Scattering of transverse-magnetic waves with a nonlinear film: Formal field solutions in quadratures

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An exact analytical study of the field solution involved in the scattering of a plane transverse-magnetic electromagnetic wave with a film having an intensity-dependent nonlinearity is presented here. By finding exact integrating factors, a formal solution expressed in terms of simple quadratures is derived. An explicit result for the reflection coefficient as a function of the incident intensity is presented for a Kerr medium.

I. INTRODUCTION

The interaction of electromagnetic waves with nonlinear dielectric media has been the subject of intense theoretical and experimental investigation in recent years.[1] A substantial amount of work has been reported in particular on guided waves. However, in contrast, there is relatively little work on scattered waves since the pioneering work by Kaplan[2] on semi-infinite nonlinear medium. This is because of the extra mathematical complexity associated with scattered waves whose field solution has an amplitude and a nontrivial phase factor, both of which are intrinsically coupled together nonlinearly through Maxwell's equations. In comparison, the guided wave solution, in general, can be represented by rather trivial phase factors. As a result, it was only quite recently that exact analytical solutions have been obtained for the scattering of plane transverse-electric waves with nonlinear thin films,[3-7] bilayers and superlattices.[8]

In the case of transverse-magnetic waves, the problem of finding the field solution is even more difficult. This comes about because, for transverse magnetic waves, the natural variable to study is the magnetic field, but the nonlinear dielectric constant may be simple, under the usual assumptions, only if it is expressed in terms of the electric field components. When it is expressed in terms of the magnetic field, one generally finds that it contains terms involving not only the magnetic field, but also spatial derivatives of the magnetic field, and the dielectric constant itself. In the case of guided waves, however, there has been considerable progress in recent years. Exact dispersion relations at a linear-nonlinear interface have been obtained either based on a first-order differential equation[9,10] developed by Berkhoer and Zakharov,[11] or by means of a conservation law[12] that is obtained by generalizing a procedure developed by Liu and Joseph.[13] However, to our knowledge, the corresponding results for scattered waves are currently not available, and as a result theoretical studies have been mostly numerical in nature.

In this paper we report some exact analytical results for the scattering of a plane transverse-magnetic (TM) wave of a single frequency with a nonlinear thin film. The nonlinearity is assumed to depend only on the intensity. This dependence need not be Kerr-like, and can be completely arbitrary. We show that Maxwell's equations can be solved exactly and the solution can be found in quadratures. This is accomplished by finding integrating factors for the various nonlinear ordinary differential equations encountered in our calculation. The general solution is found to contain a total of four integration constants. These constants can be uniquely determined from the required electromagnetic boundary conditions that must be imposed on the solution at each of the surfaces of the film. It turns out that one of these constants represents the energy flux across the film, and by setting this constant to zero, one recovers our previous solution for guided TM waves.[14] This is at least conceptually very satisfying because waves in a thin film, irrespective of whether linear or nonlinear, can always be considered as either guiding or propagating. Thus our solution offers the opportunity to treat them on an equal footing. With this solution one then calculates the internal field as well as the reflection and transmission coefficients. We then make use of our analytical results to calculate specifically for a Kerr medium the reflection coefficient, and we find that it exhibits optical bistable behavior as a function of the incident intensity.

Besides its fundamental importance, and its potential function as a mirrorless optical switch and other nonlinear optics applications, our work on the scattering of EM waves with finite NL films can also be of use in developing methods for studying NL prism couplers. And because the general results here are valid independent of the form of the dependence of the dielectric constant on the intensity, they can be used to study effects associated with the saturation of the nonlinearity.[15]

