Self-fields and critical-current-density anisotropy of high-temperature superconductors

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Hysteresis loops of high-temperature superconductors often exhibit a peak in the irreversible magnetization around zero field. A mechanism for such a peak is proposed, involving self-fields in platelet-shaped samples which bend vortices in plane, generating a contribution from a large in-plane component of critical current density. Numerical electromagnetic calculations demonstrate this effect and show that its size depends strongly on the presence of an additional isotropic component.

Hysteresis loops of high-temperature superconductors are usually understood[1–4] in terms of the Beam critical-state model.[5,6] However several features remain puzzling. One is the frequent appearance of a peak in the irreversible magnetization $M(H)$ around zero field, which, particularly at low temperatures, is superimposed on a broad background of irreversible magnetization stretching to very high fields. Good recent examples are in Fig. 1(a) of Ref. 7 and Fig. 2 of Ref. 8 and they resemble the calculated curve of Fig. 1 of this paper, which will be explained further below. In many cases this peak occurs in platelet-shaped crystals with both c axis and applied field perpendicular to the plate, although similarly shaped peaks have also been observed with field applied in the (a,b) plane.

A possible explanation for this peak or other low-field anomalies has been sought in connection with the lower critical field,[2] although simulations show such effects to be small.[1] Another possible mechanism comes from the field-dependence of the critical current density,[1] that is, a dependence of a given current density component J_c on the *amplitude* (rather than the direction) of the local field. Measurements with **H** parallel to the (a,b) plane in Y-Ba-Cu-O crystal platelets should probably be interpreted in this way.

In this paper we propose a third possible mechanism which could apply in some—though certainly not all—cases. The mechanism involves in-plane self-fields in platelet-shaped samples, combined with a highly anisotropic critical current density[9–12] which is maximum when the vortices lie in the (a,b) plane. In other words, J_c and hence the irreversible magnetization depend on the *direction* of the local field.

The idea for this mechanism for the peak emerges from recent electromagnetic calculations of the self-field distribution inside a disk-shaped superconductor with isotropic critical current density in the critical state.[13,14] At remanence, for example, these calculations reveal field lines strongly distorted from the vertical direction (the z axis) perpendicular to the plane. Over much of the sample, the lines lie horizontal, in a way which might be described crudely as a severely flattened dipole distribution, illustrated schematically in Fig. 2(a). The field lines circle around a ring lying in the center of the disk and about $0.15R$ from the edge of the disk of radius R. The self-fields are typically of size $J_c t$ (where J_c is the critical current density and t is the thickness), except for logarithmic divergences near the axis and edge of the disk. In the presence of an applied field H_a, one must add vectorially H_a to the self-fields generated by the critical currents, and thus the local field pattern "straightens out" [see Fig. 2(b)] as H_a increases.

We emphasize that these earlier calculations assumed an *isotropic* current density. However data[10–12,15,16] on the high-temperature superconductors reveal a highly anisotropic current density. In particular, for our Y-Ba-Cu-O crystals,[12] the component $J_c^{(a,b),(a,b)}$, corresponding

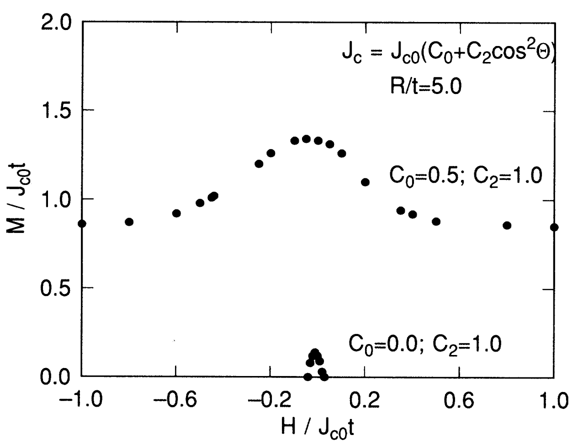

FIG. 1. Calculated normalized irreversible magnetization M vs applied field H_a, for a disk with $R/t=5$ in the critical state, with a local-field-direction-dependent critical current density, described by Eq. (1) with $C_2=1$. C_0, the strength of the isotropic component of the critical current density, is 0.5 and 0. The solutions for $C_0=0$ proved to be reproducible but unstable. The height of this peak may actually be lower than what we have shown. The comparison of the two shows the considerable enhancement of the peak in $M(H)$ with increasing C_0.

to current and field both in the (a,b) plane but perpendicular to each other, appears to be larger at low temperatures, by at least a factor of 5, than the component $J_c^{(a,b),c}$, corresponding to current along (a,b) but field along c. Transport measurements[10,11] on Y-Ba-Cu-O films suggest the same trend. $J_c^{(a,b),(a,b)}$ is the component usually associated with "intrinsic pinning."

