Gauge-invariant formulation of the intracollisional field effect including collisional broadening

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A gauge-invariant formalism is presented for describing nonperturbatively the effects of the electric field on electron-phonon scattering in a nondegenerate semiconductor. The Kadanoff-Baym nonequilibrium formulation of many-body systems is employed to treat the intracollisional field effect and collisional broadening on equal footing. We derive an analytic, gauge-invariant model for the spectral density function and for the density of energy states that accounts for both effects simultaneously in a relatively simple and rigorous way.

I. INTRODUCTION

The semiclassical description of charge-transport phenomena in semiconductors, whether based on the Boltzmann equation[1] or on Monte Carlo techniques,[2] relies on the Fermi golden rule for the calculation of the transition rates. The sharp δ function appearing in these quantities implies that both energy and momentum are well-defined observables of the system. In other words, when a particle suffers a collision with one of the crystal modes, its energy and momentum can only change by an amount equal to the energy and momentum of the phonon involved in the scattering event.

As the size of the sample decreases, however, quantum-interference effects may come into play[3] since now the charges may maintain their phase coherence over distances comparable to the characteristic length (e.g., the gate length or a depletion length) of the device. The uncertainty relations may therefore play an important role and the broadening of the electron momentum should be considered. Furthermore, the mean collision duration and the mean free time also may not be negligible compared with the transit time through the device and the long-time limit required to establish the conservation of energy in the Fermi golden rule will break down. This phenomenon is called collisional broadening (CB).[4] Now that collisions cannot be treated as instantaneous, the presence of an electric field contributes further to modifying the energy difference between the initial and final states. This is the intracollisional field effect (ICFE).[5]

A theory of transport capable of overcoming the limitations of the semiclassical approach is then needed, especially under the present-day push by the technological possibility of fabricating devices of nanometer dimensions.[6] A theory of this kind can be formulated by treating the electron energy $\hbar\omega$ and its momentum $\mathbf{k}$ as independent quantities, related to each other according to a spectral density function $A(\mathbf{k},\omega)$ of finite width, rather than through the sharp δ function of the Fermi golden rule.

In real space, the spectral function $A(1,2)$ can be defined[7] in terms of the anticommutator of the fermion field operator $\hat{\Psi}(1)$ of the particle at the space-time point $(\mathbf{r}_1,t_1)$ and the conjugate field operator $\hat{\Psi}^\dagger(2)$ at another point $(\mathbf{r}_2,t_2)$, namely

$$A(1,2)=\langle\{\hat{\Psi}(1),\hat{\Psi}^\dagger(2)\}\rangle\ . \tag{1.1}$$

Here, the angular brackets $\langle\cdots\rangle$ indicate a thermodynamic average for equilibrium situations and an average over the available states for nonequilibrium systems. One of the advantages of a spectral function defined in this way is that products of field operators of the type appearing in Eq. (1.1) can be put in a gauge-invariant form (see Sec. II), thus allowing the formulation of a quite general theory whose properties do not depend on the choice of the gauge. Another advantage is that $A(1,2)$ can be calculated within the Kadanoff-Baym formalism[8,9] based on the retarded Green's function

$$G^r(1,2)=-\frac{i}{\hbar}\langle\{\hat{\Psi}(1),\hat{\Psi}^\dagger(2)\}\rangle\Theta(t_1-t_2)\ . \tag{1.2}$$

Indeed, by comparing (1.1) and (1.2) it is clear that the spectral density is known, once G^r has been determined. The retarded Green's function satisfies Dyson's equation,

$$G^r(1,2)=G_E^r(1,2)+\int d\bar{1}\,d\bar{2}\,G_E^r(1,\bar{1})\Sigma^r(\bar{1},\bar{2})G^r(\bar{2},2)\ , \tag{1.3}$$

where G_E^r is the retarded propagator for an electron in the presence of the electric field, but without scattering.[10,11] This allows us, at least in principle, to deal with arbitrary field strengths. Σ^r is the "retarded self-energy" describing the interactions of the electrons with the crystal and, for nondegenerate electron systems, it is, to a good approximation, a functional of G^r only.[9,11]

Solving Eq. (1.3) is not an easy task, mainly because of the complications arising by the nonconservation of the electron wave vector,[12] and the subsequent mixing of space and time coordinates that appears when the interactions are modified by the electric field.

Recently,[12] a technique was proposed in order to deal with these problems and to derive a spectral density function that included both ICFE and CB simultaneously. The approach was based on the idea that scattering events do not occur between states described by the plane

waves of a free electron, as was the case with previous formulations, but between the states of an electron in the field, namely the Airy functions of the first kind. The formalism was formulated in the scalar-potential gauge in terms of the Airy coordinate s, interpreted as the quantum-mechanical analog of the classical electron turning point in the field direction z. For example, it was found that

$$G_E^r(\mathbf{k}_\perp,s,\omega)=\frac{1}{\hbar\omega-\varepsilon(\mathbf{k}_\perp)-eEs+i\eta} , \tag{1.4}$$

and, from this,

$$A_E(\mathbf{k}_\perp,s,\omega)=2\pi\delta(\hbar\omega-\varepsilon(\mathbf{k}_\perp)-eEs) , \tag{1.5}$$

where $\varepsilon(\mathbf{k}_\perp)$ is the kinetic energy of the electron in a parabolic band on the plane perpendicular to the direction of the field. In this way, Eq. (1.3) could be solved analytically and a high-field spectral density $A(\mathbf{k}_\perp,s,\omega)$ including quantum effects was derived. The method was, however, limited by not being gauge invariant, thus making the interpretation of the results uncertain because of the possibility that the approximations made could be gauge dependent and therefore physically incorrect.[7] Also in the case of exact expressions such as (1.4) and (1.5), the resulting physical picture could be dependent on the gauge as can be understood, for instance, by comparing (1.5) with its gauge-invariant counterpart

$$\tilde{A}_E(\mathbf{k},\omega)=\frac{1}{\Theta}\mathrm{Ai}(-[\hbar\omega-\varepsilon(\mathbf{k})]/\Theta) , \tag{1.6}$$

or, in the time domain,

$$\tilde{A}_E(\mathbf{k},\tau)=\frac{1}{\hbar}\exp\left[-i\frac{(eE)^2}{24\hbar m}\tau^3-i\frac{\varepsilon(\mathbf{k})}{\hbar}\tau\right] , \tag{1.7}$$

where $\varepsilon(\mathbf{k})=\hbar^2\mathbf{k}^2/2m$ and $\Theta=(\hbar eE)^{2/3}/(2m)^{1/3}$. Here, as well as in the remainder of the paper, an overtilde denotes gauge-invariant functions.

Our aim, therefore, is to implement the Airy-coordinate formalism in order to formulate a gauge-invariant theory of wider and more general validity. To accomplish this, in Sec. II we present the details of the new formalism and explain with a simple example how a function expressed in terms of Airy coordinates can be cast into gauge-invariant form. In Sec. III, by an appropriate choice of the self-energy function, we solve Dyson's equation and derive a gauge-invariant model for the spectral density $\tilde{A}(\mathbf{k},\omega)$. Finally, the density of states $\rho(\omega)$ is calculated in order to show how the concomitance of an external field and the scattering processes modifies the electron energies.

