Breakdown of conductance quantization and mesoscopic fluctuations in the quasiballistic regime

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We present an analytical theory of corrections to the quantum ballistic conductance of a channel formed in a two-dimensional electron gas (2D EG). Backscattering that causes the corrections occurs inside the channel and is due to a random potential produced by charged donors. The spatial separation of the donors from the 2D EG implies that the scattering potential is smooth and hence gives a natural scale for the width of the channel. We derive the necessary conditions for conductance quantization in both cases of narrow and wide channels. These conditions determine how many quantized steps of the conductance can be observed at a given channel length. An analysis based on our results shows that in existing experiments breakdown of the conductance quantization and a crossover to mesoscopic fluctuations occurs in the narrow-channel limit. The dominating mechanism of breakdown is backscattering within the propagating mode with the largest mode number. This conclusion is validated by a comparison with experimental data. We determine the amplitude of mesoscopic conductance fluctuations in the ballistic regime and derive the minimum temperature for which they are smeared out.

I. INTRODUCTION

It is now well established experimentally[1,2] that the conductance of an electrostatically confined two-dimensional electron gas (2D EG) in a high-mobility GaAs heterostructure reveals quantized values. Theoretically this can be explained within the framework of waveguide theory[3,4] if one assumes that an electron wave propagates through a channel with regular and sufficiently smooth[4] boundaries. However, deviations from quantized values are clearly observed at low temperatures. These deviations vary from sample to sample and can hardly be explained by any particular geometrical features of the waveguide discussed in Ref. 3. Furthermore, recent experiments by Timp *et al.*[5] demonstrate a crucial dependence of the conductance quantization on the mobility of the sample and on the length of the electrostatically formed channel. These experiments strongly suggest that the same scattering mechanisms that limit the mobility are also responsible for the deviation of the conductance from the quantized values. Deviations from the quantum ballistic propagation of electron waves through a finite-size channel were recently studied numerically by several groups. Song He and Das Sarma as well as Kander, Imry, and Sivan used[6] the Anderson model with uncorrelated diagonal disorder. However, in real high-mobility samples, which are produced by modulation doping, the potential causing scattering in the 2D plane is smooth on the scale of the Fermi wavelength.[7] A more realistic model of the potential produced by randomly distributed charged donors was used by Davies, Nixon, and Baranger.[8] They suggested that the lack of screening, especially pronounced near the channel pinchoff condition, is responsible for a strong increase in potential fluctuations and is the main cause for the subsequent breakdown of conductance quantization. Although the results of their numerical calculations look similar to experimental data,[5] the proposed framework does not give a distinct criterion for the conductance quantization. In particular, it does not provide any clear link to the properties of the unconstrained 2D electron gas.

In this paper we have studied scattering of electron waveguide modes by a weak random potential analytically. We show that the criterion for quantization significantly deviates from the naive one requiring the ratio between the length of the channel L and the transport mean-free path l_{tr} to be small.[9] There are two main reasons for this. One is that the *effective* path length can considerably exceed the length of the channel. The other is that large momentum scattering by a smooth potential is much suppressed. For a narrow channel whose width is a few Fermi wavelengths, backscattering requires a large momentum transfer.[10,11] Simultaneously, the path length in this case is generally speaking of the order of the length of the channel. This is why it is easier to observe the quantization for a small number of propagating modes. Exceptions are when the channel width is such that a new mode has just been switched on. In this case the longitudinal momentum is small compared to the transverse one and the effective path length becomes large. This enhances the role of scattering for newly switched modes and may explain the asymmetry of the steplike structure of the conductance versus gate voltage[12] observed in a number of experiments (see, e.g., Fig. 3 of Ref. 5 and Figs. 2 and 5 of Ref. 11). The situation is quite different for channels with a width exceeding the spatial scale of the random potential (given by the width of the space layer). The backscattering is no longer suppressed. Also, the effective length of the trajectory that corresponds to a mode with a high transverse wave number is large. Both these facts make the requirement for the observation of a quantized conductance rather strict. We have derived the necessary condition for conductance quantization that replaces the naive criterion $L < l_{\mathrm{tr}}$ mentioned above. This stricter criterion arises without any assumption of a change in the potential due to lack of screening and can be expressed in terms of parameters pertaining to the 2D EG in an unconstrained geometry. An analysis based on our results shows that in experiments[5] breakdown of the conductance quantization occurs already for narrow channels with a width smaller than the spatial scale of the scattering potential.

The amplitude for backscattering of an electron with definite energy depends in general on the realization of the random potential. However, at sufficiently high temperatures thermal averaging makes the corrections to the quantized conductance almost independent of the realization. We determine the minimum temperature T_{av} necessary for thermal averaging. At lower temperatures corrections do depend on the particular realization of the random potential due to interference of backscattered waves. We determine the amplitude of conductance fluctuations as a function of the temperature, width of the channel and parameters of the 2D EG. Both this amplitude and the characteristic temperature T_{av} differ from the corresponding values for conventional mesoscopic systems. This happens because here the electron motion is ballistic rather than diffusive.

In order to observe conductance fluctuations, one has to find means for changing the realization of the potential. Without changing the sample, this is normally possible only by applying a magnetic field, which changes the interference conditions. It is noteworthy that in the present system it is possible to literally change the scattering potential by shifting the position of the channel in the 2D plane.[13,11]

In Sec. II we present a qualitative discussion of different regimes of scattering inside the channel. A simplified model for the scattering cross section is used. Rigorous calculations based on the Born approximation for scattering are presented in Sec. III, while impurity averaging using a realistic scattering potential is carried out in Sec. IV. Finally, mesoscopic fluctuations of the conductance are discussed in Sec. V, and our conclusions are presented in Sec. VI.

