PHYSICAL REVIEW B VOLUME 44, NUMBER 9 1 SEPTEMBER 1991-I

Chiral spin states, hole dynamics, and superconductivity in strongly correlated electronic systems

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(Received 26 October 1990)

Relation between strongly correlated electronic systems and anyonlike systems is established. A U(1) gauge theory is developed for the frustrated Hubbard model in the limit of infinite on-site Coulomb repulsion. An effective Chern-Simons theory, which describes chiral spin liquids is obtained. The fermions (holes) are coupled to the gauge fields creating thus an anyonlike system. I show that under some circumstances the system is a superconductor for special value of the hole density.

I. INTRODUCTION

The discovery of the high-T_c superconductors[1] stimulated the search for additional mechanisms of superconductivity. In his pioneer work,[2] Anderson stressed the essentially two-dimensional (2D) character of these superconductors. This has lead to the development of theories in $2+1$ dimensions, which possess excitations with exotic quantum numbers and statistics.[3,4]

An important step was done by Kalmeyer and Laughlin.[5] They argued that the ground state of the 2D frustrated Heisenberg antiferromagnetic is well described by a fractional quantum Hall wave function for bosons. This analogy was useful to understand high-T_c superconductivity and has lead to the advent of the so-called anyon mechanism of superconductivity. One can describe the gas of anyons in terms of a system of fermions (bosons) coupled to U(1) Chern-Simons gauge fields (statistical gauge fields).[6] The anyons have fractional spin and statistics and violate the discrete symmetries P and T. Fetter *et al.*[7] showed, in the random-phase approximation, that the current-current correlation function has a Goldstone pole, and that this pole implies the Meissner effect for an electromagnetic field.

A somewhat different approximation scheme is carried out in Ref. 8. The authors describe the order parameter for superconducting phase in terms of "spontaneous breaking of commutativity of translations."

A powerful condition for superconductivity was obtained by Banks and Lykken[9] (see also Refs. 10 and 11). They proved that the Goldstone pole occurs when the induced Chern-Simons term for the statistical gauge potential exactly cancels the bare term responsible for the statistics. Then the Goldstone boson is described by the scalar Bose field dual to the massless gauge fields in $2+1$ dimensions.

If anyon models are relevant to high-T_c superconductors, the P-and T-symmetry breaking can have important experimental consequences.[12,8,10]

Many different models and approaches were used to study the superconductivity induced by topologically massive gauge fields.[13] Several attempts have been made towards a microscopic derivation of the anyon models,[14] but this remains the weakest point in the theory, which requires further elaborations. In this paper, I deal with this problem.

I start from the 2D frustrated Hubbard-like model in the limit of infinite on-site Coulomb repulsion. The underlying problem is to create an adequate field theory. Usually, the bosonic and fermionic raising and lowering operators are used to realize Hubbard's matrix (or spin) algebra. Some of these representations are true for definite spin values. In other cases it is difficult to separate the physical Hilbert space. In this work the supersymmetric extension of the holomorphic path-integral representation of systems with SU(2) dynamical symmetry[4,15] is worked out. Coherent states for SU(2|1) superalgebra are used to define the path integral. As a result, a U(1) gauge-field theory is derived. The spin fluctuations of the system are described by chargeless two-component (spin-up and -down) Bose fields and the charge of the holes is carried by spinless fermionic fields. The spin fluctuations are effectively taken into account by integrating over the Bose fields in the mean-field approximation. When the 2D antiferromagnet is sufficiently frustrated, the P and T violating phase (called chiral spin phase) is energetically preferable.[16] The corresponding states are spin-rotationally invariant, short range, and effectively can be described by a topologically massive U(1) gauge theory. The fermion fields are coupled to this gauge potential, creating, thus, an anyonlike system. I show that, under some circumstances and for some values of the holes' density, the Chern-Simons term induced by fermions cancels the topologically massive term in the effective theory of the chiral spin liquid. Hence, the Banks-Lykkem condition for superconductivity is satisfied. Following Refs. 9 and 10, I obtain an effective Ginzburg-Landau action. Some results of Ref. 17 are used.

The paper is organized as follows. In Sec. II the path integral for the large-U Hubbard-like theory is considered and the corresponding U(1) gauge-field theory is presented. In Sec. III, I derive the effective theory of chiral spin liquids. The dynamics of holes is studied and the conditions for the hole superconductivity are obtained. Section IV is devoted to the concluding remarks.

II. COHERENT STATES AND PATH-INTEGRAL REPRESENTATION

I shall consider a theory with a Hamiltonian:

$$H=-t_1\sum_{\langle i,j\rangle}[C^\dagger_{i\sigma}C_{j\sigma}+\text{H.c.}]-t_2\sum_{\langle\langle i,j\rangle\rangle}[C^\dagger_{i\sigma}C_{j\sigma}+\text{H.c.}]+J_1\sum_{\langle i,j\rangle}\mathbf{S}_i\cdot\mathbf{S}_j+J_2\sum_{\langle\langle i,j\rangle\rangle}\mathbf{S}_i\cdot\mathbf{S}_j-\mu\sum_i C^\dagger_{i\sigma}C_{i\sigma}\ , \quad (1)$$

where $C^\dagger_{i\sigma}$ ($C_{j\sigma}$) are Fermi operators of the electron on a site i (j) of a 2D square lattice which act on states with no double occupancy,

$$\{|a,i\rangle\}=\{|0,i\rangle,|\uparrow,i\rangle,|\downarrow,i\rangle\} \quad (2)$$

on a lattice site i.

By $\langle i,j\rangle$ I denote the sum over the nearest neighbors, and by $\langle\langle i,j\rangle\rangle$ I denote the sum over the next neighbors along a diagonal. The spin operators $\mathbf{S}_i$ have a representation $S^\alpha_i=\frac{1}{2}C^\dagger_i\sigma^\alpha C_i$, where σ^α are Pauli matrices and μ is the chemical potential.

It is convenient to introduce the projection operators $X^{ab}_i=|a,i\rangle\langle i,b|$. One can consider the matrices $X^{\uparrow 0}_i$, $X^{\downarrow 0}_i$ ($X^{0\uparrow}_i$, $X^{0\downarrow}_i$) as Fermi operators, and the remaining as Bose operators. Then the operators X^{ab}_i form a basis of the supersymmetrical SU(2|1) Lie algebra:[18]

$$[X^{ab}_i,X^{cd}_j]_\pm=\delta_{ij}(\delta^{bc}X^{ad}_i\pm\delta^{ad}X^{cb}_i) \quad (3)$$

and the Hamiltonian (1) can be rewritten in terms of these operators.