II. FORMALISM

A. Gauge-invariant transform

Let us consider the product $g(1,2)=\hat{\Psi}(1)\hat{\Psi}^\dagger(2)$ of field operators. In terms of the Wigner coordinates

$$\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2,\quad \tau=\tau_1-\tau_2 ,$$
$$\mathbf{R}=\frac{\mathbf{r}_1+\mathbf{r}_2}{2},\quad T=\frac{\tau_1+\tau_2}{2} ,$$

it can also be expressed as

$$g(\mathbf{r},\tau;\mathbf{R},T)=\hat{\Psi}(\mathbf{R}+\mathbf{r}/2,\,T+\tau/2)\times\hat{\Psi}^\dagger(\mathbf{R}-\mathbf{r}/2,\,T-\tau/2) . \tag{2.1}$$

We not want to prove that, for a particle of charge q, if ϕ and $\mathbf{A}$ are the scalar and vector potentials, respectively, the function $\tilde{g}(\mathbf{k},\omega,\mathbf{R},T)$ defined as

$$\tilde{g}(\mathbf{k},\omega,\mathbf{R},T)=\int\frac{d\mathbf{r}}{(2\pi)^{3/2}}\int\frac{d\tau}{\sqrt{2\pi}}e^{iw(\omega,\tau,T,\mathbf{k},\mathbf{r},\mathbf{R})}g(\mathbf{r},\tau,\mathbf{R},T) ,$$
$$w(\omega,\tau,T,\mathbf{k},\mathbf{r},\mathbf{R})=\int_{-1/2}^{1/2}d\lambda\left[\tau\left[\omega+\frac{q}{\hbar}\phi(\mathbf{R}+\lambda\mathbf{r},T+\lambda\tau)\right]-\mathbf{r}\cdot\left[\mathbf{k}+\frac{q}{\hbar c}\mathbf{A}(\mathbf{R}+\lambda\mathbf{r},T+\lambda\tau)\right]\right] \tag{2.2}$$

remains unchanged under the gauge transformation

$$\mathbf{A}\rightarrow\mathbf{A}'(\mathbf{x},t)=\mathbf{A}(\mathbf{x},t)+\nabla\chi(\mathbf{x},t),\quad \phi\rightarrow\phi'(\mathbf{x},t)=\phi(\mathbf{x},t)-\frac{1}{c}\frac{\partial\chi(\mathbf{x},t)}{\partial t} , \tag{2.3}$$

where $\chi(\mathbf{x},t)$ is an arbitrary scalar function. The proof goes as follows.

From elementary quantum mechanics[13] we know that the wave function in the new gauge is related to the wave function in the original gauge by

$$\Psi'(\mathbf{x},t)=\exp\left[i\frac{q}{\hbar c}\chi(\mathbf{x},t)\right]\Psi(\mathbf{x},t) . \tag{2.4}$$

By substituting this into Eq. (2.1), and Eq. (2.3) into Eq. (2.2), we have

$$\tilde{g}(\mathbf{k},\omega,\mathbf{R},T)=\int\frac{d\mathbf{r}}{(2\pi)^{3/2}}\int\frac{d\tau}{\sqrt{2\pi}}e^{iw'}\exp\left[i\frac{q}{\hbar c}\chi(\mathbf{R}+\mathbf{r}/2,\,T+\tau/2)\right]\exp\left[-i\frac{q}{\hbar c}\chi(\mathbf{R}-\mathbf{r}/2,\,T-\tau/2)\right]g(\mathbf{r},\tau,\mathbf{R},T) ,$$
$$w'=w-\int_{-1/2}^{1/2}d\lambda\left[\left[\frac{q}{\hbar c}\right]\tau\frac{\partial}{\partial t}\chi(\mathbf{x},t)+\left[\frac{q}{\hbar c}\right]\mathbf{r}\cdot\nabla\chi(\mathbf{x},t)\right]\equiv w+\Delta w . \tag{2.5}$$

In order to obtain a gauge-invariant $\tilde{g}$, Δw must cancel the factor $e^{+i(q\chi/\hbar c)-i(q\chi/\hbar c)}$ in (2.5). Indeed, by remembering, from (2.2), that $\mathbf{x}=\mathbf{R}+\lambda\mathbf{r}$ and $t=T+\lambda\tau$, we can write Δw as the total derivative

$$\Delta w=-\left[\frac{q}{\hbar c}\right]\int_{-1/2}^{1/2}d\lambda\frac{d\chi(\mathbf{x},t)}{d\lambda}=-\left[\frac{q}{\hbar c}\right][\chi(\mathbf{R}+\mathbf{r}/2,\,\mathbf{T}+\tau/2)-\chi(\mathbf{R}-\mathbf{r}/2,\,\mathbf{T}-\tau/2)]\ ,$$

and we see that the cancellation occurs.

Thus, we have proved that even though the wave functions and the electromagnetic potentials change with the gauge, products like (2.1), and therefore the various Green's functions, are independent of the gauge, provided that we transform them by the prescription (2.2). For an electron ($q=-e$) in homogeneous, steady-state fields, (2.2) reduces to

$$\tilde{g}(\mathbf{k},\omega,\mathbf{R},T)=\int\frac{d\tau}{\sqrt{2\pi}}e^{i(\omega+(e/\hbar)\mathbf{E}\cdot\mathbf{R})\tau}\int\frac{d\mathbf{r}}{(2\pi)^{3/2}}e^{-i\mathbf{k}\cdot\mathbf{r}}g_{\phi}(\mathbf{r},\tau,\mathbf{R},T) \tag{2.6}$$

for g calculated in the scalar-potential gauge (g_{ϕ}) with $\phi=-\mathbf{E}\cdot\mathbf{R}$, and to

$$\tilde{g}(\mathbf{k},\omega,\mathbf{R},T)=\int\frac{d\tau}{\sqrt{2\pi}}e^{i\omega\tau}\int\frac{d\mathbf{r}}{(2\pi)^{3/2}}\exp\left[-i\left[\mathbf{k}+\frac{e}{\hbar}\mathbf{E}T\right]\cdot\mathbf{r}\right]g_{\mathbf{A}}(\mathbf{r},\tau,\mathbf{R},T) \tag{2.7}$$

for g calculated in the vector-potential gauge ($g_{\mathbf{A}}$) with $\mathbf{A}=-c\mathbf{E}T$. In the Appendix, we give two simple examples of how (2.6) and (2.7) may be applied.

