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Vortex-chain states and critical currents in anisotropic high-T_c superconductors

B. I. Ivlev*
Department of Physics, University of Southern California, Los Angeles, California 90089-0484

N. B. Kopnin*
Low Temperature Laboratory, Helsinki University of Technology, SF-02150 Espoo, Finland
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The vortex state in an anisotropic superconductor with a magnetic field tilted with respect to the *ab* plane is considered. Vortices are shown to form a chain structure with large intrachain vortex density and large interchain distance. The shear energy when one chain shifts relative to the others is decreased in this state. This leads to an increase of the effective pinning force and of the critical depinning current. A qualitative comparison is given between theory and experimental data on the dependence of critical current on the tilt angle.

I. INTRODUCTION

The static behavior of magnetic flux lattices in high-T_c superconductors has drawn considerable attention.[1-8] In the papers[1,9] the triangular Abrikosov vortex lattice was observed experimently using the decoration technique. In the paper[6] a qualitatively different picture was obtained. Besides some region with the conventional triangular lattice, separate vortex chains with quite different periodicity were observed. The period of the vortices on each chain was significantly smaller than the size of the vortex lattice unit cell. The formation of vortex chains can be understood by taking into account the attractive character of the vortex interaction in an anisotropic superconductor.[10-12] Vortices attract each other if they are tilted at some angles with respect to the $a-b$ plane of the high-T_c material. The vortices penetrate inside the sample near the lower critical field H_{c1} as chains,[10] and this vortex chain structure (VCS) remains up to some much higher magnetic field.[12]

This vortex structure rearrangement leads to an interesting behavior.[12] An essential feature of the VCS is a softening of the vortex lattice. The shift of one dense chain relative to another costs only a small energy, and so the shear modulus is small.[7-12] This softening can even result in melting of the vortex state.[13-15] Anyway, the softening or melting should make weaker the collective lattice properties. In a soft state the pinning centers are more effective, because the Lorentz force acting on a pinned vortex is not summed over the large volume of a lattice. As a result the critical current increases in VCS.

The purpose of this paper is to point out the conditions on the magnetic field and temperature for the VCS existence and to analyze the experimental data[16] on critical current measurements in the framework of the VCS.

II. DESCRIPTION OF THE LATTICE

We will consider an anisotropic high-T_c superconductor like a Y-Ba-Cu-0 compound. This material can be considered as homogeneous, at least at not very low temperatures. The superconductor is supposed to be uniaxial with London penetration depths λ_{ab} for currents in the *ab* plane and λ_c for currents parallel to the *c* axis ($\lambda_c \gg \lambda_{ab}$). The magnetic induction **B** is tilted with respect to the *ab* plane at the angle θ as shown in Fig. 1.

The vortex lattice can be considered in the xz' which is perpendicular to the vortices. The triangular unit cell has sizes x_0 and z_0 in the xz' plane as shown on a Fig. 2. Within a unit cell the coordinates of the vortices are 0, qx_0, and x_0. The case $q=\frac{1}{2}$ corresponds to the isosceles triangle unit cell. The parameter q has to be found by energy minimization, due to the flux quantization rule $x_0 z_0=\phi_0/B$. Let us introduce the parameter

$$p=2\pi\frac{\lambda(\theta)}{\lambda_c}\frac{x_0}{z_0}\ , \tag{1}$$

where the angle-dependent penetration depth is determined by the equation $\lambda^2(\theta)=\lambda_{ab}^2\cos^2\theta+\lambda_c^2\sin^2\theta$. For a given value of the magnetic field the parameters p and q determine the vortex lattice. We have chosen in Fig. 2 the "natural" coordinates x' and $z'\lambda_c/\lambda(\theta)$ in which the *normal* Abrikosov lattice consists of equilateral triangles. For this normal lattice $p\approx 1$. Below we shall find the actual vortex lattice to be strongly different from the normal one in some interval of tilted angle values.

III. THE FREE ENERGY Of A VORTEX LATTICE

On the basis of the London approach one can write down an expression for the free energy density as was done in the paper:[12]

$$F=\frac{B^2}{8\pi}+\frac{\phi_0 B\lambda(\theta)}{32\pi^2\lambda_{ab}^2\lambda_c}\left[\ln\left[\frac{H_{c2}(\theta)}{B}\right]+g(p,q)\right]\ . \tag{2}$$

