Noise from backscattered electrons in the integer and fractional quantized Hall effects

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We report low-temperature experiments on noise (in equilibrium and in the presence of current flow) in Hall bars with 0.5-μm gates across them. The Hall bars were formed on high-mobility GaAs-$Al_xGa_{1-x}As$ heterostructures, and by using the gate we were able to explore the noise characteristics both in the integer quantized Hall effect and for fractional filling factors. In the integer quantized Hall effect, the noise power exhibits plateaus where the resistance does, and it exhibits plateaus in spite of the lack of clear resistance plateaus when the filling factor is fractional beneath the gate. The excess noise found in the experiments is less than that predicted by the simple shot-noise formula.

Noise in resistors has been studied for many decades,[1] and it has recently attracted some attention when the resistor exhibits the quantized Hall effect.[2] The quantized Hall effect is a particularly interesting case because it provides extremely accurate resistance values in a wide variety of two-dimensional samples. In fact it is now used as a standard for resistance and for the fine-structure constant, so that ultimately, resistance noise in this effect will limit the resolution to which the standard can be read.[3] The quantized Hall effect also forms an interesting case because the usual diffuse scattering from impurities and defects, which plays an important role in the noise from classical resistors, does not affect the resistance much (hence the very accurate quantization). The quantized Hall effect can be viewed as a situation in which the resistance is entirely due to the finite number of charge-carrying channels (where each channel is transmitted with probability of unity), so that there is no reflection of the current by impurities anywhere in the sample.[2] A potential drawn up across the current path by a gate [see inset to Fig. 1(a)] artificially reflects one or more of the current channels.[4,5] When a certain number of channels is completely reflected, one also finds quantized longitudinal resistance.[4-6] The edge states propagate not only without significant scattering, but without much interchannel equilibration.[4] Measurements of equilibration lengths indicate that they can easily exceed 100 μm, which is more than an order of magnitude greater than the equilibration length in the absence of the quantized Hall effect.[4,7-10]

Noise in this essentially nonequilibrium system has been studied at moderate temperatures,[11,12] where noise was measured in the integer-quantized Hall effect at temperatures $T \geq 4.2$ K. Related noise measurements on ballistic electrons have been made in resonant-tunneling structures[13] and in quantum point contacts[14-16] where the shot-noise power density (the mean square fluctuation amplitude divided by the measurement bandwidth) is substantially smaller than one might predict with the simple formula $S_I \equiv \langle (\Delta I)^2 \rangle / \Delta\nu = 2eI$, where I is the current through the resistor.[17,18] Rather than the standard classical approximations, a more apt reckoning of the noise comes from its relation to the transmission matrix that governs the resistance.[19] A complete description of the noise in the quantized Hall effect, which includes the effects of separate voltage and current probes, has been given recently,[20] and it too finds reduced shot noise.

We combine the barrier and quantized Hall effect experiments and extend these measurements to lower temperatures on a standard Hall bar (10 μm wide and 38 μm long between the centers of the voltage contacts) with a narrow Schottky contact gate (0.5 μm long) across it. By applying a negative voltage to the gate, we deplete the region beneath it and raise a barrier to the current flow along the sample. The barrier reflects in sequence each of the edge channels (Landau levels) where all the current flows—at least near equilibrium. As each edge channel is reflected, ever larger quantized plateaus appear in the longitudinal resistance $R_L \equiv R_{1,2;6,3} = (V_2 - V_1)/I_{3\to 6}$ (see the probe labeling in Fig. 1). In agreement with the experiments on point contacts,[14-16] we also find that the shot noise is reduced below the value one predicts by assuming that the carriers are in equilibrium and that the equilibrium noise (in the absence of current flow) approximately tracks resistance measured by the probes. In addition, we find plateaus when the filling factor beneath the gate is a rational fraction less than one, even though no plateaus appear in the longitudinal resistance. Moreover, the shot noise does not simply track the longitudinal resistance as one would predict for carriers in equilibrium.