B. Airy transform

When a potential $U(z)$ is applied, the eigenfunctions $\Psi(\mathbf{r})$ of the system can be factorized as

$$\Psi(\mathbf{r})=e^{i\mathbf{k}_{\perp}\cdot\mathbf{r}_{\perp}}\varphi_s(z)\ ,$$

with $\varphi_s(z)$ determined by

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}+U(z)+\frac{h^2\mathbf{K}_{\perp}^2}{2m}\right]\varphi_s(z)=\varepsilon_s(\mathbf{k}_{\perp})\varphi_s(z)\ ,$$

where the index s labels the eigenvalues that can constitute either a discrete or a continuous spectrum. In the Hilbert space of the eigenfunctions $\varphi_s(z)$, for any function or operator F, we can define the transformation

$$F(\mathbf{k}_{\perp},z,z',\omega)=\sum_{s,s'}\varphi_s(z)F_{s,s'}(\mathbf{k}_{\perp},\omega)\varphi_{s'}(z')\ . \tag{2.8}$$

In particular, for the uniform-field case, this becomes the Airy transform[12]

$$F(\mathbf{k}_{\perp},z,z',\omega)=\int\int\frac{ds\,ds'}{L^2}\mathrm{Ai}((z-s)/L)\times\mathrm{Ai}((z'-s')/L)F(\mathbf{k}_{\perp},s,s',\omega)\ , \tag{2.9}$$

since in this case

$$\varphi_s(z)=(1/L)\mathrm{Ai}((z-s)/L)\ ,$$

with $L=(\hbar^2/2meE)^{1/3}$ as the normalization length defined by the condition

$$\int dz\,\varphi_s^*(z)\varphi_{s'}(z)=\delta(s-s')\ .$$

C. The two transformations combined

Assume now that s, s', and ω are not independent variables and consider functions of the type (see Sec. III for an example of a function of this kind)

$$F(\mathbf{k}_{\perp},\,\omega-(eE/\hbar)s,\,\omega-(eE/\hbar)s')\ .$$

In order to put this function into gauge-invariant form, we have to apply (2.9) and then (2.6), namely

$$\tilde{F}(\mathbf{k}_{\perp},\mathbf{k}_z,\mathbf{k}_{z'},\tau)=\int\frac{d\omega}{\sqrt{2\pi}}\exp\left[i\left[\omega+\frac{eE}{\hbar}Z\right]\tau\right]\int\frac{dz}{\sqrt{2\pi}}\int\frac{dZ}{\sqrt{2\pi}}e^{ik_z(Z+z/2)}e^{ik_{z'}(Z-z/2)}\times\int\int\frac{ds\,ds'}{L^2}\mathrm{Ai}\left[\frac{(Z+z/2-s)}{L}\right]\mathrm{Ai}((Z-z/2-s')/L)\times F(\mathbf{k}_{\perp},\,\omega-(eE/\hbar)s,\,\omega-(eE/\hbar)s')\ , \tag{2.10}$$

where z and z' in (2.9) have been replaced by $Z+z/2$ and $Z-z/2$ in the center-of-mass coordinate space.

By writing

$$F(\mathbf{k}_{\perp},\,\omega-(eE/\hbar)s,\,\omega-(eE/\hbar)s')=\int\frac{dt}{\sqrt{2\pi}}\exp\left[i\left[\omega-\frac{eE}{\hbar}s\right]t\right]\int\frac{dt'}{\sqrt{2\pi}}\exp\left[-i\left[\omega-\frac{eE}{\hbar}s'\right]t'\right]F(\mathbf{k}_{\perp},t,t')$$

and using the integral representation[14]

$$\mathrm{Ai}(x)=\int_{-\infty}^{\infty}\frac{du}{2\pi}e^{iu^3/3+ixu},$$

Eq. (2.10) reduces to

$$\tilde{F}(\mathbf{k},\tau)=\sqrt{2\pi}L\frac{\hbar}{eE}F(\mathbf{k}_\perp,\hbar k_z/eE+\tau/2,\hbar k_z/eE-\tau/2)\tilde{A}_E(k_z,\tau)\ . \tag{2.11}$$

Here we notice that $\hbar k_z/eE$ can be regarded as the k_z-dependent "center-of-mass" time $T(k_z)$ since $\hbar k_z$ is the electron momentum in the field direction and eE the accelerating force due to the field. Thus, in our formalism, the unperturbed, field-dependent spectral function $\tilde{A}_E$ can be exactly factored out of all the functions (Green's functions, self-energies, correlation functions) we will be dealing with. This resembles a formal analogy to Eq. (2.21) of Ref. 11 for the definition of the "reduced functions." In the present case, however, $\tilde{A}_E$ contains information about the motion along the field direction only. This motion is also described partly [through $T(k_z)$] by the F function on the right-hand side of Eq. (2.11), which also gives us information about the motion on the plane perpendicular to the field direction.

By the same procedure, a function $F(\omega-eEs/\hbar)$, of a single $(\omega-eEs/\hbar)$ argument, can be put into gauge-invariant form. In this case we have

$$\tilde{F}(\mathbf{k},\tau)=F(\mathbf{k}_\perp,\tau)\tilde{A}_E(k_z,\tau)\ , \tag{2.12}$$

where

$$F(\tau)=\int\frac{d\Omega}{\sqrt{2\pi}}e^{-i\Omega\tau}F(\Omega)$$

with

$$\Omega=\omega-eEs/\hbar\ .$$

The k_z dependence is now carried only by the unperturbed, field-dependent spectral density function $\tilde{A}_E$ which also contains an explicit dependence on the electric field. This is an interesting result: it reveals that the transverse and parallel components of the motion are separated and can be treated independently. Equation (2.12) also implies that the Fourier transform from τ to ω of Eq. (2.12) is just the convolution product

$$\tilde{F}(\mathbf{k},\omega)=\int d\Omega\,F(\mathbf{k}_\perp,\Omega)\tilde{A}_E(k_z,\omega-\Omega)\ . \tag{2.13}$$

This property will be very useful in calculating the quantities of interest. Furthermore, remembering the expression (1.6) for $\tilde{A}_E(k_z,\omega)$ given in the Introduction, we have

$$\tilde{F}(\mathbf{k},\omega)=\int d\Omega\,F(\mathbf{k}_\perp,\Omega)\frac{1}{\Theta}\mathrm{Ai}(\{\hbar\Omega-[\hbar\omega-\varepsilon(k_z)]\}/\Theta)\ . \tag{2.14}$$

This result states that in order to transform a function f defined in (s,ω) space into gauge-invariant form, we simply have to take its single Airy transform

$$f(s)=\int dz\frac{1}{L}\mathrm{Ai}((z-s)/L)f(z)\ ,$$

with L, z, and s replaced by Θ, $\hbar\Omega$, and $\hbar\omega-\varepsilon(k_z)$, respectively.

