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Lower critical field of a Josephson-coupled layer model of high-T_c superconductors

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We calculate the lower critical field H_{c1} for a magnetic field applied parallel to the layers of a Josephson-coupled layer model of high-T_c superconductors. This result is obtained by using an expression for the vortex line energy that explicitly involves the gauge-invariant phase difference between superconducting layers. We examine the behavior of H_{c1} as a function of temperature and discuss the manifestation of the superconductor discreteness below a crossover temperature where the coherence length $\xi_c(T)$ becomes comparable to the lattice constant c. This temperature dependence is contrasted with the standard result for H_{c1} for a continuous anisotropic type-II superconductor.

I. INTRODUCTION

Measurements of the lower critical field H_{c1}, either by rf surface resistance[1-3] or magnetization[4-6] techniques, have usually been interpreted using Ginzburg-Landau (GL) theory. The determination of H_{c1}, which provides valuable information on ab-plane versus c-axis anisotropy, is all the more useful for high-T_c materials, since it can be measured directly, whereas the upper critical field H_{c2} cannot. The known high-T_c compounds are generally accepted to have a layered structure and an associated small coherence length (ξ_c) in the c direction. The anisotropic GL theory[7] provides an adequate description for H_{c1} for these materials when the applied field is oriented perpendicular to the layers. However, when the applied field is oriented parallel to the layers, the anisotropic GL theory may not be appropriate when ξ_c is of the size of the c-direction lattice constant. The present work addresses this point by using the detailed result for the vortex core structure in a layered anisotropic high-T_c superconductor.[8]

In an earlier work[8] we developed a model of anisotropic high-T_c superconductors based on an infinite periodic stack of Josephson-coupled, parallel superconducting layers. This model, which is similar to those of Bulaevskii[9-15] and Lawrence and Doniach,[16] consists of alternating superconducting and insulating layers of thickness d_s and d_i, respectively, giving a stacking periodicity of $s=d_i+d_s$. In Ref. 17 Volkov developed a Josephson-coupled layer model where the insulating regions have zero thickness. In Ref. 18 a model for anisotropic high-T_c superconductors was constructed by using an array of Josephson-coupled superconducting blocks. When the blocks are fused in the a and b directions, but weakly coupled in the c direction, a layer model is obtained which is similar to that which we use. The theory for the structure of an isolated vortex parallel to the layers developed in Ref. 8 is used in this paper to find the lower critical field H_{c1}.

For calculational purposes in the following, the layers in our model are taken to be parallel to the xy (ab) plane, with the center of the insulating layers at $z=z_n=ns$ ($n=0,\pm1,\pm2,\ldots$) (see Fig. 1). In this phenomenological description of a high-T_c superconductor, we regard the superconducting layers as corresponding to the double CuO_2 planes in compounds like Y 1:2:3 ($YBa_2Cu_3O_{7-\delta}$) or Bi 2:2:1:2 ($Bi_2Sr_2CaCu_2O_8$). The insulating regions roughly correspond to the other layers, with the values of d_i and s depending on the details of the particular material. In general, the length s is typically on the order of 10 Å. In Y 1:2:3, we take s to correspond to the lattice constant c, but in Bi 2:2:1:2, which has two formula units per unit cell, we take $s=c/2$.

We note that our theory may apply to the new organic superconductors. For instance, in the

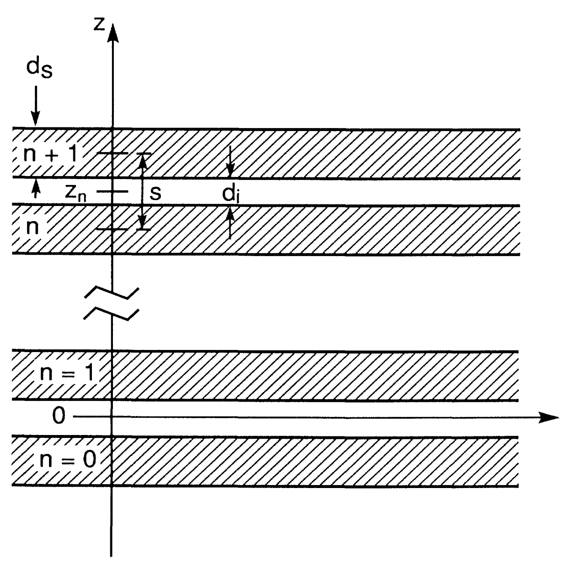

FIG. 1. Geometry of the Josephson-coupled layer model. The insulating layers of thickness d_i alternate with superconducting layers (cross hatched) of thickness d_s. The middles of the insulating layers are in the planes $z_n=ns$ ($n=0,\pm1,\pm2,\ldots$) where $s=d_i+d_s$.

bis(ethylenedithio)tetrathiafulvalene (BEDT-TTF) family, the compound κ-(BEDT-TTF)$_2$-Cu(NCS)$_2$ has a layered structure consisting of alternating sheets of BEDT-TTF molecules and Cu(NCS)$_2$ ions.[19] It has been reported that the parallel penetration depth at low temperatures is very large (on the order of several mm) in this compound and therefore Josephson coupling of the layers may be important.[19] This particular BEDT-TTF compound has a resulting anisotropy of approximately $\xi_{bc}/\xi_a \simeq 19$.[20,21] We mention that both the upper[20,21] and lower critical fields[22] have been estimated in this organic superconductor.

