is equivalent to thinking of the static orientation of the magnetization as approximately constant across the film thickness. Again, a small canting of the magnetization at the surface has a negligible effect on the resonance mode frequency,[16] and is expected to also have a small effect on the surface-mode frequency.

With these assumptions, when the applied static field is decreased to a critical field H_c, below which the magnetization leaves the plane of the film, the symmetry of the film also changes and pure dipolar modes go soft. In very thin films, however, even the dipolar dominated surface mode contains a significant exchange energy. For a mode whose amplitude is nearly constant across the film, the frequency at H_c will be nonzero only if

$$Aq_y^2 > 2\pi M_s^2 \ . \tag{1}$$

Here A is the exchange constant of the material, q_y the component of the wave vector parallel to the surface, and M_s the saturation magnetization. Equation (1) states simply that a mode will be soft at H_c unless the exchange energy contained in the mode is greater than the demagnetizing energy.

In what follows, the theory for multilayers is described and followed by a discussion of calculations for an example multilayer of uniformly magnetized, single-domain thin films. As a first approximation, the films are assumed to be coupled only through dipolar fields and the general features are examined. Next, a small interlayer exchange coupling is included and the effects on the dipole-exchange spin-wave frequencies are studied.

II. THEORY

The geometry is defined in Fig. 1. The unprimed coordinate system is that of the multilayer with the x axis

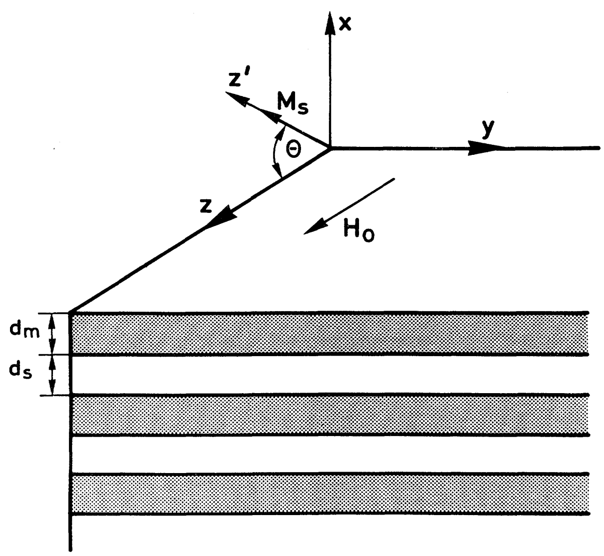

FIG. 1. Geometry. The film planes lie parallel to the y-z plane with the x axis normal to the stack. An external applied field is directed along the z axis in the film planes. In equilibrium, the magnetization $\mathbf{m}$ is assumed to be oriented the same in all magnetic layers and makes an angle θ with the z axis in the x-z plane. A primed coordinate system is defined for the magnetization with $\mathbf{m}$ along the z' axis.

normal to the film planes. There are N magnetic layers in the stack and all have the same thickness d_f. There are $N-1$ nonmagnetic layers in the stack, each of thickness d_s. The applied field H_0 lies along the z axis. The saturation magnetization per unit volume M_S lies parallel to the x-z plane in each layer and in the direction of the z' axis of a primed coordinate system. The angle θ is defined between the z' and z axes. The following theory is restricted to the Voigt geometry where propagation is perpendicular to the applied field in the y direction with wave-vector component q_y.

The equilibrium position of the magnetization is calculated under the assumption that the orientation of the magnetization is the same for each layer and the orientation of the magnetization is determined by minimizing a dipolar free energy.[15] Furthermore, each film is assumed to be uniformly magnetized without domain walls. While these assumptions are reasonable for large external applied fields, they do not necessarily hold for small applied fields when large perpendicular anisotropies are present. In fact, it will be shown below that for external field values near H_c, a reordering of the magnetization most likely occurs due to dipolar interactions.

The generalization of the spin-wave frequency calculations to a multilayer system is straightforward and so the calculations are only outlined here. The formalism is similar to that given in Refs. 15 and 17. First the magnetic torque equations, together with the appropriate Maxwell equations for the magnetic field, are written in the magnetization's frame. The magnetostatic limit is taken and a magnetic scalar potential ϕ is defined. This results in a system of three coupled equations of motion in each layer for ϕ and the time varying magnetization $\mathbf{m}$. The amplitudes of $\mathbf{m}$ and ϕ may differ from layer to layer, so in order to distinguish the different amplitudes in each layer, $\mathbf{m}$ and ϕ are indexed by the superscript n. The solutions in layer n for m_x^n, m_y^n, and ϕ^n are assumed to be propagating waves of the form $e^{i(q_y y-\omega t)}$ for propagation perpendicular to H_0. Note that $\mathbf{m}=\mathbf{0}$ in the nonmagnetic layers.

Outside the multilayer, the magnetization is zero and the potential is assumed to decay exponentially away from the surface according to $\exp(-\alpha x)$ in the vacuum above the stack, and $\exp(\alpha x)$ in the vacuum below the stack. In the nonmagnetic spacer layers, both of these increasing and decreasing solutions are allowed. The potential outside the magnetic material obeys Laplace's equation, which requires $\alpha=|q_y|$ in the spacer layers and outside the stack.

Inside the magnetic layers, the x dependence of the solutions is assumed to be of the form $\exp(iq_x x)$ and $\exp(-iq_x x)$ where q_x can be real, imaginary, or complex as determined by the equations of motion. Six possible values of q_x are found when exchange interactions are included into the equations of motion, which means there are six partial waves traveling inside the material. The index j is used to distinguish the six possible q_x and the corresponding m_x^n, m_y^n, and ϕ^n.