We can also establish a connection between Eq. (2.13) and (2.11). Let us take the average of (2.11) over the k_z dependence of the center-of-mass time $T(k_z)$. This means that we only have to average the function $F(T(k_z)+\tau/2,T(k_z)-\tau/2)$:

$$\langle F(\hbar k_z/eE+\tau/2,\hbar k_z/eE-\tau/2)\rangle=\int\frac{dk_z}{v}F(\hbar k_z/eE+\tau/2,\hbar k_z/eE-\tau/2)\ , \tag{2.15}$$

where $v=\sqrt{2\pi}L$ is the volume element in (k_z,s) space. Again by writing

$$F(\hbar k_z/eE+\tau/2,\hbar k_z/eE-\tau/2)=\int\frac{d\Omega}{\sqrt{2\pi}}\exp\left[-i\left[\frac{\hbar k_z}{eE}+\frac{\tau}{2}\right]\Omega\right]\int\frac{d\Omega}{\sqrt{2\pi}}\exp\left[i\left[\frac{\hbar k_z}{eE}-\frac{\tau}{2}\right]\Omega\right]F(\Omega,\Omega')\ ,$$

the above average (2.15) becomes

$$\langle F(\hbar k_z/eE+\tau/2,\hbar k_z/eE-\tau/2)\rangle=\frac{1}{v}\frac{eE}{\hbar}F(\tau)\ . \tag{2.16}$$

By inserting this in place of $F(\hbar k_z/eE+\tau/2,\hbar k_z/eE-\tau/2)$ in Eq. (2.11) we obtain precisely Eq. (2.12), which is valid for functions diagonal in $\omega-eEs/\hbar$ in s space.

D. An example: $\tilde{G}^r_E(\mathbf{k},\omega)$

As an example of how the above procedure is employed, let us transform G^r_E given in Eq. (1.4) into gauge-invariant form. In order to use (2.12), we need to know $G^r_E(\mathbf{k}_\perp,\tau)$. This can be calculated as follows:

$$G_E^r(\mathbf{k}_\perp,\tau)=\int \frac{d(\omega-(eE/\hbar)s)}{\sqrt{2\pi}}\exp\left[-i\left[\omega-\frac{eE}{\hbar}s\right]\tau\right]G_E^r(\mathbf{k}_\perp,s,\omega)$$

$$=\int \frac{d\Omega}{\sqrt{2\pi}}e^{-i\Omega\tau}\frac{1}{\hbar\Omega-\varepsilon(\mathbf{k}_\perp)+i\eta}=-\frac{i}{\hbar}\Theta(\tau)\exp\left[-i\frac{\varepsilon(\mathbf{k}_\perp)}{\hbar}\tau\right] .$$

By the prescription (2.12), and $\varepsilon(\mathbf{k})=\varepsilon(\mathbf{k}_\perp)+\varepsilon(k_z)$ then, we have

$$\tilde{G}_E^r(\mathbf{k},\tau)=-\frac{i}{\hbar}\Theta(\tau)\exp\left[-i\frac{(eE)^2}{24\hbar m}\tau^3-i\frac{\varepsilon(\mathbf{k})}{\hbar}\tau\right] , \tag{2.17}$$

in agreement with previous results,[10] and, by Fourier-transforming the τ variable,

$$\tilde{G}_E^r(\mathbf{k},\omega)=-\frac{\pi}{\Theta}\{\mathrm{Gi}(-[\hbar\omega-\varepsilon(\mathbf{k})]/\Theta)+i\,\mathrm{Ai}(-[\hbar\omega-\varepsilon(\mathbf{k})]/\Theta)\} , \tag{2.18}$$

where $\mathrm{Gi}(x)$ can be evaluated in terms of Airy functions.[14]

III. DYSON'S EQUATION AND ITS GAUGE-INVARIANT SOLUTION

As explained in the Introduction, the calculation of static properties such as the spectral function and the density of states of the system interacting with the environment requires the solution of the full Dyson's equation (1.3) for the retarded Green's function.

If the electric field is applied along the z direction, Dyson's equation can be written as

$$G^r(\mathbf{k}_\perp,z,z',\omega)=G_E^r(\mathbf{k}_\perp,z,z',\omega)+\int dz_1\int dz_2 G_E^r(\mathbf{k}_\perp,z,z_1,\omega)\Sigma^r(\mathbf{k}_\perp,z_1,z_2,\omega)G^r(\mathbf{k}_\perp,z_2,z',\omega) , \tag{3.1}$$

or, more simply, by using the transformation (2.3),

$$G^r(\mathbf{k}_\perp,s,s',\omega)=G_E^r(\mathbf{k}_\perp,s,\omega)\delta(s-s')+G_E^r(\mathbf{k}_\perp,s,\omega)\int ds_2\Sigma^r(\mathbf{k}_\perp,s,s_2,\omega)G^r(\mathbf{k}_\perp,s_2,s',\omega) . \tag{3.2}$$

Equation (3.1) can also be put in gauge-invariant form by the transformation (2.6) and it reads

$$\tilde{G}^r(\mathbf{k},\omega)=\tilde{G}_E^r(\mathbf{k},\omega)+\int\int\frac{dt_1dt_2}{2\pi}\int\int\frac{d\omega_1d\omega_2}{2\pi}\exp[it_1(\omega-\omega_1)-it_2(\omega_1-\omega_2)]\tilde{G}_E^r(\mathbf{k}_\perp,k_z-(eE/2\hbar)t_1,\omega+\omega_1-\omega_2)$$
$$\times\tilde{\Sigma}^r(\mathbf{k}_\perp,k_z-(eE/2\hbar)(t_1-t_2),\omega_1)\tilde{G}^r(\mathbf{k}_\perp,k_z+(eE/2\hbar)t_2,\omega_2) . \tag{3.3}$$

The solution of Eq. (3.2) requires, as a preliminary ingredient, the knowledge of the self-energy $\Sigma^r(\mathbf{k}_\perp,s,s_2,\omega)$, whereas for the solution of (3.3) we need an expression for $\tilde{\Sigma}^r(\mathbf{k}_\perp,k_z-eE(t_1-t_2)/2h,\omega_1)$.

In the Airy representation, the self-energy can be calculated by the methods described in Ref. 12. For a system of electrons weakly interacting with nonpolar-optical phonons in equilibrium, it turns out to be independent of the transverse momentum and the k_z dependence is replaced by the Airy coordinate s. It reads

$$\Sigma^r(s,s_2,\omega)=-i\sum_\eta|V|^2\left[N_0+\frac{\eta+1}{2}\right]\int\frac{dz}{L^2}\mathrm{Ai}\left[\frac{z-s}{L}\right]\mathrm{Ai}\left[\frac{z-s_2}{L}\right]$$
$$\times\int\frac{ds'}{L^2}\mathrm{Ai}^2\left[\frac{z-s'}{L}\right]\int\frac{d(\mathbf{k}_\perp-\mathbf{q}_\perp)}{(2\pi)^2}G_E^r(\mathbf{k}_\perp-\mathbf{q}_\perp,s',\omega-\eta\omega_0) , \tag{3.4}$$

where $\mathbf{q}$ and N_0 are the phonon momentum and occupation number, respectively, $|V|$ is the electron-phonon matrix element, and $\eta=+1(-1)$ corresponds to emission (absorption) of a phonon of frequency ω_0 by the electron of energy $\hbar\omega$. Equation (3.4) is exact in the sense that, within the present physical model (first Born approximation), no mathematical approximations were made to obtain it. Furthermore, Σ^r includes the electron-phonon–interaction matrix elements and it is a function of the difference of the ω and s variables, namely

$$\Sigma^r(s,s_2,\omega)=\Sigma^r(\omega-(eE/\hbar)s,\omega-(eE/\hbar)s_2) , \tag{3.5}$$

as can be immediately verified.

