Effect of fluctuations on the transport properties of type-II superconductors in a magnetic field

Salman Ullah and Alan T. Dorsey
Department of Physics, University of Virginia, McCormick Road, Charlottesville, Virginia 22901
(Received 17 January 1991; revised manuscript received 18 March 1991)

The time-dependent Ginzburg-Landau theory is used to study both transverse and longitudinal transport properties of a layered superconductor in a magnetic field near the mean-field transition temperature $T_{c2}(H)$. We evaluate the transport coefficients in the self-consistent Hartree approximation which interpolates smoothly between the high-temperature regime, dominated by Gaussian fluctuations, and the low-temperature flux-flow regime, with no intervening divergence. This behavior is in agreement with the experimental results for the Ettingshausen coefficient, Nernst coefficient, longitudinal conductivity, and Hall conductivity in high-temperature superconductors.

I. INTRODUCTION

The discovery of the high-temperature superconductors has revived interest in fluctuation effects in superconductors, both in thermodynamic properties and in transport properties.[1] In this paper we shall be concerned with the effect of fluctuations on the transport properties of a superconductor in a magnetic field in the vicinity of the transition from the normal state to the Abrikosov flux-lattice state. Here we extend our previous work on the behavior of one particular transport property, the Ettingshausen effect,[2] to the calculation of other transport coefficients.

The study of fluctuation effects in superconductors has a long history, and we refer the reader to the article by Skocpol and Tinkham[3] for a review of earlier work. Much of the early literature focused on fluctuation effects in zero magnetic field, in the vicinity of the transition from the normal state to the Meissner state. For instance, Aslamazov and Larkin[4] showed that superconducting fluctuations in the normal phase cause the electrical conductivity to diverge at the critical temperature. These fluctuations also manifest themselves as nonanalyticities in thermodynamic properties such as the specific heat and susceptibility. Implicit in most of the early work is the assumption that the fluctuations do not interact; that is, only Gaussian fluctuations are considered. Although this assumption breaks down in the critical region (typically quite small in the conventional superconductors), it at least captures the qualitative aspects of the fluctuations in zero magnetic field. If the Gaussian approximation is used to calculate the properties of a type-II superconductor near the mean-field flux-lattice phase boundary $T_{c2}(H)$, one would predict similar nonanalytic behavior in the thermodynamic and transport properties. However, in this circumstance the Gaussian approximation drastically underestimates the effects of fluctuations, since fluctuations which are transverse to the applied field are stiff [they have a length scale which is determined by the magnetic length $l_H=(\hbar c/e^*H)^{1/2}$]; hence, the fluctuations of a bulk superconductor in a magnetic field become effectively one dimensional, as noted by Lee and Shenoy.[5] Fluctuations become more important in systems with reduced dimensionality; for example, fluctuations destroy the ordered phase in one-dimensional systems with short-range interactions. Therefore, one expects that interactions between the fluctuations are important near $T_{c2}(H)$, and that these interactions remove the nonanalyticities present in zero magnetic field. Calculations of the specific heat of a superconductor in a magnetic field which treat the interaction terms within the Hartree approximation,[6,7] and extensions thereof,[8-13] find that the specific heat is smooth through the mean-field transition temperature, in accordance with the above expectations. The transport properties are also expected to be smooth in the vicinity of $T_{c2}(H)$. We recently considered the Ettingshausen coefficient (a transverse thermomagnetic effect) within the Hartree approximation, and showed that it varied smoothly from the fluctuation regime in the normal state to the mean-field regime [$T<T_{c2}(H)$], in quantitative agreement with the recent measurements on Y-Ba-Cu-O by Palstra *et al.*[14] In this paper we extend our results to calculate the Nernst effect, the thermopower, the longitudinal electrical conductivity, and the Hall conductivity. We would like to stress that our calculations are based on the time-dependent Ginzburg-Landau theory which omit certain microscopic contributions to the transport coefficients, for example, the Maki-Thompson terms.[15] The Maki-Thompson terms become less important as the magnetic field is increased due to the enhancement of pair breaking[3] and we therefore do not expect them to qualitatively change our conclusions. This consideration notwithstanding, the spirit of our approach is semiphenomenological in the sense that the parameters appearing in the Ginzburg-Landau theory are to be determined experimentally.

The plan of the paper is as follows. In Sec. II we develop the linear-response formalism required for the computation of the transport coefficients, and discuss the nature of the Hartree approximation. In Sec. III we evaluate the various thermoelectric transport coefficients, while in Sec. IV we compute diagonal components of the electrical conductivity tensor as well as the Hall coefficient. As many of the results are numerically cumbersome, the important results are summarized in Figs. 1–3, where we plot the various transport coefficients as functions of $T-T_{c2}(H)$ for one set of typical parameter values. Appendix A summarizes the definitions of transport coefficients within linear response, Appendix B provides the details of the expansion of the correlation and response functions to linear order in the external electric field, and Appendix C summarizes the behavior of the entropy and the magnetization within the Hartree approximation.

II. LINEAR RESPONSE

A. Equations of motion

In this section we shall set up the formalism necessary to compute the transport coefficients in linear-response theory. We describe the layered structure of Y-Ba-Cu-O using the Lawrence-Doniach model which consists of superconducting planes separated by a distance s, with a Josephson coupling between the planes.[16] The Hamiltonian is

$$\mathcal{H}=\sum_n \int d^2x \left[\frac{\hbar^2}{2m} \left| \left[\nabla_\perp - i\frac{e^*}{\hbar c}\mathbf{A} \right] \psi_n \right|^2 + \frac{\hbar^2}{2m_c s^2}|\psi_n-\psi_{n+1}|^2 + a|\psi_n|^2 + \tfrac{1}{2}b|\psi_n|^4 + \frac{1}{8\pi}(\nabla\times\mathbf{A})^2 \right] , \quad (2.1)$$

where e^* is twice the electron charge and $m\equiv m_{ab}$ and m_c are effective Cooper pair masses in the a-b plane and along the c axis, respectively, $a=a_0(T/T_0-1)$, with T_0 the bare transition temperature, $\nabla_\perp$ is the derivative in the a-b plane and the applied field is assumed perpendicular to the a-b plane. The sum over n is a sum over the superconducting planes. Since we are interested in transport phenomena, it is necessary to introduce some kind of dynamics for the order parameter; the simplest is a gauge-invariant version of relaxational dynamics,

$$[\Gamma_0^{-1}+i\lambda_0^{-1}] \left[\frac{\partial}{\partial t} + i\frac{e^*}{\hbar c}\Phi \right] \psi_n(\mathbf{x},t) = -\frac{\delta\mathcal{H}}{\delta\psi_n^*(\mathbf{x},t)} + \zeta_n(\mathbf{x},t) , \quad (2.2)$$

where Φ is the scalar potential. The noise term ζ is chosen to have Gaussian white-noise correlations:

$$\langle \zeta_n^*(\mathbf{x},t)\zeta_m(\mathbf{x}',t') \rangle = 2k_B T\Gamma_0^{-1}\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')\delta_{nm} , \quad (2.3)$$

where the $\langle \cdots \rangle$ denotes a noise average.[17] We have assumed that the relaxation rate has an imaginary part λ_0^{-1} in order to break the particle-hole symmetry which exists for $\lambda_0^{-1}=0$. By particle-hole symmetry we mean that, under the transformation of complex conjugation and $\mathbf{H}\to-\mathbf{H}$, the equation of motion for ψ^*, Eq. (2.2), is the same as that for ψ, provided $\lambda_0^{-1}=0$. This result would, in turn, imply that $\sigma_{xy}(\mathbf{H})=\sigma_{xy}(-\mathbf{H})$; however, we know that, on general grounds, $\sigma_{xy}(\mathbf{H})=-\sigma_{xy}(-\mathbf{H})$, so that $\sigma_{xy}(\mathbf{H})=0$ if $\lambda_0^{-1}=0$. Therefore, $\lambda_0^{-1}\neq 0$ is necessary in order that the Hall conductivity be nonzero. The thermopower also vanishes (even in zero magnetic field) for particle-hole symmetric systems. Such an imaginary relaxation rate can arise from microscopic considerations[18] or might be generated by coupling the order parameter to conserved densities, as in the critical dynamics of neutral superfluids (model F).[19] The equation of motion for the order parameter should be supplemented by an equation of motion for the vector potential of the form $\sigma_n \partial\mathbf{A}/\partial t = -\delta\mathcal{H}/\delta\mathbf{A}$,[20] with σ_n the normal-state conductivity (ignoring fluctuations of the electromagnetic field).