The complete solution to the thin-film problem requires the application of appropriate boundary conditions to match these six partial waves to the waves outside each magnetic film.[13] These must include the usual conditions on normal **B** and tangential **h** fields, where $\mathbf{B}=\mathbf{h}+4\pi\mathbf{m}$ and $\mathbf{h}=-\nabla\phi$, as well as the Rado-Weertman exchange boundary conditions.[18] With six partial waves in each magnetic film and two waves in each nonmagnetic film, there are $6N+2N$ unknown amplitudes for the total stack. The problem thus requires $8N$ boundary conditions. These are supplied by $4N$ electromagnetic boundary conditions and $4N$ Rado-Weertman boundary conditions.

The Rado-Weertman boundary conditions contain terms corresponding to surface exchange and anisotropy fields. As noted in the previous work, although surface anisotropy is assumed to result in an out-of-plane magnetization, anisotropy terms cannot be included in the Rado-Weertman boundary conditions when the surface anisotropies are treated as effective bulk anisotropies.[15]

The problem is formulated for numerical solution by writing $(m_{xj}^n)'=a_j^n\phi_j^n$, $(m_{yj}^n)'=b_j^n\phi_j^n$ where the prime indicates the component is in the magnetization frame. The equations of motion are then solved for a_j^n and b_j^n in terms of ϕ_j and the boundary conditions for the nth layer are written in terms of ϕ_j^n. The Rado-Weertman boundary conditions for the interface between the nth and the $(n+1)$th layers become

$$\sum_{j=1}^{6}\left[\frac{H_{12}q_{xj}\cos\theta+iq_y\dfrac{\omega}{\gamma}}{H_{12}H_{21}-\dfrac{\omega^2}{\gamma^2}}\right]q_{xj}\phi_j^n=0\ , \tag{2}$$

$$\sum_{j=1}^{6}\left[\frac{-H_{21}q_y+iq_{xj}\cos\theta\dfrac{\omega}{\gamma}}{H_{12}H_{21}-\dfrac{\omega^2}{\gamma^2}}\right]q_{xj}\phi_j^n=0\ . \tag{3}$$

The gyromagnetic ratio γ is defined by $\gamma=g\gamma_0$ where $\gamma_0=8.79\times10^6$ Hz/Oe and g is the g factor of the material. Also,

$$H_{12}=H_0\cos\theta-4\pi M_s\sin^2\theta+R_y+\frac{2A}{M_s}q^2\ , \tag{4}$$

$$H_{21}=H_0\cos\theta-4\pi M_s\sin^2\theta+R_x+\frac{2A}{M_s}q^2\ . \tag{5}$$

Here A is the exchange constant defined by $2JS^2/a$ for a bcc crystal of lattice constant a with nearest-neighbor exchange integral J and spin number S. The anisotropy fields are contained in R_x and R_y:

$$R_x=\frac{2}{M_s}(K_1+2k_s/d_f)(1-2\sin^2\theta)+4\frac{K_2}{M_s}(1-4\sin^2\theta)\cos^2\theta\ , \tag{6}$$

$$R_y=\frac{2}{M_s}(K_1\cos^2\theta+2k_s\sin^2\theta/d_f)+4\frac{K_2}{M_s}\cos^4\theta\ . \tag{7}$$

First- and second-order bulk anisotropy constants are represented by K_1 and K_2 which are defined for an easy direction along $\hat{z}$. The out-of-plane surface anisotropy is defined as k_s and appears here as an effective bulk anisotropy that scales inversely with film thickness d_f. We use the convention that $k_s<0$ implies an out-of-plane easy axis.

Combining the boundary conditions on continuity of normal **B** and tangential **h** into one equation, the electromagnetic boundary conditions are given by

$$\sum_{j=1}6\left[4\pi M_s\left[\frac{H_{12}q_{xj}\cos\theta+iq_y\dfrac{\omega}{\gamma}}{H_{12}H_{21}-\dfrac{\omega^2}{\gamma^2}}\right]\cos\theta-(iq_{xj}+q_y)\right]\phi_j^n+q_y\phi_j^{n+1}=0\ . \tag{8}$$

At the Nth magnetic interface, Eqs. (2) and (3) apply, but Eq. (8) is replaced by

$$\sum_{j=1}^{6}\left[4\pi M_s\left[\frac{H_{12}q_{xj}\cos\theta+iq_y\omega_\gamma}{H_{12}H_{21}-\dfrac{\omega^2}{\gamma_2}}\right]\cos\theta-(iq_{xj}-q_y)\right]\phi_j^N=0\ , \tag{9}$$

and at the first magnetic interface, Eq. (8) is replaced by

$$\sum_{j=1}^{6}\left[4\pi M_s\left[\frac{H_{12}q_{xj}\cos\theta+iq_y\dfrac{\omega}{\gamma}}{H_{12}H_{21}-\dfrac{\omega^2}{\gamma^2}}\right]\cos\theta-(iq_{xj}+q_y)\right]\phi_j^1=0\ . \tag{10}$$

The numerical procedure is as follows. First the equilibrium angle is calculated by minimizing a free energy. Next, the q_{xj} are calculated for a given ω and q_y. The boundary condition matrix is then constructed with these q_{xj}, q_y, and ω and the process is repeated by a numerical iteration routine that seeks an ω which would give a zero determinant for the boundary condition matrix.