In this paper we calculate the lower critical field H_{c1} for a magnetic field applied parallel to the layers of the above model. The lower critical field is obtained from the vortex line energy ε_1 by the usual thermodynamic relation (e.g., Refs. 23–25)

$$\varepsilon_1 = \frac{\phi_0}{4\pi} H_{c1} , \tag{1.1}$$

where $\phi_0 \equiv hc/2e$ is the flux quantum. However, ε_1 itself is found by employing an expression explicitly involving the gauge-invariant phase difference $\Delta\gamma_n$ across successive superconducting layers. In the following section, we recall the necessary results[8] for calculating ε_1. After evaluating the vortex line energy in Sec. III we discuss the consequences for the lower critical field. We finish with a brief summary and some concluding remarks in Sec. IV.

Expressions derived from GL theory that involve ξ_c cannot remain valid when ξ_c becomes smaller than the periodicity length. In general, we expect such expressions to be replaced by corresponding expressions in which ξ_c is replaced by c or the layer spacing, which becomes the smallest length in the problem. While expressions for H_{c1} based upon the anisotropic GL theory should be correct at temperatures sufficiently high that the coherence length $\xi_c(T)$ is much larger than the lattice constant c, our result applies for low temperatures, below a crossover temperature, at which $\xi_c(T)$ becomes approximately equal to the lattice constant c. We note that the existence of such a crossover temperature may be indicated by data from torque-magnetometry experiments on untwinned single crystals of Y 1:2:3.[26] Current theory based on a three-dimensional (3D) London treatment appears inadequate to explain these results at lower temperatures when the magnetic field lies close to the CuO_2 planes.[26] We expect that at low enough temperature the discreteness of the superconductor will become manifest. In the theory presented in Ref. 8, the discreteness of the superconductor, for temperatures below the crossover temperature, was reflected in the dimensions of the vortex core area, which in turn modified the result for the viscous drag coefficient and flux flow resistivity (calculated for vortex motion parallel to the layers).

Our discussion assumes some knowledge of the behavior of Josephson junctions. Background information on single Josephson junctions, including the basic relations between the gauge-invariant phase difference and electric and magnetic fields, can be found, e.g., in Refs. 27–31.

II. SUMMARY OF MODEL OF ANISOTROPIC LAYERED SUPERCONDUCTORS

Here we recall and discuss the results from Ref. 8 that we need for calculating the vortex line energy in the next section. For that purpose we assume that a single vortex is present in the central (or $n=0$) insulating layer of the model. With the vortex parallel to the x axis (a direction) and centered on the origin, the magnetic field produced, $\mathbf{b}(y,z)=\hat{\mathbf{x}}b(y,z)$, is to a high degree of accuracy when $s \ll \lambda_b$,[8]

$$b(y,z) = \frac{\phi_0}{2\pi\lambda_b\lambda_c} K_0(\tilde{R}) , \tag{2.1}$$

where K_p is a modified Bessel function of the second kind of order p and

$$\tilde{R} \equiv (\tilde{u}_0^2 + \tilde{y}^2 + \tilde{z}^2)^{1/2} ,$$
$$\tilde{y} \equiv y/\lambda_c, \quad \tilde{z} \equiv z/\lambda_b, \quad \tilde{u}_0 \equiv s/2\lambda_b . \tag{2.2}$$

The relation between the magnetic field of the vortex, the supercurrent density $\mathbf{j}(y,z)$, and the superfluid velocity $\mathbf{a}_s(y,z)$ in a superconducting layer can be expressed as

$$\mathbf{a}_s = \mathbf{a} + \frac{\phi_0}{2\pi}\nabla\gamma, \quad \mathbf{j} = -\frac{c}{4\pi\lambda_s^2}\mathbf{a}_s , \tag{2.3}$$

where $\mathbf{a}(y,z)$ is the vector potential, $\nabla\times\mathbf{a}=\mathbf{b}$, and $\gamma(y,z)$ is the phase of the order parameter. In Eq. (2.3), λ_s is the intrinsic penetration depth, taken to be isotropic, of the individual superconducting layers of thickness d_s. The penetration depths λ_b and λ_c govern the behavior of the vortex magnetic field and screening supercurrent density outside the core region, as briefly discussed below. These penetration depths are related to λ_s by[8,18,32]

$$\lambda_b^2 = \frac{s}{d_s}\lambda_s^2, \quad \lambda_c^2 = \frac{c\phi_0}{8\pi^2 s J_0} + \frac{d_s}{s}\lambda_s^2 . \tag{2.4}$$