Here $H_{c2}(\theta)$ is the upper critical field and

$$g(p,q)=G(p,q)-\frac{p}{6}\frac{\lambda_c^2\sin^2\theta}{\lambda^2(\theta)}+4\alpha\frac{\lambda_{ab}\cos\theta}{\lambda(\theta)}\left[\frac{p}{6}\right]^{1/2}\ , \tag{3}$$

where

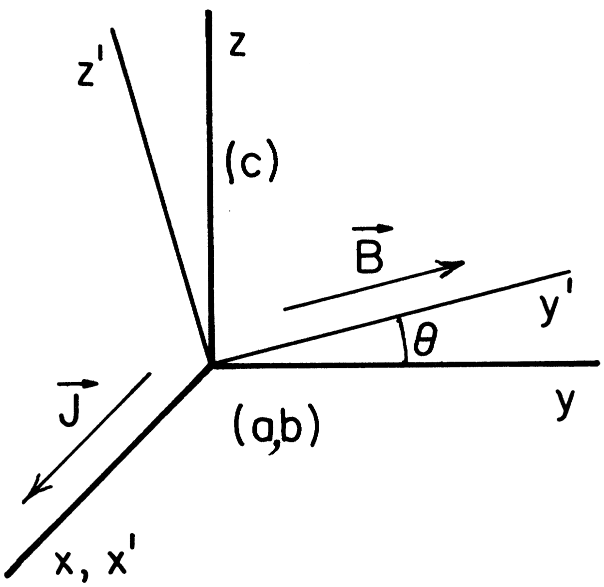

FIG. 1. Vortex cores are aligned along the y' axis.

$$\alpha^2=\frac{3\pi B}{8\phi_0}\frac{\sin^4\theta}{\cos^2\theta}\frac{\lambda_c^3}{\lambda(\theta)} .$$

The function $G(p,q)$ was introduced in the paper[7]

$$G(p,q)=\sum_{n=1}^{\infty}\frac{2}{n}\left[\frac{\sinh(pn)}{\cosh(pn)-\cos(2\pi qn)}-1\right]-\ln p+\frac{p}{6} . \tag{4}$$

Expression (3) for the function $g(p,q)$ is valid if the parameter p satisfies the inequality

$$\frac{\phi_0}{B\lambda_c\lambda_{ab}}\ll\frac{p\lambda_{ab}}{\lambda(\theta)}\ll\frac{B\lambda_c\lambda_{ab}}{\phi_0} . \tag{5}$$

Below we shall consider the interval of magnetic fields

$$H_{c1}(\theta)\ll B<H^*(\theta) , \tag{6}$$

where the lower critical magnetic field is

$$H_{c1}(\theta)=\frac{\phi_0\lambda(\theta)}{4\pi\lambda_c\lambda_{ab}^2}\ln\kappa , \tag{7}$$

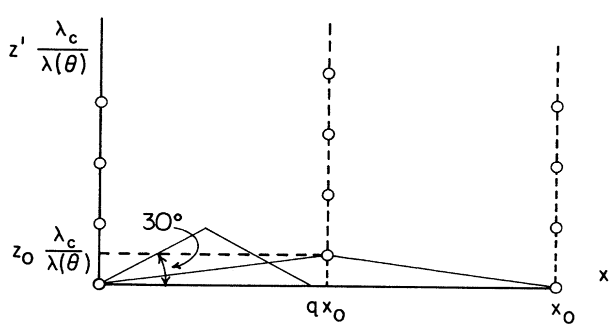

FIG. 2. The picture of the $x'z'$ plane. Vortex cores are denoted by circles. The normal lattice corresponds to the 30° triangle.

κ is the Ginzburg-Landau parameter, and

$$H^*(\theta)=K\frac{\phi_0\lambda_c}{\lambda_{ab}^2\lambda(\theta)} , \tag{8}$$

where k is a number of order unity. The tilted angle interval is supposed to satisfy the relations

$$\cos\theta\approx 1 , \; ,$$
$$\lambda(\theta)\gg\lambda_{ab} . \tag{9}$$

The relations (9) require that θ not be too close to 0 or $\pi/2$.

IV. THE VORTEX CHAIN STATE

The equilibrium lattice parameter can be obtained by minimization of the free energy (2). In the isotropic case $g=G(p,1)$, and minimization gives $q=\frac{1}{2}$ and $p=\pi\sqrt{3}$ which corresponds to equilateral triangles (the normal lattice). At these values of the parameters the function $G(p,q)$ reaches its minimum value $G\simeq-0.83$.

In our anisotropic case the minimum of g falls also at $q=\frac{1}{2}$. For minimization with respect to p we can use the asymptotic value of $G(p,\frac{1}{2})=p/6-\ln p$ for $p\gg 1$. Then the minimization of the function $g(p,\frac{1}{2})$ in Eq. (3) gives $p=p_0$, where

$$p_0=\frac{6\lambda^2(\theta)}{\lambda_{ab}^2\cos^2\theta}[(1+\alpha^2)^{1/2}-\alpha]^2 . \tag{10}$$

The minimum value of g is

$$g(p_0,\tfrac{1}{2})=1+2\alpha[(1+\alpha^2)^{1/2}-\alpha]-\ln p_0 . \tag{11}$$

For small values of $\alpha\ll 1$ (moderate fields) we get the previous result[12]

$$p_0=\frac{6\lambda^2(\theta)}{\lambda_{ab}^2\cos^2\theta} . \tag{12}$$

For large values $\alpha\gg 1$ (larger fields)

$$p_0=\frac{4\phi_0}{\pi B}\frac{\lambda^3(\theta)}{\lambda_{ab}^2\lambda_c^3\sin^4\theta} . \tag{13}$$

Note that both the cases of small and large values α lie within the limits (6).