III. RESULTS

For thick nonmagnetic spacer layers, the individual magnetic films are coupled through stray dipolar magnetic fields. These stray fields are largest for surface localized modes, and the following theory is used to describe collective excitations of a multilayer structure formed by coupling the surface modes from each individual layer. The collective modes of the system are characterized as surface-mode- or bulk-mode-like depending on whether their amplitudes vary in an oscillatory manner across the stack or whether they decay exponentially with distance into the stack.[19]

The main effect from out-of-plane anisotropies on the collective spin-wave modes of a multilayer is a lowering of the mode frequencies and a corresponding mixing of surface-mode and bulk-mode characteristics. This leads to a number of interesting features which will be discussed below. First, the general features of the collective modes of a multilayer are examined by comparison to the dipolar modes of a single film. Next, an existence criterion for the collective surface mode is examined. Finally, the effects of interlayer exchange on the multilayer modes are briefly discussed.

A. General features and single film analogies

The frequencies of the lowest frequency modes for a six-bilayer stack are shown in Fig. 2 as a function of applied field. The magnetic parameters are taken from previous experiments with thin Co layers,[6] and are $A=2.85\times10^{-6}$ ergs/cm, $4\pi M_s=14.5$ kG, and $g=2$. The bulk anisotropies K_1 and K_2 are both set to zero. The Co layers are 8.8-Å thick and the spacer layers are 7.6-Å thick. The features to be described below are very general, and should be exhibited by a variety of systems with different parameters. In general, the overall magnitude of the spin-wave energies is most sensitive to M_s and directly proportional to g. The anisotropies also serve to shift the energies, and for the surface mode, exchange interactions provide a small increase in the energy on the order of $Aq_{\parallel}^2$. The main assumption is that A is large enough, and $q_{\parallel}d_f$ small enough, that the exchange dominated standing spin-wave modes exist at frequencies above the surface-mode frequency.

The surface anisotropy is chosen as $k_s=-0.4$ ergs/cm^2 so that an applied field greater than $H_c=1.26$ kG is required to force the magnetization in-plane. For fields less than approximately 500 G, the magnetization is almost completely normal to the film plane. Since a dipolar dominated surface mode cannot exist in single thin

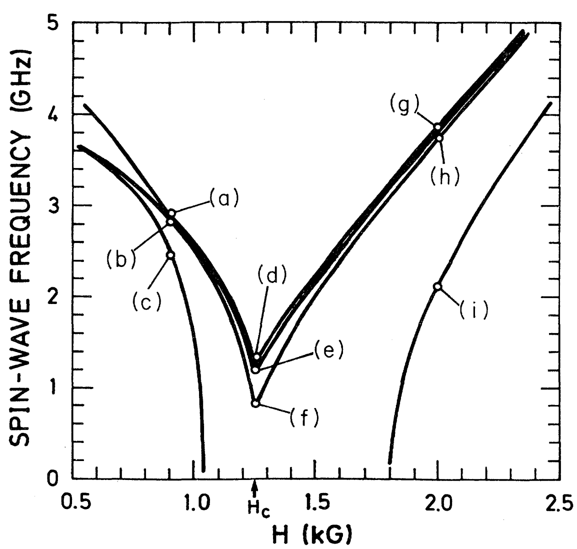

FIG. 2. Frequencies of modes for a six-bilayer stack as functions of applied field H_0. The parameters are appropriate for Co and are given in the text. The Co layers are 8.8-Å thick and the nonmagnetic layers are 7.6-Å thick. The surface anisotropy is chosen to be -0.4 ergs/cm^2 so that the critical field H_c is 1.26 kG. The circled numbers refer to the mode profiles of Fig. 3.

films or superlattices when the magnetization is completely perpendicular to the film planes, only spin-wave frequencies for applied fields above $H_0=0.5$ kG are shown.[16,20]

At fields above H_c, the magnetization is forced into the film planes and the spectrum consists of five nearly degenerate bulk modes and a lower frequency surface mode. For fields below H_c the magnetization has an out-of-plane component and the surface mode appears to cross through and rise above the bulk band as the field is lowered. Near H_c, the bulk modes take their lowest values while the surface mode goes completely soft.

The region near H_c where the surface mode is soft deserves some comment since the softening of a surface mode can often be associated with a surface phase transition.[21] In the example of Fig. 2, for applied field strengths near H_c there is a strong possibility for the direction of the magnetization to vary across the stack in order to minimize the net demagnetization energy of the structure. Although we do not attempt to formally associate the formation of such a domain structure with the softening of the surface mode in the present work, we note that such an interpretation is consistent with recent experimental measurements.[22]

With this interpretation, spin-wave frequencies for fields where the surface mode is soft may not be accurately predicted with this theory. Furthermore, even for fields well below H_c where the surface mode still exists the static orientation of the magnetization across the stack will depend strongly on the history of the sample, i.e., if the magnetization is first saturated along the stack axis and then brought into the film planes or if the magnetization is first saturated in the film planes and then allowed to rotate into an out-of-plane direction. The present theory may not be valid when domain structures form since, as previously discussed, this theory is valid only for cases where the magnetization is oriented in the same direction in all layers.

In Fig. 3, the normal component of the magnetization m_x is plotted as a function of position in the stack for various fields and frequencies. The profiled modes are identified by the circled numbers which refer to Fig. 2. The region near 1 kG is particularly interesting since here all the modes are strongly localized to one stack surface. The highest and two lowest-frequency modes are profiled at $H_0=0.9$ kG, which is the field at which the surface mode passes through the bulk band. Note how strongly the highest-frequency mode (number 1) is localized to a stack surface.

The highest- and two lowest-frequency bulk modes are also profiled at H_c in order to illustrate the characteristics of the bulk modes near H_c if the magnetization remained uniformly oriented in all layers. The highest-frequency mode (number 4) is symmetric with respect to the midplane of the stack, but this symmetry is lost as the order of the modes decreases.