The geometric factor d_s/s in Eq. (2.4) may be thought of as arising from conservation of the supercurrent density in the periodic layers. It is seen that the penetration depth λ_c is composed of two parts: one due to the intrinsic screening and the other due to the weak Josephson coupling. In the extreme limit of infinite Josephson coupling ($J_0 \to \infty$) the latter contribution vanishes, the geometric mean of λ_b and λ_c is simply λ_s, and we recover the case of a continuous anisotropic type-II superconductor. In the limit that the insulator thickness $d_i \to 0$, we again have a continuous superconductor, $d_s \to s$, $J_0 \to \infty$, and $\lambda_b \to \lambda_s$, $\lambda_c \to \lambda_s$. For further discussion along these lines, see Ref. 18. In the case of the known high-T_c materials the Josephson coupling term dominates in the penetration depth λ_c so that we typically have $\lambda_c^2 \gg \lambda_b^2 \sim \lambda_s^2$. For instance, in Y 1:2:3 (Refs. 33–35) the anisotropy ratio $\lambda_c^2/\lambda_b^2 \approx 30$ and in Bi and Tl compounds this ratio is far larger.[36]

In the following we also require expressions for the gauge-invariant phase difference,

$$\Delta\gamma_n(y)=\gamma_n(y)-\gamma_{n+1}(y)-\frac{2\pi}{\phi_0}\int_{C_n}\mathbf{a}\cdot d\mathbf{l} \ , \tag{2.5}$$

of the superconducting wave function across the junction between superconducting layers n and $n+1$. In Eq. (2.5), γ_n is the phase of the order parameter at the top of the nth superconducting layer (where $z=z_n-d_i/2$) and γ_{n+1} is the phase at the bottom of the $(n+1)$st superconducting layer (where $z=z_n+d_i/2$). The contour C_n connects these two points; i.e, it extends directly across the junction, from the bottom to the top of the nth insulating layer. By using Eq. (2.1), Ampere's law, and the Josephson relation $J_z(y,z)=J_0\sin\Delta\gamma_n(y)$, we have the phase difference (2.5) as

$$\sin\Delta\gamma_n(y)=\frac{2\tilde{u}_0K_1(\tilde{R})\tilde{y}}{\tilde{R}} \ , \tag{2.6a}$$

where the good approximation[8,18,32] $1/\lambda_c^2\simeq 8\pi^2 sJ_0/c\phi_0$ was used. Since it can be shown that the maximum of $\Delta\gamma_n$ decreases with n approximately as $1/2|n|$, we can well approximate the phase difference by using the asymptotic form $K_1(x)\simeq 1/x$, for x near zero, giving

$$\sin\Delta\gamma_n(y)=\frac{2\tilde{u}_0\tilde{y}}{\tilde{R}^2} \ . \tag{2.6b}$$

We recall that for the component of the supercurrent density $\mathbf{j}$ pointing in the b direction, the length scale for exponential decay along the z (c) axis is set by the penetration depth λ_b. Similarly, for the component pointing in the c direction, the length scale for decay along the y (b) axis is set by λ_c. The streamlines of the supercurrent, which also represent contours of constant magnetic field, are elliptical except for small zig-zags due to the intervening insulating layers (see Fig. 2). The explicit solution (2.1) does not model the straight supercurrent streamlines in the insulating layers. However, the error introduced in this approximation is small because the magnetic field outside of the vortex core changes significantly in magnitude only over a distance λ_b or λ_c, and these lengths are normally much larger than the insulator thickness d_i.

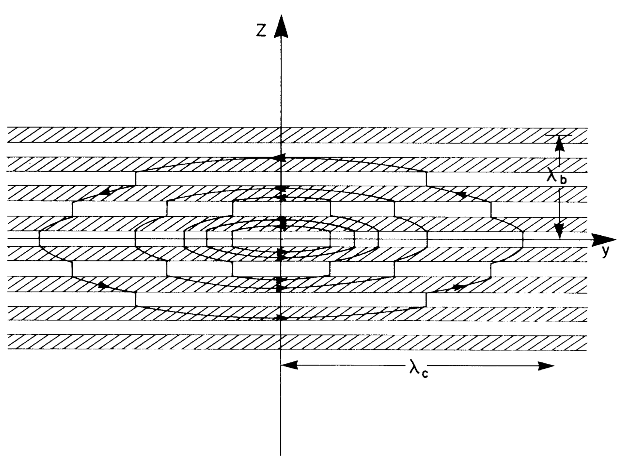

FIG. 2. A sketch of the supercurrent distribution around a single vortex in the barrier region of the central Josephson junction in an infinite layer model of an anisotropic high-T_c superconductor. The vortex is parallel to the x axis (a direction). The London penetration depths λ_b and λ_c give the scale for the decay of the supercurrent components pointing in the b and c directions, respectively. The streamlines of the supercurrent, which also represent contours of constant magnetic field, would be ellipses in the absence of the intervening insulating layers.