The field distribution arising from currents flowing around a disk in the (a,b) plane has components in both (a,b) and c directions. Thus one can expect both components $J_c^{(a,b),(a,b)}$ and $J_c^{(a,b),c}$ to contribute to the irreversible magnetization M at remanence. When the field pattern straightens out with increasing applied field along the c axis, the irreversible magnetization should become increasingly dominated by the component $J_c^{(a,b)c}$ and the contribution from $J_c^{(a,b),(a,b)}$ will decrease. If $J_c^{(a,b),(a,b)}$ is larger than $J_c^{(a,b),c}$, one can then expect the irreversible magnetization to decrease as the applied field exceeds the typical self-fields of the disk; this causes a peak in $M(H_a)$, centered near $H_a=0$.

To confirm the plausibility of this mechanism we have performed numerical electromagnetic calculations of the field and current distributions in a disk with an anisotropic critical current density. We use the same methods as in the earlier calculations[13,14] with isotropic current density. We describe the angular dependence with a power series in $\cos^2\theta$,

$$J_c = J_{c0}(C_0 + C_2\cos^2\theta + C_4\cos^4\theta + \cdots), \quad (1)$$

where θ is the net field angle out of the disk plane (vector addition of the self-fields and applied field) and the C_i are dimensionless constants. Let us consider the simplest case with only C_0 and C_2 terms; J_c is maximum when the local field is in the plane, minimum when the field is normal, along the z axis, provided $C_2>0$. C_0 represents an additional isotropic component, whose role, as we shall see, is quite significant.

Our procedure is to start with a uniform azimuthal (circumferential) current density throughout the entire disk and to calculate the field distribution by numerical integration of standard equations[14,17] for the fields around a current ring. Then, applying Eq. (1) we recalculate a current distribution through the disk, recalculate the fields, and so forth, iterating until the results converge to a stable solution within some given numerical accuracy. Details are given in Ref. 17. Then the magnetization is calculated by the usual integration of the current density over the volume which is then divided by the volume. This calculation entirely ignores forces due to vortex curvature,[18] on the assumption that in large samples such as crystals this contribution is small.

An example of the distribution of radial (H_r) and axial (H_z) fields along a radius, obtained by this method, is shown in Fig. 3(a) for the case $C_0=0$, $C_2=1$, all other $C_i=0$ and $R/t=5$. This distribution differs considerably from that found earlier for uniform current density.[13,14] Instead of a roughly uniform radial field across most of the surface, H_r is strongly peaked near the outer edge of the sample, while H_z changes sign at the same place. The current is concentrated in this same region. Thus the extreme anisotropic case appears to give something much closer to the simple dipolar field distribution from a current ring.

Integrating the current, and also adding results obtained in the presence of an applied field, we found the magnetization dependence shown in Fig. 1. All fields are normalized by $J_{c0}t$. The results for $C_0=0$ show a peak near $H=-0.01$. As explained earlier, the applied field reduces the magnetization to zero because the net field lines straighten out $(\theta\rightarrow\pi/2)$ with increasing applied field, and so $J_c\rightarrow 0$ according to Eq. (1). However, these results for $C_0=0$ are unstable and erratic with continued iteration of the program; we show here only the most frequent values.

Some insight into the ringlike current distribution can be obtained by considering the symmetry of the radial fields, which must change sign above and below the midplane and which therefore must be zero at the midplane. If $C_0=0$ and only the angle-dependent terms are present, symmetry requires that the current be zero at the midplane. But then Maxwell's equation dictates that $(dB_z/dr)-(dB_r/dz)=0$ there (we ignore here the difference between B and H, assuming that H_{c1} is small). This means that the gradients of B_z along the radial direction must be comparable to those of B_r along the z direction, and hence that the currents must be localized