Our experiments were performed on a small standard Hall bar formed by wet etching of a two-dimensional electron gas in a GaAs-$Al_xGa_{1-x}As$ heterostructure. The Hall bar had a mobility of 132 m^2/V sec and a carrier concentration of $3.63 \times 10^{15}/m^2$. The Hall bar was 10 μm wide, and it was spanned by a Schottky contact gate (made of Ni-Cr/Au) 0.5 μm long. There were opposing voltage probes (8 μm wide ports) on either side of the gate 15 μm away from its center as depicted in the inset of Fig. 1(a). Measurements of the Hall and longitudinal resistances contained clear spin splitting of the Landau levels at $B > 2.3$ T and some signature of fractional occupation for $1 < N < 2$. All measurements were made

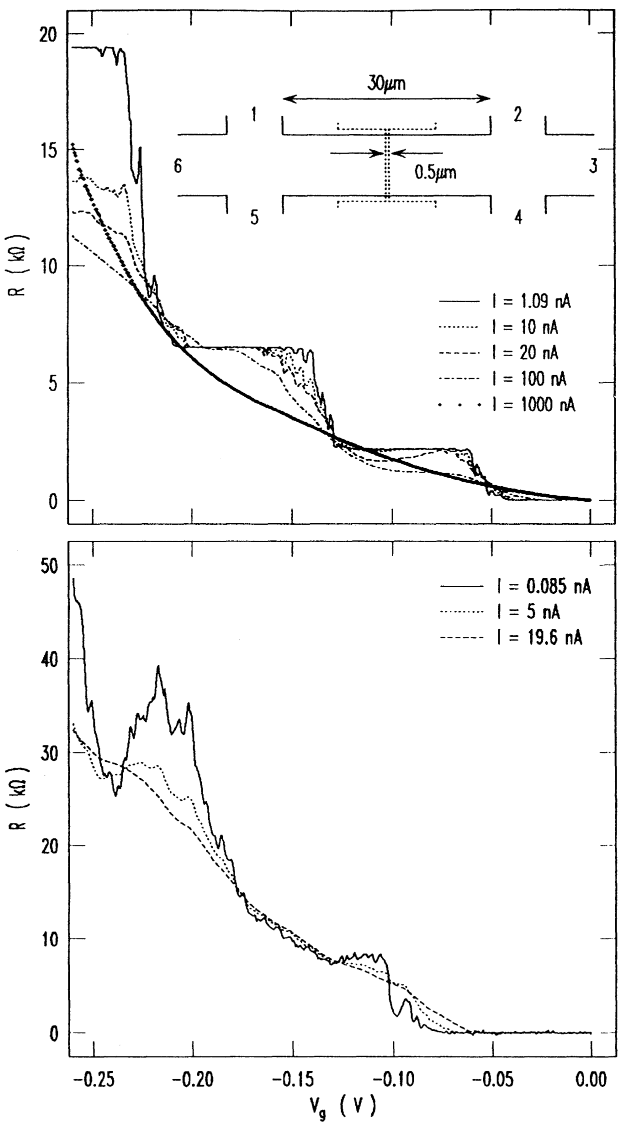

FIG. 1. (a) The longitudinal resistance as a function of the gate voltage at a field of $B=3.75$ T, where, in the bulk of the sample, the Landau occupation was $N=4.0$. As labeled, the different line types refer to different rms ac currents enforced through the channel. The inset contains a scale drawing of the Hall bar; the dotted line illustrates the location of the gate level. (b) Similar data for $B=15.0$ T, where the bulk filling factor is $N=1.0$.

inside the mixing chamber of a dilution refrigerator inside a superconducting solenoid magnet, which was inside of an rf shielded room. For all measurements reported here, the mixing chamber temperature was $T \lesssim 0.01$ K.