II. REDUCTION OF THE PROBLEM TO QUADRATURES

Let us consider here a single nonlinear film which occupies region (2) between $z=0$ and d. It is assumed that the nonlinear material can be characterized by a dielectric function which depends on the instantaneous value of the local electric field intensity, i.e.,

$$\epsilon=\epsilon(|\mathbf{E}(\mathbf{r})|^2) . \quad (1)$$

Regions (1) and (3), which correspond to $z<0$ and $z>d$, respectively, are taken to be linear, with dielectric constants ϵ_1 and ϵ_3. The interest here is in the monochromatic transverse-magnetic wave which has the form of a plane wave along x and a time dependence of the form $e^{-i\omega t}$. Therefore ϵ has no time dependence, and depends on z only through its dependence on $|\mathbf{E}(\mathbf{r})|^2$. Thus one writes, suppressing a time dependent factor,

$$\mathbf{B}(\mathbf{r})=\hat{\mathbf{y}}B_y(z)e^{ik_x x} \quad (2)$$

and

$$\mathbf{E}(\mathbf{r})=[\hat{\mathbf{x}}E_x(z)+\hat{\mathbf{z}}E_z(z)]e^{ik_x x} . \quad (3)$$

Using Eqs. (2) and (3) in Maxwell's equations we find the following equations for the field components:

$$k_x B_y=\frac{-i\omega\mu}{c}\epsilon E_z , \quad (4)$$

$$\partial_z B_y=\frac{i\omega\mu}{c}\epsilon E_x , \quad (5)$$

$$\partial_z E_x-ik_x E_z=\frac{i\omega}{c}B_y . \quad (6)$$

We choose to eliminate E_x and E_z in favor of B_y. Thus we find

$$E_x=\frac{-ic}{\omega\mu}\frac{1}{\epsilon}\partial_z B_y , \quad (7)$$

$$E_z=\frac{-c}{\omega\mu}\frac{1}{\epsilon}k_x B_y , \quad (8)$$

and B_y obeys the equation

$$\left[\frac{B'}{\epsilon}\right]'=\left[\frac{\eta^2}{\epsilon}-1\right]B , \quad (9)$$

where

$$\epsilon=\epsilon\left[\left|\frac{B'}{\epsilon}\right|^2+\left|\frac{\eta B}{\epsilon}\right|^2\right] . \quad (10)$$

In Eq. (9) we have dropped the subscript on B, have defined $k_0=\omega\sqrt{\mu}/c$, $\eta=k_x/k_0$, a dimensionless length in the z direction, $\zeta=k_0 z$, and have used the prime to denote differentiation with respect to ζ. The mathematical problem now is to find B from Eq. (9) where ϵ is given by Eq. (10). As opposed to the case of guided waves, we must look for complex solutions. It turns out that despite the complexity of these equations, B can be found in quadratures irrespective of the form of the dependence of the nonlinear dielectric constant on the intensity.

Before we start we should point out the fact that, because of gauge invariance, the solution to Eq. (9) can only be determined up to the multiplication with a constant of unit magnitude. In other words, if B is a solution then so is $Be^{i\psi_0}$, where ψ_0 is an arbitrary constant. This constant, however, is important to determine the actual fields within the film, and the reflection coefficient. Its value can be determined from boundary conditions. Now, to find $B(\zeta)$, we begin by representing it in polar form:

$$B(\zeta)=A(\zeta)e^{i\psi(\zeta)} , \quad (11)$$

where $A(\zeta)$ and $\psi(\zeta)$ are unknown real functions to be determined. Inserting this form for B in Eq. (9), assuming ϵ is real, and separating the result into real and imaginary parts yields the following two equations:

$$\frac{A''-(\psi')^2 A}{\epsilon}-\frac{\epsilon' A'}{\epsilon^2}=\left[\frac{\eta^2}{\epsilon}-1\right]A , \quad (12)$$

$$\frac{2\psi' A'+\psi'' A}{\epsilon}-\frac{\psi'\epsilon' A}{\epsilon^2}=0 . \quad (13)$$

Equation (13) can be integrated once if it is multiplied by an integrating factor A. The result can be written as

$$\psi'=\frac{c_1\epsilon}{A^2} . \quad (14)$$

The existence of the constant c_1 is a direct consequence of the conservation of energy in the z direction. To see this, we calculate the Poynting vector:

$$\mathbf{S}=\frac{c}{8\pi}\mathrm{Re}\mathbf{E}\times\mathbf{H}^* . \quad (15)$$

Using Eqs. (7), (8), (11), and (14), we find

$$\mathbf{S}=\frac{c}{8\pi\sqrt{\mu}}\left[\frac{\eta A^2(\zeta)}{\epsilon(\zeta)}\hat{\mathbf{x}}+c_1\hat{\mathbf{z}}\right] . \quad (16)$$

This confirms our interpretation of the integration constant c_1. In the case of guided waves, c_1 must be zero. This then implies that $\psi'=0$, and so ψ must be a constant. This constant is of no significance for guided waves, and is usually set to zero.