For $H_0=2$ kG, the magnetization is completely in-plane and the lowest-frequency mode and two highest-frequency bulk modes are profiled. The order (i.e., the number of nodes in the amplitude profile) of the bulk modes increases with frequency, so that the highest-order

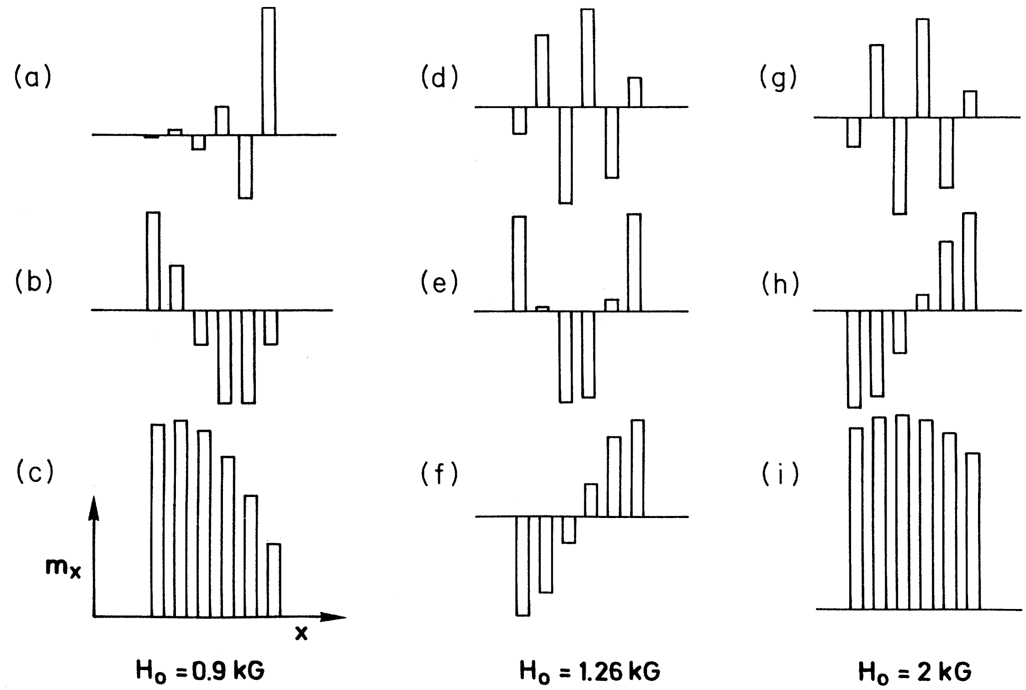

FIG. 3. The normal component of the magnetization m_x is shown as a function of position in the stack for different modes from Fig. 2 for three different applied fields: $H_0=0.9$ kG, where the static magnetization M_s has a large out-of-plane component; $H_0=1.26$ kG, where M_s begins to turn out-of-plane; and $H_0=2$ kG, where M_s is completely in-plane. The number associated with each profile identifies the corresponding mode in Fig. 2.

bulk mode also lies at the top of the bulk band. Note that the lowest-frequency mode (number 9) appears as a bulk-like mode localized to a surface of the stack. Note that this "surface" mode goes soft at a field larger than H_c.

To understand the behavior of the collective modes, it is useful to notice two features from the dipolar modes of parallel and perpendicularly magnetized single films. The first relevant feature for dipolar modes in single films is the behavior of the surface mode as the magnetization switches from out-of-plane to in-plane. For small applied fields the surface mode is bulklike and in the limit of zero applied field and completely perpendicular magnetization it merges into the resonance mode with frequency ω_b. For larger fields it becomes strongly localized to a surface of the film. As the field is increased further, the surface mode goes soft.

This feature can be observed for the multilayer stack of Fig. 2. First, the collective surface mode also changes its degree of localization as the magnetization is forced in-plane. For small applied fields the high-frequency mode (number 1) is weakly localized to a surface of the stack and as the applied field is increased, this mode becomes more strongly localized to a surface of the stack and also merges into the bulk band. This broadens the bulk-mode band and all the modes become somewhat localized to a stack surface, with the lowest-frequency modes having the strongest localization.

The second analogy that can be drawn between the dipolar spin-wave modes of a single thick film and the dipole-exchange modes of a multilayer is an inversion of bulk mode frequencies with respect to mode order. In the single thick film, dipolar bulk modes that propagate parallel to an in-plane H_0 are degenerate in frequency with the resonance mode with frequency $\omega_b=\gamma\sqrt{H_0(H_0+4\pi M_s)}$ in the absence of any anisotropies.[13] A large perpendicular anisotropy, however, removes this degeneracy and a band with finite width is formed. The modes near the high-frequency limit of this band then have the highest order and the lower-order modes exist at the lower band edge.

With H_0 perpendicular to the film, however, the ordering of the modes with frequency is reversed so that the modes at the high-frequency bulk band edge have the lowest order and the modes at the low-frequency bulk band edge have the highest order.[13] Furthermore, the bulk modes lie below ω_b. Therefore, as the magnetization changes from in-plane to out-of-plane, the ordering of the modes must reverse. This reversal has been verified with a model that neglects exchange interactions ($A=0$). Note that without exchange interactions, the problem is somewhat less complicated since the exchange boundary conditions are no longer needed.