II. QUALITATIVE DISCUSSION

For the sake of simplicity we will assume that the waveguide has hard walls. In this case the transverse wave vector for mode number n is given by $n\pi/d$, where d is the width of the channel. Because of energy conservation, the electron with Fermi momentum $\hbar k_F$ in the 2D EG acquires the longitudinal wave vector

$$k_n=\left[k_F^2-\left[\frac{n\pi}{d}\right]^2\right]^{1/2}=k_F\left[1-\left[\frac{n}{z}\right]^2\right]^{1/2}, \quad z=\frac{k_F d}{\pi} \tag{1}$$

when entering the channel in the nth mode. This mode can be considered to be the result of interference between two bouncing trajectories of the type shown in Fig. 1. The angle θ_n defining the trajectory is determined by the ratio of the longitudinal and total momenta,

$$\sin\theta_n=\frac{k_n}{k_F}=\left[1-\left[\frac{n}{z}\right]^2\right]^{1/2}. \tag{2}$$

The effective length of the trajectory S_n is therefore

$$S_n(z)=\frac{L}{\sin\theta_n}. \tag{3}$$

Backscattering from mode number n requires a momentum transfer of at least k_n. In general, the backscattering rate depends on the momentum transfer, and the requirement of ballistic propagation of mode number n can be presented in the form $S_n(z)<v_F\tau_b(k_n)$. Here $1/\tau_b(k)$ is the backscattering rate for momentum transfer k. The above equations show that the restriction on the length of the channel depends significantly on the longitudinal momentum of the mode:

$$L<\frac{k_n}{k_F}v_F\tau_b(k_n), \tag{4}$$

with k_n given by Eq. (1).

Scattering by a smooth potential is essentially of the low-angle type corresponding to small momentum transfer $q \ll k_F$. To account for this we shall for our qualitative discussion use a simplified form of the scattering cross section,

$$\sigma(\theta)=\sigma_0\Theta(\theta_c-\theta). \tag{5}$$

The truncation at the angle $\theta_c \ll \tau$ crudely models the angular dependence of the real cross section. Obviously,

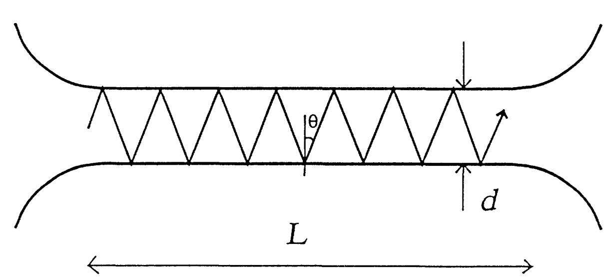

FIG. 1. Schematic picture of a quasi-one-dimensional channel of length L and width d. Each mode in the channel can be thought of as resulting from interference between two bouncing trajectories. One such trajectory, defined by the angle θ related to the ratio of the longitudinal and transverse momentum, is shown. The second trajectory corresponds to letting $\theta\to\pi-\theta$.

even scattering of this type strongly affects a newly switched-on mode for which z only slightly exceeds n. However, with increasing z, θ_n becomes larger than θ_c at a certain point. At that point all modes with numbers less than $n+1$ are unaffected by scattering. If simultaneously the $(n+1)$th mode is not switched on, then scattering has no effect at all and the conductance has its quantized value $G=n(2e^2/h)$. These two conditions can be expressed by the double inequality

$$n\left[1+\frac{\theta_c^2}{2}\right]<z<n+1\ , \tag{6}$$

where we have used the fact that θ_c is small. The inequalities (6) can only be satisfied for

$$n<n_0,\quad n_0=\frac{2}{\theta_c^2}\ . \tag{7}$$

The characteristic number n_0 separates two regions. In the region where

$$z>n_0\ , \tag{8}$$

the scattering affects the quantized conductance for all values of z, and several $(\sim z/n_0)$ propagating modes are involved in the scattering. We can crudely estimate the correction δG to the conductance in the limit $z\gg n_0$ by using the model (5) for the scattering cross section. First we note that using standard definitions of transport $(l_{\rm tr})$ and lifetime (l_s) mean free paths,

$$\frac{1}{l_{\rm tr}}=n_i\int d\theta\sigma(\theta)[1-\cos(\theta)],\quad \frac{1}{l_s}=n_i\int d\theta\sigma(\theta)\ , \tag{9}$$

one finds $l_{\rm tr}^{-1}\sim n_i\sigma_0\theta_c^3$, $l_s^{-1}\sim n_i\sigma_0\theta_c$ for the unconstrained 2D EG. The model parameters σ_0 and θ_c can therefore be related to the 2D density of scatterers n_i and the mean free paths as

$$\theta_c^2\sim\frac{l_2}{l_{\rm tr}},\quad n_i\sigma_0\theta_c^2\sim\frac{1}{\sqrt{l_{\rm tr}l_s}}\ . \tag{10}$$

Now, the typical path length for a mode subject to backscattering in the channel is $S\sim L/\theta_c$. Assuming that roughly half of all possible scattering events result in backscattering, we find that the typical mean free path for such a mode is given by the relation

$$\frac{1}{l}\sim\frac{1}{l_s}\sim n_i\sigma_0\theta_c\ . \tag{11}$$

If $S\ll l$, the transmission coefficient for a single mode deviates from unity by an amount of order S/l. Taking the number of such modes into account, one concludes that

$$\delta G\sim\frac{e^2}{h}\frac{S}{l}\frac{z}{n_0}\sim Ln_i\sigma_0\theta_c^2 z\ . \tag{12}$$

This result can be expressed in terms of experimentally accessible parameters using Eq. (10). One finds

$$\delta G\sim\frac{e^2}{h}\frac{L}{\sqrt{l_{\rm tr}l_s}}z\ . \tag{13}$$

It follows from Eq. (13) that conductance quantization is preserved for

$$z\lesssim\frac{\sqrt{l_{\rm tr}l_s}}{L}\ . \tag{14}$$

The initial assumption (8) of a large z is only compatible with the condition (14) for very short channels,

$$L\lesssim l_s^{3/2}/l_{\rm tr}^{1/2}\ . \tag{15}$$

[We have used here the definition (7) of n_0 and Eq. (10).] For longer channels that do not satisfy the requirement (15) the conductance quantization breaks down for $z\le n_0$, i.e., in the regime of *small* rather than *large* z. In this regime, where $z<n_0$, breakdown occurs mainly because of scattering within a single propagating model (with the highest mode number). Our model (5) for the scattering cross section and Eqs. (6) and (7) indicate that at $n<n_0$ each plateau in $G(z)$ has a "window,"

$$\frac{n}{n_0}<z-n<1\ , \tag{16}$$

where the quantized value of G is preserved.