In Eq. (2.1), $\tilde{u}_0$ serves as a dimensionless vortex-core radius. Typically, $\tilde{u}_0=s/2\lambda_b$ is of the order of the reciprocal of the Ginzburg-Landau parameter κ, or approximately 10^{-2} in a high-T_c superconductor. It gives the distance at which the Josephson tunneling current density reaches its maximum value (J_0) in the central junction as $\tilde{y}_{\max}=\tilde{u}_0$. For $\xi_c \lesssim s/2$, the continuum description for the magnetic field of the vortex and the gauge-invariant phase difference loses its validity, and the vortex, which fits between neighboring superconducting layers, behaves as a Josephson vortex rather than an Abrikosov vortex. This is the case with which we are concerned, where the amplitude of the order parameter is not suppressed in the vortex core, for temperatures below the crossover temperature.

It can be noted that for the Josephson-coupled layer model,[8] there are two length scales, $y_{\max}$ and λ_c, required to characterize the spatial variation of $J_z(y,z=0)$ and $b(y,z=0)$. The peak value of $J_z(y,z=0)$ occurs at $y=y_{\max}=s\lambda_c/2\lambda_b$, while for large values of y, both $J_z(y,z=0)$ and $b(y,z=0)$ are dominated by the exponential $\exp(-y/\lambda_c)$. This is in contrast to the case of a single Josephson junction where the Josephson penetration-depth λ_J is the sole length scale needed to describe J_z and b.

III. CALCULATION OF VORTEX LINE ENERGY

The free energy per unit length of vortex line, measured relative to the free energy per length in the Meissner state, can be written as a sum of electromagnetic and Josephson-coupling contributions

$$\varepsilon_1=\varepsilon_{1\,\mathrm{EM}}+\varepsilon_{1J} \ . \tag{3.1}$$

Each energy contribution is described in more detail below. We note, however, that no condensation energy term is included in Eq. (3.1) to reflect variation in the order parameter on each layer. Although we expect such variation to be small, a condensation energy term for the superconducting layers closest to the vortex axis would be needed if dimensional crossover and its effects were to be studied in detail.

In Eq. (3.1), $\varepsilon_{1\,\mathrm{EM}}$ is the electromagnetic-field energy per unit length of vortex, which is the sum of the magnetic-field energy per unit length, and the energy per unit length due to the kinetic energy of supercurrents. The respective energy densities of these two contributions are proportional to the square of b and the square of the supercurrent density. We have for the electromagnetic energy density in the superconducting layers

$$F_{\mathrm{EM}}=\frac{b^2}{8\pi}+\frac{2\pi\lambda_s^2}{c^2}(j_y^2+j_z^2) \ . \tag{3.2}$$

As a first step in writing ε_1 directly in terms of the gauge-invariant phase difference, we transform Eq. (3.2). From a vector identity and Ampere's law we have

$$\nabla\cdot(\mathbf{a}\times\mathbf{b})=b^2-\mathbf{a}\cdot\frac{4\pi}{c}\mathbf{j}\ . \tag{3.3}$$

Then by employing the expressions (2.3) for the supercurrent density and superfluid velocity, respectively, in terms of the vector potential $\mathbf{a}$ we obtain

$$F_{\mathrm{EM}}=\frac{1}{8\pi}\nabla\cdot(\mathbf{a}\times\mathbf{b})-\frac{\phi_0}{4\pi c}\nabla\cdot(\mathbf{j}\gamma_n)\ . \tag{3.4}$$

As usual, $\mathbf{b}=\nabla\times\mathbf{a}$ and $\nabla\cdot\mathbf{j}=0$ were also used. For the insulating layers in the model, only the term $b^2/8\pi$ is present in F_{EM}, which by Eq. (3.3) can be written in the form

$$\frac{b^2}{8\pi}=\frac{1}{8\pi}\nabla\cdot(\mathbf{a}\times\mathbf{b})+\frac{1}{2c}\mathbf{j}\cdot\mathbf{a}\ . \tag{3.5}$$

The term ε_{1J} in Eq. (3.1) is the Josephson-coupling energy per length. The Josephson-coupling energy per unit area of junction n is

$$F'_{Jn}=\frac{\phi_0 J_0}{2\pi c}(1-\cos\Delta\gamma_n)\ . \tag{3.6}$$

To obtain ε_{1J} we must integrate (3.6) over the junction area corresponding to the unit length of the vortex and sum over all junctions.