The value of p determined by Eq. (10) and in limiting cases by Eqs. (12) and (13) remains large $p\gg 1$. At $B=H^*(\theta)$ the value p given by Eq. (13) becomes order of unity $p\approx 1$. This means that in the interval of fields (6) p is a large quantity, and the unit cell of the vortex lattice is compressed in the z' direction as shown in Fig. 2. In other words, the vortices form chains along the z' direction. If $B>H^*(\theta)$ the difference of the function g from G is small, and the equilibrium lattice parameters are determined by minimization of $G(p,q)$. Analogously if $\lambda(\theta)\approx\lambda_{ab}$ the function g is close to $G(p,q)$.

So apart from the VCS conditions, which are $H<H^*(\theta)$ and $\lambda(\theta)>\lambda_{ab}$ the equilibrium value is $p=\pi\sqrt{3}$ and in the "natural" coordinates of Fig. 2 the lattice is hexagonal. A fragment of this lattice is shown in Fig. 2 with the angle of 30°. This state is the normal vortex lattice.

V. PINNING AND THE CRITICAL CURRENT

In accordance with the VCS conditions, $H < H^*(\theta) > \lambda_{ab}$, we can plot "the phase diagram" as shown in Fig. 3. The condition $\lambda(\theta) > \lambda_{ab}$ means $\theta \gg \lambda_{ab}/\lambda_c$. For $\theta \approx 1$ the values of H^* and H_{c1} have the same order of magnitude. So the dashed region in Fig. 3 represents the VCS. Here we ignored the demagnetizing factor because for $B \gg H_{c1}$ the induction and magnetic field are very close to each other.

In the VCS the lattice should be significantly softened. The chains of vortices are located in planes which are parallel to **c** and **B**. If the concentration of vortices in each chain increases then the shear displacement of one chain relative to another costs little energy because the magnetic field along a chain is almost homogeneous.[7]

Taking into account that $\partial U_x/\partial z \approx (q-\frac{1}{2})$ and expanding the function $G(p,q)$ near the point $q=\frac{1}{2}$ up to second order, we find the shear modulus c_{66} is proportional to $\exp(-p_0)$.[7,12] The decrease of the shear modulus makes more probable the melting of the vortex lattice.[13–15] For our consideration it is not very important whether the lattice is melted or not. More important is the general fact of system softening in the VCS. In this state the effective pinning is strong because an isolated vortex chain can move easily and be kept fixed by pinning centers more strongly than the big region of a collectively coupled lattice. This causes an increase of the critical depinning current in the VCS.

Below we shall make a quantitative analysis of the angular dependence of the critical current when a magnetic field is titled with respect to the *ab* plane in Y-Ba-Cu-O materials.[16] The essential feature of those results is a peak in the critical current in a region of width $\Delta\theta$ near $\theta=0$ when the magnetic field is almost parallel to layers. The width $\Delta\theta$ is strikingly dependent on the temperature and the magnetic field.

In Fig. 4 the angular dependence of the field $H^*(\theta)$ is plotted for four values of temperature 4.2, 40, 60, and 77. At those particular temperatures the critical currents were measured as in Ref. 16. The London penetration depth is supposed to be $\lambda_{ab}^{-2}(T)=\lambda_{ab}^{-2}(0)(T_c-T)/T_c$. Then from the relation (8) it follows that

$$H^*(\theta)=K\frac{\phi_0}{\lambda_{ab}^2(0)}\frac{T_c-T}{T_c}\left[\sin^2\theta+\frac{\lambda_{ab}^2}{\lambda_c^2}\cos^2\theta\right]^{-1/2}, \tag{14}$$

where we have chosen the values $T_c \simeq 82$ K, $K\phi_o/\lambda_{ab}^2(0) \simeq 2$ T, and $\lambda_{ab}/\lambda_c \simeq 0.15$. In order to compare our results with Ref. 16 we took three values of magnetic field 7, 3, and 0.5 T in Fig. 4 as in the experimental paper. At each fixed magnetic field the VCS corresponds to a horizontal line between the vertical dashed line and solid one. In this angle interval the critical current increases due to an increased pinning.