On the other hand, a model for Σ^r to be used in Eq. (3.3), which includes the intracollisional field effect to the lowest order, has been proposed by several authors.[15,16] In our formalism, this can be expressed as

$$\mathrm{Re}[\tilde{\Sigma}^r(\omega)]=0\ ,$$
$$\mathrm{Im}[\tilde{\Sigma}^r(\omega)]=\sum_\eta |V|^2\left[N_0+\frac{\eta+1}{2}\right]\int\frac{d\mathbf{k}}{(2\pi)^3}\mathrm{Im}[\tilde{G}^r_E(\mathbf{k},\omega-\eta\omega_0)] \tag{3.6}$$
$$=-2\pi\rho_{3D}\sum_\eta |V|^2\left[N_0+\frac{\eta+1}{2}\right]\Theta^{1/2}[\mathrm{Ai}'^2(-\xi)+\xi\,\mathrm{Ai}^2(-\xi)]\ ,$$

with $\xi=(\hbar\omega-\eta\hbar\omega_0)/\Theta$ and $\rho_{ND}=\pi V(2m/\hbar^2)^{N/2}$ the N-dimensional free-electron density-of-states factor.

This model neglects the real part of the self-energy, which determines the renormalization of the quasiparticle energies, and because of the averaging procedure over $\mathbf{k}$, ignores the dependence of the electron momentum along the field direction. In any case, however, neither (3.6) nor (3.4) allow Eq. (3.3) or Eq. (3.2) to be solved in an easy and obvious way. A better model, which does not ignore the above features, can be obtained by realizing that due to the singular nature of the self-energy (3.4) in $s-s_2$, the s_2 dependence of the product $\Sigma^r(\mathbf{k}_\perp,s,s_2,\omega)G^r(\mathbf{k}_\perp,s_2,s',\omega)$ in Eq. (3.2) is dominated by Σ^r, and that, therefore, we can move $G^r(\mathbf{k}_\perp,s_2,s',\omega)$ outside of the integral in (3.2). This results in the replacement of the self-energy (3.4) by its average over the variable s_2,

$$\Sigma^r\left[\omega-\frac{eE}{\hbar}s\right]=(\tfrac{3}{8})^{1/2}\frac{\rho_{2D}}{\pi}\left[\frac{eE}{\Theta}\right]\sum_\eta |V|^2\left[N_0+\frac{\eta+1}{2}\right]\int_0^\infty\frac{dt}{t^{3/2}}\exp\left[i\left[\frac{t^3}{12}+\zeta t+\frac{\pi}{4}\right]\right]\ , \tag{3.7}$$

where $\zeta=(\hbar\omega-eEs-\eta\hbar\omega_0)/\Theta$. Equation (3.7) can be transformed into gauge-invariant form by the prescription (2.13) and it reads

$$\tilde{\Sigma}^r(k_z,\omega)=\sum_\eta |V|^2\left[N_0+\frac{\eta+1}{2}\right]\tilde{F}(k_z,\omega)\ ,$$
$$\mathrm{Re}[\tilde{F}(k_z,\omega)]=\frac{\rho_{3D}}{(2\pi)^2}\Theta^{1/2}\left[\mathrm{Ai}'(\xi)\mathrm{Bi}'(\xi)-\xi\,\mathrm{Ai}(\xi)\mathrm{Bi}(\xi)+\frac{\sqrt{\xi}}{\pi}\Theta(\xi)\right]\ , \tag{3.8}$$
$$\mathrm{Im}[\tilde{F}(k_z,\omega)]=-\frac{\rho_{3D}}{(2\pi)^2}\Theta^{1/2}[\mathrm{Ai}'^2(\xi)-\xi\,\mathrm{Ai}^2(\xi)]\ ,$$

where now $\xi=[\hbar\omega-\varepsilon(k_z)-\eta\hbar\omega_0]/\Theta$. The general behavior of $\mathrm{Im}[\tilde{\Sigma}^r(k_z,\omega)]$ is very similar to that of $\mathrm{Im}[\Sigma^r(\omega)]$, except that now an explicit dependence on k_z has to be considered. Furthermore, the real part of Σ^r, which describes how the unperturbed energy of the electron is modified by the presence of the interactions (including the electric field), is now taken into account. The analytical and physical properties of a self-energy of the type (3.8) have been extensively discussed[12] in the Airy-coordinate representation (s,ω) and an extension to the present case is straightforward and will not be repeated here. We notice, however, that, because of the many integrations and the mixing of momentum and time variables appearing in (3.3), the self-energy (3.8) is apparently not sufficient to diagonalize this expression of Dyson's equation. Nevertheless, the form (3.7) for the self-energy allows Eq. (3.2) to be solved immediately, and we obtain

$$G^r(\mathbf{k}_\perp,s,\omega)=\frac{1}{\hbar\omega-eEs-\varepsilon(\mathbf{k}_\perp)-\Sigma^r(\omega-(eE/\hbar)s)}\ . \tag{3.9}$$

Since the full retarded Green's function above is only a function of the difference $\omega-eEs/\hbar$, we can again use Eq. (2.13) to transform it into gauge-invariant form; that is,

$$\tilde{G}^r(\mathbf{k},\omega)=\int d\Omega\, G^r(\mathbf{k}_\perp,\Omega)\tilde{A}_E(k_z,\omega-\Omega)\ . \tag{3.10}$$

The gauge-invariant spectral density can now be immediately calculated, and it is given by

$$\tilde{A}(\mathbf{k},\omega)=\int d\Omega\, A(\mathbf{k}_\perp,\Omega)\tilde{A}_E(k_z,\omega-\Omega) \tag{3.11}$$

with

$$A(\mathbf{k}_\perp,\Omega)=-2\,\mathrm{Im}G^r=\frac{-2\,\mathrm{Im}\Sigma^r(\Omega)}{[\hbar\Omega-\varepsilon(\mathbf{k}_\perp)-\mathrm{Re}\Sigma^r(\Omega)]^2+[\mathrm{Im}\Sigma^r(\Omega)]^2}\ . \tag{3.12}$$

Figure 1 shows the spectral density function $\tilde{A}(\mathbf{k},\omega)$ for different values of the electric field, as a function of the dimensionless variable $\hbar\omega/\Theta$ ($\Theta/\hbar\omega_0=0.004$, 0.12, 0.55, and 2.54 for $E=1$, 5, 50, and 500 kV/cm, respectively). Because of the approximately equal weight that $A(\mathbf{k}_\perp,\Omega)$ and $\tilde{A}_E(\mathbf{k}_z,\omega-\Omega)$ gives to the integral (3.11) at these values of the field, $\tilde{A}(\mathbf{k},\omega)$ is not the positive semidefinite quantity which is required in standard Monte Carlo simulations. However, at very high electric fields (≥ 500 kV/cm), or for higher scattering rates, where the dominating contribution in (3.11) arises from the broad Lorentzian-type shape of $A(\mathbf{k}_\perp,\Omega)$, the oscillatory behavior of $\tilde{A}$ is characterized by a rather large period (the first negative value of $\tilde{A}$ appears at electron energies $\hbar\omega\geq 10\hbar\omega_0$) with the amplitude of the oscillations decaying very slowly as the energy increases. As the field strength is reduced, the oscillations are compressed to a much smaller range of energies (the first negative value of $\tilde{A}$ appears at $\hbar\omega=0.5\hbar\omega_0$ for $E=50$ kV/cm) and their amplitude now decreases very rapidly. Finally, the oscillatory behavior dies out at very low fields, where $\tilde{A}_E$ approaches its δ-function behavior, and

the more familiar[4,15,17,18] Lorentzian-type shape of $\tilde{A}$ reappears.