Let us define the coherent states by the relation

$$|\xi,Z\rangle=\exp\sum_i[\xi_iX^{0\uparrow}_i+Z_iX^{\downarrow\uparrow}_i]|\Uparrow\rangle\ , \quad (4)$$

where Z_i are c numbers, ξ_i are Grassman complex variables, $|\Uparrow\rangle\equiv\otimes_i|\uparrow,i\rangle$, and i runs over all lattice sites. Formula (4) generalizes Radcliffe's[19] definition of SU(2) group coherent states. It is straightforward to prove the formulas

$$\langle\xi^1,Z^1|Z^2,\xi^2\rangle=\prod_j(1+\bar Z^1_jZ^2_j+\bar\xi^1_j\xi^2_j)\ , \quad (5)$$

$$\frac{1}{2\pi i}\int\prod_j d\bar Z_jdZ_jd\xi_jd\xi_j\frac{1}{\prod_j(1+\bar Z_jZ_j+\bar\xi_j\xi_j)}|Z,\xi\rangle\langle\xi,Z|=1\ . \quad (6)$$

Following the holomorphic path-integral approach,[20] I use the coherent states (4) in the evaluation of the partition function $Z(\beta)=\text{Tr}e^{-\beta H}$. It is evident from Eq. (6) that this function permits the representation

$$Z(\beta)=\frac{1}{2\pi i}\int\prod_j d\bar Z_jdZ_jd\bar\xi_jd\xi_j\frac{1}{\prod_j(1+\bar Z_jZ_j+\bar\xi_j\xi_j)}\langle-\xi,Z|e^{-\beta H}|Z,\xi\rangle\ . \quad (7)$$

Consider the operator $e^{-\beta H}$ as a multiple of many small evolutions:

$$e^{-\beta H}=\lim_{N\to\infty}\left[1-\frac{\beta}{N}H\right]^N .$$

Thus,

$$\begin{aligned}Z(\beta)=\lim_{N\to\infty}\int\prod_j(2\pi i)^{-1}d\bar Z_jdZ_jd\bar\xi_jd\xi_j\\
&\times\prod_{k=1}^{N-1}(2\pi i)^{-1}d\bar Z_j(\tau_k)dZ_j(\tau_k)d\bar\xi_j(\tau_k)d\xi_j(\tau_k)\left\langle-\xi,Z\left|\left[1-\frac{\beta}{N}H\right]\right|Z(\tau_{N-1}),\xi(\tau_{N-1})\right\rangle\\
&\times\left\langle\xi(\tau_{N-1}),Z(\tau_{N-1})\left|\left[1-\frac{\beta}{N}H\right]\right|Z(\tau_{N-1\cdot 2}),\xi(\tau_{N-2})\right\rangle\\
&\times\cdots\left\langle\xi(\tau_1),Z(\tau_1)\left|\left[1-\frac{\beta}{N}H\right]\right|Z,\xi\right\rangle\\
&\times\exp\left[-\sum_j\ln(1+\bar Z_jZ_j+\bar\xi_j\xi_j)[1+\bar Z_j(\tau_{N-1})Z_j(\tau_{N-1})+\bar\xi_j(\tau_{N-1})\xi(\tau_{N-1})]\right.\\
&\left.\times\cdots\times[1+\bar Z_j(\tau_1)Z_j(\tau_1)+\bar\xi_j(\tau_1)\xi_j(\tau_1)]\right]\ ,\end{aligned} \quad (8)$$

where Eq. (6) is used again.

Let us represent the kernel

$$\left\langle \xi(\tau_k), Z(\tau_k) \left| \left[1-\frac{\beta}{N}H \right] \right| Z(\tau_1), \xi(\tau_1) \right\rangle$$

in the form

$$\left\langle \xi(\tau_k), Z(\tau_k) \left| \left[1-\frac{\beta}{n}H \right] \right| Z(\tau_1), \xi(\tau_1) \right\rangle = \left[1-\frac{\beta}{N}h(\tau_k,\tau_1) \right] \langle \xi(\tau_k), Z(\tau_k) | Z(\tau_1), \xi(\tau_1) \rangle$$

$$\approx \exp\left[-\frac{\beta}{N}h(\tau_k,\tau_1) + \sum_j \ln[1+\bar{Z}_j(\tau_k)Z_j(\tau_1)+\bar{\xi}_j(\tau_k)\xi_j(\tau_1)] \right] . \quad (9)$$

In the limit $N\to\infty$, one obtains the path-integral representation of the partition function

$$Z(\beta) = c\int \prod_{j,\tau} d\bar{Z}_j(\tau) dZ_j(\tau) d\bar{\xi}_j(\tau) d\xi_j(\tau) e^{-A(\bar{Z},Z,\bar{\xi},\xi)} , \quad (10)$$

where

$$A(\bar{Z},Z,\bar{\xi},\xi) = \int_0^\beta d\tau \left[\sum_i \frac{\bar{Z}_i(\tau)\dot{Z}_i(\tau)+\bar{\xi}_i(\tau)\dot{\xi}_i(\tau)}{1+\bar{Z}_i(\tau)Z_i(\tau)+\bar{\xi}_i(\tau)\xi_i(\tau)} + h(\tau) \right] . \quad (11)$$

$Z_i(\tau)$ are complex fields, subject to periodic boundary conditions $Z_i(0)=Z_i(\beta)$, $\xi_i(\tau)$ are Grassman complex fields subject to antiperiodic conditions $\xi_i(0)=-\xi_i(\beta)$, and c is a normalization constant.

The Bose fields $Z_i(\tau)$ are chargeless and describe the spin fluctuations of the system [see Eq. (4)]. The Fermi fields $\xi_i(\tau)$ carry the charge of the holes. The Hamiltonian can be found from Eqs. (1), (4), and (5):

$$h(\tau) = -t_1 \sum_{\langle i,j\rangle} \frac{\xi_i\bar{\xi}_j+\xi_j\bar{\xi}_i+\bar{Z}_iZ_j\xi_i\bar{\xi}_j+\bar{Z}_jZ_i\xi_j\bar{\xi}_i}{(1+\bar{Z}_iZ_i+\bar{\xi}_i\xi_i)(1+\bar{Z}_jZ_j+\bar{\xi}_j\xi_j)} - t_2 \sum_{\langle\langle i,j\rangle\rangle} [\cdots]$$
$$+\frac{J_1}{4}\sum_{\langle i,j\rangle} \frac{(Z_i+\bar{Z}_i)(Z_j+\bar{Z}_j)-(Z_i-\bar{Z}_i)(Z_j-\bar{Z}_j)+(1-\bar{Z}_iZ_i)(1-\bar{Z}_jZ)}{(1+\bar{Z}_iZ_i+\bar{\xi}_i\xi_i)(1+\bar{Z}_jZ_j+\bar{\xi}_j\xi_j)}$$
$$+\frac{J_2}{4}\sum_{\langle\langle i,j\rangle\rangle}[\cdots] - \mu\sum_i \frac{1+\bar{Z}_iZ_i}{1+\bar{Z}_iZ_i+\bar{\xi}_i\xi_i} , \quad (12)$$

where the ellipsis in the second sum stands for the expression in the first one, and that in the fourth sum stands for the expression in the third.