One of the essential simplifications which we shall make is to consider the magnetic field to be given by the external magnetic field: this simplification precludes the Abrikosov flux-lattice solution and determination of the nature of the transition. However, we must include the contribution of the supercurrent to the magnetic field since this gives rise to a term of the same order as $|\psi|^4$. If we restrict ourselves to the lowest Landau level then the supercurrent is included by introducing a renormalized coupling,[9] $b_\kappa=b(1-2/\kappa^2)$, where κ is the Ginzburg-Landau parameter.[21]

B. Linear-response equations

The definitions of the transport coefficients in linear response are summarized in Appendix A. In order to determine the transport coefficients we have to evaluate the heat current and the electric current to linear order in the applied electric field. In terms of the full, nonequilibrium correlation function

$$C_{nm}(\mathbf{x},t;\mathbf{x}',t') = \langle \psi_n(\mathbf{x},t)\psi_m^*(\mathbf{x}',t') \rangle , \quad (2.4)$$

the heat current (which is obtained from energy conservation) is given by[22]

$$\langle \mathbf{J}^h \rangle = -\frac{\hbar^2}{2m} \left[\left[\nabla - i\frac{e^*}{\hbar c}\mathbf{A}(\mathbf{x}) \right] \left[\frac{\partial}{\partial t'} - i\frac{e^*}{\hbar c}\Phi(\mathbf{x}') \right] + \left[\nabla' + i\frac{e^*}{\hbar c}\mathbf{A}(\mathbf{x}') \right] \left[\frac{\partial}{\partial t} + i\frac{e^*}{\hbar c}\Phi(\mathbf{x}) \right] \right] C_{nn'}(\mathbf{x},t;\mathbf{x}',t')|_{\mathbf{x}=\mathbf{x}';t=t',n=n'} \quad (2.5)$$

while the electric current is given by

$$\langle \mathbf{J}^e \rangle = \left[\frac{\hbar e^*}{2mci}(\nabla-\nabla') - \frac{e^{*2}}{mc^2}\mathbf{A}(\mathbf{x}) \right] C_{nn'}(\mathbf{x},t;\mathbf{x}',t')|_{\mathbf{x}=\mathbf{x}';t=t';n=n'} . \quad (2.6)$$

(Note that, for the z component of the currents, the mass m is replaced by the effective mass m_c). In order to determine the *linear response* transport coefficients, we have to compute the correlation function C to linear order in the electric field; this expansion for the linearized equation of motion (see Sec. II C below) is carried out in Appendix B. Once the correlation function is determined, the transport coefficients follow by evaluating the currents; for example, the electric conductivity σ_{yy} is given by $\langle J_y^e \rangle = \sigma_{yy} E_y$ while the Ettingshausen coefficient α_{yx} is given by $\langle J_y^h \rangle = \alpha_{yx} E_x$.

C. The Hartree approximation

In order to calculate the transport coefficients it is necessary to employ some approximation to deal with the cubic term in the equation of motion. The Gaussian approximation neglects this term entirely; however, as discussed in the Introduction, the Gaussian approximation is inadequate. A simple approximation which captures the interesting fluctuation effects is the Hartree approximation, in which the cubic term in the equation of motion $b_\kappa \psi |\psi|^2$ is replaced by $b_\kappa \langle |\psi|^2 \rangle \psi$ (which is equivalent to replacing the quartic term $|\psi|^4$ in the Hamiltonian by $2\langle |\psi_n|^2 \rangle |\psi_n|^2$). Whence, the new renormalized coefficient of the linear term in the equation of motion $\tilde{a}$ is given by

$$\tilde{a} = a + b_\kappa \langle |\psi_n|^2 \rangle \ . \tag{2.7}$$

The quantity $\langle |\psi_n(\mathbf{x},t)^2 \rangle = C(\mathbf{x},t;\mathbf{x},t)$, in general, depends upon the electric field to linear order. We shall neglect this electric-field dependence in the self-consistent equation, and evaluate $\langle |\psi_n(\mathbf{x},t|^2 \rangle$ in equilibrium, which is equivalent to neglecting vertex corrections when calculating the transport coefficients. With this approximation, we calculate $\tilde{a}$ self-consistently from Eq. (2.7). Thus, the time-dependent Ginzburg-Landau theory in the Hartree approximation is defined by Eqs. (2.1)–(2.3) and (2.7). The Hartree approximation applied to the time-dependent Ginzburg-Landau equation has been used by several authors to study the zero-field conductivity,[23–26] and recently more sophisticated approximation methods have been developed by Ikeda *et al.*[27–29] to calculate the longitudinal conductivity in a magnetic field. Our purpose here is to use the simple Hartree approximation to understand some of the qualitative features of transport in a magnetic field.

We shall first study the solutions of the self-consistent equation, Eq. (2.7). The evaluation of $\langle |\psi_n(\mathbf{x},t)|^2 \rangle$ in equilibrium [see Appendix B, Eqs. (B7)–(B9), for the appropriate expressions] gives

$$\epsilon_H = \tilde{\epsilon}_H - \frac{(2\kappa^2-1)}{4\gamma^2 d}\frac{s}{\Lambda_T} h \times \sum_{n=0}^{N} \frac{1}{(\tilde{\epsilon}_H + 2hn)^{1/2}} \times \frac{1}{[1+d^2(\tilde{\epsilon}_H+2hn)]^{1/2}} \ , \tag{2.8}$$

where $\epsilon = a/a_0$ is a reduced temperature, $\epsilon_H = \epsilon + h$ (with similar expressions for $\tilde{\epsilon}$ and $\tilde{\epsilon}_H$), with $h = H/H_{c2}^{ab}(0)$ a dimensionless magnetic field, $H_{c2}^{ab}(0) = \phi_0/2\pi\xi_{ab}^2(0)$ is the zero-temperature critical field, $\phi_0 = 2\pi\hbar c/e^*$ is the flux quantum, $\Lambda_T = \phi_0^2/16\pi^2 k_B T$ is a thermal length, $d = s/2\xi_c(0)$ is a dimensionless interplanar spacing, the zero-temperature coherence length in the a-b plane is $\xi_{ab}(0) = (\hbar^2/2ma_0)^{1/2}$, while that along the c axis is $\xi_c(0) = (\hbar^2/2m_c a_0)^{1/2}$, and the anisotropy parameter is $\gamma = \xi_c(0)/\xi_{ab}(0)$. Notice that the sum in Eq. (2.8) has a physical cutoff $N = l_H^2/\xi_{ab}(0)^2$, where $l_H = (\hbar c/e^* H)^{1/2}$ is the magnetic length. The cutoff reflects the fact that the Ginzburg-Landau theory is not valid on length scales less than the zero-temperature coherence length $\xi_{ab}(0)$. This ultraviolet cutoff prevents the sum in Eq. (2.8) from diverging. Finally, we have used the mean-field expression for the coefficient of the quartic term $b = 2\pi\kappa^2/(\hbar e/mc)^2$.[30] We shall now discuss the solutions of Eq. (2.8) in both the low-field and high-field limits, in order to clarify the main features of the Hartree approximation.

In the limit of low magnetic fields ($\tilde{\epsilon}_H \gg 2h$), the sum in Eq. (2.8) may be evaluated using the Euler-Maclaurin summation formula.[31] In the three-dimensional limit ($d^2\tilde{\epsilon}_H \ll 1$) we obtain

$$\epsilon_R = \tilde{\epsilon} + \frac{(2\kappa^2-1)s}{4\gamma^2 d \Lambda_T}\tilde{\epsilon}^{1/2}\left[1 - \frac{1}{2}\left[\frac{h}{\tilde{\epsilon}}\right] + O\left[\left[\frac{h}{\tilde{\epsilon}}\right]^2\right]\right] , \tag{2.9}$$

where $\epsilon_R \equiv \epsilon + \Lambda$ is the shifted temperature variable with $\Lambda \propto (hN)^{1/2}$ a cutoff. For zero field $h=0$, as $\epsilon_R \to 0$ the second term in (2.9) dominates so that $\tilde{\epsilon} \sim \epsilon_R^2$, signaling a phase transition at $\epsilon_R = 0$ (the order-parameter susceptibility diverges as $\tilde{\epsilon} \to 0$). Thus, in zero field the Hartree approximation leads to a shift in the mean-field transition temperature T_0 and a change in the correlation length exponent, defined by $\xi \sim \epsilon_R^{-\nu}$, from $\nu = \frac{1}{2}$ in the Gaussian case to $\nu = 1$, which is the familiar zero-field Hartree result.[32] Next, consider the two-dimensional ($d^2\tilde{\epsilon}_H \gg 1$), low-field limit of (2.8),

$$\epsilon_R = \tilde{\epsilon} + \frac{(2\kappa^2-1)s}{8\gamma^2 d^2 \Lambda_T}\left[\ln\tilde{\epsilon} - \left[\frac{h}{\tilde{\epsilon}}\right] + O\left[\left[\frac{h}{\tilde{\epsilon}}\right]^2\right]\right] , \tag{2.10}$$

where ϵ_R is the shifted temperature variable. In this case, even for zero field there is no solution to Eq. (2.10) with $\tilde{\epsilon} = 0$. The lack of a transition to a state with conventional long-range order in two-dimensional systems with continuous symmetry is in accordance with the Mermin-Wagner theorem.[33] However, if the magnetic field is not zero, then Eqs. (2.9) and (2.10) do not have solutions with $\tilde{\epsilon} = 0$. Therefore, in the Hartree approximation even a small magnetic field prevents the order-parameter susceptibility from diverging, thereby preventing a transition to a phase with conventional long-range order.

Next, we consider the high-field limit of the self-consistent equation, such that $\tilde{\epsilon}_H \ll 2h$. In this limit the sum in Eq. (2.8) is dominated by the contribution from the lowest Landau level ($n=0$), so we have

$$\epsilon_H=\tilde{\epsilon}_H-\frac{(2\kappa^2-1)}{4\gamma^2 d}\frac{s}{\Lambda_t}h\frac{1}{\tilde{\epsilon}_H^{1/2}}\frac{1}{(1+d^2\tilde{\epsilon}_H)^{1/2}} . \quad (2.11)$$

There are no solutions of this equation with $\tilde{\epsilon}_H=0$ and, hence, there is no phase transition at any nonzero temperature in this approximation. The lack of a phase transition at any temperature may be understood in terms of the reduction of the dimensionality of the system by the magnetic field: since the order-parameter correlations transverse to the magnetic field have length scale set by the magnetic length $l_H=(\hbar c/e^*H)^{1/2}$, which is always finite, the magnetic field reduces the effective dimension of the system by two. Hence, the fluctuations in three-dimensional systems are effectively one dimensional;[10] a one-dimensional system with short-range interactions will not undergo a transition to an ordered phase at any nonzero temperature. It follows that the transport coefficients in a magnetic should be smooth functions of the temperature, in contradistinction to the results based on the Gaussian approximation.[2]

We note that our discussion is restricted to the case of a transition to a phase with a spatially uniform order parameter. Our assumption that $\langle|\psi(\mathbf{x},t)|^2\rangle$ is independent of $\mathbf{x}$ is clearly incorrect for an Abrikosov flux lattice. This approximation precludes discussion of the existence and nature of the transition to the flux-lattice state within our Hartree approximation. A more careful calculation would include the spatial variation of the magnetic field (recall our assumption that the induction field is the same as the externally applied field, which should be reasonable for high-κ superconductors), and would allow the order parameter to be spatially inhomogeneous.[34]

In the remainder of the paper we shall use the results of this section to compute, in turn, the Ettingshausen and Nernst coefficients, the thermopower and finally some of the components of the electrical conductivity tensor. The calculations are straightforward but rather tedious. In an effort to spare the reader a ceremony of unenlightening algebra, we have omitted most of the intermediate expressions but have indicated the steps required to obtain the various results.