At a certain magnetic field, in the bulk of the sample (that part not beneath the gate) a certain number of Landau levels cross the Fermi surface, and these give rise to the transport current.[2] For integer occupation number (when there are no extended states at the Fermi energy in the bulk of the sample), the Hall resistance is quantized at $R_H \equiv R_{2,4;6,3} = Nh/e^2$, and the longitudinal resistance is zero. Any potential barrier raised by the application of voltage to the gate reflects the edge states back toward the source from which they were injected. If K edge states are completely reflected, then the longitudinal resistance measured across the gate is no longer zero, but instead it too is quantized at[2]

$$R_L = \frac{hK}{e^2 N(N-K)} .$$

For a magnetic field $B=3.75$ T, the bulk occupation was $N=4$, and the application of a gate voltage generated a series of plateaus in R_L at rational fractions $\frac{1}{12}$, $\frac{1}{4}$, and $\frac{3}{4}$ as expected. Examples of such data are displayed as the continuous line in Fig. 1(a). As the current impressed on the channel increased, the quantization was gradually suppressed until the classical Hall resistance was recovered. The spin-split quantization is more readily destroyed by the high currents; for instance, the plateaus at $V_g=-0.09$ V and $V_g<-0.25$ V are killed by currents of the order of 10 or 20 nA (and the Hall voltage is $\simeq 0.1$ mV), but there is still some vestige of the plateau at $V_g=-0.175$ V for currents greater than 100 nA. There are also more or less periodic oscillations between the plateaus, which we suppose arise from Aharonov-Bohm effects and backscattering beneath or near the gate[21] as the gate draws the Fermi level up around some propitiously located impurity.

When the bulk occupation is $N=1$, there are also current-dependent features in $R_L(V_g)$, but there are no plateaus. Examples of these data appear in Fig. 1(b). We associate the two main features (which are centered at $V_g=-0.21$ and -0.11 V) with partial reflection of edge states having occupancy numbers $\frac{2}{3}$ and $\frac{1}{3}$.[22,6] We suppose that the fractional plateaus are not completely formed because the transmission or equilibration of the edge states is not ideal, and we further suppose that this distinction from the integer case is a result of the many-body correlation that results in the fractional states. The gaps for the fractional effects are much smaller than for integer filling factors, so potential variations beneath the gate due to ionized donors might be more of a problem.[23] Given, however, that the fractional reflections have been observed in larger samples, we doubt that this is the explanation of our results. The fractional $R_L(V_g)$ features are much more sensitive to current than for the integer reflections, the classical resistance being nearly restored at smaller Hall voltages (1 mV rather than 5–6 mV). Reducing the current to $I=0.01$ nA (where the voltage drop is less than the nominal temperature of the dilution refrigerator) caused further changes in the detailed shape of the features, but no plateaus developed. It has been argued[24] that there is no equilibrium fractional quantized Hall effect in any quantum system. It may be that our small sample size (effectively only the area beneath the gate) allows enough quantum-mechanical interference to disrupt the fractional quantization that is present in larger samples where phase coherence of the carriers is not important. The equilibration mentioned here is different from the inter-Landau-level mixing, which occurs on much longer length scales (100 μm) in the integer quantized Hall effect.

For the noise measurements, two of the voltage probes [1 and 2 in the inset of Fig. 1(a)] on the same side of the sample, but on either side of the gate, were connected to the input of a room-temperature, battery-powered voltage preamplifier (Brookdeal 5004), and the output of the preamplifier went through filters and a buffer to a commercial spectrum analyzer (HP3582a) located outside of the shielded room. The spectrum analyzer would only measure up to 25 kHz and the nominal noise floor of the preamplifier was 6×10^{-19} V^2/Hz. Because of the bandwidth limits, the spectrometer never reached the white-noise regime of the noise from either the sample or the room-temperature calibration resistor (15 kΩ wire wound). Gate bias and current bias for the noise measurements were applied by batteries, and these circuits were isolated from all other electronics.