We now find the electromagnetic line energy $\varepsilon_{1\,\mathrm{EM}}$ for an infinite stack of Josephson junctions, obtained from Eqs. (3.4) and (3.5). By integrating over all superconducting and insulating layers and summing, to obtain an integration over all space, and applying the divergence theorem (assuming that $\mathbf{b}$ vanishes at infinity), the first term on the right-hand side (RHS) of (3.4) and (3.5) contributes nothing to $\varepsilon_{1\,\mathrm{EM}}$. The contribution of the second term on the RHS of (3.4) would similarly be zero except that the gauge-invariant phase difference across the junction does not vanish. By using (3.4) and (3.5) and applying the divergence theorem to each junction of our model, we thus obtain

$$\varepsilon_{1\,\mathrm{EM}}=-\frac{\phi_0}{4\pi c}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}dy\ j_z(y)\left[\gamma_n(y)-\gamma_{n+1}(y)-\frac{2\pi}{\phi_0}\int_{C_n}dz\ a_z(y)\right]\ . \tag{3.7a}$$

Since $j_z=J_z=J_0\sin\Delta\gamma_n$, the Josephson tunneling current density, Eq. (3.7a) can be written in terms of the gauge-invariant phase difference $\Delta\gamma_n$, Eq. (2.5), as

$$\varepsilon_{1\,\mathrm{EM}}=-\frac{\phi_0 J_0}{4\pi c}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}dy\ \Delta\gamma_n(y)\sin\Delta\gamma_n(y)\ . \tag{3.7b}$$

Combining Eq. (3.7b) with Eq. (3.6) integrated and summed over all junctions, we have for the vortex line energy

$$\varepsilon_1=\frac{\phi_0 J_0}{2\pi c}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}dy[1-\cos\Delta\gamma_n(y)-\tfrac{1}{2}\Delta\gamma_n(y)\sin\Delta\gamma_n(y)]\ . \tag{3.8a}$$

Because of the symmetry of the phase difference in each junction of the stack,

$$\Delta\gamma_0(y)=2\pi-\Delta\gamma_0(-y),$$

$$\Delta\gamma_l(y)=-\Delta\gamma_l(-y),\quad l\neq 0\ .$$

Eq. (3.8a) can be written in terms of the current per unit length flowing around the vortex, $I'_0=\int_0^\infty J_z(y,z=0)dy$, as

$$\varepsilon_1=\frac{\phi_0 I'_0}{2c}+\frac{\phi_0 J_0}{\pi c}\sum_{n=-\infty}^{\infty}\int_0^{\infty}dy[1-\cos\Delta\gamma_n(y)-\tfrac{1}{2}\Delta\gamma_n(y)\sin\gamma_n(y)]\ . \tag{3.8b}$$

Alternatively, $I'_0=cb(0,0)/4\pi$ by Faraday's law, so that the vortex line energy becomes

$$\varepsilon_1=\frac{\phi_0 b(0,0)}{8\pi}+\frac{\phi_0 J_0}{\pi c}\sum_{n=-\infty}^{\infty}\int_0^{\infty}dy[1-\cos\Delta\gamma_n(y)-\tfrac{1}{2}\Delta\gamma_n(y)\sin\Delta\gamma_n(y)]\ , \tag{3.9}$$

which is a general expression showing ε_1 as a dominant electromagnetic line energy term plus a sum over all junctions of Josephson and electromagnetic line energy contributions. For the case of a single superconducting film, the electromagnetic line energy was calculated by the above means in Ref. 37. However, Eq. (3.9) is an expression for the vortex line energy in a layered, Josephson-coupled type-II superconductor.

As applied to our layer model, separating out the contribution of the central junction, Eq. (3.9) yields

$$\varepsilon_1=\frac{\phi_0^2}{16\pi^2\lambda_b\lambda_c}K_0(\tilde{u}_0)+\frac{\phi_0^2}{16\pi^2\lambda_b\lambda_c}(1-\ln 2)+\frac{2\phi_0 J_0}{\pi c}\sum_{n=1}^{\infty}\int_0^{\infty}dy[1-\cos\Delta\gamma_n(y)-\tfrac{1}{2}\Delta\gamma_n(y)\sin\Delta\gamma_n(y)]\ , \tag{3.10}$$

where Eq. (2.1) was used for b and Eq. (2.6b) with $\tilde{z}=0$, to very good approximation, for $\sin\Delta\gamma_0$. Since $\Delta\gamma_n$ is fairly small for layers with $n\neq 0$, there is a great deal of cancellation in the integrand in the last term of Eq. (3.10):

$$1-\cos\Delta\gamma_n(y)-\tfrac{1}{2}\Delta\gamma_n(y)\sin\Delta\gamma_n(y)=\frac{\Delta\gamma_n^4(y)}{4!}[1+O(\Delta\gamma_n^2(y))]\ . \tag{3.11}$$