III. THERMOELECTRIC TRANSPORT COEFFICIENTS

In this section we shall use the formalism introduced in the previous section to compute the Ettingshausen coefficient and the thermopower. The Nernst coefficient is obtained from the Ettingshausen coefficient using the linear-response transport equations—see Appendix A. We shall mention once more that the Ettingshausen coefficient, Nernst coefficient, and the electrical conductivity are nonzero to zeroth order in λ_0^{-1}, whereas the thermopower and Hall conductivity are nonzero to first order in λ_0^{-1}. We shall find that the thermoelectric transport coefficients increase with magnetic field. This result is easy to understand given that the vortices transport entropy, or heat (the entropy of the normal vortex core is higher than that of the superconducting state) so that the magnitude of the heat current increases with the density of vortices. Finally, the density of vortices increases with the applied magnetic field so that the thermoelectric transport coefficients are indeed expected to increase with magnetic field.

A. Ettingshausen effect

The Ettingshausen effect is a transverse thermomagnetic effect in which a magnetic field is applied in the z direction, a constant current is supplied in the x direction, and the temperature gradient is measured in the y direction. The temperature gradient in the y direction is a result of the transverse motion of the vortices due to the Lorentz force. In order to determine the Ettingshausen coefficient, we need to compute the y component of the heat current to first order in the electric field E_x. (We set λ_0^{-1} to zero.) The heat current follows from Eqs. (2.5), (B6), (B7), and (B9).

$$\begin{aligned}\langle J_y^h\rangle=16\pi\frac{(e^*)^2}{mc^2}E_x\Gamma_0 Hk_BT\\ \times\sum_{n,m}\frac{1}{2^n n!}\frac{1}{2^m m!}\int_{-\pi/s}^{\pi/s}\frac{dq}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\omega^2\frac{1}{(\omega^2+\Gamma_0^2\varepsilon_{nq}^2)}\frac{1}{(\omega^2+\Gamma_0^2\varepsilon_{mq}^2)}\\ \times\int_{-\infty}^{\infty}d\xi_x\int_{-\infty}^{\infty}d\xi_1\xi_1(\xi-\xi_x)\exp[-(\xi-\xi_x)^2-(\xi_1-\xi_x)^2]\\ \times H_n(\xi-\xi_x)H_n(\xi_1-\xi_x)H_m(\xi-\xi_x)H_m(\xi_1-\xi_x) ,\end{aligned} \quad (3.1)$$

where ε_{nq} is given by Eq. (B8). The integrals are best done in the following order: the ξ integrals first, then the ω integral, and finally the q integral. Whence, the Ettingshausen coefficient $\alpha_{yx}=\langle J_y^h\rangle/E_x$ is

$$\alpha_{yx}=-\frac{\phi_0}{8\pi^2\Lambda_T}\frac{d}{s}h\sum_{n=0}^{N}\left[\frac{n}{[\mu_{n-1/2}(1+d^2\mu_{n-1/2})]^{1/2}}-\frac{n+\frac{1}{2}}{[\mu_n(1+d^2\mu_n)]^{1/2}}\right] , \quad (3.2)$$

where

$$\mu_n=\tilde{\epsilon}_H+2hn , \quad (3.3)$$

and $\tilde{\epsilon}_H\equiv\tilde{\epsilon}+h$. The three-dimensional limit is obtained by taking $d^2\mu_n\to 0$, and the two-dimensional limit is $d^2\mu_n\gg 1$. [Note that the sum in (3.2) is written in a slightly more useful manner than the corresponding expression in Ref. 2.]

The results for the Ettingshausen coefficient given by

(2.8) and (3.2) were extensively discussed in Ref. 2. The main conclusion of that paper was that the Ettingshausen coefficient α_{yx} is a smooth function of the reduced temperature ϵ_H. The Ettingshausen coefficient does not diverge because the denominator in (3.2) is always nonzero for all values of the reduced temperature ϵ_H. We reiterate that $\tilde{\epsilon}_H$ is nonzero simply because there is no value of the temperature ϵ_H for which the self-consistent equation (2.8) has solution with $\tilde{\epsilon}_H=0$.[35]

In Ref. 2 we compared the lowest-Landau-level expression for α_{yx} with the experimental results of Palstra *et al.*[14] and found good agreement. There we fit the lowest-Landau-level results to the data in the mean-field region far below $\epsilon_H=0$. Inclusion of the higher Landau levels merely changes the mean field $T_{c2}(H)$ and the overall magnitude of the Ettingshausen coefficient.[36] In Ref. 2 we pointed out that the experimental results show a pronounced magnetic field dependence of the mean-field slopes of the Ettingshausen coefficient plotted as a function of ϵ_H (By mean-field slopes we refer to the temperature region for which the Ettingshausen coefficient is a linear function of ϵ_H.) It is possible that this discrepancy is removed by including vertex corrections in the same manner as in Ref. 13.

The self-consistent equations predict that the transport coefficients should exhibit scaling behavior as a function of magnetic field and reduced temperature. We expect this scaling behavior to hold even in a higher-order approximation[8–10] thereby making it useful in analyzing data with regard to the fluctuation theory. The scaling functions are valid only in either the two- or three-dimensional limits (not for the general Lawrence-Doniach model) and are easily obtained in the case of the lowest Landau level (high field). However, the scaling forms are not restricted to the high-field case since, as we have discussed above, the inclusion of higher Landau levels in the Hartree approximation simply renormalizes the critical temperature and the magnitude of the transport coefficient, but not its functional form.

For $n=0$, the Ettingshausen coefficient given by (3.2) reduces to

$$\alpha_{yx}=\frac{\phi_0}{16\pi^2\Lambda_T}\frac{d}{s}\frac{h}{[\tilde{\epsilon}_H(1+d^2\tilde{\epsilon}_H)]^{1/2}}\ , \tag{3.4}$$

while the self-consistency equation is given by Eq. (2.11). There is a dimensional crossover implicit in Eqs. (2.11) and (3.4). For temperatures well above the mean field $T_{c2}(H)$ the correlation length is small compared to the interplanar separation: $d^2\tilde{\epsilon}_H \gg 1$, hence, $\alpha_{yx}\sim\epsilon_H^{-1}$, characteristic of two dimensional (2D) Gaussian fluctuations. As the temperature is lowered, the correlation length grows and eventually becomes comparable to the interplanar separation: $d^2\tilde{\epsilon}_H \ll 1$, so that $\alpha_{yx}\sim\epsilon_H^{-1/2}$, characteristic of three-dimensional Gaussian fluctuations. Similar considerations apply to the other transport coefficients (see below).

We may write Eq. (3.4) in a scaling form: in two dimensions

$$\alpha_{yx}^{2\mathrm{D}}=\frac{k_BT}{\phi_0 s}h\left[\frac{s\Lambda_T}{(2\kappa^2-1)\xi_{ab}^2(0)h}\right]^{1/2}\times F_{2\mathrm{D}}\left[\left[\frac{s\Lambda_T}{(2\kappa^2-1)\xi_{ab}^2(0)h}\right]^{1/2}\epsilon_H\right]\ , \tag{3.5}$$

while in three dimensions

$$\alpha_{yx}^{3\mathrm{D}}=\frac{k_BT}{\phi_0 s}dh\left[\frac{4\gamma^2 d\Lambda_T}{(2\kappa^2-1)sh}\right]^{1/3}\times F_{3\mathrm{D}}\left[\left[\frac{4\gamma^2 d\Lambda_T}{(2\kappa^2-1)sh}\right]^{2/3}\epsilon_H\right]\ . \tag{3.6}$$

When the reduced temperature ϵ_H is large and negative, the self-consistent equation is easily solved by noting that the first term on the right-hand side of Eq. (2.11) is much smaller than the second term, and therefore the first term may be dropped. Substituting the resulting expression for $\tilde{\epsilon}_H$ into our expression for the Ettingshausen coefficient in the lowest Landau level, Eq. (3.4), we obtain the mean-field (MF) result,

$$\alpha_{yx}^{\mathrm{MF}H}=\frac{1}{8\pi}\frac{(H_{c2}-H)}{(2\kappa^2-1)}\ , \tag{3.7}$$

which is correct in either two or three dimensions. This expression differs slightly from Maki's result[37] obtained from the mean-field Abrikosov solution:

$$\alpha_{yx}^{\mathrm{MF}}=\frac{1}{4\pi\beta_A}\frac{(H_{c2}-H)}{(2\kappa^2-1)}\ , \tag{3.8}$$

where $\beta_A=\overline{\langle|\psi|^4\rangle}/\overline{\langle|\psi^2\rangle}^2$ (the overbar denotes a spatial average), which for a triangular flux lattice $\beta_A\approx 1.16$. Thus, although the functional form of (3.7) and (3.8) is identical, the coefficients differ slightly: $\alpha_{yx}^{\mathrm{MF}H}/\alpha_{yx}^{\mathrm{MF}}=\beta_A/2$. This discrepancy is expected since in the Hartree approximation, $\beta_A=2$. From this calculation, we see that the scaling functions have the asymptotic forms $F_{2\mathrm{D}}(x)$, $F_{3\mathrm{D}}(x)\sim -x$ for large negative values of x. When ϵ_H is large and positive, the first term on the right-hand side of Eq. (2.11) dominates and we have $\tilde{\epsilon}_H\approx\epsilon_H$, corresponding to Gaussian fluctuations. Therefore, the scaling functions have the limiting forms $F_{2\mathrm{D}}(x)\sim x^{-1}$ and $F_{3\mathrm{D}}(x)\sim x^{-1/2}$ for large positive values of x. In the intermediate regime ($\epsilon_H\approx 0$), the coherence length is much larger than the interplanar separation $\xi\gg d$, so that three-dimensional Gaussian fluctuations dominate. The Ettingshausen coefficient for the full range of reduced temperatures and for several magnetic fields is plotted in Fig. 1. In Ref. 2 we compared the experimental results with the two- and three-dimensional scaling forms and concluded that the two-dimensional form gave a better fit. Of course, the scaling functions did not collapse the data onto a single curve in the mean-field region due to the field dependence of the slopes.