We will use Eq. (14) to eliminate ψ' from all our equations. Thus we obtain from Eq. (12) a single equation for A:

$$\frac{A''}{\epsilon}-\frac{c_1^2\epsilon}{A^3}-\frac{\epsilon' A'}{\epsilon^2}=\left[\frac{\eta^2}{\epsilon}-1\right]A . \quad (17)$$

From Eqs. (10), (11), and (14) we find

$$\epsilon=\epsilon\left[\left[\frac{A'}{\epsilon}\right]^2+\left[\frac{\eta A}{\epsilon}\right]^2+\left[\frac{c_1}{A}\right]^2\right] . \quad (18)$$

What we have accomplished so far is to uncouple ψ and A. Our next problem is to solve Eq. (17) for A with ϵ obeying Eq. (18). It should be noted that even in the case of a Kerr medium in which the nonlinear part of the dielectric constant is simply proportional to the intensity, the dielectric constant is an exceedingly complicated function of A and A'. To proceed further, we separate out from ϵ a linear part, ϵ_0, and a nonlinear part, ϵ_2, where $\epsilon_2(I)\to 0$ as $I\to 0$. We also assume that $\epsilon_2(I)$ is a monotonic function of I, and so has a unique inverse function $I(\epsilon-\epsilon_0)$. From Eq. (18) we now can write

$$\left[\frac{A'}{\epsilon}\right]^2=I(\epsilon-\epsilon_0)-\left[\frac{\eta A}{\epsilon}\right]^2-\left[\frac{c_1}{A}\right]^2 . \quad (19)$$

We then differentiate this equation with respect to ζ, and use Eq. (17) to eliminate the factor $(A'/\epsilon)'$ from the resulting equation. This yields the following equation:

$$\frac{2\eta^2 A A'}{\epsilon^2}-\frac{A A'}{\epsilon}-\frac{\eta^2 A^2\epsilon'}{\epsilon^3}=\frac{\epsilon'}{2}\frac{dI}{d\epsilon} . \tag{20}$$

This equation can be integrated if it is multiplied by ϵ. The result is

$$\left[\frac{2\eta^2-\epsilon}{\epsilon}\right]A^2=\epsilon I(\epsilon-\epsilon_0)-\int_0^{\epsilon-\epsilon_0}du\, I(u)+c_2 , \tag{21}$$

where c_2 is another integration constant. This is a first integral of Eq. (17), because it involves no higher than a first-order derivative of A.

Now in order to find A as a function of ζ, we need a second integral. It is rather hopeless if we try to proceed directly with this equation, because it is an extremely complicated first-order nonlinear differential equation in A. A way to get around this problem is to make a change of dependent variable from A to ϵ. Physically, since the field A behaves only according to the effective value of ϵ that it sees, a more appropriate dependent variable is ϵ, and not A. Instead of trying to solve the differential equation in Eq. (21) we use Eq. (19) to write

$$(A')^2=\epsilon^2 I(\epsilon-\epsilon_0)-(\eta A)^2-\left[\frac{c_1\epsilon}{A}\right]^2 , \tag{22}$$

and consider Eq. (21), not as a first integral, but as an equation giving A as a function of ϵ. Thus we write

$$A^2(\epsilon)=\left[\epsilon^2 I(\epsilon-\epsilon_0)-\epsilon\int_0^{\epsilon-\epsilon_0}du\, I(u)+c_2\epsilon\right]\Big/(2\eta^2-\epsilon) . \tag{23}$$