The field-theoretical realization, Eqs. (10)–(12), of the model, Eqs. (1) and (2), is quite complicated. It is desirable to introduce fields, making the theory more convenient for further calculations. Let us make a change of the fields,

$$\varphi_{i,1}(\tau)=a_i(\tau)Z_i(\tau), \quad \varphi^\dagger_{i,1}(\tau)=a_i(\tau)\bar{Z}_i(\tau) ,$$
$$\psi_i(\tau)=a_i(\tau)\xi_i(\tau), \quad \psi^\dagger_i(\tau)=a_i(\tau)\bar{\xi}_i(\tau) , \quad (13)$$

where the real field $a_i(\tau)$ is subject to the condition

$$\varphi^\dagger_{i,1}(\tau)\varphi_{i,1}(\tau)+a_i(\tau)a_i(\tau)+\psi^\dagger_i(\tau)\psi_i(\tau)=1 . \quad (14)$$

I introduce a second scalar complex field $\varphi_{i,2}(\tau)$ so that $\varphi_{i,2}(\tau)=a_i(\tau)$ if

$$\arg\varphi_{i,2}(\tau)=0 . \quad (15)$$

Then, making use of (14) and (15), one can rewrite, in terms of the fields $\varphi_{i\sigma}(\tau)$ $(\sigma=1,2)$ and $\psi_i(\tau)$, the kinetic term in the action, Eq. (11),

$$A_{\rm kin} = \int_0^\beta d\tau \sum_i \left[\sum_{\sigma=1}^{2} \varphi^\dagger_{i\sigma}(\tau)\dot{\varphi}_{i\sigma}(\tau)+\psi^\dagger_i(\tau)\dot{\psi}(\tau) \right] ,$$

the Hamiltonian

$$h(\tau) = t_1 \sum_{\langle i,j\rangle} [\psi^\dagger_i\psi_j(\varphi_{i\sigma}\varphi^\dagger_{j\sigma})+\text{H.c.}]$$
$$+t_2 \sum_{\langle\langle i,j\rangle\rangle} [\psi^\dagger_i\psi_j(\varphi_{i\sigma}\varphi^\dagger_{j\sigma})+\text{H.c.}]$$
$$+J_1\sum_{\langle i,j\rangle} \mathbf{S}_i\cdot\mathbf{S}_j + J_2\sum_{\langle\langle i,j\rangle\rangle}\mathbf{S}_i\cdot\mathbf{S}_j - \mu\sum_i(1-\psi^\dagger_i\psi_i) , \quad (16)$$

where $S^\alpha_i=\frac{1}{2}\varphi^\dagger_i\sigma^\alpha\varphi_i$, and the constraint, Eq. (14),

$$\varphi^\dagger_{i,1}(\tau)\varphi_{i,1}(\tau)+\varphi^\dagger_{i,2}(\tau)\varphi_{i,2}(\tau)+\psi^\dagger_i(\tau)\psi_i(\tau)=1 . \quad (17)$$

Introducing a Lagrange multiplier $B_i(\tau)$ for the constraint (17) and summing up all that is mentioned above, one obtains a final expression for the partition function:

$$Z(\beta)=c\int\prod_{i,\tau}\prod_{\sigma}d\varphi_{i\sigma}^{\dagger}(\tau)d\varphi_{i\sigma}(\tau)d\psi_i^{\dagger}(\tau)d\psi_i(\tau)dB_i(\tau)\times\prod_{i,\tau}\delta[\arg\varphi_{i2}(\tau)]e^{-A}\ , \tag{18}$$

where the action

$$A=\int_0^{\beta}d\tau\left\{\sum_j\left[\varphi_{j\sigma}^{\dagger}(\tau)\left[\frac{d}{d\tau}-iB_j(\tau)\right]\varphi_{j\sigma}(\tau)+\psi_j^{\dagger}(\tau)\left[\frac{d}{d\tau}-iB_j(\tau)\right]\psi_j(\tau)+iB_j(\tau)\right]+h(\tau)\right\} \tag{19}$$

is invariant under the gauge transformations

$$\varphi_{j\sigma}(\tau)\rightarrow e^{i\alpha(j,\tau)}\varphi_{j\sigma},\quad \psi_j(\tau)\rightarrow e^{i\alpha(j,\tau)}\psi_j(\tau)\ ,\qquad B_j(\tau)\rightarrow B_j(\tau)+\frac{d}{d\tau}\alpha(j,\tau) \tag{20}$$

if $\alpha(j,0)=\alpha(j,\beta)$. The connection between Eqs. (10)–(12) and (16)–(19) needs further elucidation.

Let us start from the theory, Eqs. (16)–(19). Imposing the gauge-fixing condition (15) and solving the constraint (17), we obtain Eqs. (10)–(12). The fields $Z_j(\tau)$ and $\xi_j(\tau)$ are the physical degrees of freedom of the theory, Eqs. (1) and (2), and they build up the physical Hilbert space. Following the standard procedure[21] one can go to an arbitrary gauge condition. A convenient one is the temporal gauge condition imposed on the field $B_j(\tau)$. The gauge invariance ensures the connection with the theory, Eqs. (1), (2), and (10)–(12).

It is convenient, sometimes, to introduce dynamical collective coordinates. The gauge-invariant one can be dropped in calculations, but the gauge-field excitations must be accounted for because they make a contact with the starting theory, Eqs. (1), (2), and (10)–(12).

One can write the action (19) from (1) by hand making use of the slave-boson (-fermion) technique.[22] The procedure is correct if the principles of gauge theory are observed.

III. *T*- AND *P*-SYMMETRY BREAKING, HOLE DYNAMICS, AND SUPERCONDUCTIVITY

A. *T*- and *P*-symmetry breaking and chiral spin liquids

Let us consider the theory, Eqs. (16)–(19). The spin term $\mathbf{S}_i\cdot\mathbf{S}_j$ can be rewritten in the form

$$\mathbf{S}_i\cdot\mathbf{S}_j=\sum_{\sigma=1}^{2}[(\varphi_{i\sigma}\varphi_{j\sigma}^{\dagger})(\varphi_{j\sigma}\varphi_{i\sigma}^{\dagger})-\tfrac{1}{2}(|\varphi_{i1}|^2+|\varphi_{i2}|^2)(|\varphi_{j1}|^2+|\varphi_{j2}|^2)]\ .$$