Finally, we note that, in the limit of high magnetic fields, the Ettingshausen coefficient is closely related to the equilibrium entropy $S(T,H)$, which is calculated in

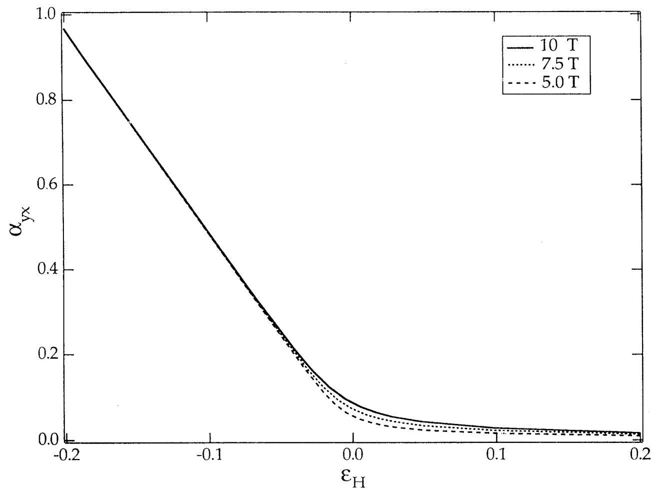

FIG. 1. The dimensionless high-field Ettingshausen coefficient α_{yx} in units of $k_B T/\phi_0 s$ vs the reduced temperature $\epsilon_H = [1/H_{c2}^{ab}(0)](dH_{c2}/dT)[T - T_{c2}(H)]$ for different magnetic fields. The slope of the 7.5-T curve in the mean-field region $\epsilon_H < -0.1$ is set equal to the experimental value of Ref. 14. Furthermore, $H_{c2}^{ab}(0) = 400$ T, $s = 12$ Å, and $\xi_c(0) = 2$ Å. This procedure gives $\kappa \approx 20$. Notice that the fluctuation effect increases with the magnetic field (in agreement with experiment results) while the slopes in the mean-field region are independent of the magnetic field contrary to the experimental results (see Sec. III A).

Appendix C [see Eq. (C4)],

$$\alpha_{yx} = -\left[\frac{dH_{c2}}{dT}\right]^{-1}\left[\frac{\partial \epsilon_H}{\partial \tilde{\epsilon}_H}\right] S(T,H) , \tag{3.9}$$

where $(\partial \epsilon_H / \partial \tilde{\epsilon}_H)$ is given by Eq. (C3). This result is in accordance with the notion that the Ettingshausen effect is simply due to the transport of entropy by the vortices.

B. Nernst effect

The Nernst effect is the inverse of the Ettingshausen effect. Here, one applies a temperature gradient in the x direction which, in turn, drives the vortices in the x direction. Faraday's law implies that the longitudinal motion of the vortices in a magnetic field in the z direction will induce an electric field in the y direction. This induced voltage is called the Nernst voltage. The Onsager transport equations give the precise relationship between the Nernst and Ettingshausen coefficients. One finds (see Appendix A)

$$\nu \equiv \frac{E_y}{H(\partial T/\partial x)} = -\frac{1}{HT}\frac{\alpha_{xy}}{\sigma_{xx}} . \tag{3.10}$$

Here, the longitudinal resistivity is simply $\rho_{xx} = 1/\sigma_{xx}$ because the Hall conductivity is negligible; the σ_{xx} is the measured conductivity, which includes the contribution from the normal electrons. The relation (3.10) between the Nernst coefficient and the Ettingshausen coefficient has been verified experimentallly by Hagen *et al.*[38]

C. Thermopower

The thermopower is the longitudinal version of the Ettingshausen coefficient: one measures the induced temperature gradient in the y direction due to an electric current flowing in the y direction. Unlike the Ettingshausen effect, there is a substantial contribution to the thermopower from the normal electrons. The thermopower perpendicular to the magnetic field is defined by

$$\langle J_y^h \rangle = \alpha_{yy} E_y \tag{3.11}$$

and we may compute the superconducting contribution following the method outlined in Sec. II. We find for the thermopower perpendicular to the magnetic field

$$\alpha_{yy} = \frac{\phi_0}{32\pi^2 \Lambda_T \xi_c(0)} \Gamma_0 \lambda_0^{-1} h^2 \times \sum_{n=0}^{N} \frac{(n+1)(1+2d^2\mu_{n+1/2})}{\mu_{n+1/2}^{3/2}(1+d^2\mu_{n+1/2})^{3/2}} , \tag{3.12}$$

where μ_n is given by (3.3). In a similar fashion we find, for the thermopower along the field direction,

$$\alpha_{zz} = \frac{\phi_0}{32\pi^2 \Lambda_T \xi_c(0)} \Gamma_0 \lambda_0^{-1} \gamma^2 h \times \sum_{n=0}^{N} \frac{[1 - 2d\mu_n^{1/2}(1+d^2\mu_n)^{1/2} + 2d^2\mu_n]}{\mu_n^{1/2}(1+d^2\mu_n)^{1/2}} \tag{3.13}$$

The *zero*-field Gaussian result has been previously obtained by Maki[39] and may be derived from either Eq. (3.12) or (3.13) by using the Euler-Maclaurin formula, in the limit $h \to 0$, and by replacing $\tilde{\epsilon}_H$ by ϵ. Thus, in three dimensions ($d\epsilon \to 0$), the thermopower is a cusplike function of ϵ:

$$\alpha_{yy}^{3D}(h=0) = \frac{\phi_0 \Gamma_0 \lambda_0^{-1}}{32\pi^2 \Lambda_T \xi_c(0)} (\Lambda^{1/2} - \epsilon^{1/2}) , \tag{3.14}$$

where Λ is a cutoff, and $\alpha_{zz}^{3D}(h=0) = \gamma^2 \alpha_{yy}^{3D}(h=0)$. In two dimensions the thermopower has a logarithmic divergence at $\epsilon = 0$:

$$\alpha_{yy}^{2D}(h=0) = \frac{\phi_0 \Gamma_0 \lambda_0^{-1}}{64\pi^2 \Lambda_T \xi_c(0) d} (\ln\Lambda - \ln\epsilon) , \tag{3.15}$$

and $\alpha_{zz}^{2D}(h=0) = \gamma^2 \alpha_{yy}^{2D}(h=0)$. The results in the Hartree approximation in three dimensions are obtained by replacing ϵ by $\tilde{\epsilon} \sim \epsilon_R^2$ in Eq. (3.14). However, measurements of the thermopower of Y-Ba-Cu-O in zero magnetic field[40] suggest that the thermopower diverges as $|T - T_c|^{-1/2}$. Recently, Lu and Patton[41] have computed the zero-field thermopower from a microscopic theory and found that there is indeed a power-law divergence of this form in the *normal-state* thermopower due to the interaction with the superconducting fluctuations. Therefore, our result for the thermopower in zero field based on the Ginzburg-Landau theory may be of marginal relevance for understanding the data.

For nonzero field, Howson *et al.*[40] find that the peak in the thermopower rapidly vanishes with increasing field. This result is clearly in agreement with our general expectations: there is no divergence in the thermopower, (3.12) and (3.13), in nonzero magnetic field. It would be interesting to generalize the microscopic calculation of Lu and Patton to a nonzero magnetic field, in order to study the evolution of the thermopower from zero field to high fields.

IV. ELECTRICAL CONDUCTIVITY IN A MAGNETIC FIELD

The original studies of the Gaussian fluctuation conductivity in a magnetic field predicted a divergence at $T_{c2}(H)$.[42] This predicted divergence is *not* observed. The absence of a divergent contribution to the fluctuation conductivity in a magnetic field is easy to understand on physical grounds: the motion of vortices in a superconductor provides a dissipation mechanism and, hence, a finite flux-flow conductivity. The theoretical resolution of this problem has been provided by Ikeda *et al.*[27] who have shown that the divergence is eliminated by treating the problem in the Hartree approximation. Therefore, just as in the case of the Ettingshausen effect, the Hartree approximation gives a qualitatively different prediction for the electrical conductivity (in a magnetic field) than does the Gaussian theory. We emphasize that we calculate only the contribution of the superconducting order parameter to the conductivity so that comparison with experiment necessitates the addition of the normal-state contribution to our results. The high-field results for σ_{yy}, σ_{yx}, and σ_{zz} are summarized in Figs. 2 and 3.