To estimate the corrections to the conductance in the intervals given by Eq. (16) a theory based on a more realistic model for the scattering is necessary. We will show below that in typical experimental situations, considerable scattering and breakdown of quantization occur already in the region of $z\ll n_0$.

III. CORRECTIONS TO THE CONDUCTANCE IN THE BORN APPROXIMATION

Consider a long channel as in Fig. 1, with *smooth* entrance and exit that do not produce backscattering and longitudinal resonances. The possible deviation from exponentially sharp steps in the conductance is then due to scattering of the electrons inside the channel. In order to derive a more precise condition than above for observing the steps in the function $G(d)$ we shall consider such scattering events in the Born approximation. Adiabatic wave functions[4] are used to form an orthogonal basis for the unperturbed problem. In the presence of a scattering potential $U(x,y)$ the solution of the Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(x,y)+U(x,y)\Psi(x,y)=E\Psi(x,y) \tag{17}$$

can be expanded as

$$\Psi_n(x,y)=\sum_m c_{mn}(x)\phi_{mx}(y)\ , \tag{18}$$

where $\phi_{mx}(y)$ is the wave function of the transverse motion[4] and the coefficients $c_{mn}(x)$ are determined by inserting Eq. (18) in (17). Neglecting derivatives with respect to the index x of the transverse wave function ϕ_{mx}, consistent with the adiabatic approximation, one finds

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+[\varepsilon_l(x)-E]\right]c_{ln}(x)+\sum_m U_{lm}(x)c_{mn}(x)=0 \ . \qquad (19)$$

Here $\varepsilon_l(x)$ is the energy associated with the transverse motion in the channel and

$$U_{lm}(x)=\int_{-\infty}^{\infty} dy\,\phi_{lx}^*(y)\phi_{mx}(y)U(x,y) \ . \qquad (20)$$

To zeroth order in $U(x,y)$, the functions $c_{mn}=c_{mn}^{(0)}$ are[4]

$$c_{mn}^{(0)}(x)=\delta_{mn}\psi_n(x) \ , \qquad (21)$$

where for a forward propagating wave

$$\psi_n(x)=\left[\frac{k_n(-\infty)}{k_n(x)}\right]^{1/2}\exp\left[i\int^x k_n(x')dx'\right] \ , \qquad (22)$$

and $k_n(x)=\{2m[E-\varepsilon_n(x)]\}^{1/2}/\hbar$ is the corresponding longitudinal wave vector. The lower limit of integration in Eq. (22) only contributes an arbitrary phase to $\psi_n(x)$ and is left unspecified. Note, however, that the pre-exponential factor is important for conserving the current. The normalization was chosen so that the prefactor goes to unity as $x\to-\infty$. Hence, well before the channel, the usual normalization factor of a plane wave is recovered.

The corrections to the quantized values of the conductance are due to backscattering. Hence we are interested in the coefficients $c_{ln}(x\to-\infty)$, which give the reflection amplitudes from an incoming wave in mode n to back propagating modes l. To first order in perturbation theory, the total reflection coefficient can then be determined as (see Appendix A)

$$R_n=\sum_l |c_{ln}^{(1)}(x\to-\infty)|^2 = \left[\frac{m}{\hbar^2}\right]^2\sum_l\left|\int_{-\infty}^{\infty}dx'\frac{U_{ln}(x')}{\sqrt{k_l(x')k_n(x')}}\exp\left[i\int^{x'}dx''[k_l(x'')+k_n(x'')]\right]\right|^2 . \qquad (23)$$

We want to calculate the correction δG to the quantized values of the conductance. At finite temperature the conductance can be written as

$$G=e^2\int_{-\infty}^{\infty}dE\,g(E)v(E)T(E)\left[-\frac{\partial f(E)}{\partial E}\right] ,$$

where g is a density of states, v is a velocity, $T(E)$ is the total transmission coefficient, and f is the Fermi function. Using the identity $T_n=1-R_n$ valid for each mode, we find from Eq. (23)

$$\delta G=\frac{2e^2}{h}\left[\frac{m}{\hbar^2}\right]^2\int_{-\infty}^{\infty}dE\left[-\frac{\partial f(E)}{\partial E}\right]\sum_n\frac{1}{k_n}\sum_l\frac{1}{k_l}\left|\int_{-\infty}^{\infty}dx\,e^{i(k_n+k_l)x}\int_{-\infty}^{\infty}dy\,U(x,y)\phi_n(y)\phi_l(y)\right|^2 , \qquad (24)$$

where the sums are over propagating modes in the channel. To obtain the result in the form of Eq. (24), we have used simplifications applicable to a long channel, where in the main part of the channel the confining potential is x independent and hence k_l,k_n are constants and the transverse wave functions ϕ_l,ϕ_n do not depend on x.

Equation (24) contains effects caused by interference between waves backscattered at different points inside the channel. The typical phase difference between such waves, $\eta(E)=(k_n+k_l)L$, is determined by the length of the channel. The interference terms, being dependent on $\exp[i\eta(E)]$, oscillate with energy and hence, after thermal averaging, vanish if the temperature is not too small. The width of the Fermi function restricts the energy integration in Eq. (24) to an interval $\Delta E\sim k_BT$. This energy range corresponds to a range $\Delta k=\Delta E/\hbar v_F$ of wave vectors. One finds therefore that the interference terms are averaged out if

$$\Delta kL\gtrsim 1\Longrightarrow T\gtrsim T_{\text{av}},\quad T_{\text{av}}=\frac{\hbar^2k_F}{k_Bm_L} \ . \qquad (25)$$

In this case, i.e., for $T\gg T_{\text{av}}$, the value of Eq. (24) can be replaced by the impurity averaged value $\langle\delta G\rangle$ that corresponds to neglecting the interference terms.

Without scattering, the steplike structure in the function $G(d)$ is preserved if the thermal smearing is smaller than the energy difference between transverse modes,

$$k_BT<\varepsilon_{n+1}-\varepsilon_n\Longrightarrow T<\frac{\hbar^2\pi k_F}{k_Bmd} \ . \qquad (26)$$

For a long channel, the requirements (25) and (26) determine a wide interval of temperature. For sufficiently low temperatures within this region, we can replace the derivative of the Fermi function in Eq. (24) by a δ function. In the following section we will be working in this limit.