In the experiment however the critical current proceeds to increase in the region of small angles $\theta \le \lambda_{ab}/\lambda_c$ where the vortex lattice is normal. This can be explained by the fact that for small angles one cannot completely ignore the layered structure of this material. At $\theta=0$ the critical current reaches its maximum value due to intrinsic interlayered pinning.[17,18] Our approximation of the anisotropic superconductor is sufficient in general at relatively large angles $\theta > \lambda_{ab}/\lambda_c$. The role of the layers in the formation of a critical current peak for the normal vortex lattice was analyzed in Ref. 19. For this reason an increase of critical current at angles

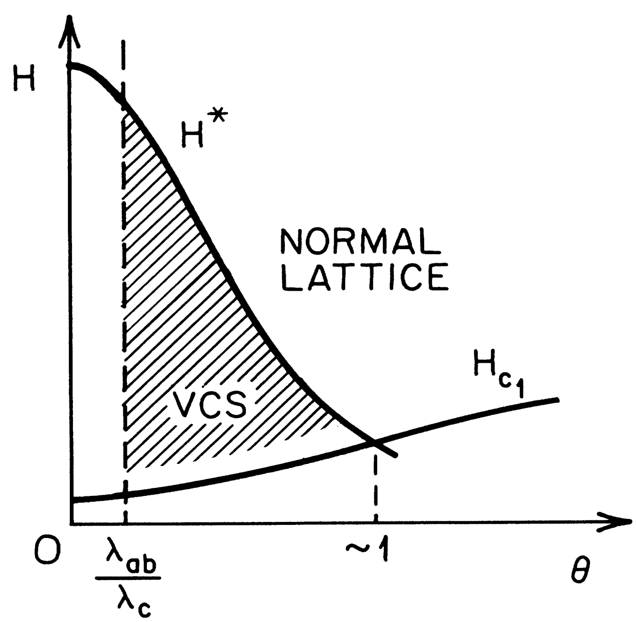

FIG. 3. The H-θ diagram. There is no sharp boundary between the VCS and the normal lattice, this transition is smooth.

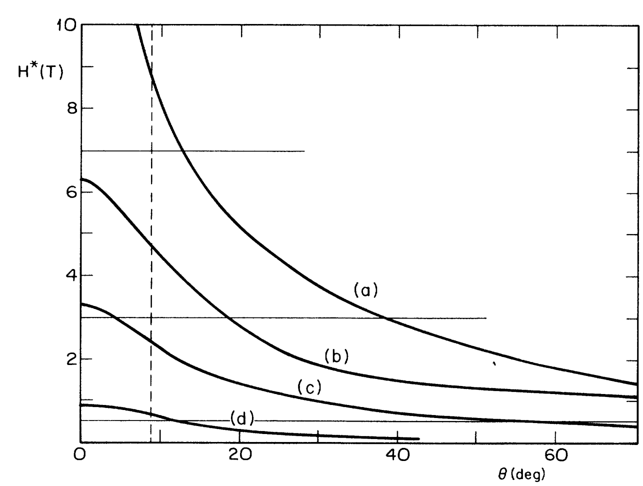

FIG. 4. The H-θ diagram for different temperatures; (a) 4.2 K, (b) 40 K, (c) 60 K, and (d) 77 K. The horizontal lines denote the values of magnetic fields 7, 3, and 0.5. The vertical dashed line corresponds to the angle in radians $\theta=\lambda_{ab}/\lambda_c=0.15$.

$\theta > \lambda_{ab}/\lambda_c$ can be explained by the VCS formation, and at smaller angles by the effects of the layered structure.

As an example let us analyze the case of $H=3$ T in Fig 4. The VCS region has a maximum value for $T=4.2$ and it is smaller for $T=40$ K. For $T=60$ K the VCS region disappears, and only a narrow peak remains caused by the layer effects. We see that the angular dependence of the peak in the critical current sharpens with *increasing* temperature in accordance with the experimental results.[16]

As another example we may consider the case of fixed temperature $T=4.2$ K (curve 1 in Fig. 4). At the magnetic field $H=0.5$ T, as can be seen in Fig. 4, the VCS range is broad. For $H=3$ T it is smaller, for $H=7$ T it becomes even smaller so the peak in the angular dependence of the critical current also sharpens with an increase of the magnetic field. The sharpening of the peak in the angular dependence of the critical current both for increasing temperature[16] and magnetic field were observed experimentally.

In the framework of the normal vortex lattice in a layered superconductor the sharpening of the critical current peak with increase of the magnetic field can also be explained at low temperatures ($T=4.2$ k).[19] In order to go from the qualitative considerations presented here to a more precise description one has to unify the two approaches: the VCS formation and the effects of layers on pinning.

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*Permanent address: L. D. Landau Institute for Theoretical Physics, U.S.S.R. Academy of Sciences, 117334 Moscow, U.S.S.R.

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