Faraday rotation in quasi-two-dimensional electron systems in the quantized Hall regime

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Considering the significance of the finite thickness of a quasi-two-dimensional electron system, we have studied the Faraday rotation when a polarized electromagnetic wave is incident perpendicular to the layer in the quantized Hall regime. It is shown that in the high-frequency regime the Faraday rotation is frequency-independently quantized to the fine-structure constant $\alpha = e^2/hc$ irrespective of details of the layer, whereas in the low-frequency regime one circular polarization of radiation is transparent while another is absorbed.

There is much interest in the properties of high-mobility two-dimensional electron systems since the discoveries of the integer quantum Hall effect (IQHE) and the fractional quantum Hall effect (FQHE) in the metal-oxide-semiconductor inversion layers and in the GaAs-$Al_{1-x}Ga_xAs$ heterojunctions.[1,2] Previous investigations of the quantum Hall effect (QHE) emphasized its two-dimensional nature,[3] whereas effects associated with the finite thickness of the layer have been somewhat neglected. However, experiments in quasi-two-dimensional (Q2D) electron systems on the electronic spectrum and ac conductance have provided evidence that two-dimenensionality of an electronic system is not a prerequisite for the observation of the quantum Hall effect.[4] Even more interesting is the observation of QHE-like behavior in the Bechgaard salts (TMTSF)X,[5–9] where X is the anion ClO_4, ReO_4, or PF_6 and TMTSF is tetramethyltetraselenafulvalene. These bulk materials are quasi-two-dimensional open-orbit metals when no magnetic field is applied. As to magneto-optical properties, cyclotron resonance of volume carriers in Q2D electron systems has been extensively studied in metal-oxide-semiconductor inversion layers,[10–12] and in very-high-mobility GaAs-$Al_{1-x}Ga_xAs$ heterojunctions.[13,14] In the Faraday configuration, anomalous structures in the cyclotron resonance linewidth were found at or near the filling factors where the FQHE occurred.[14] In Voigt configuration, novel results have been found that the cyclotron resonance peak position ω_{res} does depend on the areal density of electrons. It looks like a contradiction to the well-known plasma shift of cyclotron resonance in the Voigt configuration: $\omega_{\text{res}}^2 = \omega_C^2 + \omega_P^2$.[10] In the present paper we emphasize its quasi-two-dimensional nature and study the Faraday rotation, physically a three-dimensional effect, in Q2D electron systems in which the quantum Hall effects are observable.

Faraday rotation is the most familiar dispersion effect. It describes the rotation of the polarization of radiation passing through a substance in the direction of an applied magnetic field. It was observed in semiconductors first at a microwave frequency[15] and then at higher frequencies in the infrared region.[16] The experimental results were related to the effective mass; therefore, they were quantitatively connected to the electronic band structure.[17] This technique has proven to be versatile for determining effective masses under low magnetic-field conditions where cyclotron resonance cannot be carried out.

Let us consider a high-mobility Q2D electron system in a strong magnetic field B applied perpendicular to the layer (Fig. 1). The electron system may be regarded as two dimensional, since the electrons are confined within the slab of thickness d and move relatively freely along the xy plane. Physically the Faraday rotation arises from the differences in propagation of the two types of circular polarization into which the plane-polarized beam may be resolved. The Faraday rotation ϑ is defined as

$$\vartheta = (\omega d/2c)(\eta_- - \eta_+) , \tag{1}$$

where η_+ and η_- are the indices of refraction of right and left circularly polarized radiation of frequency ω. Expressions for $\eta_\pm$ can be easily obtained by use of Maxwell's equations,

$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}}/c, \quad \nabla \times \mathbf{H} = 4\pi \mathbf{J}/c + \dot{\mathbf{D}}/c \tag{2}$$

which, combined with constitutive equations,

$$\mathbf{J} = \overleftrightarrow{\sigma} \cdot \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H}, \quad \mathbf{D} = \epsilon \mathbf{E} , \tag{3}$$

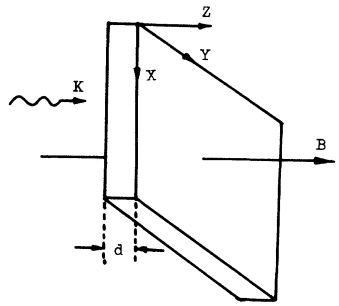

FIG. 1. A quasi-two-dimensional electron system of thickness d with an applied magnetic field B perpendicular to the layer. The incident radiation is propagating along the applied magnetic field.