For very small fields, the magnetization is aligned perpendicular to the film. The degeneracy of the bulk modes is broken by the strong k_s, as described above, and for fields below H_c, the frequency of the modes decreases as their order increases. As the field is increased, the magnetization is pulled into the film plane and all the modes go soft. Before the magnetization is completely in-plane, however, the bulk modes cross each other as the low-order modes soften at fields smaller than the high-order modes. An examination of the equations of motion shows that a negative contribution to the energy of the modes comes from the perpendicular anisotropy, and this contribution increases as the square of the wave vector of the modes.[16] This in turn leads to an order-dependent softening of the modes since the order of a mode is proportional to the normal component of the wave vector.

This inversion of the mode order as a function of frequency can also be observed for the collective bulk modes of the multilayer structure as a function of nonmagnetic layer thickness. In Fig. 4 the mode frequencies are shown as functions of the nonmagnetic layer thickness d_s for the six-bilayer stack of Fig. 2. The applied field is 500 G and k_s is -0.4 ergs/cm^2 so that the magnetization has a large out-of-plane component. Not shown in this figure is the highest-frequency mode which is the surface mode. Instead, it is interesting to observe the bulk band which for small d_s has the lowest-order modes with the lowest frequencies. As d_s is increased, the mode order inverts so that the highest order modes have the lowest frequencies. This means that the greatest density of bulk states lies at the top of the bulk band for small d_s and at the bottom of the bulk band for d_s greater than about 12 Å.

This crossing of modes can also be observed when the easy axis is in-plane. For the in-plane case, however, the crossing occurs for a d_s an order of magnitude larger than that of the out-of-plane case and is strongly dependent on the strength of the out-of-plane anisotropy.[23]

A similar feature can also be found in superlattice models where exchange interactions are neglected. Calculations were performed for a multilayer without exchange and an analogous inversion was found when the frequencies of the collective bulk modes approached the single film ferromagnetic resonance frequency

$$\omega_B^2 = \gamma H_0 (H_0 + 4\pi M_s + 4k_s / d_m M_s) \ .$$

At this frequency in the dipolar limit of no exchange, the decay constant in the material α changes from real to complex. For frequencies above ω_B, the decay constant in the magnetic layers is real and the collective surface mode of the stack is composed of surface modes from the individual layers. For frequencies at and below ω_B, the decay constant in the layers is imaginary and a new band of collective surface modes appears which are composed of the bulk modes in the individual layers.[19] One then finds that the collective bulk modes for the stack interact with this band of collective surface modes for certain values of k_s and d_s and lift the degeneracy of this band of surface modes.

When exchange is included, however, this band of collective surface modes does not exist at ω_B since the bulk modes of the ultrathin films are dominated by exchange interactions and exist at higher frequencies. Instead, for large d_s, the bulk modes are not appreciably localized to a stack surface, while for small d_s below the inversion point, the lowest-order bulk modes become localized to a stack surface.

The mode inversion can also be seen in Fig. 5, where the mode frequencies for the six-bilayer stack of Fig. 2 are calculated as a function of k_s. Here $H_0 = 1$ kG, $d_f = 10$ Å, and $d_s = 50$ Å and the inversion occurs at about $k_s = -0.32$ ergs/cm^2. Note that for these k_s

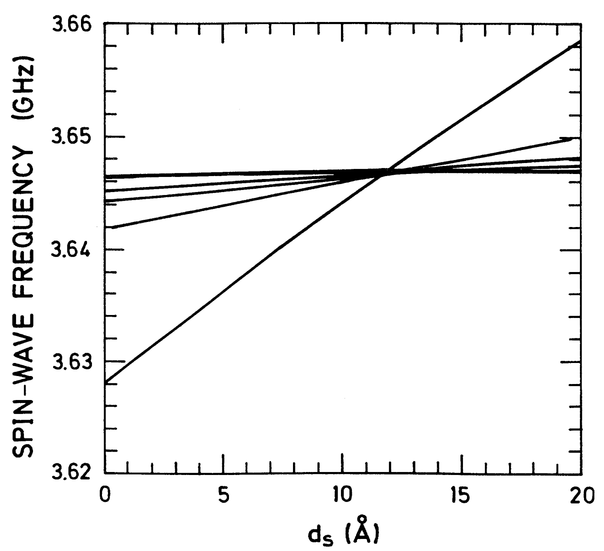

FIG. 4. The frequencies for modes of the six bilayer stack of Fig. 2 are shown as functions of nonmagnetic layer thickness d_s for $H_0 = 500$ G. Here, only the bulk band is shown and $k_s = -0.4$ ergs/cm^2 so that the static magnetization has a large out-of-plane component. At approximately $d_s = 10$ Å the bulk modes cross and the mode order reverses with respect to frequency.

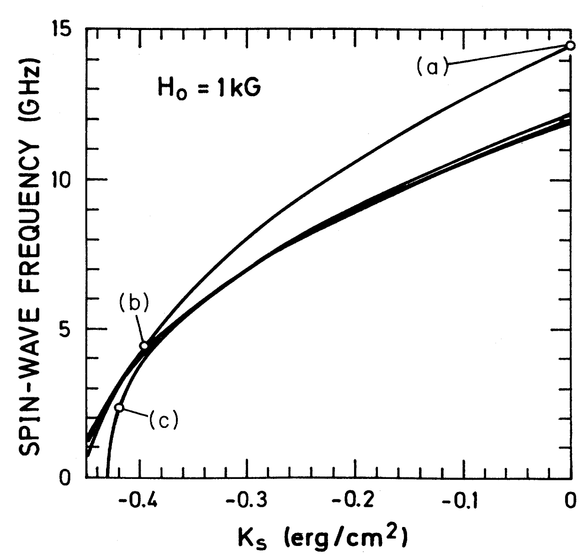

FIG. 5. Frequencies for modes of the six-bilayer stack as functions of k_s with H_0 set at 1 kG. $d_s = 50$ Å and $d_m = 10$ Å. Note that the surface mode goes soft for $k_s = -0.43$ ergs/cm^2 and passes through the bulk band for k_s near -0.4 erg/cm^2. The circled numbers refer to the profiles of Fig. 6.

values and this magnetic-field strength, the magnetization lies in the film plane. For $k_s > -0.32$ ergs/cm^2, the frequency of the bulk modes decreases with increasing order. For $k_s < -0.32$ ergs/cm^2, the frequency of the bulk modes increases with increasing order.