Because $A'=\epsilon' dA(\epsilon)/d\epsilon$, Eq. (22) can be rewritten as

$$\tfrac{1}{2}(\epsilon')^2+V(\epsilon)=0 , \tag{24}$$

where the "potential" is given by

$$V(\epsilon)=\frac{1}{2}\left[\frac{dA(\epsilon)}{d\epsilon}\right]^{-2}\left[\epsilon^2 I(\epsilon-\epsilon_0)-[\eta A(\epsilon)]^2-\left[\frac{c_1\epsilon}{A(\epsilon)}\right]^2\right] , \tag{25}$$

where $A^2(\epsilon)$ is given by Eq. (23). With ζ as the "time" and ϵ as the "position," we can interpret this equation as the conservation of "energy" equation for a classical particle moving in a one-dimensional "potential" V, with zero total mechanical energy. This equation can easily be integrated to give

$$\zeta-\zeta_0=\pm\int_{\epsilon(\zeta_0)}^{\epsilon(\zeta)}\frac{d\epsilon}{\sqrt{-2V(\epsilon)}} , \tag{26}$$

where ζ_0 is the third integration constant. Now, inverting this equation gives $\epsilon(\zeta)$, which then gives $A(\zeta)$ when Eq. (23) is used. Therefore $A(\zeta)$ can be found for any $\epsilon_2(I)$ by performing two integrations, one in Eq. (23) and one in Eq. (26). The phase function, $\psi(\zeta)$, can then be found from Eq. (14) by an additional integration. This will introduce a fourth integration constant. For a wave incident on the $z=0$ surface we find it convenient to write

$$\psi(\zeta)=\int_0^{\zeta}d\zeta\,\Psi(\zeta)+\psi_0 , \tag{27}$$

and so we have

$$\Psi(\zeta)=c_1\epsilon/A^2(\zeta) . \tag{28}$$

This last constant, ψ_0 is the one we expect to be present because of gauge invariance.

Our complete solution for $B(\zeta)$ involves a total of four real integration constants. These can be uniquely determined by imposing the required electromagnetic boundary conditions on the solution.

III. IMPOSING BOUNDARY CONDITIONS

Now we consider the boundary conditions on the fields, again suppressing the time dependent factor $e^{-i\omega t}$ in our equations. We consider a TM plane wave incident at an angle θ from the $z<0$ region. We let $k_i=k_0\sqrt{\epsilon_i}$, $k_{ix}=k_i\sin\theta$, $k_{iz}=k_i\cos\theta$, where $i=1$ or 3. We also use dimensionless parameters $\eta_{ix}=\sqrt{\epsilon_i}\sin\theta$ and $\eta_{iz}=\sqrt{\epsilon_i}\cos\theta$. Here $\epsilon_{1,3}$ are assumed to be positive.

For $z<0$ we let

$$\mathbf{B}=\hat{\mathbf{y}}B_0(e^{i\eta_{1z}\zeta}+re^{-i\eta_{1z}\zeta})e^{ik_x x} , \tag{29}$$

where B_0 is a real constant and r is the complex reflection coefficient. Within the film we let

$$\mathbf{B}=\hat{\mathbf{y}}A(\zeta)\exp\left[i\int_0^{\zeta}d\zeta\Psi(\zeta)\right]e^{\psi_0}e^{ik_x x} , \tag{30}$$

and for $z>d$ we let

$$\mathbf{B}=\hat{\mathbf{y}}tB_0e^{i\eta_{3z}(\zeta-\zeta_d)}\exp\left[i\int_0^{\zeta_d}d\zeta\Psi(\zeta)\right]e^{\psi_0}e^{ik_x x} , \tag{31}$$

where $\zeta_d=k_0 d$ and t is the complex transmission coefficient.

At $z=0$, continuity of the tangential field components, E_x and B_y, implies

$$\eta_{1z}(1-r)B_0\epsilon(0)=\epsilon_1[\Psi(0)A(0)-iA'(0)]e^{i\psi_0} \tag{32}$$

and

$$(1+r)B_0=A(0)e^{i\psi_0} , \tag{33}$$

respectively. Similarly, by applying the boundary conditions at $z=d$, we obtain

$$\epsilon_3[\Psi(\zeta_d)A(\zeta_d)-iA'(\zeta_d)]=\eta_{3z}tB_0\epsilon(\zeta_d) \tag{34}$$

and

$$A(\zeta_d)=tB_0 . \tag{35}$$

We therefore have four complex equations, Eqs. (32)–(35), for the determination of eight real quantities, namely, the real and imaginary parts of r and t, and ζ_0, c_1, c_2, and ψ_0.

