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Quantum oscillation in vortex states of type-II superconductors

Kazumi Maki*

Groupe de Physique des Solides, Université Paris 7, Tour 23, 2, Place Jussieu, 75251 Paris CEDEX, France

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We examine theoretically magnetic quantum oscillations like the de Haas–van Alphen and Subnikov–de Haas effects in vortex states in type-II superconductors. For a magnetic field B not very small compared with $H_{c2}(T)$, the main effect of superconductivity is introduction of the extra quasiparticle scattering rate proportional to $|\Delta|^2$, the square of the superconducting order parameter. Therefore the detectability of the magnetic quantum oscillation is determined from the condition that the extra Dingle temperature due to $|\Delta|^2$ is not too large. For example, in a magnetic field ~ 10 T, the superconducting transition temperature T_c has to be less than 30 K in order to have a reasonable signal.

As is well known the quantum oscillation in magnetization and magnetoresistance will provide unique information on the shape of the Fermi surface in the normal metals. On the other hand, whether the similar oscillation exists in the vortex states in type-II superconductors is not clear, though the de Haas–van Alphen (dHvA) effect in the vortex states of $2H$-$NbSe_2$ has been reported already.[1] In particular, this possibility allows a unique way to explore the Fermi surface of high-T_c superconducting cuprates, since their upper critical field $H_{c2}(T)$ at low temperatures is extremely large.

In this paper we shall examine the Landau quantum oscillation in a vortex state of type-II superconductors. For definiteness we consider a Fermi surface that is a modulated cylinder, as found in many organic superconductors,[2] and a magnetic field $B(\sim 10\text{ T})$ tilted by an angle θ from the c axis. Since the size of the Landau orbit at the Fermi surface is still large ($\sim 10\ \mu$m) for $B \sim 10$ T, coarse-grain homogeneity of the magnetic field in the scale of 1 μm suffices for the quantization of the quasiparticle orbit.[3] Therefore our problem reduces to formulation of the quantization of the quasiparticle orbit in the presence of the superconducting order parameter in the vortex state of a type-II superconductor.

When the orbital quantization is negligible, the Green's function of the quasiparticle in the vortex state in a magnetic field B close to the upper critical field in the clean limit is given by[4,5]

$$g^{-1}(i\omega_n,\mathbf{p}) = i\tilde{\omega}_n - \xi - \Delta^2 \int_{-\infty}^{\infty} \frac{\rho(u)\,du}{i\tilde{\omega}_n + \xi - u} \tag{1}$$

where

$$\rho(u) = (\sqrt{\pi}\alpha)^{-1} e^{-(u/\alpha)^2}, \quad \alpha = v_F(2eB\cos\theta)^{1/2},$$
$$\xi = \frac{1}{2m} p_\perp^2 - 2t_c \cos(cp_3), \tag{2}$$

and

$$\tilde{\omega}_n = \omega_n \left(1 + \frac{1}{2\tau|\omega_n|}\right) \tag{3}$$

and ω_n to the Matsubara frequency and τ is the quasiparticle lifetime in the normal state. The above Green's function is a generalization of the one in an isotropic system to an anisotropic system.

From the pole of the Green's function the energy and the damping constant of the quasiparticle are given by

$$E \cong \xi[1+2(\Delta/\alpha)^2], \quad \text{for } |\xi| < \alpha, \tag{4}$$

$$\Gamma = \tau^{-1} + 2\sqrt{\pi}\Delta^2/\alpha, \tag{5}$$

and

$$\Delta^2 \cong \Delta^2(T)[1 - B/H_{c2}(T,\theta)], \tag{6}$$

at low temperatures and $\Delta(T)$ is the temperature dependent BCS energy gap.

In the normal state the quasiparticle energy in a tilted magnetic field is quantized[6] as

$$\xi = \xi(n,p_3) = \omega_c(\theta)(n-n_0) - A\cos(cp_3), \tag{7}$$

where

$$\omega_c = cB\cos\theta/m, \quad n_0 = p_F^2/2m\omega_c(\theta)$$
$$A = 2t_c J_0(cp_F \tan\theta). \tag{8}$$