Examples of power spectra recorded when the bulk of the sample was in the $N=4$ state are contained in Fig. 2. Throughout most of the available region of the spectrometer, the spectrum maintains approximately a $1/f$ dependence, crossing over to an approximately white spectrum only above 20 kHz. A $1/f$ dependence in this frequency range is consistent with other measurements.[12] Strictly speaking, this places the data outside of the realm of the available theoretical predictions, which are concerned with the white region of the spectrum at higher frequencies. The various spikes and corruptions of the $1/f$ dependence seemed to result from external noise sources and were picked up from the power connections of the spectrum analyzer or by the antennae formed by the circuit wiring. It is clear that the barrier raised by the gate affects the noise level more or less equally throughout the frequency range. We attribute this to an increase in the (white) Johnson noise which underlies the flicker noise. As the barrier height increases, so does the noise power, and this is typical of all of our measurements. The characteristic noise power density S_V is computed from each such spectrum by averaging through the (relatively) quiet region between 20 and 25 kHz, and these are plotted below to indicate the Johnson or excess noise levels in the sample under the particular conditions (gate voltage or current) being studied. There is a small remnant frequency dependence of the spectrum in the region over which we have averaged, but the contribution of this frequency is about an order of magnitude smaller than the changes in the white part of spectrum that interests us. Since the spectrum is crossing over from $1/f$ to flat in this range, and the crossover depends on the gate voltage, we have not attempted to extract the pure Johnson noise by fitting and subtracting the V_g-dependent flicker noise. It was also not possible to fit Hooge's rule[25] to the $1/f$ spectrum and remove it, since the $1/f$ part of the spectrum did not increase as fast as R^2. Given all of these open questions about the nature of the flicker noise, we think that it is wiser to display the raw data and issue this caution that the noise plotted below is biased a few percent higher as the gate voltage goes more negative.

The gate voltage dependence of the Johnson noise power density was measured for bulk filling factors $N=4.0$ and 1.0, and the results appear in Fig. 3. In each case there are plateaus in $S_V(V_g)$ approximately where the longitudinal resistance contains plateaus. The absolute calibration of the spectrometer and the noise temperature of the sample were made by substituting a wire wound, 15 kΩ commercial resistor for the sample under the assumption that its noise spectrum in the same frequency range would indicate the Johnson noise from 15 kΩ at 292 K. Since the noise spectrum in the calibration spectrum was not white (in fact, the $1/f$ roll-off was slower than from the sample), the temperature calibration is on the high side. Applying this calibration and the Johnson noise formula to the sample, we infer a sample

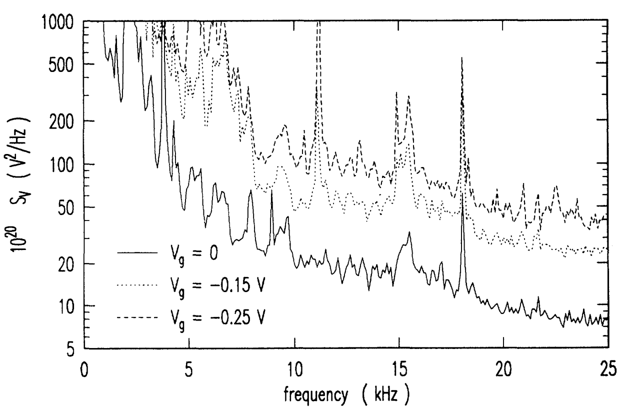

FIG. 2. Noise spectra at $T=1.2$ K between two voltage probes with no imposed current at several gate voltages. The gate lies between the probes.