I introduce collective complex fields $U_{ij}(\tau)$ and $V_{ij}(\tau)$ by a Hubbard-Stratanovich transformation. Then, the Hamiltonian, Eq. (16), reads

$$\begin{aligned}h(\tau)=\int_0^{\beta}d\tau\Bigg[&-\frac{2t_1}{J_1}\sum_{\langle i,j\rangle}[\psi_j^{\dagger}U_{ji}\psi_i+\psi_i^{\dagger}U_{ij}\psi_j]+\frac{2t_1^2}{J_1}\sum_{\langle i,j\rangle}\psi_i^{\dagger}\psi_i\psi_j^{\dagger}\psi_j\\&-\sum_{\langle i,j\rangle}\left[\frac{2}{J_1}U_{ij}^{\dagger}U_{ij}-U_{ij}^{\dagger}\varphi_{i\sigma}\varphi_{j\sigma}^{\dagger}-U_{ij}\varphi_{i\sigma}^{\dagger}\varphi_{j\sigma}\right]-\frac{2t_2}{J_2}\sum_{\langle\langle i,j\rangle\rangle}[\psi_j^{\dagger}V_{ji}\psi_i+\psi_i^{\dagger}V_{ij}\psi_j]\\&+\frac{2t_2^2}{J_2}\sum_{\langle\langle i,j\rangle\rangle}\psi_i^{\dagger}\psi_i\psi_j^{\dagger}\psi_j-\sum_{\langle\langle i,j\rangle\rangle}\left[\frac{2}{J_2}V_{ij}^{\dagger}V_{ij}-V_{ij}^{\dagger}\varphi_{i\sigma}\varphi_{j\sigma}^{\dagger}-V_{ij}\varphi_{i\sigma}^{\dagger}\varphi_{j\sigma}\right]-\mu\sum_i(1-\psi_i^{\dagger}\psi_i)\Bigg]\ .\end{aligned} \tag{21}$$

The terms $(|\varphi_{i1}|^2+|\varphi_{i2}|^2)(|\varphi_{j1}|^2+|\varphi_{j2}|^2)$ are omitted because they lead to renormalization of the chemical potential and four-fermion terms.

The gauge-symmetry properties of the theory remain unchanged if the gauge transformations of the collective coordinates are

$$U_{ij}(\tau)\rightarrow e^{i\alpha(i,\tau)}U_{ij}(\tau)e^{i\alpha(j,\tau)} \tag{22}$$

and likewise for $V_{ij}(\tau)$. The purpose now is to take into account the spin fluctuations. Integrating over the Bose fields $\varphi_{i\sigma}$ ($\varphi_{i\sigma}^{\dagger}$) one obtains, in some approximation, an effective action for the collective fields. There are many mean-field approximations (phases).[22,16] I am interested in the phase with broken P and T symmetry. It is energetically preferable when $J_2/J_1>0.5$.[16]

Let us represent the collective coordinate in the form

$$U_{ij}(\tau)=\rho_1\exp i[\delta_{ij}+B_{ij}(\tau)]\ ,\qquad V_{ij}(\tau)=\rho_2\exp i[\alpha_{ij}+B_{ij}(\tau)]\ , \tag{23}$$

where ρ_1 and ρ_2 are variational parameters. The phases $\delta_{ij}=\int_i^j\mathbf{B}\cdot d\mathbf{r}$, where $\mathbf{B}$ determines a uniform "magnetic" field perpendicular to the lattice plane and with a flux through each plaquette equal to π. The phases α_{ij} are given by the relations[23–25]

$$\alpha(i_x,i_y;i_x+a,i_y+a)=\alpha(i_x,i_y;i_x-a,i_y-a)=\pi(i_x/a+\tfrac{1}{2}),$$
$$\alpha(i_x,i_y;i_x+a,i_y-a)=\alpha(i_x,i_y;i_x-a,i_y+a)=-\pi(i_x/a+\tfrac{1}{2}), \quad (24)$$

where a is the lattice spacing.

The excitations B_{ij} around the mean field must be taken into account since they are described by gauge fields defined on the 2D square lattice, while the modules of the collective fields U_{ij}, V_{ij} are gauge invariant and their fluctuations can be dropped.

The mean-field effective Hamiltonian for the Bose fields (without gauge-field fluctuations) reads

$$\begin{aligned} h_{\rm eff}(\tau)=\sum_{\sigma=1}^{2}\Bigg[&\rho_1\sum_i\{\varphi_\sigma^\dagger(i_x,i_y)\varphi_\sigma(i_x+a,i_y)+\varphi_\sigma^\dagger(i_x+a,i_y)\varphi_\sigma(i_x,i_y)\\ &+(-1)^{i_x/a}[\varphi_\sigma^\dagger(i_x,i_y)\varphi_\sigma(i_x,i_y+a)+\varphi_\sigma^\dagger(i_x,i_y+a)\varphi_\sigma(i_x,i_y)]\}\\ &+i\rho_2\sum_i(-1)^{i_x/a}[\varphi_\sigma^\dagger(i_x,i_y)\varphi_\sigma(i_x+a,i_y+a)-\varphi_\sigma^\dagger(i_x,i_y)\varphi_\sigma(i_x+a,i_y-a)\\ &-\varphi_\sigma^\dagger(i_x+a,i_y+a)\varphi_\sigma(i_x,i_y)+\varphi_\sigma^\dagger(i_x+a,i_y-a)\varphi_\sigma(i_x,i_y)]\Bigg]. \end{aligned} \quad (25)$$

In terms of new fields,[25,26] $\Phi_\sigma(i_x,i_y)$ related to $\varphi_\sigma(i_x,i_y)$ by

$$\varphi(i_x,i_y)=i^{(i_x+i_y)/a}(\sigma_2)^{i_y/a}(\sigma_1)^{i_x/a}\Phi(i_x,i_y),$$

where σ_1 and σ_2 are Pauli matrices, the Hamiltonian takes the form

$$\begin{aligned} h_{\rm eff}(\tau)=i\rho_1\sum[&\Phi^\dagger(i_x,i_y)\sigma_1\Phi(i_x+a,i_y)-\Phi^\dagger(i_x+a,i_y)\sigma_1\Phi(i_x,i_y)\\ &+\Phi^\dagger(i_x,i_y)\sigma_2\Phi(i_x,i_y+a)-\Phi^\dagger(i_x,i_y+a)\sigma_2\Phi(i_x,i_y)]\\ -\rho_2\sum[&\Phi^\dagger(i_x,i_y)\sigma_3\Phi(i_x+a,i_y+a)+\Phi^\dagger(i_x,i_y)\sigma_3\Phi(i_x+a,i_y-a)\\ &+\Phi^\dagger(i_x+a,i_y+a)\sigma_3\Phi(i_x,i_y)+\Phi^\dagger(i_x+a,i_y-a)\sigma_3\Phi(i_x,i_y)]. \end{aligned} \quad (26)$$

In momentum space,

$$\begin{aligned} h_{\rm eff}=-\int\frac{d^2k}{(2\pi)^2}\{&2\rho_1[\Phi^\dagger(k)\sigma_1\Phi(k)\sin(k_xa)+\Phi^\dagger(k)\sigma_2\Phi(k)\sin(k_ya)]\\ &+4\rho_2\Phi^\dagger(k)\sigma_3\Phi(k)\cos(k_xa)\cos(k_ya)\}=\int\frac{d^2k}{(2\pi)^2}\Phi^\dagger H\Phi, \end{aligned} \quad (27)$$

where $-\pi/a\le k_x,k_y\le\pi/a$.