To determine these quantities we start with Eq. (33) to find

$$r=\frac{A(0)}{B_0}e^{i\psi_0}-1\ . \tag{36}$$

Substituting r in Eq. (32) gives the result

$$[\eta_{1z}\epsilon(0)+\epsilon_1\Psi(0)A(0)-i\epsilon_1 A'(0)]e^{i\psi_0}=2\epsilon(0)\eta_{1z}B_0\ . \tag{37}$$

Next, we take the square of the absolute value of this equation, get rid of Ψ using Eq. (28) and A' using Eq. (22), and express A in terms of ϵ using Eq. (23) to arrive at the result

$$4\eta_{1z}^2B_0^2=2\eta_{1z}c_1\epsilon_1+\frac{1}{2\eta_x-\epsilon(0)}\left[\eta_{1z}^2-\frac{\eta_x\epsilon_1^2}{\epsilon^2(0)}\right]\left[\epsilon^2(0)I(0)-\epsilon(0)\int_0^{\epsilon(0)-\epsilon_0}du\ I(u)+c_2\epsilon(0)\right]+\epsilon_1^2I(0)\ , \tag{38}$$

where $I(0)=I[\epsilon(0)-\epsilon_0]$ is the intensity at $z=0$. It should be noted that the intensity is not continuous at the two surfaces of the film. We also eliminate t between Eqs. (34) and (35) to get

$$\epsilon_3[\Psi(\zeta_d)A(\zeta_d)-iA'(\zeta_d)]=\eta_{3z}A(\zeta_d)\epsilon(\zeta_d)\ . \tag{39}$$

Equating the real and imaginary parts yields the results

$$\epsilon_3\Psi(\zeta_d)=\eta_{3z}\epsilon(\zeta_d) \tag{40}$$

and

$$A'(\zeta_d)=0\ . \tag{41}$$

By eliminating Ψ with Eq. (28), Eq. (40) gives

$$c_1=\frac{\eta_{3z}A^2(\zeta_d)}{\epsilon_3}\ . \tag{42}$$

The main results are contained in three real algebraic equations, Eqs. (38), (41), and (42). From Eqs. (25) and (23), it is clear that $V(\epsilon)$ depends on parameters c_1 and c_2, and so $\epsilon(\zeta)$, $A(\zeta)$, and $I(\zeta)$ depend on the three real parameters, ζ_0, c_1, and c_2. These parameters can be determined at a given incident angle and intensity, i.e., a given value of θ and B_0, by solving the set of three equations, Eqs. (38), (41), and (42).

Once these parameters have been determined, we can calculate the internal field distribution, and the effective dielectric constant within the film. The fourth integration constant, the phase constant ψ_0, can be calculated from Eq. (37). The result gives

$$\cos\psi_0=\frac{c_1\epsilon_1+\eta_{1z}A^2(0)}{2B_0\eta_{1z}A(0)} \tag{43}$$

and

$$\sin\psi_0=\frac{\epsilon_1A'(0)}{2B_0\eta_{1z}A(0)}\ . \tag{44}$$

The reflectivity can then be obtained from Eqs. (36) and (43) as

$$|r|^2=1-\frac{c_1\epsilon_1}{\eta_{1z}B_0^2}\ . \tag{45}$$

From Eq. (35) it is clear that t is real. Moreover, with the help of Eqs. (28) and (35) we find

$$t^2=\frac{c_1\epsilon_3}{\eta_{1z}B_0^2}\ . \tag{46}$$

Equations (45) and (46) combine to yield

$$\epsilon_3(1-|r|^2)=\epsilon_1t^2\ . \tag{47}$$

This is clearly the equation for the conservation of flux in the z direction.