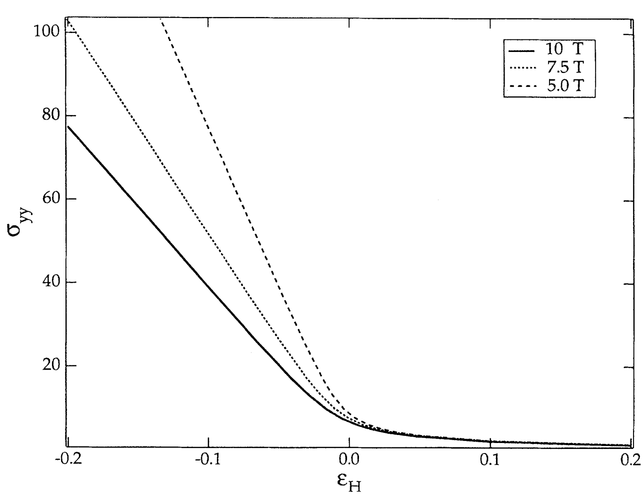

FIG. 2. The dimensionless high-field conductivity perpendicular to the magnetic field σ_{yy} in units of $\Gamma_0^{-1} m_c \xi_c(0)/(8\pi\hbar^2\Lambda_T)$ vs the reduced temperature ϵ_H. The same set of parameters are used as in Fig. 1. Notice the substantial dependence of the magnetic field of the mean-field slopes which is expected from the mean-field flux-flow result: $\sigma_{yy} \sim \epsilon_H/h$. The dimensionless high-field Hall conductivity is closely related: $\sigma_{yx} = (\lambda_0^{-1}/\Gamma_0^{-1})\sigma_{yy}$.

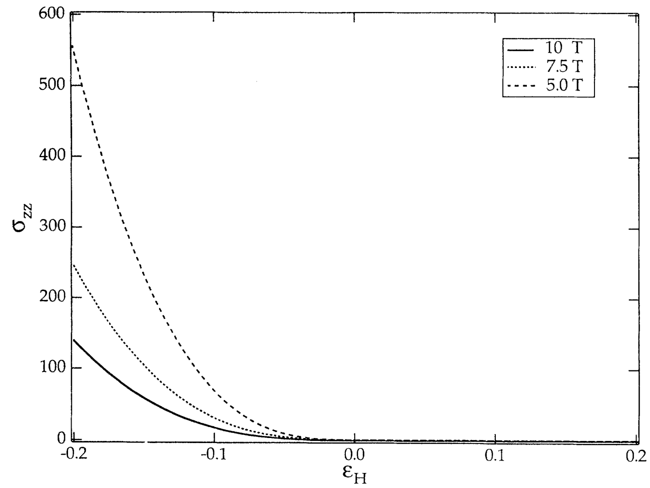

FIG. 3. The dimensionless high-field conductivity parallel to the magnetic field σ_{zz} in units of $\Gamma_0^{-1} m_c \xi_c(0)/(8\pi\hbar^2\Lambda_T)$ vs the reduced temperature ϵ_H. The same set of parameters are used in Fig. 1. In the mean-field region the conductivity $\sigma_{zz} \sim \epsilon_H^3/h^2$.

A. Longitudinal conductivity

For completeness, we shall state the results for the electrical conductivity in the a-b plane in the presence of a magnetic field applied perpendicular to the a-b plane. In the Hartree approximation

$$\sigma_{yy} = \Gamma_0^{-1} \frac{m_c \xi_c(0)}{8\pi\hbar^2\Lambda_T} \sum_{n=0}^{N} (n+1)(A_n + A_{n+1} - 2A_{n+1/2}) , \tag{4.1}$$

where $A_n = [\mu_n(1+d^2\mu_n)]^{1/2}$ and μ_n is given by Eq. (3.3). In the high-field limit we only keep the most divergent term in the sum in Eq. (4.1), which is

$$\sigma_{yy} = \Gamma_0^{-1} \frac{m_c \xi_c(0)}{8\pi\hbar^2\Lambda_T} \frac{1}{[\tilde{\epsilon}_H(1+d^2\tilde{\epsilon}_H)]} . \tag{4.2}$$

When ϵ_H is large and positive, Eq. (4.2) reproduces the results of the Gaussian fluctuation calculations.[42] On the other hand, in the mean-field regime, when ϵ_H is large and negative, we obtain from the solution of the self-consistent equation, Eq. (2.11),

$$\sigma_{yy}^{\mathrm{MF}H} = \frac{\Gamma_0^{-1} m}{\hbar^2} \frac{1}{4\pi(2\kappa^2-1)} \frac{H_{c2}-H}{H} . \tag{4.3}$$

The flux-flow resistivity calculated by Schmid[20] for the mean-field Abrikosov flux-lattice solution is

$$\sigma_{yy}^{\mathrm{MF}} = \frac{\Gamma_0^{-1} m}{\hbar^2} \frac{1}{2\pi\beta_A(2\kappa^2-1)} \frac{H_{c2}-H}{H} , \tag{4.4}$$

which again differs from the corresponding expression in the Hartree approximation Eq. (4.3) by the factor $\beta_A/2$. The high-field conductivity has a scaling form in either two or three dimensions: in two dimensions,

$$\sigma_{yy}^{2D}=\frac{\Gamma_0^{-1}m_c\xi_c(0)}{8\pi\hbar^2\Lambda_T d}\left[\frac{s\Lambda_T}{(2\kappa^2-1)\xi_{ab}^2(0)h}\right]^{1/2}\times F_{2D}\left[\left[\frac{s\Lambda_T}{(2\kappa^2-1)\xi_{ab}^2(0)h}\right]^{1/2}\epsilon_H\right], \tag{4.5}$$

while in three dimensions

$$\sigma_{yy}^{3D}=\frac{\Gamma_0^{-1}m_c\xi_c(0)}{8\pi\hbar^2\Lambda_T}\left[\frac{4\gamma^2 d\Lambda_T}{(2\kappa^2-1)sh}\right]^{1/3}\times F_{3D}\left[\left[\frac{4\gamma^2 d\Lambda_T}{(2\kappa^2-1)sh}\right]^{2/3}\epsilon_H\right], \tag{4.6}$$

where the scaling functions $F_{2D}(x)$ and $F_{3D}(x)$ are the same as those obtained for the Ettingshausen coefficient (see Sec. III A).

In order to compute the conductivity parallel to the magnetic field we have to calculate the electric current in the z direction which involves taking derivatives with respect to the discretized coordinate z. We note that, for example,

$$\partial\psi_n/\partial z\equiv\psi_n[\exp(isq)-\exp(-isq)]/2s .$$

Thus,

$$\sigma_{zz}=\Gamma_0^{-1}\frac{m_c\xi_c(0)}{32\pi\hbar^2\Lambda_T}\gamma^2 h\sum_{n=0}^{N}\frac{1}{\mu_n^{3/2}}\frac{1}{(1+d^2\mu_n)^{3/2}} . \tag{4.7}$$

The high-field limit of the conductivity in the z direction in two dimensions has the scaling form

$$\sigma_{zz}^{2D}=\frac{\Gamma_0^{-1}m_c\xi_c(0)\gamma^2 h}{32\pi\hbar^2\Lambda_T d}\left[\frac{s\Lambda_T}{(2\kappa^2-1)\xi_{ab}^2(0)h}\right]^{3/2}\times G_{2D}\left[\left[\frac{s\Lambda_T}{(2\kappa^2-1)\xi_{ab}^2(0)h}\right]^{1/2}\epsilon_H\right], \tag{4.8}$$

where $G_{2D}(x)\sim x^{-3}$ for large positive x, and $G_{2D}(x)\sim -x^3$ for large negative x. In three dimensions

$$\sigma_{zz}^{3D}=\frac{\Gamma_0^{-1}m_c}{16\pi\hbar^2(2\kappa^2-1)}G_{3D}\left[\left[\frac{4\gamma^2 d\Lambda_T}{(2\kappa^2-1)sh}\right]^{2/3}\epsilon_H\right], \tag{4.9}$$

where $G_{3D}(x)\sim x^{-3/2}$ for large positive x, and $G_{3D}(x)\sim -x^3$ for large negative x.

In the zero-field limit, the conductivities in Eqs. (4.1) and (4.7) may be evaluated by applying the Euler-Maclaurin formula to the sums. The Aslamazov-Larkin fluctuation conductivity[4] in the Hartree approximation[26] is then obtained: in spatial dimension d, $\sigma_{yy}\sim\epsilon_R^{-\nu(4-d)}$, where $\nu=1/(d-2)$ (ϵ_R is the renormalized temperature; see Sec. III).[32]

B. Hall conductivity

The fluctuation Hall conductivity in the Gaussian approximation was first discussed by Fukuyama *et al.*[43] and in the flux-flow regime by Maki[44] and by Ebisawa.[45] Here we extend their results to the Hartree case for the Lawrence-Doniach model. The calculation of the Hall effect is more tedious because we have to keep terms of order λ_0^{-1}. We compute the electric current in the y direction due to an electric field in the x direction. The general expressions are very cumbersome so we shall restrict ourselves to the high- and low-field results. For reference, however, we shall state a general, intermediate expression for the Hall coefficient:

$$\sigma_{yx}=\left[\frac{e^*}{2\pi\hbar c}\right]^2 2\pi k_B T\lambda_0^{-1}(\hbar\omega_H)^3\times\sum_{n=0}^{N}\int_{-\pi/s}^{\pi/s}\frac{dq}{2\pi}\frac{(n+1)}{\varepsilon_{nq}\varepsilon_{n+1,q}(\varepsilon_{nq}+\varepsilon_{n+1,q})^2} . \tag{4.10}$$

where ε_{nq} is given by Eq. (B8) and $\omega_H=e^*H/mc$. In the high-field limit, Eq. (4.10) reduces to

$$\sigma_{yx}=\lambda_0^{-1}\frac{m_c\xi_c(0)}{8\pi\hbar^2\Lambda_T}\frac{1}{[\tilde{\epsilon}_H(1+d^2\tilde{\epsilon}_H)]^{1/2}} , \tag{4.11}$$

with $\tilde{\epsilon}_H$ given by Eq. (2.11). The scaling functions for the high field σ_{yx} are the same as those for σ_{yy}, apart from the replacement of Γ_0^{-1} by λ_0^{-1}. In the mean-field regime, $\epsilon_H\ll 0$, the high-field result (4.11) reduces to

$$\sigma_{yx}^{\mathrm{MF}H}=\frac{\lambda_0^{-1}m}{\hbar^2}\frac{1}{4\pi(2\kappa^2-1)}\frac{H_{c2}-H}{H} . \tag{4.12}$$