The eigenvalues of the matrix H are $E^\pm=\pm\varepsilon(k)$,

$$\varepsilon(k)=2\{\rho_1^2[\sin^2(k_xa)+\sin^2(k_ya)]+4\rho_2^2\cos^2(k_xa)\cos^2(k_ya)\}, \quad (28)$$

and the free energy of the spin fluctuations in the mean-field approximations

$$F=-\frac{4\rho_1^2}{J_1}-\frac{4\rho_2^2}{J_2}+\frac{2a^2}{\beta}\int\frac{d^2k}{(2\pi)^2}\ln(e^{\beta\varepsilon/2}-e^{-\beta\varepsilon/2}). \quad (29)$$

The mean-field equations

$$\frac{\partial F}{\partial\rho_1}=0,\quad \frac{\partial F}{\partial\rho_2}=0 \quad (30)$$

have a nontrivial ($\rho_1\neq0$, $\rho_2\neq0$) solution. Numerical calculations show, for example, that in the region $0,1<\rho_2/\rho_1<2,5$, a solution exists if $1<J_1/J_2<1,7$, and that $\rho_2/\rho_1<1$ if $1,1<J_1/J_2<1,7$.

To obtain the effective action for the gauge fields $B(\tau)$ and $B_{ij}(\tau)$, one has to integrate over the Bose fields φ ($\varphi^\dagger$). Let us denote the result of integration as

$$\exp\{-W(B)\}. \quad (31)$$

By definition, W is equal to Tr ln$D$, where D is the operator in the quadratic form of the fields φ [see Eqs. (21) and (23)]. The minus sign is due to the Bose statistics of the fields. I am interested in the terms in W which break the T and P symmetry. To obtain them, it is enough to consider the continuum limit of the theory in the vicinity of the degenerate points in the Brillouin zone.[25,26] In the case under consideration, we have four such points:[25]

$$(k_x^*,k_y^*)=(0,0),\ (0,\pi/a),\ (\pi/a,0),\ \text{and}\ (\pi/a,\pi/a)\ . \tag{32}$$

Expanding the eigenvalues of the Hamiltonian, Eq. (28), around these points, one obtains the dispersion law of a relativistic particle with mass $m=2\rho_2/a\rho_1$,

$$\varepsilon(k)=2a\rho_1[k_x^2+k_y^2+m^2]^{1/2}\ .$$

Therefore, effectively, there are four relativistic particles in the theory and W can be represented as a sum of four terms. Each of them is a $\mathrm{Tr}\ln D^*$, where D^* is the operator D in the vicinity of the degenerate points. It is straightforward to see[25,26,17] that, near any one of the points (32), the operator D is exactly the Dirac operator in 3D Euclidean space:

$$D^0=\tilde{\gamma}_\mu(\partial_\mu-iB_\mu)+m\ , \tag{33}$$

where $\tilde{\gamma}_1=\sigma_1$, $\tilde{\gamma}_2=\sigma_1$, $\tilde{\gamma}_3=v\sigma_3$, $v^{-1}=2a\rho_1$, $B=B_3$, and

$$B_{ij}=\int_i^j\sum_{k=1}^{2}B_k dx_k\ .$$

Making use of the well-known result for D^0 one obtains[27]

$$W=4\,\mathrm{Tr}\ln D^0=\frac{i}{2\pi}\int d^3x\ \epsilon_{\mu\nu\lambda}B_\mu\partial_\nu B_\lambda+\tilde{W}(B)\ .$$

The last term is gauge invariant and preserves the T and P symmetry. When the gauge-field fluctuations are smaller than the mass ($|\partial_\mu B_\nu-\partial_\nu B_\mu|\ll m$), one can calculate $\tilde{W}$ in the large-m limit. The final result is

$$W(B)=\int d^3x[1/g^2(c^2E_i^2+H^2)+(i/2\pi)\epsilon_{\mu\nu\lambda}B_\mu\partial_\nu B_\lambda]\ , \tag{34}$$

where $c=v^{-1}=2a\rho_1$, $g^2=6\pi|m|$, $E_i=\partial_3B_i-\partial_iB_3$ $(i=1,2)$, and $H=\partial_1B_2-\partial_2B_1$.

I want to stress the Bose statistics of the φ (Φ) fields which reflects in the minus sign in Eq. (31). This fact, being technical, has far-reaching consequences and I shall elucidate it more deeply. As we know,[28] the kinetic term of the gauge fields induced by fermions has a sign opposite to the sign in the Maxwell theory leading to the destabilization of the gauge-field fluctuations. In contrast with this, in the case under consideration, the induced kinetic term has a true sign, due to the Bose statistics, and the effective theory, Eq. (34), describes stable short-range excitations (scalar particle with mass $M\equiv 2g^2/\pi$).[29]

B. Hole dynamics and superconductivity

The fermions (holes) ψ_i are coupled to chiral spin excitations and create an anyonlike system with effective action

$$\begin{aligned}A_{\text{eff}}=W(B)+\int d\tau\Bigg\{&\sum_j\left[\psi_j^\dagger(\tau)\left[\frac{d}{d\tau}-iB_j(\tau)\right]\psi_j(\tau)+iB_j(\tau)\right]\\&-\frac{2t_1}{J_1}\sum_{\langle i,j\rangle}\rho_1[\psi_j^\dagger(\tau)e^{-i\delta_{ij}}\psi_i(\tau)e^{-iB_{ij}(\tau)}+\psi_i^\dagger(\tau)e^{i\delta_{ij}}\psi_j(\tau)e^{iB_{ij}(\tau)}]\\&-\frac{2t_2}{J_2}\sum_{\langle\langle i,j\rangle\rangle}\rho_2[\psi_j^\dagger(\tau)e^{-i\alpha_{ij}}\psi_i(\tau)e^{-iB_{ij}(\tau)}+\psi_i^\dagger(\tau)e^{i\alpha_{ij}}\psi_j(\tau)e^{iB_{ij}(\tau)}]\\&+\frac{2t_1^2}{J_1}\sum_{\langle i,j\rangle}\psi_i^\dagger(\tau)\psi_i(\tau)\psi_j^\dagger(\tau)\psi_j(\tau)+\frac{2t_2^2}{J_2}\sum_{\langle\langle i,j\rangle\rangle}\psi_i^\dagger(\tau)\psi_i(\tau)\psi_j^\dagger(\tau)\psi_j(\tau)-\mu\sum_i[1-\psi_i^\dagger(\tau)\psi_i(\tau)]\Bigg\}\ .\end{aligned} \tag{35}$$

where $W(B)$ is given by Eq. (34).

Varying with respect to $\mathbf{B}_j(\tau)$ $(=B_3)$, we find one of the field equations which, in terms of the Green's functions, has the form

$$\tfrac{1}{2}\langle\psi_j^\dagger\psi_j-1\rangle=\frac{1}{2\pi}\langle\partial_1B_2-\partial_2B_1\rangle-i\frac{c^2}{g^2}\langle\partial_iE_i\rangle\ . \tag{36}$$

This equation implies that the path integral over the gauge fields B_μ is defined in a space of functions $B_\mu(x)$ which determine nonzero "magnetic" strength $\partial_1B_2-\partial_2B_1$ when the space coordinates go to infinity $\partial_1B_2-\partial_2B_1\to\mathbf{F}\neq 0$.