give

$$\nabla(\nabla\times\mathbf{E})-\nabla^2\mathbf{E}=-(4\pi\mu/c^2)\overleftrightarrow{\sigma}\cdot\dot{\mathbf{E}}-(\mu\epsilon/c^2)\ddot{\mathbf{E}} . \quad (4)$$

For the Faraday effect one takes the propagation along the applied dc magnetic field. Assuming a plane-wave solution of the form

$$\mathbf{E}=\mathbf{E}_0\exp[i(\omega t-Kz)] \quad (5)$$

leads to

$$K^2\mathbf{E}_0=\mu\epsilon(\omega/c)^2[\overleftrightarrow{1}-(4\pi i/\omega\epsilon)\overleftrightarrow{\sigma}]\cdot\mathbf{E}_0 , \quad (6)$$

where $\overleftrightarrow{1}$ is a unit tensor. Assuming a circular polarized wave $\mathbf{E}_0=\mathbf{E}_{0x}\pm i\mathbf{E}_{0y}$, then

$$K_{\pm}^2=\mu\epsilon(\omega/c)^2[1-(4\pi i/\omega\epsilon)\sigma_{\pm}^{(3\mathrm{D})}] , \quad (7)$$

where $\sigma_{\pm}^{(3\mathrm{D})}=\sigma_{xx}^{(3\mathrm{D})}\pm i\sigma_{xy}^{(3\mathrm{D})}$. The complex index of refraction $(\eta-i\kappa)$ is then obtained from

$$(\eta_{\pm}-i\kappa_{\pm})^2=\mu\epsilon[1-(4\pi i/\omega\epsilon)\sigma_{\pm}^{(3\mathrm{D})}] . \quad (8)$$

By the quantized Hall effect, we mean that, at a temperature low enough, the two-dimensional conductivity tensor takes the form

$$\overleftrightarrow{\sigma}^{(2\mathrm{D})}=\begin{bmatrix}\sigma_{xx} & \sigma_{xy}\\ -\sigma_{xy} & \sigma_{yy}\end{bmatrix} \quad (9)$$

with $\sigma_{xx}=\sigma_{yy}=0$ and $\sigma_{xy}=fe^2/h$ in a quantum Hall regime. Here the Landau filling factor f is an integer for the IQHE or a rational fraction for the FQHE. Considering the finite thickness d of the layer, the three-dimensional conductivity tensor is related to the two-dimensional conductivity tensor by the following expression:

$$\sigma^{(3\mathrm{D})}=\sigma^{(2\mathrm{D})}/d . \quad (10)$$

The diagonal 3D conductivity σ_{zz} makes no contribution to the Faraday rotation. In a real Q2D electron system the three-dimensional density may change gradually along the z direction arising from bending of the conduction band. So d may be regarded as the effective thickness of the layer. Therefore, in the quantized Hall regime we have

$$\sigma_{\pm}^{(3\mathrm{D})}=\pm ife^2/hd \quad (11)$$

which combined with Eq. (8) yields

$$(\eta_{\pm}-i\kappa_{\pm})^2=\mu(\epsilon\pm f\omega_0/\omega) \quad (12)$$

with $\omega_0=4\pi e^2/hd$. For typical Q2D electron systems, the host material is nonmagnetic ($\mu=1$), the layer thickness d is of order 10^2 Å, so that ω_0 is of order 10^{14}/sec which is comparable with frequencies of the near infrared region.

We first assume an incident electromagnetic wave of very high frequency $\omega\gg\omega_0$; it is evident from Eq. (12) that

$$\kappa_{\pm}\approx 0 \quad (13)$$

which means the absorption for both types of circulation is zero, and

$$\eta_{\pm}^2=\epsilon\pm f\omega_0/\omega \quad (14)$$

from which the Faraday rotation is determined,

$$|\vartheta|=(\omega d/2c)[(\epsilon+f\omega_0/\omega)^{1/2}-(\epsilon-f\omega_0/\omega)^{1/2}]$$
$$\approx 2\pi f\alpha/\eta , \quad (15)$$

where $\alpha=e^2/hc$ is the fine-structure constant, and $\eta=\sqrt{\epsilon}$ is the refraction index of the host material. It can be seen that the Faraday rotation is frequency independent and shows quantized plateaus as a function of the Landau filling factor since the Hall effect is quantized, irrespective of details of the thickness and effective mass of the Q2D electron system.