As in the case of the Ettingshausen coefficient and the longitudinal conductivity, this result differs by a factor $\beta_A/2$ from the mean-field result computed by Maki using the mean-field Abrikosov solution.[44]

In the low-field limit, Eq. (4.10) gives

$$\sigma_{yx}=\lambda_0^{-1}\frac{m_c\xi_c(0)}{96\pi\hbar^2\Lambda_T}\phi_0 h\left[\frac{1+3d^2\tilde{\epsilon}+2d^4\tilde{\epsilon}^2}{\tilde{\epsilon}^{3/2}(1+d^2\tilde{\epsilon})^{5/2}}\right] . \tag{4.13}$$

If we replace $\tilde{\epsilon}$ by the reduced temperature ϵ in Eq. (4.13), then we obtain the Gaussian results of Fukuyama *et al.*[43] in the two- and three-dimensional limits: $\sigma_{yx}\sim h\epsilon^{-2}$ in two dimensions and $\sigma_{yx}\sim h\epsilon^{-3/2}$ in three dimensions. However, in the Hartree approximation, $\tilde{\epsilon}\sim\epsilon_R^2$ in three dimensions, with ϵ_R the renormalized temperature (see Sec. II C), so that $\sigma_{yx}\sim h\epsilon_R^{-3}$ in three dimensions in low fields. Thus, the low-field Hall conductivity is more singular than the zero-field longitudinal conductivity.

Recently, Hagen *et al.*[46] have measured the Hall resistivity in Y-Ba-Cu-O thin films near the mean field H_{c2}. They found pronounced fluctuation effects above H_{c2} which reduced the Hall resistance from its normal-state value, and also found that the Hall resistance changes sign below H_{c2}. These results may have the following explanation. The Hall conductivity may be written in terms of the resistivities ρ using $\sigma_{yx}=-\rho_{yx}/(\rho_{yy}^2+\rho_{yx}^2)$ (assuming $\rho_{xx}=\rho_{yy}$). Given that, experimentally, $\rho_{yy}\gg\rho_{yx}$, it follows that $\sigma_{yx}\approx-\rho_{yx}/\rho_{yy}^2$. Hence, a sign change in ρ_{yx} will be accompanied by a corresponding sign change in σ_{yx}. Now, the total Hall conductivity is the sum of the normal-state Hall conductivity σ_{yx}^n and the superconducting contribution which is given by Eq. (4.11):

$$\sigma_{yx}^{\rm tot}=\sigma_{yx}^{n}+\lambda_0^{-1}\frac{m_c\xi_c(0)}{8\pi\hbar^2\Lambda_T}\frac{1}{[\tilde{\epsilon}_H(1+d^2\tilde{\epsilon}_H)]^{1/2}} . \quad (4.14)$$

Consequently, a sign change in σ_{yx} may result if the normal-state Hall conductivity and the superconducting (vortex) conductivity have opposite signs. Whether such a sign change is possible will depend upon the sign of the particle-hole asymmetry parameter λ_0^{-1}. This sign is not determined from any fundamental considerations but rather depends on the details of the microscopic mechanism.[43] This observation also accounts for the suppression of the Hall resistance due to fluctuations above the mean field H_{c2}. The importance of fluctuations far above the mean field H_{c2} is consistent with the Ettingshausen-effect data of Palstra *et al.*[14] who found large fluctuations 5–10 K above $T_{c2}(H)$.

V. CONCLUSIONS

We have solved the time-dependent Ginzburg-Landau equation in the Hartree approximation and have computed various transport coefficients in a magnetic field near the mean-field transition temperature $T_{c2}(H)$. We have emphasized the absence of any divergence in the transport coefficients at $T_{c2}(H)$ in nonzero magnetic field. Moreover, the lack of a divergence at $T_{c2}(H)$ is not restricted to the transport coefficients but is also reflected in equilibrium quantities such as the specific heat[9] and the magnetization. We mention one crucial advantage of the Ettingshausen effect: the theoretical value of the Ettingshausen coefficient is easier to compare with experimental results since the normal-state contribution is negligible and, hence, no subtraction procedure is required. This is in sharp contrast to the specific heat, conductivity, and thermopower. Much work remains to be done especially on the two outstanding problems that we have not considered: the effect of pinning on these results[1] and the nature of the transition (if any) from the normal state to the Abrikosov flux-lattice state.[47] Finally, a microscopic calculation is clearly necessary in order to understand the interplay between Maki-Thompson contributions to the transport coefficients, and the order-parameter fluctuations in a magnetic field.

ACKNOWLEDGMENTS

It is a pleasure to thank T. T. M. Palstra and P. Kes for providing us with their data prior to publication, M. P. A. Fisher for useful discussions on the Hall effect, and D. A. Huse for numerous helpful discussions. We would like to thank R. Ikeda for sending us Refs. 13 and 29 prior to their publication. This work was supported by National Science Foundation (NSF) Grant No. DMR 89-14051. S.U. received partial support from the Virginia Center for Innovative Technology, Grant No. CIT SUP-89-004.

APPENDIX A: DEFINITION OF TRANSPORT COEFFICIENTS

In this appendix we derive the relation between the Ettingshausen coefficient and the three experimentally measured quantities: the transverse temperature gradient, the applied voltage, and the thermal conductivity. We shall also obtain an expression for the Nernst coefficient in terms of the Ettingshausen coefficient. We begin with the linear-response equations for the electric and heat currents, $\mathbf{J}^e$ and $\mathbf{J}^h$, respectively,[48]

$$J_x^h=\alpha_{xx}E_x+\alpha_{xy}E_y+K_{xx}\left[-\frac{\partial T}{\partial x}\right]+K_{xy}\left[-\frac{\partial T}{\partial y}\right] , \quad (A1)$$

$$J_y^h=\alpha_{yy}E_y+\alpha_{yx}E_x+K_{yy}\left[-\frac{\partial T}{\partial y}\right]+K_{yx}\left[-\frac{\partial T}{\partial x}\right] , \quad (A2)$$

$$J_x^e=\sigma_{xx}E_x+\sigma_{xy}E_y+\frac{\alpha_{xx}}{T}\left[-\frac{\partial T}{\partial x}\right]+\frac{\alpha_{xy}}{T}\left[-\frac{\partial T}{\partial y}\right] , \quad (A3)$$

$$J_y^e=\sigma_{yy}E_y+\sigma_{yx}E_x+\frac{\alpha_{yy}}{T}\left[-\frac{\partial T}{\partial y}\right]+\frac{\alpha_{yx}}{T}\left[-\frac{\partial T}{\partial x}\right] , \quad (A4)$$

where $\alpha_{xx}=\alpha_{yy}$, $\sigma_{xx}=\sigma_{yy}$, $K_{xx}=K_{yy}$ and the off-diagonal transport coefficients are antisymmetric in their indices due to the applied magnetic field: $\alpha_{xy}(\mathbf{H})=-\alpha_{yx}(\mathbf{H})$ and similarly for K and σ. [We have used the Onsager reciprocity relations which give $\alpha_{xy}(\mathbf{H})=\alpha_{yx}(-\mathbf{H})$ and the fact that transverse transport coefficients are odd in the magnetic field; that is, $\alpha_{xy}(-\mathbf{H})=-\alpha_{xy}(\mathbf{H})$, due to the $\mathbf{J}\times\mathbf{H}$ form of the Lorentz force.]

Now, the boundary conditions for the Ettingshausen experiment are zero transverse currents and zero longitudinal temperature gradient. Thus, for $J_y^e=0$, $J_y^h=0$, and $\partial T/\partial x=0$, Eq. (A2) gives

$$\alpha_{yx}=\frac{K_{xx}}{E_x}\frac{\partial T}{\partial y}-\frac{\alpha_{xx}}{E_x}E_y , \quad (A5)$$

while Eq. (A4) gives

$$E_y=\frac{1}{\sigma_{xx}}\left[\frac{\alpha_{xx}}{T}\frac{\partial T}{\partial y}-\sigma_{xy}E_x\right] . \quad (A6)$$

We can eliminate E_y from these two equations, (A5) and (A6), to give an expression for the Ettingshausen coefficient in terms of the measured quantities

$$\alpha_{xy}=\frac{K_{xx}}{E_x}\frac{\partial T}{\partial y}-\frac{\alpha_{xx}^2}{\sigma_{xx}E_xT}\frac{\partial T}{\partial y}+\frac{\sigma_{xy}}{\sigma_{xx}}\alpha_{xx} . \quad (A7)$$

Empirically, the thermopower α_{xx} is small enough to render the second and third terms in Eq. (A7) negligible compared to the first term.[49] Therefore, the Ettingshausen coefficient is indeed equal to the first term in Eq. (A7) as stated in Sec. II B.