IV. PARAMETRIZATION AND COMPUTATION OF PHYSICAL QUANTITIES

Because of the apparent complexity of the above derived results, we will describe in detail here how these results can be used to compute physical quantities of interest. As we have pointed out before in our work on the corresponding TE problem, computation can be made relatively easier if one chooses an appropriate parameter. For TE waves, the best choice for the parameter is the value of the electric field at the film-substrate interface. For TM waves here, we find that the best choice is to use the value of the dielectric function, $\epsilon(\zeta_d)$, as ζ approaches the film-substrate interface from within the film.

With this value we can calculate $A(\zeta_d)$ from Eq. (23). The two integration constants, c_1 and c_2 are then determined, respectively, from Eqs. (42) and (21) by setting $\zeta=\zeta_d$. The form of the potential in Eq. (25) is then completely known. By integrating Eq. (26) with the choice $\zeta_0=\zeta_d$ we obtain $\epsilon(\zeta)$, which then yields the magnetic field within the film from Eq. (23). By integrating up to $\zeta=0$, we can also get $\epsilon(0)$ and $A(0)$. The incident magnetic field that gives rise to this internal field is then obtained from Eq. (38). The reflection coefficient can be computed as a function of the incident intensity using Eq. (45).

V. REFLECTION COEFFICIENT FOR KERR MEDIUM

We will apply our analytical results to a Kerr medium in which the dielectric function is given by

$$\epsilon=\epsilon_0+\alpha I \ . \tag{48}$$

The first integral in Eq. (21) then becomes

$$A^2=\frac{\epsilon(\epsilon^2-\epsilon_0^2)+2\alpha c_2\epsilon}{2\alpha(2\eta^2-\epsilon)} \ . \tag{49}$$

Using this result we can easily derive the potential function using Eq. (25). In order to make the resulting expression appear less complicated, we introduce new variables and parameters. The new variable x, defined so that

$$\epsilon=\epsilon_0(1+x) \ , \tag{50}$$

measures the deviation of the dielectric function from the linear value. A parameter a, which measures the angle of the incident wave, is defined by

$$a=\frac{\eta^2}{\epsilon_0} \ . \tag{51}$$

The two integration constants, c_1 and c_2, are replaced by p and q so that

$$p=\frac{2\alpha c_1}{\epsilon_0^{3/2}}, \quad q=\frac{2\alpha c_2}{\epsilon_0^2} \ . \tag{52}$$

In these definitions we have assumed that ϵ_0 is positive. In addition, if we redefine ζ as $\zeta/\sqrt{\epsilon_0}$ then the potential function can be expressed as

$$V(x)=-(2a-1-x)^2(1+x)^2\frac{2x(1+x)(2a-1-x)[(1+x)^2-1+q]-a[(1+x)^2-1+q]^2-p^2(2a-1-x)^2}{2[3a(1+x)^2-(1+x)^3-a(1-q)]^2} \ . \tag{53}$$

In Fig. 1 we show the result for the reflection coefficient as a function of the incident field intensity. The parameters we have chosen are $\epsilon_1=\epsilon_3=3.0$, $\epsilon_0=2.5$, $d/\lambda=2$, and an incident angle such that $a=1.59$. The incident intensity is normalized to the value of the nonlinear coefficient α, which we have taken to be positive. The reflection coefficient clearly shows optical bistable behavior as the incident field intensity is varied.

VI. LINEAR REGIME

It is interesting to see how the classical Fresnel coefficients can be obtained within our present framework in the linear limit. In this limit x is small. Clearly, p and q are of the same order as x. For the potential we only need it up to second order in x. The result from Eq. (53) is

$$V=-\tfrac{1}{2}(a-1)x^2+qx-aq^2-p^2(2a-1)^2 \ . \tag{54}$$

With V only quadratic in x, Eq. (54) can easily be integrated analytically. We consider the case where $a<1$, which means that we are not in the total internal reflection regime. The solution is

$$x=\frac{q}{4(1-a)}\{(2a-1)\beta\cos[2\sqrt{1-a}\,(\zeta-\zeta_d)]^{-1}\} \ , \tag{55}$$

where

$$\beta\equiv\left[1-4(1-a)\left[\frac{p}{q}\right]^2\right]^{1/2} \ . \tag{56}$$