Therefore, using Eq. (2.6b), for $n \geq 1$, we can well approximate

$$\Delta\gamma_n(y) \simeq \sin\Delta\gamma_n(y) = \frac{2\tilde{u}_0\tilde{y}}{\tilde{u}_0^2 + n^2\tilde{s}^2 + \tilde{y}^2} \quad (\tilde{s} \equiv s/\lambda_b,\ \ n \neq 0)\ . \tag{3.12}$$

When Eqs. (3.11) and (3.12) are used, the last term of Eq. (3.10) becomes

$$\frac{\phi_0^2}{384\pi^2\lambda_b\lambda_c}\Sigma_1\ , \tag{3.13a}$$

where we define the infinite sum

$$\Sigma_1 \equiv \sum_{n=1}^{\infty} \frac{1}{(1+4n^2)^{3/2}} \simeq 0.113\,08\ . \tag{3.13b}$$

By using expression (3.13a) for the last term of Eq. (3.10) we find that the total energy per unit length of vortex line (3.1) can be written as

$$\varepsilon_1 = \frac{\phi_0^2}{16\pi^2\lambda_0\lambda_c}[K_0(\tilde{u}_0) + 1 - \ln 2 + \tfrac{1}{24}\Sigma_1]\ . \tag{3.14}$$

From Eqs. (1.1) and (3.14) we then have the lower critical field for vortices parallel to the layers as

$$H_{c1} = \frac{\phi_0}{4\pi\lambda_b\lambda_c}\left[K_0\left[\frac{s}{2\lambda_b}\right] + 1 - \ln 2 + \frac{1}{24}\Sigma_1\right]\ . \tag{3.15}$$

The result (3.15) can be expanded for very small arguments of the zeroth-order modified Bessel function, due to the smallness of $\tilde{u}_0$, using $K_0(z) \approx -\ln(z/2) - C$, $C \simeq 0.5772$ being Euler's constant, as

$$H_{c1} = \frac{\phi_0}{4\pi\lambda_b(T)\lambda_c(T)}\left[\ln\frac{\lambda_b(T)}{s} + 1.12\right]\ , \tag{3.16}$$

where the approximate numerical values of Σ_1 and C have been used and we have explicitly indicated the temperature dependence of the penetration depths. In Ref. 11 Bulaevskii considered a discrete, Josephson-coupled superconductor and calculated H_{c1} to logarithmic accuracy. This paper refines that result by including the line-energy contributions from the core region.

The result (3.16) can be compared to the standard result[23–25,38–40]

$$H_{c1} = \frac{\phi_0}{4\pi\lambda_b\lambda_c}(\ln\kappa_a + 0.50) \tag{3.17}$$

for a continuous anisotropic type-II superconductor where the Ginzburg-Landau parameter $\kappa_a = \lambda_b(T)/\xi_c(T)$ is only weakly dependent on temperature. Because the stacking periodicity s has replaced the coherence length in expression (3.17), Eq. (3.16) will yield a modified temperature dependence for H_{c1}. This effect occurs at and below a crossover temperature[26,41–43] where the coherence length $\xi_c(T)$ becomes comparable with the lattice constant c.

An idea of the temperature dependence of the lower critical field (3.16) can be found by using, for instance, the result of the "two-fluid" approximation[24]

$$\lambda(T) = \lambda(0)(1-t^4)^{-1/2} \tag{3.18}$$

for λ_b. (Here $t \equiv T/T_c$ is the reduced temperature.) Using expression (3.18) in Eq. (3.16) we have

$$H_{c1} = \frac{\phi_0}{4\pi\lambda_b(T)\lambda_c(T)}\left[-\frac{1}{2}\ln(1-t^4) + \ln\left[\frac{\lambda_b(0)}{s}\right] + 1.12\right]\ . \tag{3.19}$$

In Fig. 3(a) we have plotted expression (3.19) for $4\pi\lambda_b(0)\lambda_c(0)H_{c1}/\phi_0$ versus reduced temperature for the ratio $\lambda_b(0)/s = 100$, approximating Y 1:2:3. Also plotted (dashed) for comparison is the corresponding expression from (3.17) with $\kappa_a = 330$. Below some (reduced) cross-