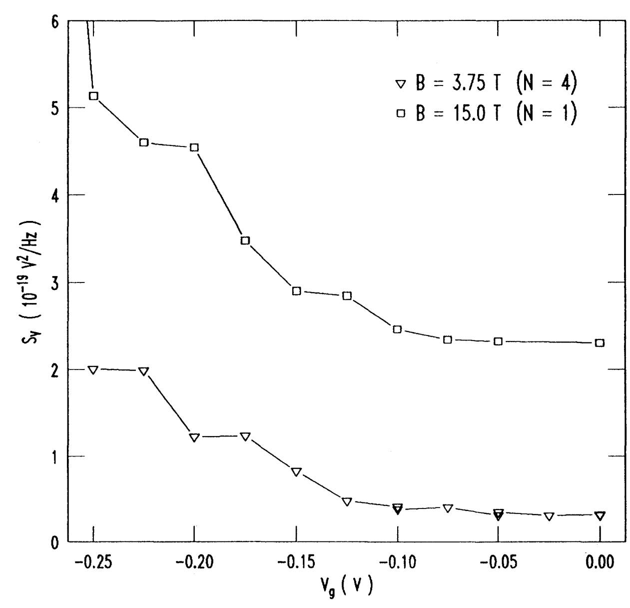

FIG. 3. Noise power densities averaged over the range between 20 and 25 kHz for the bulk filling factors of 4.0 (▽) and 1.0 (○). There was no net current in the sample.

temperature of about 0.4 K, which is rather high as expected.

The $S_V(V_g)$ approximately parallels $R_L(V_g)$, which is essentially the prediction for the equilibrium noise given by Büttiker.[20] The Johnson noise density is proportional to the *two-probe* resistance

$$S_V=4kTR_{1,2;1,2}\propto h/e^2[N-(K+r)] \ ,$$

where $R_{1,2;1,2}=R_H(V_g=0)+R_L(V_g)$ is the two-probe resistance between the voltage probes. The formula allows for the partial reflection of the highest Landau level beneath the gate with probability r. The ratios of the noise power densities between successive plateaus are approximately 2 for both steps. For $N=4.0$ in the bulk and $r=0$, the formula leads to fractions $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$, which do not describe the triangles in Fig. 3 with great quantitative accuracy. (Neither does the noise power track the bare four-probe resistance.) Nevertheless, the increases in the noise density are of the right order of magnitude, and we attribute them to mechanisms discussed by Büttiker.[20]

For bulk filling factor 1.0, there are also plateaus in the noise *in spite of the absence of such plateaus in the longitudinal resistance.* This is astonishing in view of the formula above, but strictly speaking, the formula applies only to the integer-quantized Hall effect. Taking the formula as a guide, we would have expected the Johnson noise to have about the observed amplitude and to have features similar to the resistance curve. Instead of the complicated structure present in the resistance, the noise density is relatively smooth and indicates successively reflected edge states of fractional filling factor—we suppose $\frac{2}{3}$ and $\frac{1}{3}$. The reason for the difference between $R_L(V_g)$ and $S_V(V_g)$ is not obvious. We would have expected that the reflection coefficient in the resistance measurement was very near to the equilibrium value, and that the further reduction of the measurement current would not have lead to clearly defined plateaus at filling factors $\frac{1}{2}$ and 2 that appear in samples with different configurations.[6,22]

We also measured the shot noise between the voltage probes in the presence of drive current. The usual formula for the noise density $S_I=2eI$ does not describe our data well at all. According to this the noise S_V should be approximately proportional to the square of the four-probe resistance, because $R_{1,2;1,2}=R_H(V_g=0)+R_L(V_g)$. Examples of the contradiction in the case of $N=4$ bulk filling factor are presented in Fig. 4, where the open symbols are $S_V(I)$, and the solid symbols are $R_L(I)$. Clearly the proportion between the resistance and the shot noise is violated. The classical shot noise formula does not even provide an order of magnitude estimate; for instance, it predicts $S_V\simeq 3\ \mathrm{nV^2/Hz}$ for $V_g=-0.05$ and $I=200$ nA, which is more than 2 orders of magnitude too large. We have found many instances where $S_V\propto R_L^2$ remains approximately true, but certainly there are many where it is not. As is obvious in Fig. 1, frequently the resistance decreases as the current increases owing to hot electron effects: the quantized Hall effect becomes the classical Hall effect. It is rare that the noise power drops