It is also possible to measure $\langle\delta G\rangle$ directly even at temperatures lower than T_{av} (i.e., in the mesoscopic regime) by averaging δG over positions of the channel (using a method of shifting the position of the channel proposed in Ref. 13).

IV. IMPURITY-AVERAGED CONDUCTANCE

We suppose that the potential felt by electrons in the conducting channel is due to randomly distributed impurities,

$$U(\mathbf{r})=\sum_i u(\mathbf{r}-\mathbf{r}_i)\ . \tag{27}$$

The impurity-averaged correction $\langle \delta G \rangle$ can be expressed in terms of a correlation function for the potential. If the positions of the impurities are uncorrelated, one has

$$\langle U(\mathbf{r})U(\mathbf{r}')\rangle = n_i \int \frac{d^2q}{(2\pi)^2} |u(\mathbf{q})|^2 \exp[i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')]\ . \tag{28}$$

Here $u(\mathbf{q})$ is the Fourier transform of the potential $u(\mathbf{r})$ produced by a single impurity and n_i is the 2D density of impurities.

Using Eq. (28) to calculate $\langle \delta G \rangle$ with δG given by Eq. (24), we find

$$\langle \delta G \rangle = \frac{2e^2}{h}\left[\frac{m}{\hbar^2}\right]^2 L n_i \times \sum_{nl} \frac{1}{k_n k_l} \int_{-\infty}^{\infty} \frac{dq_y}{2\pi} |F_{nl}(q_y) u(k_n+k_l, q_y)|^2\ , \tag{29}$$

with

$$F_{nl}(q_y)=\int_{-\infty}^{\infty} dy\, \phi_n(y)\phi_l(y) e^{iq_y y}\ . \tag{30}$$

If the impurity potential is smooth on the length scale determined by the Fermi wavelength $2\pi/k_F$, it is clearly possible to distinguish between two opposite limits of narrow and wide channels. For a narrow channel, typical values of q determined by $u(q)$ satisfy the condition $qd/\pi \ll 1$, where d is the channel width. In this case the form factor F_{nl} can be replaced by

$$F_{nl} \rightarrow \delta_{n,l}\ , \tag{31}$$

and backscattering occurs essentially within a mode. Furthermore, because of the dominance of small-angle scattering, a newly switched-on mode will be the most susceptible to scattering [obviously the most recently switched-on mode has the smallest value of $2k_n$, the argument of the potential function u in Eq. (29)]. This means that only one term in Eq. (29) will contribute:

$$\langle \delta G \rangle = \frac{2e^2}{h}\left[\frac{m}{\hbar^2}\right]^2 n_i L \frac{1}{k_n^2} \int_{-\infty}^{\infty} \frac{dq_y}{2\pi} |u(2k_n, q_y)|^2\ . \tag{32}$$

Here k_n is the longitudinal wave vector for the propagating mode with the highest mode number.

In order to discuss the case of a wide channel, we further specify our model supposing that the electrons are confined by a hard-wall potential in the y direction. In this case

$$\phi_m(y)=\left[\frac{2}{d}\right]^{1/2} \sin\left[\frac{m\pi}{2d}(2y+d)\right]\ . \tag{33}$$

It is obvious from Eqs. (30) and (33) that the form factor $F_{nl}(q_y)$ is large only for

$$q_y \approx \frac{\pi}{d}(\pm n \pm l)\ .$$

However, for $q_y \sim \pm(\pi/d)(n+l)$, the momentum transfer is of order k_F and large. The corresponding contributions to Eq. (29) can be neglected because the Fourier components of u at these wave vectors are small. For the important small wave vectors, the form factor is sharply peaked around $|q_y|=(\pi/d)|n-l|$. The integration over q_y can then be done, which brings us the result

$$\langle \delta G \rangle = \frac{2e^2}{h}\left[\frac{m}{\hbar^2}\right]^2 \frac{n_i L}{\pi k_F z} \times \sum_{nl} \frac{k_F^2}{k_n k_l} \left| u\left[k_n+k_l, \frac{\pi}{d}(n-l)\right]\right|^2\ . \tag{34}$$

Because of the z dependence of k_n, the correction Eq. (34) has a singularity each time a new mode is switched on. In this sense the situation is similar for wide and narrow channels. Away from the switching-on condition, on the other hand, the situation is significantly different for the two cases. For a wide channel, a large number of terms in the sum over n,l in Eq. (34) are important (as mentioned in Sec. II). This leads to a regular part of $\langle \delta G \rangle$ that grows with d. To calculate this regular part we replace the integers n and l by continuous variables, introducing the angles $\theta_{n(l)}=\arcsin(k_{n(l)}/k_F)$. Because of the dominance of low-angle scattering, mostly small values of $\theta_{n(l)}$ will contribute, which allows us to replace $\sin\theta_{n(l)}$ by $\theta_{n(l)}$ and to extend the range of integration from π to infinity. Changing variables and performing one integration gives

$$\langle \delta G \rangle = \frac{2e^2}{h}\left[\frac{m}{\hbar^2}\right]^2 \frac{z n_i L}{\pi k_F} \int_0^{\infty} d\theta\, \theta |u(k_F\theta)|^2\ . \tag{35}$$

Up to this point the discussion has been valid for a general potential. In order to be more specific, we now assume a screened Coulomb potential of the form[14]

$$u(\mathbf{q})=\frac{2\pi e^2}{\kappa}\frac{\exp(-q|z_0|)}{q+q_s},\quad q_s=\frac{2me^2}{\hbar^2\kappa}\ . \tag{36}$$

Here κ is the effective dielectric constant at the heterojunction and q_s is the 2D Thomas-Fermi screening wave vector. The smoothness of this potential is due to the fact that the impurities are located a distance z_0 away from the 2D EG. In high-mobility heterostructures produced by modulated doping this distance is large and $q_s z_0 > 1$. This allows us to neglect q in the denominator of Eq. (36). Further, if the parameter $k_F z_0$ is sufficiently large, the scattering is of the low-angle type and a standard calculation (see Appendix B) gives lifetime (l_s) and transport (l_{tr}) mean free paths as

$$l_s=\frac{2}{\pi}\frac{k_F}{n_i}(k_F z_0),\quad l_{\text{tr}}=\frac{8}{\pi}\frac{k_F}{n_i}(k_F z_0)^3 . \tag{37}$$