In the vortex state the energy spectrum are changed according to Eqs. (4) and (5). Now the density of states is given by

$$\begin{aligned} N(E) &= \frac{eH\cos\theta}{(2\pi)^2} \int_0^{2\pi/c} dp_3 \sum_n \frac{\Gamma}{\pi}[(E_n - E)^2 + \Gamma^2]^{-1} \\ &= \frac{m}{2\pi c}[1+2(\Delta/\alpha)^2]^{-1} \left\{ 1 + 2\sum_{k=1}^{\infty} J_0\left(\frac{2\pi k A}{\omega_c}\right) \cos\left[\frac{k}{\omega_c}\left(\frac{S}{m} + 2\pi E[1+2(\Delta/\alpha)^2]^{-1}\right)\right] \right. \\ &\qquad \left. \times \exp\left(-\frac{2\pi k \Gamma}{\omega_c}[1+2(\Delta/\alpha)^2]^{-1}\right) \right\}; \end{aligned} \tag{9}$$

J_0 is the Bessel function and $S=\pi p_F^2$ is the cross section of the Fermi surface. Therefore the Dingle temperature[7] in the vortex state is given by

$$T_D=\frac{\Gamma}{\pi}[1+2(\Delta/\alpha)^2]^{-1}=\pi^{-1}\left(\frac{1}{\tau}+2\sqrt{\pi}\Delta^2/\alpha\right)[1+2(\Delta/\alpha)^2]^{-1}. \tag{10}$$

For $B=10$ T, α is of the order of 10^3 K, for $v_F\sim 10^8$ cm/sec, therefore, for superconductors with $T_c<20$ K, $(\Delta/\alpha)^2$ is always negligible. Then the standard method[8] yields the oscillation of magnetization and the magnetoresistance for the electric current parallel to the c axis

$$\delta M\cong VTN_0B^{-1}\left(\frac{S}{m}\right)\sum_{k=1}^{\infty}J_0\left(\frac{2\pi kA}{\omega_c}\right)\sin\left(\frac{kS}{eH\cos\theta}\right)\left[\sinh\left(\frac{2\pi^2}{\beta\omega_c}k\right)\right]^{-1}\exp\left(-\frac{2\pi k\Gamma}{\omega_c}\right) \tag{11}$$

and

$$\delta\rho_{zz}=-\rho_{zz}(0)\frac{2\pi^2T}{\omega_c}\sum_{k=1}^{\infty}kJ_0\left(\frac{2\pi kA}{\omega_c}\right)\cos\left(\frac{kS}{eH\cos\theta}\right)\left[\sinh\left(\frac{2\pi^2}{\beta\omega_c}k\right)\right]^{-1}\exp\left(-\frac{2\pi k\Gamma}{\omega_c}\right), \tag{12}$$

though in order to detect the magnetoresistance the vortex state has to be in the flux-flow regime. In particular for $B>H_{c2}(T,\theta)$ where $\Delta=0$, the above expression describes the giant quantum oscillation in de Haas–van Alphen and Subnikov–de Haas effect in the normal state of the organic superconductors.[2] Further in the vortex state Eq. (10) accounts for the increase of the Dingle temperature as observed,[1] in $2H$-$NbSe_2$. Also Eqs. (11) and (12) indicate the main effect of the superconducting order parameter is the increase in the Dingle temperature. Other corrections are of the order of $(\Delta/\alpha)^2$, which are completely negligible in the clean limit compared with $\tau(\Delta^2/\alpha)$. Though we have obtained Eqs. (11) and (12) for a model with a particular Fermi surface, the expression for the Dingle temperature [Eq. (10)] is valid in a more general circumstance.

Therefore, in order that the quantum oscillation is detectable we have to have $2\pi\Gamma/\omega_c\leq 1$, or in the vortex state

$$4\pi^{3/2}[\Delta(0)]^2\left(1-\frac{B}{H_{c2}(T)}\right)\Big/\alpha\omega_c\leq 1. \tag{13}$$

For $B\cong 10$ T and $v_F=10^8$ cm/sec and m same as the electron mass, we obtain

$$\Delta(0)\left(1-\frac{B}{H_{c2}}\right)^{1/2}\leq 10^2\,\mathrm{K}. \tag{14}$$

Therefore in a magnetic field of the order of 10 T, the superconducting transition temperature has to be less than 60 K and application of the present technique for high-T_c cuprates appear to be difficult. Perhaps in a magnetic field of 30 T the present technique is certainly applicable for high-T_c cuprates. On the other hand, there will be no difficulty in seeing the giant quantum oscillation predicted here in the vortex states of organic superconductors, since their highest T_c is still around 11 K.

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*Permanent address: Department of Physics, University of Southern California, Los Angeles, CA 90089-0484.

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