$\tilde{A}(\mathbf{k},\omega)$ satisfies the normal sum rules because (3.12) does[12] and because of the normalization properties of the Airy functions [see Eq. (1.6)].

The zero-field, zero-scattering limit can also be evaluated. From (3.12) and (3.7) we have

$$\lim_{E\to 0} A(\mathbf{k}_\perp,\Omega)=\sqrt{2\pi}\delta(\hbar\Omega-\varepsilon(\mathbf{k}_\perp)) ,$$

and from Eq. (1.7),

$$\lim_{E\to 0} \tilde{A}_E(k_z,\omega-\Omega)=\sqrt{2\pi}\delta(\hbar\omega-\hbar\Omega-\varepsilon(k_z));$$

therefore,

$$\tilde{A}_{\text{free}}(\mathbf{k},\omega)=2\pi\int d\Omega\,\delta(\hbar\Omega-\varepsilon(\mathbf{k}_\perp))\delta(\hbar\omega-\hbar\Omega-\varepsilon(k_z))$$
$$=2\pi\delta(\hbar\omega-\varepsilon(\mathbf{k})) , \tag{3.13}$$

as in the semiclassical theory.

Once the spectral density is known, we can calculate the density of states per spin,

$$\tilde{\rho}(\omega)=\int\frac{d\mathbf{k}}{(2\pi)^3}\tilde{A}(\mathbf{k},\omega) ,$$

as follows:

$$\tilde{\rho}(\omega)=\int\frac{d\mathbf{k}}{(2\pi)^3}\int d\Omega\,A(\mathbf{k}_\perp,\Omega)\tilde{A}_E(k_z,\omega-\Omega)$$
$$=\int d\Omega\,\rho_\perp(\Omega)\rho_\parallel(\omega-\Omega) , \tag{3.14}$$

where

$$\rho_\perp(\Omega)=\int\frac{d\mathbf{k}_\perp}{(2\pi)^2}A(\mathbf{k}_\perp,\Omega)$$
$$=\frac{\rho_{2D}}{2\pi^2}\left[\frac{\pi}{2}+\tan^{-1}\left[\frac{\hbar\Omega-\mathrm{Re}\Sigma^r(\Omega)}{\mathrm{Im}\Sigma^r(\Omega)}\right]\right] \tag{3.15}$$

and

$$\rho_\parallel(\omega-\Omega)=\int\frac{dk_z}{2\pi}\tilde{A}_E(k_z,\omega-\Omega)$$
$$=\frac{\rho_{1D}}{(2\pi)^2}\frac{1}{\sqrt{\Theta}}\mathrm{Ai}^2\left[-\frac{\hbar(\omega-\Omega)}{\Theta}\right] . \tag{3.16}$$

Figure 2 shows the density of states $\rho_\parallel$. This is the density of states for a one-dimensional system corresponding to the component of the motion along the field direction. As the electron energy $\hbar\omega$ is increased, $\rho_\parallel$ oscillates between zero and twice the free-electron form of the density of states,

$$\rho_\parallel=\frac{\rho_{1D}}{(2\pi)^3}\frac{1}{\sqrt{\hbar\omega-\hbar\Omega}} .$$

Explicitly,

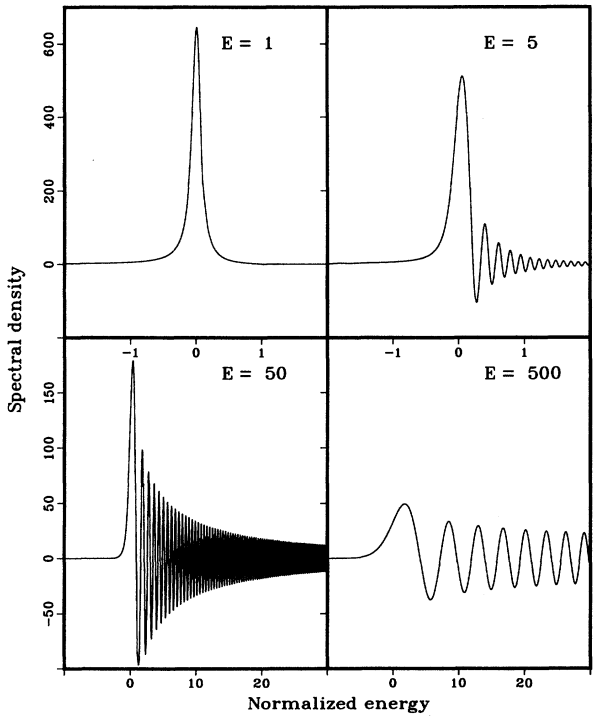

FIG. 1. The spectral density function vs the dimensionless variable $\hbar\omega/\Theta$ for phonon-emission processes with $\mathbf{k}=\mathbf{0}$ for four different values of the electric field E given in kV/cm. We use parameters appropriate to Si: an optical-phonon energy $\hbar\omega_0=0.039$ eV, $m=0.295m_0$, where m_0 is the free-electron mass. The electron-phonon interaction is given by a deformation-potential parameter $D_t=6.85\times10^{10}$ eV m^{-1}.