Using the potential Eq. (36) for a channel with hard walls, we can give explicit results for $\langle \delta G \rangle$ in the limiting cases. For a narrow channel, Eqs. (32) and (36) give

$$\langle \delta G \rangle = 2\pi\frac{e^2}{h}n_i L\frac{1}{1-n^2/z^2}\frac{1}{2k_F^2 z_0} \times \exp[-4k_F z_0(1-n^2/z^2)^{1/2}] , \tag{38}$$

or in terms of the mean free paths of Eq. (37)

$$\langle \delta G \rangle \approx \frac{e^2}{h}\frac{L}{l_s}\frac{1}{1-n/z}\exp(-a\sqrt{l_{\text{tr}}l_s}\sqrt{1-n/z}\,) , \quad n<z<n+1 . \tag{39}$$

Here a can be regarded as a constant, $a=2\sqrt{1+n/z}\approx 3$. In agreement with the qualitative treatment in Sec. II, we find that the deviations from the sharp steplike structure should be largest at the beginning of the steps, when $z-n<(l_s/l_{\text{tr}})n$ [cf. Eq. (16)]. At the same time, the main part of the quantum plateau is insensitive to scattering if the exponential factor in Eq. (39) is small.

In the limiting case of a wide channel, an evaluation of the integral in Eq. (35) gives

$$\langle \delta G \rangle = \frac{2\pi e^2}{h}n_i L\frac{z}{k_F}\frac{1}{(2k_F z_0)^2} , \tag{40}$$

or in terms of the mean free paths

$$\langle \delta G \rangle = 2\frac{e^2}{h}z\frac{L}{\sqrt{l_s l_{\text{tr}}}} . \tag{41}$$

Here the presence of the numerical coefficient is the only difference compared to the previous, qualitative estimate (13).

To determine the characteristic number n_0 separating the region where backscattering within a single mode dominates (small z) from the region of strong mode mixing (large z), we equate at $z=n_0+\frac{1}{2}$ the two limiting expressions (39) and (41) valid for $z \ll n_0$ and $z \gg n_0$, respectively. This gives the crossover parameter

$$n_0 \approx \frac{18 l_{\text{tr}}/l_s}{\left[\ln\left[\frac{l_{\text{tr}}}{l_s}\right]\right]^2} . \tag{42}$$

Although the differential scattering cross section for the potential Eq. (36) deviates from the truncated form (5), we can introduce $\theta_c^2 \sim l_s/l_{\text{tr}}$ as a measure of the second moment of the scattering cross section $\sigma(\theta)$ [cf. Eqs. (9) and (10)]. In this way a link is established between the estimate (7) and the result (42). As was already mentioned in Sec. II, the correction (41) can be *small* at $z>n_0$ only for a very short channel. With the help of Eq. (42) we can replace the requirement (15) by the more rigorous criterion

$$L<L_0,\quad L_0=\frac{1}{36}\left[\ln\left[\frac{l_{\text{tr}}}{l_s}\right]\right]^2 l_s^{3/2}/l_{\text{tr}}^{1/2} . \tag{43}$$

Only for $l<L_0$ does the breakdown occur in an essentially multimode scattering regime. If $L>L_0$, substantial deviations from a steplike pattern in the function $G(d)$ occur already for $z \ll n_0$ where Eq. (39) is valid. We now determine how many steps n can be seen for a channel of given length by using the criterion that $\langle \delta G \rangle$, as given by Eq. (39), is less than half a quantum unit of conductance (e^2/h) in the middle of the plateau (where $z=n+\frac{1}{2}$):

$$\frac{L}{l_s}(2n+1)\exp\left[-3\left[\frac{l_{\text{tr}}}{l_s}\right]^{1/2}\left[\frac{1}{2n+1}\right]^{1/2}\right] \lesssim 1 . \tag{44}$$

After some rearrangement of Eq. (44) we find for the number of well-resolved quantum steps the relation

$$n_q \lesssim \frac{4.5}{\ln^2(2n_q L/l_s)}\frac{l_{\text{tr}}}{l_s} , \tag{45}$$

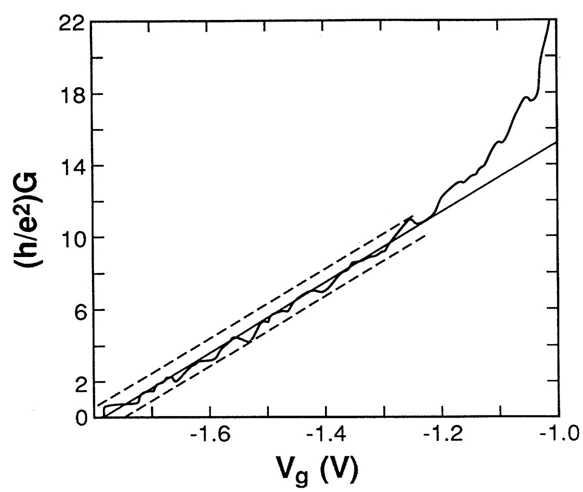

FIG. 2. Two-terminal conductance G measured in Ref. 5 as a function of gate voltage V_g for a channel with the estimated parameters $L=0.9$ μm and $l_{\text{tr}}=7$ μm. The straight solid line demonstrates that the averaged conductance $\langle G \rangle$ is a linear function of the channel width d even in the absence of conductance quantization. This is consistent with the single-mode mechanism for quantization breakdown proposed in Sec. IV. The dashed lines separated by $\approx 1.4e^2/h$ indicate the limits for fluctuations in G. The upturn of the experimental curve at $G \gtrsim 10e^2/h$ is due to the nonlinear dependence $d(V_g)$ for weak depletion and is not related to the breakdown of quantization.

which has to be solved for (the maximum value of) n_q by iteration. It is easy to check with the help of Eq. (43) that $n_q < n_0$ for $L > L_0$ and that the breakdown of quantization therefore occurs in a single-mode regime.