It is convenient, in this case, to redefine the gauge fields

$$B_\mu\to B_\mu+B_\mu^0\ , \tag{37}$$

so that these gauge fields have a zero asymptote at infinity (up to gauge transformations). I choose $B_3^0=0$, B_1^0, B_2^0 time independent and $\partial_1 B_2 - \partial_2 B_1 = \mathbf{F}$, where $\mathbf{F}$ is a nonzero constant. Under this transition, the gauge-field action (34) acquires an unessential linear term and the phases δ_{ij} and α_{ij} in Eq. (35) get the additions $\delta_{ij} \to \delta_{ij} + \delta_{ij}^0$, $\alpha_{ij} \to \alpha_{ij} + \alpha_{ij}^0$.

I should like to determine δ_{ij}^0 in such a way that the Chern-Simons term, induced by the fermions (holes), cancels that in the action (34). It is known from the theory of 2D electron hopping in a magnetic field (Hofstader's problem[30,31]) that the energy spectrum of the hopping Hamiltonian has $\mathbf{p}$ isolated zeros (degenerate points) if the magnetic flux through each plaquette is $2\pi\mathbf{q}/\mathbf{p}$ with $\mathbf{p}$ even. Each zero corresponds to a (2+1)-dimensional Dirac fermion in the continuum limit. In the case under consideration, the problem is involved due to the presence of frustrations, but one expects that the general statement about the number of fermion spaces remains true.

Let us make a change of variables (37) in the action (36) with $B^0=(0, -\pi x_1/2a^2)$. Then,

$$\delta_{ij}+\delta_{ij}^0 = \begin{cases} 0, & j_x = i_x + a \text{ and } j_y = i_y\ , \\ \dfrac{\pi}{2a} i_x, & j_x = i_x \text{ and } j_y = i_y + a\ , \end{cases}$$

and

$$\alpha(i_x,i_y;i_x+a,i_y+a)+\alpha^0(i_x,i_y;i_x+a,i_y+a) = \frac{\pi}{2}(i_x/a+\tfrac{1}{2})\ ,$$

$$\alpha(i_x,i_y;i_x+a,i_y-a)+\alpha^0(i_x,i_y;i_x+a,i_y-a) = -\frac{\pi}{2}(i_x/a+\tfrac{1}{2})\ .$$

The effective Hamiltonian of the fermions (holes), without four-fermion terms and gauge-field fluctuations, reads

$$h_f = -\frac{2t_1\rho_1}{J_1}\sum_{\langle i,j\rangle}[\psi_j^\dagger e^{-i(\delta_{ij}+\delta_{ij}^0)}\psi_i + \psi_i^\dagger e^{i(\delta_{ij}+\delta_{ij}^0)}\psi_j] - \frac{2t_2\rho_2}{J_2}\sum_{\langle\langle i,j\rangle\rangle}[\psi_j^\dagger e^{-i(\alpha_{ij}+\alpha_{ij}^0)}\psi_i + \psi_i^\dagger e^{i(\alpha_{ij}+\alpha_{ij}^0)}\psi_j]\ . \tag{38}$$

The term with chemical potential is dropped because μ can be regarded as a constant shift of the field $B_i(\tau)$ in zero-temperature case.

In the momentum space,

$$h_f = -\int\frac{d^2k}{(2\pi)^2}\left\{\theta_1\left[2\cos(ak_x)\psi^\dagger(k_x,k_y(\psi(k_x,k_y)+\psi^\dagger\left(k_x+\frac{\pi}{2a},k_y\right)\psi(k_x,k_y)\exp(iak_y)+\text{H.c.}\right]\right.$$
$$+\theta_2\left[\psi^\dagger\left(k_x+\frac{\pi}{2a},k_y\right)\psi(k_x,k_y)\exp(iak_x+iak_y+i\pi/4)+\text{H.c.}\right.$$
$$\left.\left.+\psi^\dagger\left(k_x-\frac{\pi}{2a},k_y\right)\psi(k_x,k_y)\exp(iak_x-iak_y-i\pi/4)+\text{H.c.}\right]\right\}\ , \tag{39}$$

where $-\pi/a \le k_x, k_y \le \pi/a$, $\theta_1 = 2t_1\rho_1/J_1$, and $\theta_2 = 2t_2\rho_2/J_2$. I introduce a four-component "spinor" $\psi_s(k)$,

$$\begin{aligned} \psi_1(k_x,k_y) &= \psi\left(k_x+\frac{\pi}{2a},k_y\right)\ , \\ \psi_2(k_x,k_y) &= \psi\left(k_x+\frac{\pi}{2a},k_y\right) \\ \psi_3(k_x,k_y) &= \psi\left(k_x+\frac{3\pi}{2a},k_y\right)\ , \\ \psi_4(k_x,k_y) &= \psi\left(k_x+\frac{2\pi}{a},k_y\right)\ . \end{aligned} \tag{40}$$

In terms of the fields $\psi_s(k)$, the Hamiltonian has the form

$$h_f = \int\frac{d^2k}{(2\pi)^2}\psi^\dagger(k)\mathbf{H}\psi(k)\ , \tag{41}$$

where k_x and k_y run over the reduced Brillouin zone

$$-\pi/4a \le k_x \le \pi/4a,\quad -\pi/a \le k_y \le \pi/a\ , \tag{42}$$

and a 4×4 matrix $\mathbf{H}$ is given by

$$\mathbf{H} = \begin{bmatrix} -a_1 & z^*b_1^- & 0 & zb_2^+ \\ zb_1^- & -a_2 & z^*b_2^- & 0 \\ 0 & ab_2^- & a_1 & z^*b_1^+ \\ z^*b_2^+ & 0 & zb_1^+ & a_2 \end{bmatrix}\ , \tag{43}$$

where $z=\exp(iak_y)$, $a_1=2\theta_1\sin(ak_x)$, $a_2=2\theta_1\cos(ak_x)$,

$$b_1^{\pm}=\theta_1\pm 2\theta_2\cos(ak_x-\pi/4)\ ,$$

and

$$b_2^{\pm}=\theta_1\pm 2\theta_2\cos(ak_x+\pi/4)\ .$$

The secular equation

$$E^4-8(\theta_1^2+\theta_2^2)E^2+16\sqrt{2}\theta_1^2\theta_2 E -16\theta_2^4\sin^2(2ak_x)\sin^2(2ak_y) +4(\theta_1^2-2\theta_2^2)^2[\sin^2(2ak_x)+\sin^2(2ak_y)]=0 \quad (44)$$

has four roots for fixed values of k_x and k_y. Therefore, the original band for the tight-binding model is split into four bands.