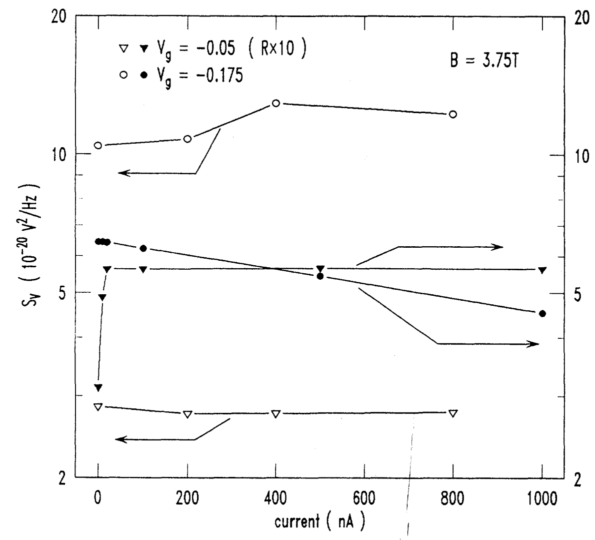

FIG. 4. Selected comparisons of the excess (shot) noise power density (open symbols) and the dynamic resistance measurements (solid symbols) as a function of the dc current through the sample. The classical shot noise is so much larger than the observed signals that it is off the top of the graph even for $V_g=-0.05$ V.

as I increases, and on those occasions where it does it always increases as I approaches very large values and the Hall resistance approaches the classical regime. Even when the electron heating increases the resistance, there is no guarantee that the shot noise will track as is demonstrated by the $V_g=-0.05$ V data in Fig. 4.

Some sort of oddity in the shot noise is not too surprising in view of recent theoretical predictions that shot noise should depend delicately on the transmission coefficients in the quantized Hall effect. Typically it is supposed to be below the classical value; this is in agreement with our results. The precise form that was predicted,[20]

$$S_V=\frac{2h}{e}\frac{r(1-r)}{[N-(K+r)]^2}(V_3-V_6) \ ,$$

suggests that the open circles in Fig. 4 should show no effect at low current, because at $V_g=-0.175$ an integer number of Landau levels are reflected and the partial reflection coefficient $r=0$ for all currents less than $\simeq 100$ nA. This is reasonable and consistent with Fig. 4, where nothing much happens until $I>200$ nA. In contrast, for the $V_g=-0.05$ V results, we would have expected $S_V\sim 1/r$ to track the inverse of $R_L\sim r$. (Since these data are from the riser to the first plateau $K=0$.) In the low-current regime $I<20$ nA, we do not expect the linear response equations for the resistances to break down in any serious way, so evidently, there is some other physics to consider.

We have observed equilibrium excess noise in the quantized Hall effect from a high mobility GaAs heterostructure Hall bar with a short gate. Clear integer plateaus in $R_L(V_g)$ and the absence of their fractional counterparts indicate that the equilibrium or coupling range for the integer-quantized Hall effect is different from that for the fractional effect. The equilibrium noise power densities in both the integer and fractional regimes are consistent with recent predictions[20] and with the notion of edge states controlling the transport physics in both regimes. The shot noise from the samples violates predictions based on classical formulas. The shot noise results, however, are partly in agreement with these theoretical predictions,[20] but one or two puzzles remain to be resolved.