A curious observation noted above was that the surface mode sometimes lies below the bulk band. This is due to the perpendicular anisotropy which, like spin "pinning" at surfaces, lowers the frequency of the surface mode.[19] This is illustrated in Fig. 5. For some k_s values greater than about -0.37 ergs/cm^2, the highest-frequency mode is localized to a surface of the stack and is properly the stack surface mode. For small k_s values this is no longer true, and will be discussed further below. For k_s values less than about -0.37 ergs/cm^2, however, the surface mode merges with the top of the bulk band and the lowest-order bulk mode becomes localized to a surface of the stack and eventually goes soft at approximately -0.43 ergs/cm^2. The k_s value at which the surface mode passes through the bulk band is significant: For k_s values more negative than this, the film has an out-of-plane easy axis. In the single thick-film analogy, where exchange is neglected, at this point the dipolar surface mode is degenerate with the ferromagnetic resonance mode, i.e., when $2k_s/d_f + 2\pi M_s^2 = 0$.

B. Existence criteria for the surface mode

A very surprising feature can also be found for the collective surface mode. In Fig. 6, the magnitude of the x component of the time varying magnetization m_x is shown as a function of position in the stack for three values of k_s. The circled numbers identify the profiles with the modes of Fig. 5. All other parameters are the same as in Fig. 4, in particular $d_s = 50$ Å and $d_f = 8.8$ Å.

When $k_s = -0.4$ ergs/cm^2 in this example, the surface mode lies above the bulk band and one can see from Fig. 6 that it is strongly localized to a surface of the stack. When $k_s = 0$, however, the "surface mode" resembles a bulk mode that is weakly localized to the stack surface. When $k_s = -0.42$ ergs/cm^2, the surface mode lies beneath the bulk band and resembles a bulk mode that is strongly localized to a stack surface. This is in contrast to usual superlattice theory which predicts that a collective surface mode should only exist when $d_s \le d_f$.[19]

When out-of-plane anisotropies are present, however, a surface mode can exist even when $d_s > d_f$. The departure from the $d_s < d_f$ rule can be understood by deriving the $d_s < d_f$ rule for semi-infinite magnetic superlattices with perpendicular anisotropies. As in Ref. 19, one begins with solutions for the magnetic potential ϕ that satisfy $\nabla\cdot\mathbf{B} = 0$ everywhere (as remarked above, for $A = 0$ the only boundary conditions involved are on normal $\mathbf{B}$ and tangential $\mathbf{h}$). A dispersion relation for the collective surface mode is then derived:

$$\omega_s = \gamma \left[H_0 + 2\frac{k_s}{M_s d_f} + 2\pi M_s \right] . \tag{11}$$

As in the $k_s = 0$ case, the frequency of the collective surface mode is identical to that of a single semi-infinite film surface mode. Next, the surface mode is required to decay exponentially away from the surface of the stack which in turn requires the ratio of magnetic to nonmagnetic layer thicknesses to satisfy the following inequality:

$$\frac{d_f}{d_s} > \frac{q_y}{\alpha} . \tag{12}$$

Here α is the decay constant inside the magnetic layers.

The decay constant α is determined by $\nabla\cdot\mathbf{B} = 0$. Using appropriate solutions for ϕ and $\mathbf{m} = \chi\mathbf{h}$ where χ is the magnetic susceptibility tensor, this is written in terms of the components of χ in the magnetostatic limit as

$$(\alpha^2 - q_y^2) + 4\pi(\chi_{xx}\alpha^2 - \chi_{yy}q_y^2) = 0 . \tag{13}$$

Without perpendicular anisotropies, $\chi_{xx} = \chi_{yy}$ and this reduces to the usual $\alpha^2 = q_y^2$ condition. With perpendicular anisotropies, however,