Let us denote the ratio θ_2/θ_1 by λ. At the points $\lambda^*=0$, $k^*=(0,0)$, and $(0,\pm\pi/2a)$ of the three-dimensional space (λ,k_x,k_y), two bands touch. In the vicinity of any of these points the corresponding eigenvalues of matrix (43) have a form of the energy of the relativistic particle with mass $M=\lambda/a$,

$$E(q)=\pm\hat{c}(M^2+q_x^2+q_y^2)^{1/2}\ ,$$

where $\hat{c}=3a/\sqrt{2}$ and $q_i=k_i-k_i^*$. Therefore, in the continuum limit for small λ, one obtains an effective theory of four[32] Dirac fermions coupled to the gauge fields B_μ. In the presence of an electromagnetic field A_μ, the effective action reads

$$L_f=\sum_{\alpha=1}^{4}[\bar{\psi}_\alpha\hat{\gamma}_\mu(\partial_\mu-iB_\mu+i\mathbf{e}A_\mu)\psi_\alpha+M\bar{\psi}_\alpha\psi_\alpha]\ , \quad (45)$$

where $\hat{\gamma}_i=\gamma_i$ $(i=1,2)$, $\hat{\gamma}_3=\hat{c}^{-1}\gamma_3$, γ_μ are 3D Euclidean Dirac matrices, and $\mathbf{e}$ is the electromagnetic charge of the holes.

From Eqs. (34), (35), and (45) and integrating out the fermions, one obtains an effective Lagrangian for the statistical gauge fields B_μ and the electromagnetic potential A_μ:

$$L_{\text{eff}}=(1/g^2)[c^2E_i^2(B)+H^2(B)]+\frac{i}{2\pi}\epsilon_{\mu\nu\lambda}B_\mu\partial_\nu B_\lambda+iB_3 -(1/\hat{g}^2)[\hat{c}^2E_i^2(B-\mathbf{e}A)+H^2(B-\mathbf{e}A)] -\frac{i\kappa}{2\pi}\epsilon_{\mu\nu\lambda}(B_\mu-\mathbf{e}A_\mu)\partial_\nu(B_\lambda-\mathbf{e}A_\lambda) +L_M(A)+iR\partial_iB_i\ , \quad (46)$$

where $\hat{g}^2=6\pi|M|$, the Chern-Simons coefficient is $\kappa=\text{sgn}\lambda$, and the Maxwell Lagrangian $L_M(A)$ for the electromagnetic field is added. The last term in (46) is the gauge-fixing term. It was systematically dropped in previous formulas. I use the Coulomb gauge $\partial_iB_i=0$, and R is the corresponding Lagrange multiplier.

It is clear that when $\lambda>0$, the Chern-Simons term induced by fermions exactly cancels the Chern-Simons term in Eq. (35). Hence, the Banks-Lykken condition for superconductivity is satisfied.

Equation (36) yields the connection between the holes' density and B^0. With the above choice of B^0 (respectively, δ_{ij}^0 and α_{ij}^0), the number of holes per lattice site is equal to $\frac{1}{2}$ [the filling factor $\nu=\frac{1}{2}(1-\langle\psi^\dagger\psi\rangle)$ is equal to $\frac{1}{4}$]. Therefore, a system with a filling factor $\frac{1}{4}$ and small positive λ is a superconductor.

It is easy to check that, at the points $(\lambda^*)^2=\frac{1}{4}(2\pm\sqrt{2})$ and $k^*=(\pm\pi/4a,\pm\pi/4a)$, where the $\pm$ signs are not correlated, two bands touch too. Following the same scheme as above, one obtains, in the continuum limit and for small $\Delta\lambda=\lambda-\lambda^*$, the effective action (45) with mass

$$M^2=(2\mp\sqrt{2})(\Delta\lambda)^2/a^2\ .$$

The Chern-Simons coefficient is $\kappa=\text{sgn}\Delta\lambda$; however, the filling is not the same when the Fermi energy is just at this degenerate mode.

The above discussion is a special case of Ref. 33. To obtain all values of λ for which $\kappa=1$, one needs more refined methods.

One can take into account the four-fermion terms also. For that purpose, it is convenient to introduce the collective coordinates $b_i(\tau)=\psi_i^\dagger(\tau)\psi_i(\tau)$, and the corresponding Lagrange multiplier $d_i(\tau)$. Then the four-fermion terms can be rewritten in the form

$$L_{\text{ff}}=\frac{2t_1^2}{J_1}\sum_{\langle i,j\rangle}b_i(\tau)b_j(\tau)+\frac{2t_2^2}{J_2}\sum_{\langle\langle i,j\rangle\rangle}b_i(\tau)b_j(\tau) +\sum_i d_i(\tau)[b_i(\tau)-\psi_i^\dagger(\tau)\psi_i(\tau)]\ .$$

One can set the collective coordinate equal to their mean-field value

$$\langle d_i(\tau)\rangle=\mathbf{b}(-1)^{(i_x+i_y)/a}\ ,$$

$$\langle b_i(\tau)\rangle=\mathbf{b}(-1)^{(i_x+i_y)/a}\ ,$$

where $\mathbf{b}$ and $\mathbf{d}$ are constants. Then the effective Hamiltonian (38) gets an additional term

$$\mathbf{d}\sum_i(-1)^{(i_x+i_y)/a}\psi_i^\dagger\psi_i\ .$$

This term makes the calculations technically more difficult, but the final result is qualitatively the same.

Let us choose the parameters of the theory in such a way that the Chern-Simons coefficient in (46) $\kappa=1$. The mean-field approximation is correct if the statistical gauge fluctuations are small and stable. The last condition is implemented if

$$\frac{1}{2\epsilon}=\frac{c^2}{g^2}-\frac{\hat{c}^2}{\hat{g}^2}$$

and (47)

$$\frac{\chi}{2}=\frac{1}{g^2}-\frac{1}{\hat{g}^2}$$

are positive. Make a transformation $B_\mu\to B_\mu+\mathbf{e}A_\mu$ in the Lagrangian (46). Then one obtains

$$L_{\rm eff}=\frac{1}{2\epsilon}E_i^2(B+{\rm e}A)+\frac{\chi}{2}H^2(B+{\rm e}A)+\frac{i{\rm e}}{\pi}\epsilon_{\mu\nu\lambda}B_\mu\partial_\nu A_\lambda+iB_3+iR\partial_iB_i+\frac{2{\rm e}}{\hat{g}^2}[\hat{c}^2E_i^2(A)E_i(B)+H(A)H(B)]$$
$$+L_M(A)+\frac{{\rm e}^2}{\hat{g}^2}[\hat{c}^2E_i^2(A)+H^2(A)]+\frac{i{\rm e}^2}{2\pi}\epsilon_{\mu\nu\lambda}A_\mu\partial_\mu A_\lambda+i{\rm e}D\partial_iA_i\ , \tag{48}$$

where ϵ and χ are given by Eqs. (47).