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[1]J. B. Johnson, Phys. Rev. **29**, 367 (1927).
[2]M. Büttiker, Phys. Rev. B **38**, 9375 (1988), and references cited therein.
[3]F. Delahaye, Metrologia **25**, 73 (1988); F. Delahaye and D. Bournard (unpublished).
[4]S. Komiyama, H. Hirai, S. Sasa, and S. Hiamizu, Phys. Rev. B **40**, 12 566 (1989).
[5]R. J. Haug, A. H. MacDonald, P. Streda, and K. von Klitzing, Phys. Rev. Lett. **61**, 2797 (1988); S. Washburn, A. B. Fowler, H. Schmid, and D. Kern, *ibid.* **61**, 2801 (1988).
[6]A. M. Chang and J. E. Cunningham, Solid State Commun. **72**, 651 (1989); A. M. Chang, *ibid.* **74**, 871 (1990); L. P. Kouwenhoven, B. J. van Wees, N. C. van der Vaart, C. J. P. M. Harmans, C. E. Timmering, and C. T. Foxon, Phys. Rev. Lett. **64**, 685 (1990).
[7]B. W. Alphenaar, P. L. McEuen, R. G. Wheeler, and R. N. Sacks, Phys. Rev. Lett. **64**, 677 (1990); P. L. McEuen, A. Szafer, C. A. Richter, B. W. Alphenaar, J. K. Jain, A.D. Stone, R. G. Wheeler, and R. N. Sacks, *ibid.* **64**, 2062 (1990).
[8]B. J. van Wees, E. M. M. Willems, L. P. Kouwenhoven, C. J. P. M. Harmans, H. G. Williamson, C. T. Foxon, and J. J. Harris Phys. Rev. B **39**, 8066 (1989).
[9]R. J. Haug, K. von Klitzing, K. Ploog, and P. Streda, Surf. Sci. **229**, 229 (1990).
[10]G. Müller, D. Weiss, S. Koch, K. von Klitzing, H. Nickel, W. Schlapp, and R. Lösch, Phys. Rev. B **42**, 7633 (1990).
[11]B. E. Kane, D. C. Tsui, and G. Weimann, Surf. Sci. **196**, 183 (1988).
[12]A. J. Kil, R. J. J. Zijlstra, M. F. H. Schuurmans, and J. P. André, Phys. Rev. B **41**, 5169 (1990).
[13]Y. P. Li, A. Zavlasky, D. C. Tsui, M. Santos, and M. Shayegin, Phys. Rev. B **41**, 8388 (1990).
[14]G. Timp, R. E. Behringer, and J. E. Cunningham, Phys. Rev. B **42**, 9259 (1990).
[15]Y. P. Li, D. C. Tsui, J. J. Heremans, J. A. Simmons, and G. Weimann, Appl. Phys. Lett. **57**, 774 (1990).
[16]C. Dekker, A. J. Scholten, F. Liefrink, R. Eppenga, H. van Houten, and C. T. Foxon, Phys. Rev. Lett. **66**, 2148 (1991).
[17]G. B. Lesovik, Pis'ma Zh. Eksp. Teor. Fiz. **49**, 515 (1989) [JETP Lett. **49**, 594 (1989)].
[18]B. Yurke and G. P. Kochanski, Phys. Rev. B **41**, 8184 (1990).
[19]R. Landauer, Physica D **38**, 226 (1989); R. Landauer and T. Martin (unpublished); C. W. J. Beenakker and H. van Houten, Phys. Rev. B **43**, 12 066 (1991).
[20]M. Büttiker, Phys. Rev. Lett. **65**, 2901 (1990).
[21]J. K. Jain and S. A. Kivelson, Phys. Rev. Lett. **60**, 1542 (1988); Phys. Rev. B **37**, 4276 (1988).
[22]C. W. J. Beenakker, Phys. Rev. Lett. **64**, 216 (1990); A. H. MacDonald, *ibid.* **64**, 220 (1990).
[23]J. H. Davies and J. A. Nixon, Phys. Rev. B **39**, 3423 (1989).
[24]D. J. Thouless, Phys. Rev. B **40**, 12 034 (1989).
[25]F. N. Hooge, Physica B **83**, 14 (1976).