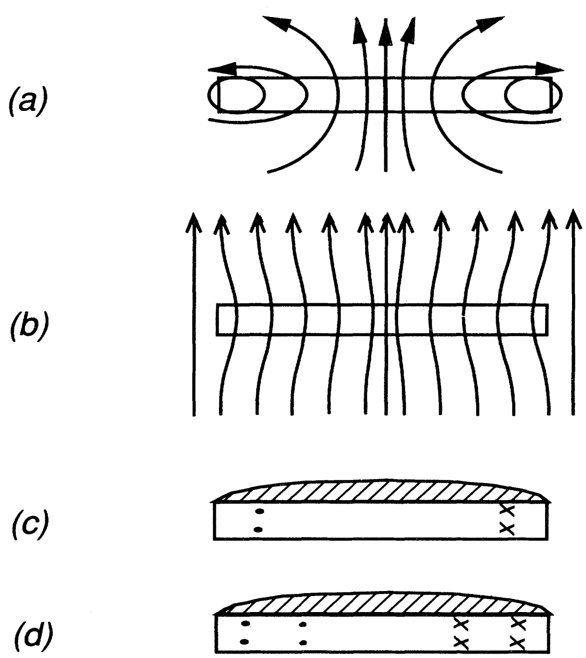

FIG. 2. Schematic field lines for a vertical cross section of a disk (a) at remanence and (b) with an applied field H_a, for the case of uniform current density circulating circumferentially around the disk ($C_0=1$, $C_2=0$). The applied field causes the net local field in the sample to straighten out parallel to the z axis along the disk normal. (c) and (d) show schematic current distributions for a disk with an extreme anisotropic current density ($C_0=0$, $C_0=1$), for smaller and larger aspect ratios, respectively.

on a comparable scale in the two directions, namely, on the scale of the disk thickness t. Thus we can expect two loops, symmetrically displaced on either side of the midplane, as shown schematically in Fig. 2(c). From the outside, this distribution will resemble a single current loop, just the type of solution that the numerical calculations come up with.

It is interesting to note that for higher sample aspect ratios, one would expect other solutions with two (or more) pairs of current loops to appear [Fig. 2(d)]. Indeed these do tend to arise in the numerical calculations for larger radii, as shown for the case $C_0=0$, $C_2=1$, and $R/t=10$ in Fig. 3(b). This solution does not stabilize but seems to cycle between one and two current loops as we iterate our program. We have not yet attempted a more complete investigation of this unstable regime. But there is an appealing, though perhaps superficial, analogy between the possible chaotic behavior of this problem and the well-known chaotic behavior of Rayleigh-Benard rolls,[19] which suggests that the phenomena we calculate deserve further work.

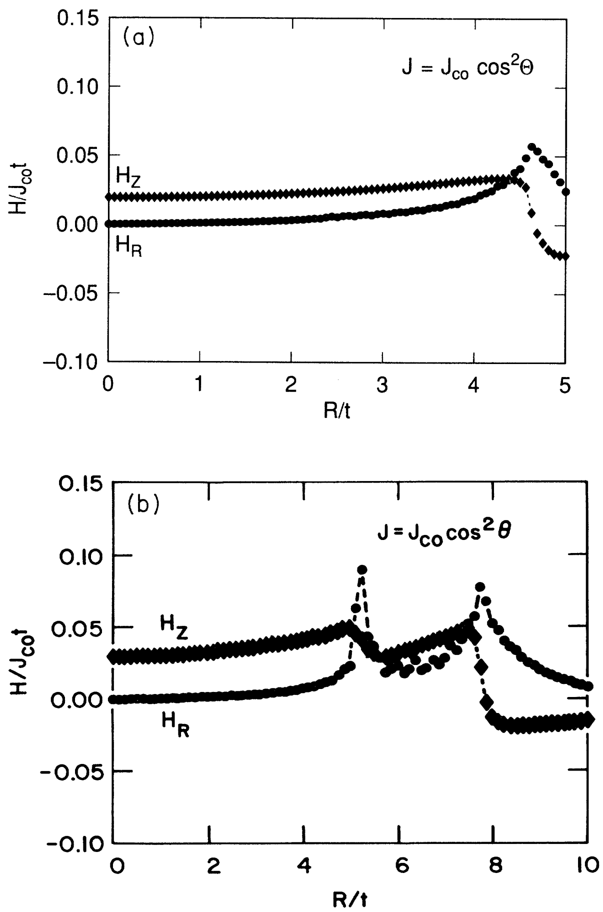

FIG. 3. Calculated field distributions for a disk in its critical state, with current density going as $\cos^2\theta$, where θ is the angle of the net local field direction out of the disk plane (no isotropic component, $C_0=0$, $C_2=1$). Triangles show the z-field component at the center plane of the disk, and circles show the radial field at the surface of the disk. In (a) the results are for aspect ratio $R/t=5$, in (b) for $R/t=10$. The results in (a) are stable, those in (b) are unstable, with the solution jumping between single- and double-peaked states as the iterations of the calculation proceed.

Comparing to experiment, the result of Fig. 1 for $C_0=0$ is not very satisfactory. The magnetization is very small; at $H_a=0$, it is 0.13 in our normalized units, while the usual uniform Bean critical state formula would give $J_cR/3$ or $\frac{5}{3}$ in the same units ($R/t=5$). The reason is clearly the reduced current flow arising from the formation of a localized current ring, as described above. We also attempted to investigate the dependence on radius, and while we had difficulty achieving convergence at higher radii as mentioned already, the calculations indicate a sublinear dependence on radius, while linearity is usually observed in experiment.[20,21] We note that the magnetization of a ring, calculated with the volume inside the ring, is independent of radius.