The Nernst effect is measured under the following boundary conditions: $\partial T/\partial y=0$, $J_y^h=0$, and $J_y^e=0$. Then (A4) gives

$$E_y=\frac{\alpha_{yx}}{\sigma_{xx}}\frac{1}{T}\left[\frac{\partial T}{\partial x}\right] , \tag{A8}$$

where we have omitted a term proportional to σ_{xy}. We shall define the Nernst coefficient ν as in Ref. 38,

$$\nu\equiv\frac{1}{H}\frac{E_y}{(\partial T/\partial x)} . \tag{A9}$$

Therefore, from (A8) and (A9) we find a relation between the Ettingshausen coefficient and the Nernst coefficient

$$\nu=\frac{1}{HT}\alpha_{yx} . \tag{A10}$$

APPENDIX B: CORRELATION AND RESPONSE FUNCTIONS IN LINEAR RESPONSE

In this appendix we outline the method for obtaining the correlation function to linear order in the electric field for the linearized equation of motion. We start with the equation of motion for the order parameter in the Hartree approximation,

$$(i\lambda_0^{-1}+\Gamma_0^{-1})\left[\frac{\partial}{\partial t}+i\frac{e^*}{\hbar c}\Phi\right]\psi_q=\left[\frac{\hbar^2}{2m}\left[\nabla_\perp-i\frac{e^*}{\hbar c}\mathbf{A}\right]^2-\frac{\hbar^2\gamma^2}{ms^2}[1-\cos(qs)]-\tilde{a}\right]\psi_q+\zeta_q , \tag{B1}$$

where $\tilde{a}=a+b_\kappa\langle|\psi|^2\rangle$, and where we have introduced Bloch wave states along the z direction with wave vector q. The solution of this equation is (we omit the wave-vector index q and note that all position vectors $\mathbf{x}$ lie in the xy plane)

$$\psi(\mathbf{x},t)=\int d\mathbf{x}'\int dt' R(\mathbf{x},t;\mathbf{x}',t')\zeta(\mathbf{x}',t') , \tag{B2}$$

where the response function R is the Green's function for Eq. (B1). From the definition of the correlation function, Eq. (2.4), and the noise averages, Eq. (2.3), it follows that

$$C(\mathbf{x},t;\mathbf{x}',t')=2\Gamma_0^{-1}k_BT\int d\mathbf{x}_1\int dt_1 R(\mathbf{x},t;\mathbf{x}_1,t_1)R^*(\mathbf{x}',t',\mathbf{x}_1,t_1) . \tag{B3}$$

To find the response function to linear order in the electric field we write $R=R_0+R_1$, where R_0 is the equilibrium response function and R_1 is first order in the electric field. Substituting this expansion into the equation of motion, Eq. (B1), we find, for R_1,

$$R_1(\mathbf{x},t;\mathbf{x}',t')=-i\frac{e^*}{\hbar c}(i\lambda_0^{-1}+\Gamma_0^{-1})\int d\mathbf{x}_1\int dt_1\Phi(\mathbf{x}_1)R_0(\mathbf{x},t;\mathbf{x}_1,t_1)R_0(\mathbf{x}_1,t_1;\mathbf{x}',t') , \tag{B4}$$

where the equilibrium response function R_0 satisfies

$$\left[(i\lambda_0^{-1}+\Gamma_0^{-1})\frac{\partial}{\partial t}-\frac{\hbar^2}{2m}\left[\nabla_\perp-i\frac{e^*}{\hbar c}\mathbf{A}\right]^2+\frac{\hbar^2\gamma^2}{ms^2}[1-\cos(qs)]+\tilde{a}\right]R_0(x,t;x',t')=\delta(\mathbf{x}-\mathbf{x}')\delta(t-t') . \tag{B5}$$

Finally, we may express the correlation function C in terms of the response function R_0 using Eqs. (B3) and (B4),

$$\begin{aligned}C(\mathbf{x},t;\mathbf{x}',t')=C_0(\mathbf{x},t;\mathbf{x}',t')&-i\frac{e^*}{\hbar c}\Gamma_0^{-1}\int d\mathbf{x}_1\int dt_1\Phi(\mathbf{x}_1)[R_0(\mathbf{x},t;\mathbf{x}_1,t_1)C_0^*(\mathbf{x}',t';\mathbf{x}_1,t_1)-R_0^*(\mathbf{x}',t';\mathbf{x}_1,t_1)C_0(\mathbf{x},t;\mathbf{x}_1,t_1)]\\&+\frac{e^*}{\hbar c}\lambda_0^{-1}\int d\mathbf{x}_1\int dt_1\Phi(\mathbf{x}_1)[R_0(\mathbf{x},t;\mathbf{x}_1,t_1)C_0^*(\mathbf{x}',t';\mathbf{x}_1,t_1)+R_0(\mathbf{x}',t';\mathbf{x}_1,t_1)C_0(\mathbf{x},t;\mathbf{x}_1,t_1)] ,\end{aligned} \tag{B6}$$

where C_0 is the equilibrium correlation function which does not contribute to the currents. The problem is now reduced to finding the equilibrium response function R_0, which is most easily accomplished by expanding R_0 in terms of the Landau eigenfunctions in the x-y plane,[50] and Bloch waves along the z direction. Working in the Landau gauge, $\mathbf{A}=(-Hy,0,0)$, we find

$$\begin{aligned}R_0(\xi,\xi';\omega,q,\xi_0)=&\left[\frac{m\omega_H}{\hbar\pi}\right]^{1/2}\\&\times\exp[-(\xi-\xi_0)^2/2-(\xi'-\xi_0)^2/2]\\&\times\sum_{n=0}^{\infty}\frac{1}{2^n n!}\frac{H_n(\xi-\xi_0)H_n(\xi'-\xi_0)}{(i\Gamma_0^{-1}\omega-\lambda_0^{-1}\omega+\varepsilon_{nq})} ,\end{aligned} \tag{B7}$$

where the arguments of R_0 are the x coordinates in dimensionless units $\xi=(m\omega_H/\hbar)^{1/2}x$ with $\omega_H=e^*H/mc$, and

$$\xi_0=-p_y/(m\omega_H/\hbar)^{1/2}$$

with p_y the y component of the momentum, ω is the frequency variable, ε_{nq} is the energy eigenvalue

$$\varepsilon_{nq}=\hbar\omega_H(n+\tfrac{1}{2})+\tilde{a}+\frac{\hbar^2\gamma^2}{ms^2}[1-\cos(qs)] , \qquad (B8)$$

and the functions $H_n(x)$ are the Hermite polynomials. We also note that the fluctuation-dissipation theorem gives

$$C_0(\xi,\xi';\omega,q,\xi_0)=\frac{2k_BT}{\omega}\mathrm{Im}R_0(\xi,\xi';\omega,\xi_0) , \qquad (B9)$$

which may be verified by using (B3) and (B7).

APPENDIX C: THERMODYNAMIC PROPERTIES

In this Appendix we shall extend the well-known calculation of the low-field Gaussian fluctuation magnetization and entropy[51] to the Hartree case. This calculation is completely analogous to that of the specific heat in the Hartree approximation—see, for example, Ref. 10. Similar results have also been obtained recently by Ikeda and Tsuneto.[13]

The free energy in the Hartree approximation is[51]

$$F=-k_BT\frac{e^*H}{2\pi\hbar c}\sum_{n=0}^{N}\int_{-\pi/s}^{\pi/s}\frac{dq}{2\pi}\ln\left[\frac{\pi k_BT}{\varepsilon_{nq}}\right] , \qquad (C1)$$

where ε_{nq} is the energy variable given by Eq. (B8). The magnetization follows from $M=-\partial F/\partial H$. In three dimensions, the integration over q is divergent, a reflection of the fact that the theory is valid only on length scales greater than the zero-temperature coherence length: the momentum integrals are, therefore, cut off at high momenta Q. The resulting Q-dependent terms may be absorbed into the normal-state free energy and by shifting the bare transition temperature T_0.[10] Thus, we find, for the magnetization,

$$M=-\frac{\phi_0}{32\pi^2\Lambda_T\xi_c(0)}h\sum_{n=0}^{N}\frac{(\partial\tilde{\epsilon}_H/\partial h)}{[\mu_n(1+d^2\mu_n)]^{1/2}} . \qquad (C2)$$

The derivative $(\partial\tilde{\epsilon}_H/\partial h)=(\partial\tilde{\epsilon}_H/\partial\epsilon_H)(\partial\epsilon_H/\partial h)$, where $(\partial\epsilon_H/\partial h)=1$. (Note that

$$\epsilon_H=[1/H_{c2}^{ab}(0)](dH_{c2}/dT)[T-T_{c2}(H)]$$

follows from the determination of the temperature at which the order-parameter susceptibility diverges.[42]) Using the self-consistent equation in the lowest Landau level, Eq. (2.11), we obtain

$$\left[\frac{\partial\epsilon_H}{\partial\tilde{\epsilon}_H}\right]=1+\frac{(2\kappa^2-1)}{4\gamma^2\Lambda_T}\xi_c(0)h\frac{1+2d^2\tilde{\epsilon}_H}{[\tilde{\epsilon}_H(1+d^2\tilde{\epsilon}_H)]^{3/2}} . \qquad (C3)$$

The second term on the right-hand side of Eq. (C3) is generally small compared to one. Again, there is no divergence of the magnetization in finite magnetic field and this theoretical expectation is borne out by experiment.[52–54] The high-field scaling form is the same as that for the Ettingshausen coefficient.

Finally, the entropy, $S(T,H)=-\partial F/\partial T$ is

$$S(T,H)=-\frac{\phi_0}{32\pi^2\Lambda_T\xi_c(0)}H_{c2}(0)h \times\sum_{n=0}^{N}\frac{(\partial\tilde{\epsilon}_H/\partial T)}{[\mu_n(1+d^2\mu_n)]^{1/2}} , \qquad (C4)$$

where $(\partial\tilde{\epsilon}_H/\partial T)=(\partial\tilde{\epsilon}_H/\partial\epsilon_H)(\partial\epsilon_H/\partial T)$, with the first factor given by Eq. (C3), and the second factor

$$(\partial\epsilon_H/\partial T)=[1/H_{c2}(0)](dH_{c2}/dT) .$$

The high-field scaling form of the entropy is also the same as that for the Ettingshausen coefficient.