In the low-frequency limit, $\omega\ll\omega_0$, from Eq. (12) we have

$$(\eta_{\pm}-i\kappa_{\pm})^2=\pm f\omega_0/\omega \quad (16)$$

whose solutions are of the form

$$\eta_{+}=(f\omega_0/\omega)^{1/2} , \quad (17a)$$

$$\kappa_{+}\approx 0 , \quad (17b)$$

for one circulation of radiation which is transparent through the layer, and

$$\eta_{-}\approx 0 , \quad (18a)$$

$$\kappa_{-}=(f\omega_0/\omega)^{1/2} , \quad (18b)$$

for another which is absorbed by the layer. In the intermediate-frequency regime $\omega\approx\omega_0$, the dependences of the Faraday rotation and the absorption on the frequency of radiation and thickness of the layer are more complicated. In general, the dependences, attributed to quantization of the Hall resistance and concomitant vanishing of the diagonal resistance, are quite different from those expected for the free-electron Faraday effect.[15–17]

It should be noticed that the high-frequency condition $\omega>\omega_0$ is satisfied in the ultraviolet regime. So a promising candidate for experimentation to test the prediction for the quantized Faraday rotation Eq. (15) is the ultraviolet ray, possibly from a mercury vapor lamp. The low-frequency condition is satisfied at microwave frequency. In this regime of frequency one circulation of radiation is transparent while another is opaque. The selectivity of circular polarization in a Q2D electron system may be of practical importance with potential applications.

In conclusion, we have demonstrated theoretically that the finite thickness of a Q2D electron system in the quantized Hall regime may have important physical significance. As consequences of the quantized Hall effect the rotation of polarized radiation passing through the Q2D electron system shows quantized behavior in the high-frequency regime, whereas in the low-frequency regime one circularly polarized radiation is transparent while another is absorbed.

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[1]K. von Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980).

[2]D. C. Tsui, H. L. Stomer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).

[3]For reviews, see K. von Klitzing, Rev. Mod. Phys. **58**, 59 (1986), and H. Aoki, Rep. Prog. Phys. **50**, 655 (1987).

[4]T. W. Hickmott, Phys. Rev. Lett. **57**, 751 (1986).

[5]D. Jerome and H. J. Schltz, Adv. Phys. **31**, 299 (1982).

[6]P. M. Chaikin, Mu-Yong Choi, J. F. Kwak, J. S. Brooks, K. P. Martin, M. J. Naughton, E. M. Engler, and R. L. Greene, Phys. Rev. Lett. **51**, 2333 (1983).

[7]B. Piveteau, L. Brossard, F. Creuzet, D. Jerome, R. C. Lacoe, A. Moradpour, and M. Mibault, J. Phys. C **19**, 4483 (1986).

[8]J. R. Cooper, W. Kang, P. Auban, G. Montambaux, D. Jerome, and K. Bechgaard, Phys. Rev. Lett. **63**, 1984 (1989).

[9]S. T. Hannahs, J. S. Brooks, W. Kang, L. Y. Chiang, and P. M. Chaikin, Phys. Rev. Lett. **63**, 1988 (1989).

[10]Wen-qin Zhao, C. Mazure, F. Koch, X. Ziegler, and H. Maier, Surf. Sci. **142**, 400 (1984).

[11]H. Reisinger and F. Koch, Surf. Sci. **170**, 397 (1986).

[12]S. Oelting, U. Merkt, and J. P. Kotthaus, Surf. Sci. **170**, 402 (1986).

[13]Th. Englert, J. C. Maan, Ch. Uihlein, D. C. Tsui, and A. C. Gossard, Solid State Commun. **46**, 545 (1983).

[14]G. L. J. A. Rikken, H. W. Myron, C. J. G. M. Langerak, and H. Sigg, Surf. Sci. **170**, 160 (1986).

[15]R. R. Rau and M. E. Caspari, Phys. Rev. **100**, 632 (1955).

[16]H. Von Kimmel, Z. Naturforsch **12A**, 1016 (1957).

[17]R. N. Brown and B. Lax, Bull. Am. Phys. Soc. **4**, 133 (1959).