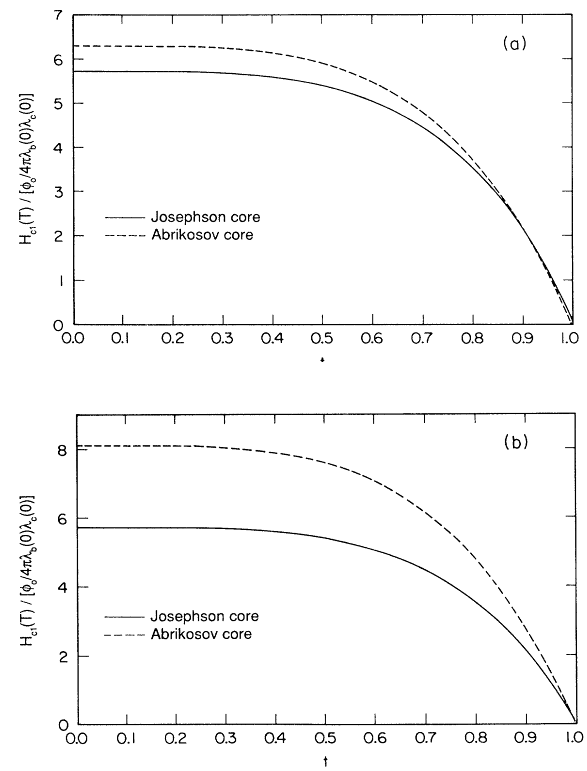

FIG. 3 (a) Lower critical field $H_{c1}(T)/[\phi_0/4\pi\lambda_b(0)\lambda_c(0)]$ for a vortex along the a axis vs reduced temperature $t = T/T_c$ for parameters corresponding to $YBa_2Cu_3O_7$. The solid curve shows the Josephson core model result derived in this paper, Eq. (3.19), shown here for $\lambda_b(0)/s = 100$. The dashed curve shows the Abrikosov core model, Eq. (3.17), for $\kappa_a = 330$. At low temperatures, below a (reduced) crossover temperature of the order of $t^* \approx 0.9$, the temperature dependence of H_{c1} should follow the Josephson core result, while at high temperatures above t^*, H_{c1} should follow the Abrikosov core result. (b) Same as (a), but for parameters corresponding to $Bi_2Sr_2CaCu_2O_8$: $\lambda_b(0)/s = 100$ for the Josephson core result and $\kappa_a = \lambda_b/\xi_c = 2000$ for the Abrikosov core result.

over temperature t^*, H_{c1} will follow the temperature dependence of (3.19) instead of (3.17).

In order to obtain a rough estimate of the crossover temperature,[26,41–43] we note that $\xi_c(0)=3$ Å, $T_c=92.5$ K, and $c=11.9$ Å for the Y 1:2:3 compound.[33] If we use the rough temperature dependence $\xi_c(T)=\xi_c(0)(1-t)^{-1/2}$, we have $\xi_c(T^*)=c\approx s$ when $T^*=T_c\{1-[\xi_c(0)/c]^2\}$ or $T^*\approx 0.94T_c\approx 87$ K. Therefore, the crossover temperature can be quite near T_c. Of course, the precise value of t^* depends on the criteria used to obtain it. In turn, the condition for crossover to occur should be found by a detailed theory taking into account variation of the order parameter.

Shown in Fig. 3(b) are similar results corresponding to the Bi 2:2:1:2 compound, which has an effective-mass anisotropy ratio of 3000.[36] Using $\xi_c(0)\approx 0.6$ Å and $c=30.6$ Å,[44] and $\xi_c(T^*)=s=c/2$ we obtain $t^*\approx 0.9985$, so that the crossover temperature is very close to T_c. The Abrikosov core result applies only for a very limited temperature range near T_c.

IV. SUMMARY

We have determined the lower critical field H_{c1} for a magnetic field applied parallel to the layers of a Josephson-coupled layer model of high-T_c superconductors, based on the theory of the structure of an isolated vortex.[8] In so doing, we developed an expression (3.9) involving the gauge-invariant phase difference for the vortex line energy in a layered, Josephson-coupled type-II superconductor. Our result (3.16), like that of Ref. 11, takes into account the discreteness of a high-T_c superconductor.

The orders of magnitude of the H_{c1} expressions for the Josephson-core case [Eq. (3.16)] and the Abrikosov-core case [Eq. (3.17)] are the same because of the common prefactor $\phi_0/4\pi\lambda_b\lambda_c$. The strong similarities between the two expressions arise because the magnetic field and supercurrent density distributions for the two cases look identical at distances r well outside the core region. Differences arise, however, in the logarithmic factors; their arguments can be thought of as the ratios of upper to lower cutoffs of logarithmically divergent integral. As we explain below, although the upper cutoffs are the same for both Eqs. (3.16) and (3.17), the lower cutoffs are not, because the dimensions of the Josephson and Abrikosov cores differ.