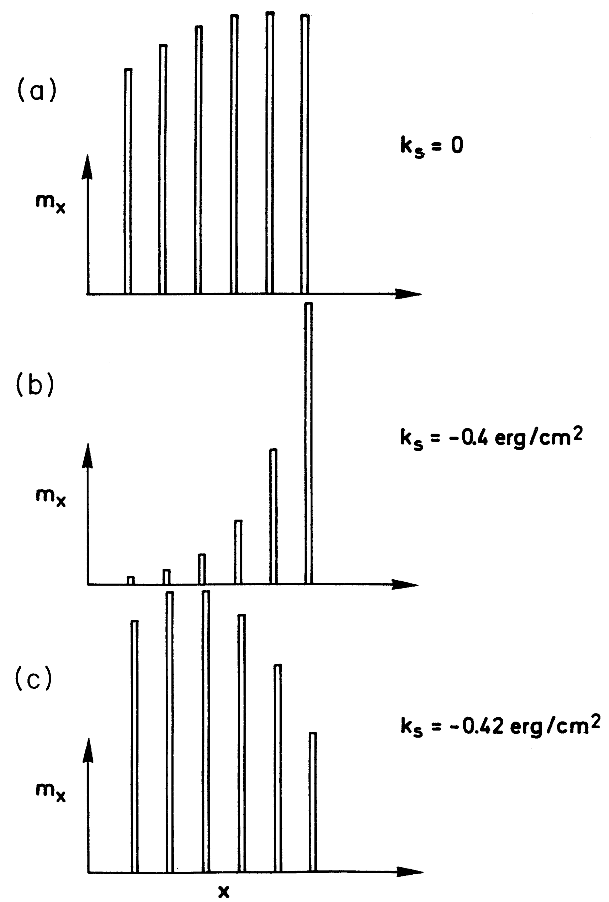

FIG. 6. The magnitude of the x component of the time varying magnetization m_x for the surface mode is shown as a function of position in the six-bilayer stack. The parameters are the same as in Fig. 5 with $d_s > d_m$. The surface-mode- or bulk-mode-like character of the mode depends strongly on the magnitude of k_s. The circled numbers identify which modes of Fig. 4 are profiled.

$$\chi_{xx}=\frac{H_0 M_s}{H_0\left[H_0+4\frac{k_s}{M_s d_f}\right]-\left[\frac{\omega}{\gamma}\right]^2}, \tag{14}$$

$$\chi_{yy}=\frac{H_0 M_s\left[H_0+4\frac{k_s}{M_s d_f}\right]}{H_0\left[H_0+4\frac{k_s}{M_s d_f}\right]-\left[\frac{\omega}{\gamma}\right]^2}. \tag{15}$$

Using Eqs. (11), (14), and (15), the existence condition for a surface mode of the semi-infinite stack [Eq. (12)] is written in terms of H_0, M_s, and k_s. The condition is put into a particularly simple form with the definitions

$$E_d=2\pi M_s^2, \tag{16}$$

$$E_a=-2\frac{k_s}{d_f}, \tag{17}$$

which are the shape anisotropy and perpendicular anisotropy energies, respectively. The resulting expression is then

$$\frac{d_f}{d_s}>\left|\frac{E_d-E_a}{E_d+E_a}\right|. \tag{18}$$

This states that in order for a surface mode to exist, the ratio of the magnetic to nonmagnetic thicknesses must be larger than the fractional difference between the shape and perpendicular anisotropy energies. The multilayer of Fig. 5, for example, supports a surface mode for $k_s<-0.1$ ergs/cm^2. Note that when E_a is zero, Eq. (18) reduces to the familiar $d_f \geq d_s$ rule. Also, it is interesting that this generalization is solely due to the lower symmetry of the film where $\chi_{xx}\neq\chi_{yy}$.

C. Interlayer exchange

Experimental evidence exists for exchange coupling across thin nonmagnetic layers, and so it is of interest to study the effects of interlayer exchange for the multilayers described above.[24] The proper inclusion of interlayer exchange interactions into the preceding theory would entail a new calculation of the static orientation of the magnetization. This is not a trivial problem since a number of configurations are possible that could drastically reduce the net demagnetization energy of the structure by the formation of domains across the stack. The assumption of uniformity, however, is still reasonable with ferromagnetic interlayer coupling since then the magnetization of neighboring layers prefer to lie in the same direction. The inclusion of interlayer exchange is accomplished by using the Hoffman boundary conditions.[25] These differ from the Rado-Weertman boundary conditions by the inclusion of terms of the form

$$A_{n,n+1}\mathbf{m}_n\times\mathbf{m}_{n+1} \tag{19}$$

into the boundary conditions given by Eqs. (2)–(4). The strength of the exchange interaction between layers n and $n+1$ is specified by $A_{n,n+1}$.

This coupling between the magnetizations in adjacent layers produces a parallel alignment for positive $A_{n,n+1}$. A large value of $A_{n,n+1}$ binds the spins together across thin spacer layers and shifts the frequencies of the collective bulk modes towards those of the bulk modes for a single film whose thickness is that of the total stack. In what follows, the interlayer exchange is assumed constant between all layers and the interlayer exchange constant is referred to as A_{12}.

In multilayer systems with interlayer exchange coupling, one also finds crossings and repulsions as the stack surface mode crosses through the bulk band. As seen above, the collective bulk modes lie in a narrow band. This band is broadened by interlayer exchange interactions, as can be seen in Fig. 7 where a small value of $A_{12}=0.005$ ergs/cm^2 is included for the multilayer stack of Fig. 2. The positive (ferromagnetic) interlayer exchange increases the energy of the collective bulk modes. For fields below 2 kG in the case shown in Fig. 2, where the surface-mode lies above the bulk band, interlayer exchange leads to interactions between the surface and bulk modes. These interactions are clearly visible in Fig. 7 as the surface mode passes through the bulk band. Again, the frequencies shown here for fields where the surface mode is soft may not be accurate since the softening of the surface mode may signal a reordering of the magnetization. Finally, as in the single film case, the ordering of the bulk modes is such that the lowest-order modes have the lowest energies.