[1]For an overview of fluctuation effects in high-T_c superconductors, see D. S. Fisher, M. P. A. Fisher, and D. A. Huse, Phys. Rev. B **43**, 130 (1991).
[2]S. Ullah and A. T. Dorsey, Phys. Rev. Lett. **65**, 2066 (1990).
[3]W. J. Skocpol and M. Tinkham, Rep. Prog. Phys. **38**, 1049 (1975).
[4]L. G. Aslamazov and A. I. Larkin, Phys. Lett. **26A**, 238 (1968).
[5]P. A. Lee and S. R. Shenoy, Phys. Rev. Lett. **28**, 1025 (1972).
[6]R. F. Hassing, R. R. Hake, and L. J. Barnes, Phys. Rev. Lett. **30**, 6 (1973).
[7]K. F. Quader and E. Abrahams, Phys. Rev. B **38**, 11 977 (1988).
[8]A. J. Bray, Phys. Rev. B **9**, 4752 (1974).
[9]D. J. Thouless, Phys. Rev. Lett. **34**, 946 (1975).
[10]G. J. Ruggeri and D. J. Thouless, J. Phys. F **6**, 2063 (1976).
[11]E. Brézin, A. Fujita, and S. Hikami, Phys. Rev. Lett. **65**, 1949 (1990).
[12]S. Hikami and A. Fujita, Prog. Theory. Phys. **83**, 443 (1990); Phys. Rev. B **41**, 6379 (1990).
[13]R. Ikeda and T. Tsuneto, J. Phys. Soc. Jpn. (to be published).
[14]T. T. M. Palstra, B. Batlogg, L. F. Schneemeyer, and J. V. Waszczak, Phys. Rev. Lett. **64**, 3090 (1990).
[15]K. Maki, Prog. Theor. Phys. **39**, 897 (1968); **40**, 193 (1968); R. S. Thompson, Phys. Rev. B **1**, 327 (1970); Physica **55**, 296 (1971).
[16]W. Lawrence and S. Doniach, in *Proceedings of the 12th International Conference on Low Temperature Physics,* edited by E. Kanda (Academic Press of Japan, Kyoto, 1971), p. 361.
[17]This is "model A" in the classification scheme of P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. **49**, 435 (1977); in the absence of fluctuations it reduces to the time-dependent Ginzburg-Landau equation derived for superconductors by A. Schmid, Phys. Kondens. Mat. **5**, 302 (1966) and E. Abrahams and T. Tsuneto, Phys. Rev. **152**, 416 (1966). Such a phenomenological approach does not catpure the Maki-Thompson terms, which also contribute to, e.g., the fluctuation conductivity.
[18]H. Ebisawa and H. Fukuyama, Prog. Theor. Phys. **46**, 1042 (1971). These authors show that λ_0^{-1} is very small, of order

T_c/T_F smaller than Γ_0^{-1}. See also, A. G. Aronov and S. Hikami, Phys. Rev. B **41**, 9548 (1990), for another microscopic mechanism which breaks particle-hole symmetry.

[19]C. De Dominicis and L. Peliti, Phys. Rev. B **18**, 353 (1978).

[20]A. Schmid, Phys. Kondens. Mat. **5**, 302 (1966).

[21]In mean-field theory, sufficiently close to H_{c2} it is necessary to retain the lowest Landau level only (as per the Abrikosov mean-field solution)—see Ref. 30. Away from H_{c2}, higher Landau levels become important: they will renormalize b_κ, resulting in a magnetic field dependent coupling for the quartic term. The precise form of this renormalization is an open problem.

[22]C. Caroli and K. Maki, Phys. Rev. **164**, 591 (1967). This expression for the heat current is not modified by the presence of an imaginary part of the relaxation time. The overall minus sign was omitted in Ref. 2.

[23]J. R. Tucker and B. I. Halperin, Phys. Rev. B **3**, 3768 (1971).

[24]S. Marcelja, Phys. Lett. **28A**, 180 (1968); E. Masker,, S. Marcelja, and R. D. Parks, Phys. Rev. **188**, 745 (1969).

[25]K.Kajimura, N. Mikoshiba, and K. Yamaji, Phys. Rev. B **4**, 209 (1971).

[26]A. T. Dorsey, Phys. Rev. B **43**, 7575 (1991).

[27]R. Ikeda, T. Ohmi, and T. Tsuneto, J. Phys. Soc. Jpn. **58**, 1377 (1989).

[28]R. Ikeda, T. Ohmi, and T. Tsuneto, J. Phys. Soc. Jpn. **59**, 1397 (1990).

[29]R. Ikeda, T. Ohmi, and T. Tsuneto, J. Phys. Soc. Jpn. (to be published).

[30]A. L. Fetter and P. C. Hohenberg, in *Superconductivity, Vol. 2,* edited by R. D. Parks (Dekker, New York, 1969), p. 902.

[31]C. M. Bender and S. A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers* (McGraw-Hill, New York, 1978), Chap. 6.

[32]The correlation-length exponent in the zero-field Hartree approximation in dimension $2<d<4$ is $\nu=1/(d-2)$. The Hartree approximation in zero field is equivalent to the $N\rightarrow\infty$ limit of an O(N) model (the "spherical model" limit). For further discussion, see S.-k. Ma, in *Phase Transitions and Critical Phenomena,* edited by C. Domb and M. S. Green (Academic, London, 1976), Vol. 6.

[33]N. D. Mermin and H. Wagner, Phys. Rev. Lett. **17**, 1133 (1966).

[34]Some steps in this direction have been taken by I. Affleck and E. Brézin, Nucl. Phys. **B257**, 451 (1985), who formulate an O(N) version of the Ginzburg-Landau Hamiltonian, which they attempt to solve in the $N\rightarrow\infty$ limit. If the order parameter is assumed to be uniform, then they produce the simple Hartree approximation discussed above. However, their formulation also allows for the possibility of nonuniform solutions, although they were unable to exhibit such solutions.

[35]See also, K. Maki, Phys. Rev. B **43**, 1252 (1991), for the Gaussian expression of the Ettingshausen coefficient.

[36]Inclusion of the higher Landau levels is equivalent to the renormalization of the Ginzburg-Landau coefficients a and b. See Ref. 10.

[37]K. Maki, Physica (Utrecht) **55**, 125 (1981).

[38]S. J. Hagen, C. J. Lobb, R. L. Greene, M. G. Forrester, and J. Talvacchio, Phys. Rev. B **42**, 6777 (1990).

[39]K. Maki, J. Low. Temp. Phys. **14**, 419 (1974).

[40]M. A. Howson, T. A. Friedmann, S. E. Inderhees, J. P. Rice, D. M. Ginsberg, and K. M. Ghiron, J. Phys.: Condens. Matter **1**, 465 (1989).

[41]Y. Lu and B. R. Patton (unpublished).

[42]D. R. Tilley and J. B. Parkinson, J. Phys. C **2**, 2175 (1969); K. D. Usadel, Z. Phys. **227**, 260 (1969); K. Maki, J. Low. Temp. Phys. **1**, 513 (1969); H. J. Mikeska and H. Schmidt, Z. Phys. **230**, 239 (1970); R. A. Klemm, J. Low. Temp. Phys. **16**, 381 (1974).

[43]H. Fukuyama, H. Ebisawa, and T. Tsuzuki, Prog. Theor. Phys. **46**, 1028 (1971).

[44]K. Maki, Phys. Rev. Lett. **23**, 1223 (1969).

[45]H. Ebisawa, J. Low. Temp. Phys. **9**, 11 (1972).

[46]S. J. Hagen, C. J. Lobb, R. L. Greene, M. G. Forrester, and J. H. Kang, Phys. Rev. B **41**, 11 630 (1990).

[47]E. Brézin, D. R. Nelson, and A. Thiaville, Phys. Rev. B **31**, 7124 (1985); D. R. Nelson and S. Seung, *ibid.* **39**, 9153 (1989); A. Houghton, R. A. Pelcovits, and A. Sudbo, *ibid.* **40**, 6763 (1989); M. A. Moore (unpublished).

[48]J. M. Ziman, *Electrons and Phonons* (Oxford University, Oxford, 1960), Chap. 7.

[49]In the superconducting state, the thermopower α_{xx} is smaller by a factor of order T_c/T_F than the Ettingshausen coefficient α_{xy}. In the normal state, α_{xx} is at least as small as the normal state α_{xy} and os there is only a negligible normal-state contribution to the Ettingshausen coefficient. Moreover, in both the normal and superconducting state, the Hall conductivity is much smaller than the longitudinal conductivity, $\sigma_{xy}/\sigma_{xx}\ll 1$.

[50]L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Pergamon, Oxford, 1977), Chap. 15.

[51]L. D. Landau and E. M. Lifshitz, *Statistical Physics Part 2* (Pergamon, Oxford, 1986), Sec. 49.

[52]U. Welp, W. K. Kwok, G. W. Crabtree, K. G. Vandervoort, and J. Z. Liu, Phys. Rev. Lett. **62**, 1908 (1989).

[53]P. Kes (private communication).

[54]A reexamination of the magnetization data taken on low-temperature superconductors by J. P. Gollub, M. R. Beasley, R. Callarotti, and M. Tinkham, Phys. Rev. B **7**, 3039 (1973) also shows that the finite-field magnetization does not diverge at $T_{c2}(H)$.