In numerical estimates we shall use[15] $l_{\rm tr} \sim 10$ μm. It is difficult to extract the value of l_s from experiments; a ratio $l_{\rm tr}/l_s \sim 50$ seems to be reasonable.[16] These parameters give $L_0 \sim 0.01$ μm and $n_0 \sim 59$. So, even for moderately long channels ($L \gtrsim 0.1$ μm, say), the breakdown of conductance quantization occurs in a *single-mode backscattering* regime at $n \sim n_q \ll n_0$. For $n > n_q$ the function $G(z)$ fluctuates randomly around an average conductance $\overline{G}(z)$, which grows linearly with z. The reason is that only a single mode is being backscattered. For the same reason, the slope of $\overline{G}(z)$ should be the same as the averaged steplike conductance has in the ballistic regime, and the shift between these two averaged functions should not exceed $2e^2/h$. Ultimately, of course, more and more modes will be involved in the scattering, which will tend to decrease the slope of $\overline{G}(z)$ in the range of quite large z, i.e., at $z > n_0$.

An example of experimental data from[15] for the conductance of a long channel is presented in Fig. 2. We attribute the observed large linear portion of $\overline{G}(V_g)$, which starts from $G \sim 2e^2/h$, to the single-mode mechanism for the breakdown of conductance quantization. Very similar behavior of $\overline{G}(V_g)$ was found in numerical calculations[8] that did not assume mode mixing in the channel to be weak.

V. AMPLITUDE OF CONDUCTANCE FLUCTUATIONS

As was mentioned in Sec. II, the value of δG depends on the particular realization of the distribution of impurities that produce the random potential. This realization can be changed by means of a lateral shift of the channel. When the channel is shifted, both the number of impurities causing scattering and the interference between backscattered waves are subject to change leading to fluctuations in δG. The change in number of impurities results in a temperature-independent part of the conductance fluctuations. This part, however, turns out to be small because of the large average number of acting impurities. The most important part is caused by interference and becomes large at low temperatures. We will be able to estimate both these contributions by calculating [with the use of Eq. (24)] the rms value of δG,

$$\Delta_G \equiv [\langle(\delta G)^2\rangle - \langle\delta G\rangle^2]^{1/2} . \tag{46}$$

As detailed in Appendix C, one finds

$$\begin{aligned}\Delta_G^2 = \left[\frac{2e^2}{h}\left[\frac{m}{\hbar^2}\right]^2\right]^2 n_i L \int_{-\infty}^{\infty} dE \left[-\frac{\partial f(E)}{\partial E}\right] \int_{-\infty}^{\infty} dE' \left[-\frac{\partial f(E')}{\partial E'}\right] \sum_{n,l,m,s} \frac{1}{k_n k_l k'_m k'_s} \\ \times \left[\int_{-\infty}^{\infty} dy\,|V_{nl}(k_n+k_l,y)|^2\,|V_{ms}(k'_m+k'_s,y)|^2 + 2\pi n_i \delta(k_n+k_l-k'_m-k'_s)\right. \\ \left.\times \int_{-\infty}^{\infty} dy |V_{nl}(k_n+k_l,y)|^2 \int_{-\infty}^{\infty} dy' |V_{ms}(k'_m+k'_s,y')|^2\right] .\end{aligned} \tag{47}$$

Here the first and second terms in the last two lines of (47) correspond to the fluctuations in the number of scatterers (the "incoherent" part) and to interference effects (the "coherent" part) respectively, and

$$V_{nl}(q_x,y) = \int_{-\infty}^{\infty} dq_y e^{-iq_y y} F_{nl}(q_y) u(q_x,q_y) . \tag{48}$$

As in the previous section, we shall analyze the general expression (47) for the limiting case of narrow and wide channels with the assumption that the temperature satisfies the condition (26); (i.e., the step structure is not smeared out by thermal averaging).

For a narrow channel, backscattering occurs mainly within the propagating mode with the highest mode number. Hence we can put $n = l = m = s$ in Eq. (47), which then reduces to

$$\begin{aligned}\Delta_G^2 = \left[\frac{2e^2}{h}\left[\frac{m}{\hbar^2}\right]^2\right]^2 \frac{n_i L}{k_n^4} \\ \times \left[\int_{-\infty}^{\infty} dy\,|V_{nn}(2k_n,y)|^4\right. \\ \left. + \frac{\pi}{6}\frac{\hbar^2 k_n}{k_B m T} n_i \left|\int_{-\infty}^{\infty} dy |V_{nn}(2k_n,y)|^2\right|^2\right] .\end{aligned} \tag{49}$$

Equation (49) demonstrates the temperature dependence of the coherent part of Δ_G. One can make a crude estimate of the relative importance of this contribution and the one arising from fluctuations in the number of impurities. Calculating their ratio, one finds

$$(r_c k_F)\frac{\hbar^2 \pi k_F}{m d k_B T}(n_i k_F^2)\sqrt{z-n} \ . \tag{50}$$

Because of the smoothness of the potential, its range r_c is large compared with $1/k_F$. The second term in Eq. (50) is also large for temperatures where there is no thermal smearing [cf. Eq. (26)]. The remaining terms are of order one. Neglecting the incoherent term and using Eqs. (31), (32), and (25), we finally find that

$$\Delta_G^2 = \frac{\pi}{6}\frac{T_{\mathrm{av}}}{T}\left[1-\left[\frac{n}{z}\right]^2\right]^{1/2}\langle \delta G(z)\rangle^2, \quad n<z<n+1 \ . \tag{51}$$

It is obvious that the relative fluctuations $\Delta G/\langle \delta G\rangle$ are small for all z if $T \gg T_{\mathrm{av}}$. At low temperatures, fluctuations of the conductance are limited by the fact that only one mode is involved in backscattering. Hence deviations of $G(z)$ from the average value $\bar{G}(V_g)$ are smaller than e^2/h. This is in agreement with the observed[5,15] dependence of $G(z)$; see Fig. 2.