The contours of constant magnetic field and the current-density streamlines for both cases are ellipses for which the ratio of the semimajor axis to the semiminor axis is $\lambda_c/\lambda_b=(m_c/m_b)^{1/2}$, where m_c and m_b are the dimensionless effective masses in the anisotropic Ginzburg-Landau theory.[45] Outside the cores but within a distance λ_c along the y axis (b axis) and within λ_b along the z axis (c axis) (see Fig. 2), the main contributor to the logarithmic factor in Eq. (3.17) is the supercurrent kinetic energy density, which varies roughly as $1/r^2$, where r is the distance from the vortex axis. The logarithmic factor in Eq. (3.16), on the other hand, arises from a combination of the supercurrent kinetic energy density parallel to the layers and the Josephson-coupling energy associated with currents perpendicular to the layers. Well outside the core, the Josephson term can be linearized, the two terms can be combined, and the result can be written in a form identical to the supercurrent kinetic energy contribution with an Abrikosov core. Thus the effective upper cutoffs are the same for the two cases.

The Abrikosov core has an elliptical shape for which the ratio of the semimajor axis to the semiminor axis is $\xi_b/\xi_c=(m_c/m_b)^{1/2}$, the same as the ratio for the elliptical magnetic-field contours. The supercurrent kinetic energy thus contributes a term involving a logarithm, whose argument is the ratio of the upper cutoff to the lower cutoff. This ratio is, aside from numerical factors of order unity,

$$\lambda_c/\xi_b=(\lambda/\xi)(m_b m_c)^{1/2}=(\lambda/\xi)/m_a^{1/2}=\kappa_a$$

along the y axis (b axis). The same result is obtained along the z axis (c axis), where the corresponding ratio is

$$\lambda_b/\xi_c=(\lambda/\xi)(m_b m_c)^{1/2}=(\lambda/\xi)/m_a^{1/2}=\kappa_a \; .$$

As discussed in Ref. 8, the Josephson core can be approximated as an ellipse with semimajor axis

$$y_{\max}=(s/2)\lambda_c/\lambda_b=(s/2)(m_c/m_b)^{1/2}$$

and semiminor axis $s/2$ (half the periodicity length). Here $y_{\max}$ is the distance from the axis along the y direction at which the gauge-invariant phase difference $\Delta\gamma_0(y)$ across the central junction becomes equal to $\pi/2$ and the Josephson current density $J_z(y,0)=J_0\sin\Delta\gamma_0(y)$ reaches its maximum value, J_0.[46] The ratio of the semimajor axis to the semiminor axis of the Josephson core is then $y_{\max}/(s/2)=(m_c/m_b)^{1/2}$, the same ratio as for the Abrikosov core. The contributions that play the role of the supercurrent kinetic energy density again contribute a term involving a logarithm whose argument can be thought of as the ratio of an upper cutoff to a lower cutoff. This ratio is $\lambda_c/y_{\max}=(2\lambda/\xi)m_b^{1/2}=2\lambda_b/s$ along the y axis (b axis); the corresponding ratio along the z axis (c axis) is $\lambda_b/(s/2)$, the same result. We stress that, although a description in terms of cutoffs is helpful in understanding the origin of the arguments of the logarithmic factors, we actually used more sophisticated methods to obtain the logarithmic and constant terms in Eqs. (3.16) and (3.17).

In our calculation we ignored the small condensation energy contribution to the total line energy of a Josephson vortex. Inclusion of this term would be required to precisely determine the crossover temperature, below which our type of result should hold. In Sec. III, we illustrated the behavior of H_{c1} as a function of temperature for parameters corresponding to Y 1:2:3 and Bi 2:2:1:2. Referring to Fig. 3(a), we can note that there is surprisingly little difference between the Josephson core result (3.16) and the Abrikosov core result (3.17) for Y 1:2:3. Because the c-direction coherence length $\xi_c(T)$ can be so small (in comparison with c) in a high-T_c superconductor, the crossover temperature can be very near the transition temperature. A case in point is provided by the Bi 2:2:1:2 compound, results for which are shown in Fig. 3(b), where the Josephson core model applies over nearly the entire range of temperatures.

Our model assumes Josephson-coupled superconducting layers separated by insulating material only. It is possible that a more suitable model for some high-T_c compounds should include normal metal layers, in which case proximity effects will occur in addition to Josephson tunneling. Such a model might be appropriate in Bi and Tl compounds, which have a similar layered structure. We mention that a result for H_{c1} in a layer model with proximity effect has been given in Ref. 47. Additionally, in the study of magnetization data, the possible breakdown of an intrinsic proximity effect has been discussed in Ref. 48.

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