Make a dual transformation[9,10] introducing the fields $e_i(x)$ $(i=1,2)$ and $b(x)$. The Lagrangian reads

$$L_{\rm eff}=\frac{\epsilon}{2}e_i^2+\frac{1}{2\chi}b^2+ie_iE_i(B+{\rm e}A)+ibH(B+{\rm e}A)+iR\partial_iB_i$$
$$+iB_3+i\epsilon_{\mu\nu\lambda}B_\mu\partial_\nu A_\lambda+\frac{2{\rm e}}{\hat{g}^2}[\hat{c}^2E_i(A)E_i(B)+H(A)H(B)]$$
$$+L_M+\frac{{\rm e}^2}{\hat{g}^2}[\hat{c}^2E_i^2(A)+H^2(B)]+i\frac{{\rm e}^2}{2\pi}\epsilon_{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda+{\rm e}iR\partial_iA_i\ . \tag{49}$$

Now, let us make the transformations

$$e_i\to e_i+i\frac{2{\rm e}\epsilon\hat{c}^2}{\hat{g}^2}E_i(A)-\frac{{\rm e}}{\pi}\epsilon_{ij}A_j\ ,$$
$$b\to b+i\frac{2{\rm e}}{\hat{g}^2}H(A)-\frac{{\rm e}}{\pi}A_3\ , \tag{50}$$

in the Lagrangian (49). The result is

$$L_{\rm eff}=\frac{\epsilon}{2}\left[e_i-\frac{{\rm e}}{\pi}\epsilon_{ij}A_j\right]^2+\frac{1}{2\chi}\left[b-\frac{{\rm e}}{\pi}A_3\right]^2+i{\rm e}\beta_1\left[e_i-\frac{{\rm e}}{\pi}\epsilon_{ij}A_j\right]E_i(A)+i{\rm e}\beta_2\left[b-\frac{{\rm e}}{\pi}A_3\right]H(A)$$
$$+i\frac{{\rm e}^2}{2\pi}\epsilon_{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda+{\bf eR}\partial_iA_i+L_M^r+B_i(-i\partial_3e_i-i\partial_iR-\epsilon_{ij}\partial_jb)+iB_3(\partial_ie_i+1)\ , \tag{51}$$

where $\beta_1=1+2{\rm e}\epsilon\hat{c}^2/\hat{g}^2$, $\beta_2=1+2{\rm e}/\hat{g}^2\chi$, and L_M^r is the renormalized Maxwell Lagrangian

$$L_M^r=L_M+\frac{1}{2\epsilon_r}E_i^2(A)+\frac{\chi_r}{2}H(A)\ ,$$

where $1/(2\epsilon_r)=({\rm e}^2\hat{c}^2/\hat{g}^2)(1-2\epsilon\hat{c}^2/\hat{g}^2)$ and

$\chi_r/2=({\rm e}^2/\hat{g}^2)(1-2/\chi\hat{g}^2)$.

The effective Lagrangian (51) depends on B_μ linearly and the gauge fields can be integrated out, yielding the constraints on the fields e_i, b, and R:

$$\partial_3e_i-\epsilon_{ij}\partial_ib+\partial_iR=0\ ,$$
$$\partial_ie_i+1=0\ . \tag{52}$$

Let us represent e_i and b in the form

$$e_i=\epsilon_{ij}\partial_j\omega+\frac{\partial_i}{\Delta}\eta\quad(\Delta=\partial_1^2+\partial_2^2)\ ,$$
$$b=\partial_3\omega+f\ . \tag{53}$$

Putting (53) in (52), I obtain three equations for the functions R, f, and η:

$$\eta+1=0,\quad -\partial_2f+\partial_1R=0,\quad \partial_1f+\partial_2R=0\ . \tag{54}$$

In the space of function with a zero asymptote at infinity, the system only has the solution $R=f=0$ and one obtains $e_i=\epsilon_{ij}\partial_j\omega$ and $b=\partial_3\omega$. Substituting into Eq. (51) the expressions for e_i, b, and R, I obtain the effective Ginzburg-Landau Lagrangian in 3D Euclidean space:

$$L_{\rm eff}=\frac{1}{2\chi}\left[\partial_3\omega+\frac{e}{\pi}A_3\right]^2+\frac{\epsilon}{2}\left[\partial_i\omega+\frac{e}{\pi}A_i\right]^2$$
$$-i{\rm e}\left[\beta_1\epsilon_{ij}E_i(A)\left[\partial_j\omega+\frac{{\rm e}}{\pi}A_j\right]+\beta_2H(A)\left[\partial_3\omega+\frac{{\rm e}}{\pi}A_3\right]\right]$$
$$+i\frac{{\rm e}^2}{2\pi}\epsilon_{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda+L_M^r\ , \tag{55}$$

where $\omega(x)$ is the Goldstone boson, and the effects of the broken P and T symmetry are due to the third and fourth terms.

IV. CONCLUSIONS

In this paper the relation between the strongly correlated electronic systems and the anyons was explicitly established. I studied the frustrated Hubbard-like model. An adequate U(1) gauge theory [Eqs. (16)–(19)] was obtained making use of the coherent states for SU(2|1) superalgebra. Different parametrizations of the coherent space (different representations of the electron operators) favor different mean-field schemes. I chose Bose fields to describe the spin fluctuations. In the flux phase, when the system is sufficiently frustrated, they generate stable, short-range excitations around this mean field. An effective topologically massive gauge theory [Eq. (34)] was obtained. Another peculiarity arises in the finite-temperature case. It is well known[34] that fermions generate a Chern-Simons term with a coefficient which depends on the temperature as $\tanh(M/T)$. Hence, when the temperature T goes to infinity, the coefficient goes to zero. In contrast of this, in the case under consideration, the Chern-Simons term depends on the temperature as $\coth(M/T)$ (due to the Bose statistic of the spin fluctuations), and it increases when T increases.

I studied a flux phase with a flux through each plaquette equal to π. Most generally, one can put it equal to $2\pi\mathbf{p}/\mathbf{q}$. Then, following the same consideration, one obtains an effective Chern-Simons theory with a Chern-Simons coefficient $2\mathbf{q}/8\pi$ (spin-up and spin-down excitations are taken into account). But, the coefficient in front of the kinetic term linearly depends on $\mathbf{q}$ too. Hence, the correlation length of the chiral spin excitations (the mass of the scalar boson), which is proportional to the ratio of the two coefficients, is independent of the choice of the flux. Nevertheless, there are arguments[35] that the chiral spin states are also characterized by an integer appearing in front of the Chern-Simons term ($2\mathbf{q}$). This integer can be measured by measuring the vacuum degeneracy of the states on a torus.

I also studied the dynamics of the holes. Coupled to the statistical gauge potential, they form an anyonlike gas. I showed that a system with filling factor $\frac{1}{4}$ is a superconductor. The fact that superconductivity appears only for a separate value of the hole density confuses, but this is a consequence of the simplification of the model. A more significant problem is the finite-temperature behavior of the system. In this case, the spin fluctuations generate an effective gauge theory with Chern-Simons coefficient which depends on the temperature. It is easy to see from Eq. (36) that, for a fixed hole density, the phases δ_{ij}^0 and α_{ij} depend on the temperature. Then, the hole dynamics is described by an effective hopping Hamiltonian in a "magnetic" field which depends on a parameter (temperature). The solution of this problem is not clear.

After submission of this manuscript, I learned of Wiegmann's paper Ref. 36. This author is concerned with the microscopic background of anyonic superconductivity and its relations with the doped Mott system too. Using a somewhat different approach, the author comes to the same conclusions about the superconductivity of the system and the same Ginzburg-Landau effective action is obtained.

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