Results which seem more applicable to experiment were obtained with calculations including a finite value for C_0, the isotropic component in Eq. (1). The field-dependent magnetization, calculated for the case $C_0=0.5$, $C_2=1$, and $R/t=5$, is shown in Fig. 1. The field and current distribution (not shown here) which gives rise to this magnetization is much closer to the conventional distribution for isotropic current (see Refs. 13 and 14). Once again we find a peak in $M(H_a)$, but now it is much larger and wider, and it sits on top of a field-independent contribution from the isotropic current density, of magnitude $0.5\times5/3=0.8333$, as expected from the usual Bean formula $M=J_cR/3$ for this case (here 0.5 comes from the choice $C_0=0.5$). The peak is also slightly off-center; this is to be expected given the asymmetry of the z-self-field distribution with respect to positive and negative applied z fields. The applied-field asymmetry of the peak decreases with increasing aspect ratio; this will be the topic of future work.

Physically, we can interpret this result as follows. The isotropic component creates radial fields over much of the sample, and these fields dictate a contribution from the in-plane current density according to Eq. (1). In effect, the isotropic component stabilizes and strengthens the in-plane contribution. Figure 4 demonstrates this effect by plotting the height of the $M(H)$ peak above the high-field value, versus C_0, for constant $C_2=1$. The width of the peak increases accordingly.

The experimental angle dependence of critical current[10,11] is considerably more peaked around the in-plane direction than $\cos^2\theta$. To investigate the effect of this sharper angle dependence, we performed a series of calculations with a constant $C_0=0.5$ and $R/t=5$, and with progressively higher members of the series in Eq. (1), i.e., first with $C_2=1$ (and all other $C_{i>0}$ zero), then with $C_4=1$ (and all other $C_{i>0}$ zero), and so forth. We found that at $H_a=0$, the normalized magnetization decreased from 1.33 to 1.13 to 1.05 for $i=2,4,6$, respectively. These changes represent a significant decrease of the peak height, since the high-field level for the irreversible magnetization is 0.833, as described above.

In summary, numerical electromagnetic calculations of a disk with anisotropic current density show a zero-field

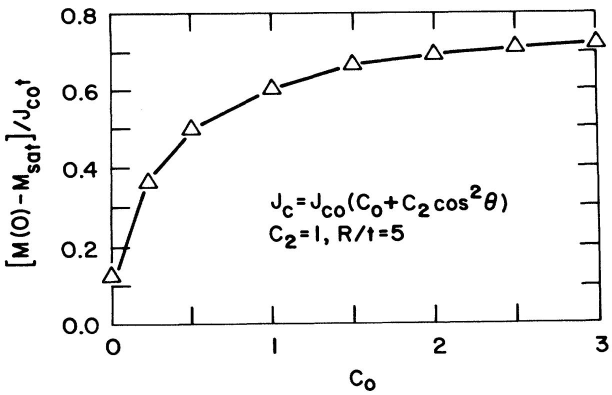

FIG. 4. Peak height above the high-field saturation value (see Fig. 1), as a function of strength of the isotropic component C_0 in Eq. (1). Although the peak actually occurs at slightly negative applied field, we approximate the peak height by $M(0)-M_{sat}$, where $M(0)$ is the remanent magnetization at $H_a=0$ and where M_{sat} is the value at high H_a.

peak in the irreversible magnetization versus field. The peak is found even when the critical current density is assumed to be completely independent of the amplitude of the applied field. Our mechanism for the peak arises specifically in platelet-shaped samples with a large in-plane current density component $J_c^{(a,b),(a,b)}$. We have shown that the peak is enhanced by the presence of an isotropic current density and depends on the breadth of the angular dependence of the current density. In principle a study of this peak may ultimately permit a determination of the magnitude of $J_c^{(a,b),(a,b)}$, which has otherwise proved difficult to measure directly[12] in crystals. This phenomenon should also be relevant in interpreting transport data[10,11] on thin films at low fields; this is another aspect for future study.

We emphasize that this proposed new mechanism for the peak does not rule out others and can occur in combination with them. Indeed, as mentioned above, peaks have been observed even when the sample geometry is not plateletlike, that is even when there are no in-plane self-fields. It must also be said that so far there is no direct experimental proof of this mechanism, which would require crystals large enough to cut a rod-shaped specimen out of a platelet and to compare the hysteresis loops. Nevertheless, though it may be hidden by other effects, it is a mechanism which seems likely to occur, given the overwhelming tendency of these superconductors to grow in a platelet shape, and given the growing evidence for a large in-plane critical current density, possibly arising from intrinsic pinning.

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