Conductance fluctuations in a wide channel, $z \gg n_0$, are due to scattering in many modes. The incoherent part of Δ_G^2 caused by the fluctuations in the number of scatterers in the channel can be easily estimated as

$$(\Delta_G^2)_{\mathrm{incoh}} \sim \frac{1}{n_i L d}\langle \delta G\rangle^2 \ . \tag{52}$$

We shall calculate the coherent part of Δ_G^2 using the same approximations as were used in deriving Eq. (35). At low temperatures [see Eq. (26)] the main contribution arises from terms with $n=m$ and $l=s$ or $n=s$ and $l=m$. Taking into account only these terms, we find for this part of Δ_G^2 in analogy with Eq. (34)

$$(\Delta_G^2)_{\mathrm{coh}} = \left[\frac{2e^2}{h}\left[\frac{m}{\hbar^2}\right]^2\right]^2 \frac{32\pi}{3}\frac{n_i^2 L \hbar^2}{k_B T d^2} \times \sum_{n,l}\frac{1}{k_n k_l (k_n+k_l)}\left| u\left[k_n+k_l, \frac{\pi}{d}(n-l)\right]\right|^2 . \tag{53}$$

Calculating the regular part following the same procedure as in the previous section, we replace sums by integrals to find

$$(\Delta_G^2)_{\mathrm{coh}} = \frac{32\pi^3}{3}\frac{1}{(k_F d)^2}\frac{\hbar v_F}{k_B T} \times \frac{1}{L}\frac{\int_0^\infty d\theta |u(\theta)|^4}{\left|\int_0^\infty d\theta\theta |u(\theta)|^2\right|^2}\langle \delta G\rangle^2 \ . \tag{54}$$

As a model for u we shall again use the screened Coulomb potential (36) with $q_s z_0 > 1$. We can now perform the angle integrations in Eq. (54):

$$(\Delta_G^2)_{\mathrm{coh}} = \frac{16\pi}{3}\left[\frac{l_{\mathrm{tr}}}{l_s}\right]^{3/2}\frac{1}{z^2}\frac{T_{\mathrm{av}}}{T}\langle \delta G\rangle^2 \ . \tag{55}$$

Within our assumptions, the ratio between the incoherent and coherent parts—Eqs. (52) and (55)—of the rms value,

$$\frac{(\Delta_G^2)_{\mathrm{incoh}}}{(\Delta_G^2)_{\mathrm{coh}}} \sim \frac{3}{16\pi^3}\frac{k_B T_{md}}{\hbar^2 \pi k_F}\left[\frac{l_s}{l_{\mathrm{tr}}}\right]^{3/2}\frac{k_F^2}{n_i} , \tag{56}$$

is obviously small. Therefore, Eq. (55) represents the dominating contribution to Δ_G. Because $z \gg n_0 \sim l_{\mathrm{tr}}/l_s$, Eq. (55) implies that fluctuations of the conductance are definitely small in the temperature range $T > T_{\mathrm{av}}$.

VI. CONCLUSIONS

It is well established from experiments that conductance quantization can be observed only in very high mobility heterostructures.[2,5,15] In point-contact geometries a large number of steps can be observed. However, when the geometry is changed to a longer channel the quantization at higher values of the conductance deteriorates, and only the first few steps can be seen. In this paper we analytically determine corrections to the quantized value of the conductance arising from impurity scattering within the channel. We find the conditions for the crossover from a region of conductance quantization to a region showing mesoscopic fluctuations. Quantization occurs only for restricted values of the channel width (which limits the ballistic conductance) and of its length. The advantage of our analytical approach is that we are able to express the criterion for conductance quantization in terms of the transport and lifetime mean free paths of the unconstrained 2D EG.

Our approach, based on lowest-order perturbation theory, is valid only when the corrections to the ballistic conductance are small, $\delta G < e^2/h$. Furthermore, we used the rms value Δ_G to characterize fluctuations in the conductance. This approach is reasonable only if $\Delta_G < \delta G$. The latter requirement is fulfilled starting from quite low temperatures (a few degrees Kelvin). As follows from the results of the previous section [see Eqs. (51) and (55)], the restriction on the temperature becomes weaker as the conductance becomes larger.

We have introduced in this paper the distinction between narrow and wide channels based on the assumption that small-angle scattering is dominating in the 2D EG. The large ratio between observed transport and lifetime mean free paths supports this assumption. To establish an explicit relation between these two parameters of the 2D EG and the scattering properties in a channel we used a screened Coulomb potential as the source of scattering. The donors producing this potential were taken to be spatially separated from the 2D EG.

Inside the channel the density of electrons is lower than in the unperturbed 2D EG.[17] Although this can affect the screening,[8] there is no guidance from experiment allowing us to take this properly into account.

We studied scattering only from impurities inside the channel assuming perfect matching conditions for wave functions at the ends. Matching requires a gradual change of the channel width over some length R at both entrances. Nevertheless, the length L of the channel should be large ($L \gg R$), so that it is well defined. In this situation it is also possible to neglect waves backscattered into the channel from points outside.

Even from the qualitative arguments in Sec. II, one expects significant deviations from conductance quantization for wide channels, where backscattering accompanied by mode mixing occurs. Our analytical calculation verifies this suggestion. It is quite surprising, however, that for parameters corresponding to the high-mobility heterostructures used in current experimental work,[5,15] backscattering becomes important in a regime where only one propagating mode (with the highest mode number) is affected. In fact, backscattering within this single mode leads to the breakdown of conductance quantization. The number of observable conductance steps determined by the proposed mechanism [see Eq. (45)] is in reasonable agreement with experiment.[5] For the same parameters, the crossover to multimode backscattering occurs for much larger conductances. This implies that there is a wide region of channel widths d for which the conductance is not quantized, but depends on width almost linearly. The reason is that all modes except one propagate ballistically, and the mode with the highest mode number adds random fluctuations. The implication is, furthermore, that in this regime conductance fluctuations are restricted by the value of the quantum unit e^2/h. Experimental data[5] are consistent with the linear dependence $G(d)$ as demonstrated in Fig. 2. Numerical modeling[8] also reveals a quasilinear $G(d)$ dependence. For fluctuations of the ballistic conductance studied in Ref. 11 the maximum value of Δ_G was estimated to be $1.2e^2/h$. This is also in agreement with our analysis of conductance fluctuations.

Finally, we remark that in our opinion the single-mode mechanism of conductance quantization breakdown is quite general. Our estimate shows that only for point contacts ($L \sim d$) does multimode scattering contribute to the breakdown.