The broadening of the bulk band is very sensitive to the magnitude of the interlayer exchange constant and can be understood intuitively with the help of the mode

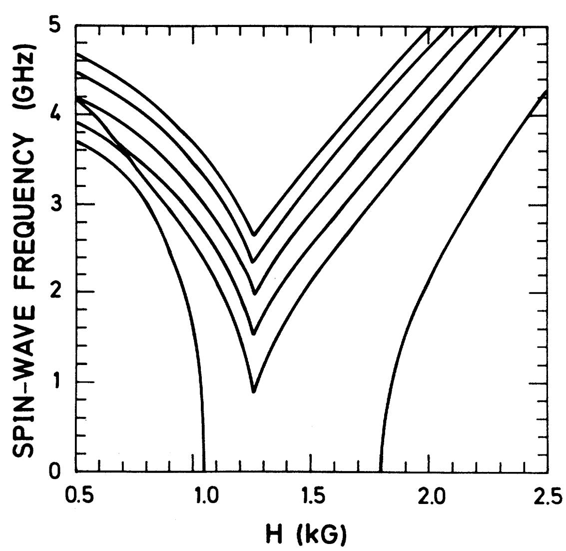

FIG. 7. The case of Fig. 2 is repeated except with a small interlayer exchange interaction ($A_{12}=0.005$ ergs/cm^2). The bulk band is broadened as the modes are shifted up in frequency. Note the repulsions below H_c as the surface mode becomes soft and passes through the bulk band.

profiles of Fig. 3. In this figure, the phase of the x component of the time varying magnetization differs by 180° from layer to layer for the highest-order bulk mode. This means that the transverse components of **m** are antiparallel between adjacent magnetic layers for the highest-order mode. With a ferromagnetic interlayer exchange coupling, the high-order modes thus have a much larger interlayer exchange energy than the low-order modes. Note that if the interlayer exchange coupling were antiferromagnetic, the reverse would be true: Then the low-order modes, for which magnetizations between neighboring magnetic layers are mostly in-phase, would have larger interlayer exchange energies than the high-order modes.

This is illustrated in Fig. 8, where the bulk-mode frequencies are shown as a function of A_{12} for the stack of Fig. 2. Here the applied field is chosen to be 500 G so that the magnetization lies out-of-plane. For $A_{12}=0$, the surface mode lies above the bulk modes. The surface mode is not strongly influenced by A_{12} since its amplitude varies relatively slowly across the spacer layers and therefore does not introduce a strong exchange interaction. As A_{12} increases, the bulk-mode frequencies increase and interact with the surface mode forming small repulsions that depend on the magnitude of the interlayer exchange A_{12}.

An important point is the sensitive dependence of the frequency broadening of the bulk band on A_{12}. An interlayer exchange constant of only 0.005 ergs/cm^2, for example, introduces a broadening of more than 1 GHz in the bulk band. In contrast, the interlayer exchange constant corresponding to $d_s=0$ would be $A_{12}=2A/a$ for bcc lattices, which here is on the order of 285 ergs/cm^2. Even with an estimated reduction of A_{12} by $\exp[-d_s/(10\ \text{Å})]$, A_{12} is still on the order of 133 ergs/cm^2 for the multilayers of Figs. 7 and 8.[17] Such a strong coupling between the magnetic layers would shift the collective bulk-mode frequencies up in frequency, and their frequencies would saturate at values corresponding to the bulk modes of a single film whose thickness equals that of the total stack.

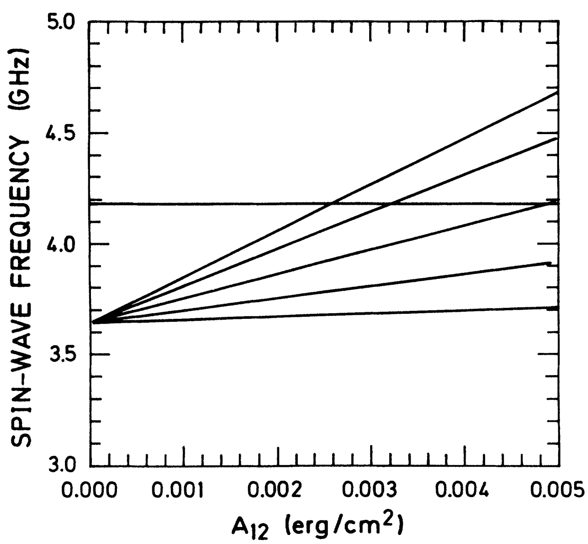

FIG. 8. The sensitive dependence of the bulk-mode frequencies on the interlayer exchange constant A_{12} is shown. The stack is that of Fig. 2 for $H_0=500$ G. As A_{12} is increased the bulk modes spread apart and increase in frequency while the surface-mode frequency is essentially unchanged.

IV. SUMMARY

The effects of out-of-plane anisotropies on the collective spin-wave modes of a multilayer stack of ferromagnetic films have been investigated. Both dipolar and exchange interactions were taken into account and the following features were found.

(i) The strong perpendicular anisotropy results in collective modes that have both bulk-mode and surface-mode characteristics. For small fields where the magnetization has a large component perpendicular to the film plane, a mode exists which is weakly localized to a surface of the stack when the magnetization is nearly perpendicular and becomes strongly localized to a surface of the stack as the magnetization is pulled into the film plane by the external applied field.

(ii) In the presence of perpendicular anisotropies, the collective surface mode can exist even when the thickness of the nonmagnetic layers is greater than the thickness of the magnetic layers. This is a generalization of the well-known $d_f > d_s$ rule for superlattices without anisotropies.

(iii) When interlayer exchange interactions are small, the surface mode of the stack goes soft at an applied field below H_c. It reappears at a field above H_c but its frequency remains below that of the bulk band of collective modes for certain values of the perpendicular anisotropy.

(iv) A small interlayer exchange coupling causes a very large broadening of the bulk band. For applied fields below H_c, crossings and repulsions appear as the surface mode passes through the bulk band.

By mixing the surface- and bulk-mode characteristics of the collective excitations of the multilayer system, perpendicular anisotropies have an extremely large effect on the collective surface mode by modifying both its frequency and its degree of localization to the stack surface. Both of these effects are readily studied with Brillouin light scattering and observation of the features predicted here, such as the position of the surface mode relative to the bulk band, should give unique information over the magnitude of out-of-phase surface anisotropies.

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