From Eq. (49) the magnetic field within the film is found to be given by

$$A^2=\frac{q}{4\alpha(1-a)}\{\beta\cos[2\sqrt{1-a}\,(\zeta-\zeta_d)]\} \ . \tag{57}$$

The constant, β, can be calculated using the above equation and Eq. (56). The result is

$$\beta=\frac{\epsilon_0^2k_{3z}^2-\epsilon_3^2k_{2z}^2}{\epsilon_0^2k_{3z}^2+\epsilon_3^2k_{2z}^2} \ . \tag{58}$$

Using this result we can write

$$p=-\frac{q\epsilon_0\epsilon_3k_{3z}k_{2z}}{\epsilon_0^2k_{3z}^2+\epsilon_3k_{2z}^2} \ . \tag{59}$$

The incident magnetic field can be calculated using Eq. (38). The result, to the lowest nontrivial order is

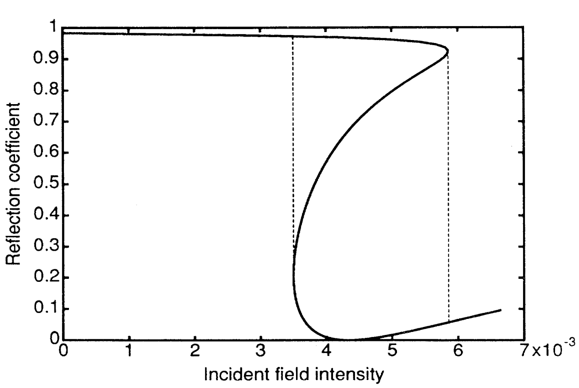

FIG. 1. The reflection coefficient for TM plane wave incident on a film with a positive Kerr nonlinearity. The parameters chosen are: $\epsilon_1=\epsilon_3=3.0$, $\epsilon_0=2.5$, $d/\lambda=2$, and $a=1.59$. The incident intensity is normalized to the value of the nonlinear coefficient α.

$$4k_{1z}^2\alpha B_0^2=\frac{q}{2(\epsilon_0^2k_{3z}^2+\epsilon_3k_{2z}^2)} \tag{60}$$

$$\times\left[2\epsilon_0\epsilon_1\epsilon_3k_{1z}k_{3z}-\frac{1}{\epsilon_0k_{2z}^2}[\epsilon_0^2k_{2z}^2(\epsilon_3^2k_{1z}^2+\epsilon_1^2k_{3z}^2)+(\epsilon_0^2k_{1z}^2-\epsilon_1^2k_{2z}^2)(\epsilon_0^2k_{3z}^2-\epsilon_3^2k_{2z}^2)\sin^2(\sqrt{1-a}\,\zeta_d)]\right]. \tag{61}$$

Finally, using Eqs. (45), (59), and (60), we obtain the classical Fresnel formula

$$|r|^2=1-\frac{4\dfrac{k_{1z}}{\epsilon_1}\dfrac{k_{3z}}{\epsilon_3}\dfrac{k_{2z}^2}{\epsilon_0^2}}{\dfrac{k_{2z}^2}{\epsilon_0^2}\left[\dfrac{k_{1z}}{\epsilon_1}+\dfrac{k_{3z}}{\epsilon_3}\right]^2+\left[\dfrac{k_{1z}^2}{\epsilon_1^2}-\dfrac{k_{2z}^2}{\epsilon_0^2}\right]\left[\dfrac{k_{3z}^2}{\epsilon_3^2}-\dfrac{k_{2z}^2}{\epsilon_0^2}\right]\sin^2(\sqrt{1-a}\,\zeta_d)}. \tag{62}$$

The corresponding result for the reflection coefficient in the total internal reflection regime can also be derived.

VII. SUMMARY AND CONCLUSION

We have shown that the problem of the scattering of TM plane waves with a nonlinear film having an intensity dependent dielectric constant can be solved in quadratures, without even having to specify the precise form of the dependency. The integration constants can all be determined unambiguously through the boundary condition. Results for the internal field distribution, the reflectivity, and other interesting quantities, can then be calculated. The computation is especially easy if all the physical variables are parametrized in terms of $\epsilon(\zeta_d)$. Application of our derived results to the case of a Kerr medium is also presented.

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