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APPENDIX A

In this appendix we derive the reflection coefficient R_n given by (23). We first note that to first order in the scattering potential the coefficients $c_{mn}=c_{mn}^{(1)}$ are determined by the equations

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+[\varepsilon_l(x)-E]\right]c_{ln}^{(1)}(x)=-U_{ln}(x)\psi_n(x)\ . \tag{A1}$$

They can be found with the help of the adiabatic Green's function $G_l(x,x')$:

$$c_{ln}^{(1)}(x)=-\int_{-\infty}^{\infty}dx'G_l(x,x')U_{ln}(x')\psi_n(x')\ , \tag{A2}$$

where

$$G_l(x,x')=i\frac{m}{\hbar^2}\frac{1}{[k_l(x)k_l(x')]^{1/2}}\exp\left[i\left|\int_{x'}^{x}dx''k_l(x'')\right|\right] \tag{A3}$$

for propagating modes.

The corrections to the quantized values of the conductance are due to backscattering. Hence we are interested in the coefficient $c_{ln}(x\to-\infty)$; from Eqs. (22), (A2), and (A3) we have

$$c_{ln}^{(1)}(x)=-i\frac{m}{\hbar^2}\left[\frac{k_l(-\infty)}{k_l(x)}\right]^{1/2}\exp\left[-i\int^{x}dx''k_l(x'')\right]\int_{-\infty}^{\infty}dx'\frac{U_{ln}(x')}{[k_l(x')k_n(x')]^{1/2}}\exp\left[i\int^{x'}dx''[k_l(x'')+k_n(x'')]\right]\ . \tag{A4}$$

Here x denotes a point outside, and x' a point inside the narrow channel ($x<x'$). As we take the limit $x\to-\infty$ we can replace the coefficient $[k_l(-\infty)/k_l(x)]^{1/2}$ by unity.

The coefficients c_{ln} give the reflection amplitudes from an incoming wave in mode n to back propagating modes l. Using Eq. (A4), the total reflection coefficient is therefore found to be given by Eq. (23).

APPENDIX B

For the sake of completeness, we shall in this appendix provide the few steps necessary to derive the result given in Eq. (37) for the single particle or lifetime mean free path (l_s) and the transport mean free path (l_{tr}). Starting from the definition, one has[14]

$$l_s = v_F \tau_s \ ,$$
$$\frac{1}{\tau_s} = \frac{2\pi}{\hbar} \int \frac{d^2k}{(2\pi)^2} \int_{-\infty}^{\infty} dz N_i(z) |v(\mathbf{k}-\mathbf{k}')|^2 \delta(E_{\mathbf{k}} - E_{\mathbf{k}'}) \tag{B1}$$
$$= n_i \frac{m}{\hbar^3} \int_0^{2\pi} \frac{d\theta}{2\pi} |u(2k_F \sin(\theta/2)|^2 \ ,$$

and

$$l_{\rm tr} = v_F \tau_{\rm tr}$$
$$\frac{1}{\tau_{\rm tr}} = n_i \frac{m}{\hbar^3} \int_0^{2\pi} \frac{d\theta}{2\pi} (1-\cos\theta) |u(2k_F \sin(\theta/2)|^2 \ . \tag{B2}$$

Here $2k_F \sin(\theta/2) = q$ is the transferred momentum and $u(q)$ is the screened Coulomb potential given by Eq. (36). The 3D impurity density $N_i(z)$ will be taken to be a sheet of impurities located a distance z_0 from the 2D EG. The 2D impurity density is denoted n_i.

If the distance z_0 is not too small, forward scattering (small momentum transfer; $q < 1/z_0 \ll k_F$) will dominate and we can use the small-angle (small-q) limits of Eq. (B1), (B2), and (36). Replacing the integration limit 2π by ∞, straightforward integration leads to the result (37).

APPENDIX C

Here we derive the expression (47) for Δ_G^2, which was defined in (46). Substituting δG from Eq. (24) in (46) we find

$$\Delta_G^2 = \left[\frac{2e^2}{h} \left[\frac{m}{\hbar^2} \right]^2 \right]^2 \int_{-\infty}^{\infty} dE \left[-\frac{\partial f(E)}{\partial E} \right] \int_{-\infty}^{\infty} dE' \left[-\frac{\partial f(E')}{\partial E'} \right] \sum_{n,l,m,s} \frac{1}{k_n k_l k'_m k'_s}$$
$$\times \left[\left\langle \sum_{i,j,r,t} S_{nl}(k_n + k_l, \mathbf{r}_i, \mathbf{r}_j) S_{ms}(k'_m + k'_s, \mathbf{r}_r, \mathbf{r}_t) \right\rangle - \left\langle \sum_{i,j} S_{nl}(k_n + k_l, \mathbf{r}_i, \mathbf{r}_j) \right\rangle \left\langle \sum_{r,t} S_{ms}(k'_m + k'_s, \mathbf{r}_r, \mathbf{r}_t) \right\rangle \right] , \tag{C1}$$

where

$$S_{nl}(k_n + k_l, \mathbf{r}_i, \mathbf{r}_j) = e^{i(k_n + k_l)(x_i - x_j)} V_{nl}(k_n + k_l, y_i) \times V^*_{ms}(k_n + k_l, y_j) \ , \tag{C2}$$
$$V_{nl}(q_x, y) = \int_{-\infty}^{\infty} dq_y e^{-iq_y y} F_{nl}(q_y) \ , \tag{C3}$$

and $F_{nl}(q_y)$ is the form factor defined in Eq. (30). The longitudinal wave vectors k_n and k'_m depend on the total energies E and E', respectively. For a long channel, $(k_n + k_l)L \gg 1$, Eq. (C1) can be simplified; after averaging over impurity positions, the only surviving terms have either $i = j$ and $r = t$ or $i = t$ and $r = j$. These two sets of terms correspond to fluctuations in the number of scatterers (the "incoherent" part) and to interference effects (the "coherent" part). They combine to give Eq. (47). In carrying out the average over impurity positions, we have used a relation of type

$$\left\langle \sum_{i,j} S_1(y_i) S_2(y_j) \right\rangle - \left\langle \sum_i S_1(y_i) \right\rangle \left\langle \sum_j S_2(y_j) \right\rangle = n_i L \int_{-\infty}^{\infty} dy S_1(y) S_2(y) \ , \tag{C4}$$

which is valid for large numbers of acting impurities.[18]

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