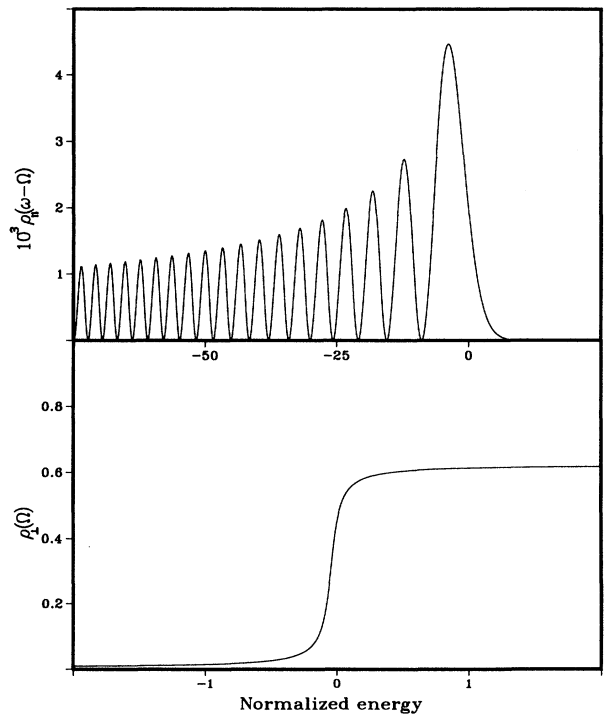

FIG. 2. The densities of states $\rho_\parallel$ and $\rho_\perp$ as functions of the dimensionless variable $\hbar\Omega/\Theta$.

$$\lim_{\hbar\omega\to\infty}\rho_{\parallel}(\omega-\Omega)=\frac{\rho_{1D}}{4\pi^3}\frac{1}{\sqrt{\hbar\omega-\hbar\Omega}}\times\sin^2\left[\frac{2}{3}\left[\frac{\hbar\omega-\hbar\Omega}{\Theta}\right]^{3/2}+\frac{\pi}{4}\right].$$

On the other hand, at very small energies, namely for vanishing electric fields, $\rho_{\parallel}$ does show free-electron behavior since

$$\lim_{E\to 0}\rho_{\parallel}(\omega-\Omega)=\frac{\rho_{1D}}{(2\pi)^3}\frac{1}{\sqrt{\hbar\Omega-\hbar\omega}}\Theta(\hbar\Omega-\hbar\omega)\ ,$$

where $\Theta(\hbar\Omega-\hbar\omega)$ is the step function.

Figure 2 also shows the density of states $\rho_{\perp}$ corresponding to the component of the motion on the plane perpendicular to the field direction. This is the density of states for a two-dimensional system. Here, however, the sharp step-function behavior, typical of the free-electron system, is smeared by the presence of the interactions.

Finally, Fig. 3 shows the total density of states (3.14) for different values of the electric fields. Here, as well as in $\rho_{\parallel}$ of Fig. 2, the oscillations denote the existence of preferred energies (rather, energy subbands) for the electron, caused by the presence of the electric field. These subbands, however, are compressed to a very small energy range as the electric field decreases, until they collapse to form a continuous spectrum at $E\leq 5$ kV/cm with the zero-field behavior restored at high electron energies. On the other hand, the negative-energy tail, also present at low fields where the oscillations have already disappeared, shows the effect of the collisional broadening. All this, together with the features of the real part of the gauge-invariant self-energy (3.8), essentially confirms, at least qualitatively, the results obtained previously[12] within the scalar-potential-gauge formulation.

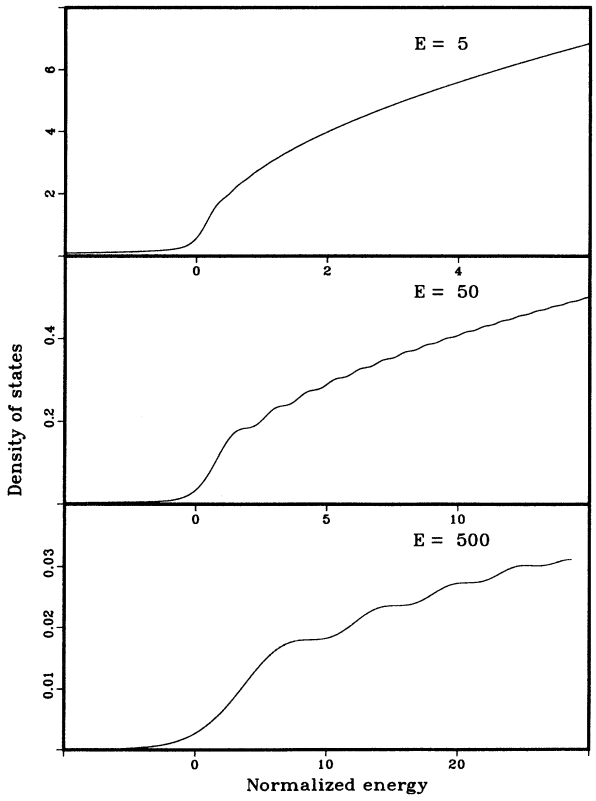

FIG. 3. The gauge-invariant density of states $\tilde{\rho}$ as given in Eq. (3.14) vs $\hbar\omega/\Theta$ for three different values of the electric field.

IV. CONCLUSIONS

Spectral density is an object of central interest in theories of interacting many-body systems. On one hand, it gives information about the quasiparticle spectrum of the system, such as densities of states or lifetimes. On the other hand, quantum kinetic theories based on nonequilibrium Green's-function techniques often require the knowledge of the spectral density as a prerequisite. Solutions for the spectral density for nontrivial systems under highly nonequilibrium conditions, such as those encountered in many semiconductor microstructures, are scarce, and in most cases restricted to limiting cases only (e.g., weak fields or weak scattering). The recent results of Bertoncini *et al.*[12] are an exception: By introducing a convenient set of variables ("Airy coordinates"), a solution for the spectral density is found that treats scattering and field effects on equal footing. The solution was obtained by making use of the singular nature of the self-energy function in the Airy representation, and it preserves the sum rules that any proposal for the spectral density must obey. However, the interpretation of results given in Airy coordinates is not straightforward because of the explicit gauge dependence. Therefore, in the present paper, we have undertaken a general analysis of the interrelationship of Airy-coordinate results and their counterparts in a gauge-invariant formulation. As an illustration of the techniques, we consider a number of examples, the results of some of which are already known. Our main formal results are contained in Eqs. (2.11) and (2.13), which allow one to transform any function found in Airy coordinates to a gauge-invariant form. As a nontrivial application, we consider the model electron-phonon system studied in Ref. 12, and analyze the resulting spectral densities and densities of states as a function of the applied field (Figs. 1–3). We find a transition from a collision-dominated regime for low fields (pure collisional broadening) to a field-dominated regime for high fields.

Our results provide a starting point for further work. The ultimate goal is to find a solution for the quantum distribution function, such as the Wigner function, from which observables such as number density or current can be extracted. There are at least three possible ways to proceed. The first would be to construct a joint spectral density, as suggested by Reggiani *et al.*,[18] and apply the quantum Monte Carlo technique.[19] However, the nonpositive definiteness of the spectral density requires either an approximation scheme or nonstandard simulation tech-

niques. Another possibility is to construct a simulation technique directly in the Airy-coordinate representation; this approach is presently being pursued.[20] Finally, the techniques developed in the present work can be applied to the Kadanoff-Baym quantum kinetic equations in order to derive a gauge-invariant distribution function $\tilde{f}(\mathbf{k})$ that will replace the "distribution function," $f(s,\omega)$, of the Airy representation,[12] from which a straightforward evaluation of the current is not possible. Work along these lines is in progress, and we hope to report our results in the near future.

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APPENDIX

Example 1: Let us apply the transformation (2.2) to the retarded Green's function,[10]

$$G^r_\phi(\mathbf{k}_1,t_1;\mathbf{k}_2,t_2)=-\frac{i}{\hbar}\Theta(t_1-t_2)\delta(\mathbf{k}_1-\mathbf{k}_2+(e/\hbar)E(t_1-t_2))\exp\left[-\frac{i}{\hbar}\int_0^{t_1-t_2}dt'\varepsilon\left[\mathbf{k}_1+\frac{e}{\hbar}\mathbf{E}t'\right]\right], \tag{A1}$$

calculated in the scalar potential gauge with $\phi=-\mathbf{E}\cdot\mathbf{R}$.

First, let us Fourier-transform (A1),

$$\begin{aligned}G^r_\phi(\mathbf{k}_1,t_1;\mathbf{k}_2,t_2)&=-\frac{i}{\hbar}\Theta(t_1-t_2)\int\frac{d\mathbf{k}_1d\mathbf{k}_2}{(2\pi)^3}e^{i\mathbf{k}_1\cdot\mathbf{r}_1-i\mathbf{k}_2\cdot\mathbf{r}_2}\delta(\mathbf{k}_1-\mathbf{k}_2+(e/\hbar)E(t_1-t_2))\\&\quad\times\exp\left[-\frac{i}{\hbar}\int_0^{t_1-t_2}dt'\varepsilon\left[\mathbf{k}_1+\frac{e}{\hbar}\mathbf{E}t'\right]\right]\\&=-\frac{i}{\hbar}\Theta(t_1-t_2)\int\frac{d\mathbf{k}_1}{(2\pi)^3}\exp\left[i\mathbf{k}_1\cdot\mathbf{r}_1-i\left[\mathbf{k}_1+\frac{e}{\hbar}\mathbf{E}(t_1-t_2)\right]\cdot\mathbf{r}_2\right]\cdot\exp\left[-\frac{i}{\hbar}\int_0^{t_1-t_2}dt'\varepsilon\left[\mathbf{k}_1+\frac{e}{\hbar}\mathbf{E}t'\right]\right],\end{aligned}$$

or, using the center-of-mass coordinates,

$$G^r_\phi(\mathbf{r},\tau;\mathbf{R},T)=-\frac{i}{\hbar}\Theta(\tau)\int\frac{d\mathbf{k}_1}{(2\pi)^3}\exp\left[i\mathbf{k}_1\cdot\mathbf{r}-i\frac{e}{\hbar}\mathbf{E}\cdot\left[\mathbf{R}-\frac{\mathbf{r}}{2}\right]\tau\right]\cdot\exp\left[-\frac{i}{\hbar}\int_0^{\tau}dt'\varepsilon\left[\mathbf{k}_1+\frac{e}{\hbar}\mathbf{E}t'\right]\right].$$

Now we can apply (2.6):

$$\begin{aligned}\tilde{G}^r(\mathbf{k},\omega,\mathbf{R},T)&=\int_{-\infty}^{\infty}\frac{d\tau}{\sqrt{2\pi}}\exp\left[i\left[\omega+\frac{e}{\hbar}\mathbf{E}\cdot\mathbf{R}\right]\tau\right]\int\frac{d\mathbf{r}}{(2\pi)^{3/2}}e^{-i\mathbf{k}\cdot\mathbf{r}}G^r_\phi(\mathbf{r},\tau;\mathbf{R},T)\\&=-\frac{i}{\hbar}\int_0^{\infty}\frac{d\tau}{\sqrt{2\pi}}e^{i\omega\tau}\int d\mathbf{k}_1\delta(\mathbf{k}_1+(e/2\hbar)\mathbf{E}\tau-\mathbf{k})\exp\left[-\frac{i}{h}\int_0^{\tau}dt'\varepsilon\left[\mathbf{k}_1+\frac{e}{\hbar}\mathbf{E}t'\right]\right]\\&=-\frac{i}{\hbar}\int_0^{\infty}\frac{d\tau}{\sqrt{2\pi}}e^{i\omega\tau}\exp\left[-\frac{i}{\hbar}\int_0^{\tau}dt'\varepsilon\left[\mathbf{k}_1-\frac{e}{2\hbar}\mathbf{E}\tau+\frac{e}{2\hbar}\mathbf{E}t'\right]\right]\\&=-\frac{i}{\hbar}\int_0^{\infty}\frac{d\tau}{\sqrt{2\pi}}e^{i\omega\tau}\exp\left[-\frac{i}{\hbar}\int_{-\tau/2}^{\tau/2}dt'\varepsilon\left[\mathbf{k}_1+\frac{e}{2\hbar}\mathbf{E}t'\right]\right]\\&=\tilde{G}^r(\mathbf{k},\omega)\ .\end{aligned} \tag{A2}$$

Example 2: Let us now transform the Green's function,[10]

$$G^r_A(\mathbf{p}_1,t_1;\mathbf{p}_2,t_2)=-\frac{i}{\hbar}\Theta(t_1-t_2)\delta(\mathbf{p}_1-\mathbf{p}_2)\exp\left[-\frac{i}{\hbar}\int_{t_2}^{t_1}dt'\varepsilon\left[\mathbf{p}_1-\frac{e}{\hbar c}\mathbf{A}(t')\right]\right],$$

calculated in the vector-potential gauge.

Using the same procedure as used in example 1, and applying (2.7), we can write

$$\tilde{G}^{r}(\mathbf{k},\omega;\mathbf{R}_2,T)=-\frac{i}{\hbar}\int_0^\infty\frac{d\tau}{\sqrt{2\pi}}e^{i\omega\tau}\int\frac{d\mathbf{r}}{(2\pi)^{3/2}}\exp\left[-i\left[\mathbf{k}+\frac{e}{\hbar}\mathbf{E}T\right]\cdot\mathbf{r}\right]\cdot\int\frac{d\mathbf{p}}{(2\pi)^3}e^{i\mathbf{p}_1\cdot\mathbf{r}}\exp\left[-\frac{i}{\hbar}\int_{T-\tau/2}^{T+\tau/2}dt'\varepsilon\left[\mathbf{p}_1-\frac{e}{\hbar}\mathbf{E}t'\right]\right]$$

$$=-\frac{i}{\hbar}\int_0^\infty\frac{d\tau}{\sqrt{(2\pi)}}e^{i\omega\tau}\int d\mathbf{p}_1\delta(\mathbf{p}_1-\mathbf{k}-(e/\hbar)\mathbf{E}T)\cdot\exp\left[-\frac{i}{\hbar}\int_{T-\tau/2}^{T+\tau/2}dt'\varepsilon\left[\mathbf{p}_1-\frac{e}{\hbar}\mathbf{E}t'\right]\right]$$

$$=-\frac{i}{\hbar}\int_0^\infty\frac{d\tau}{\sqrt{(2\pi)}}e^{i\omega\tau}\exp\left[-\frac{i}{\hbar}\int_{-\tau/2}^{\tau/2}dt'\varepsilon\left[\mathbf{k}-\frac{e}{\hbar}\mathbf{E}t'\right]\right]$$

$$=\tilde{G}^{r}(\mathbf{k},\omega)\ , \tag{A3}$$

as in Eq. (A2).

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