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Dynamical response of a two-dimensional electron system in a strong magnetic field

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A dynamical response of a two-dimensional electron system to an in-plane ac electric field has been observed in a strong magnetic field at frequencies f which are smaller than known characteristic frequencies. The effect is shown to be caused by the dynamical accumulation of charge and the formation of a dynamical potential near the edge of the sample. Its spatial distribution is characterized by a length l_E which depends on f and the conductivity σ_{xx} but not on σ_{xy}. This response offers a contactless method to measure very small values of σ_{xx} in the quantum Hall regime.

In a two-dimensional electron system (2DES) of finite area, which is exposed to a perpendicular magnetic field B, the dynamics of electrons at low frequencies $f \ll eB/2\pi m^* c$ is governed by edge plasma oscillations—the so-called edge magnetoplasmons (EMP) (see Ref. 1 and references therein). Here we report a low-frequency response of a 2DES to an in-plane ac electric field. This response appears above a characteristic frequency f_0 which depends on the longitudinal conductivity σ_{xx} and the size of the sample and where f_0 is small compared to the frequency of the fundamental EMP mode f_p. We relate this effect to the dynamical accumulation of charge and the formation of a dynamical edge potential. Its spatial distribution is characterized by a length l_E which depends on f and σ_{xx}. At $f < f_0$ electrons over the entire area of a 2DES respond to an exciting electric field. Thus the amplitude V_{ext} of this external exciting ac potential is compensated by the induced potential V_{ind} in the 2DES. At $f > f_0$ this behavior is drastically altered. In this case electrons only within a small region close to the sample edge, which is characterized by a length l_E, can respond at the frequency f to an acting field. Consequently, the external potential is not screened in the inner region of the sample, only in this edge region. We will demonstrate that this effect can be used for a contactless measurement of very small values of σ_{xx} in the quantum Hall regime.

We have investigated 2DES in modulation-doped GaAs-AlGaAs heterostructures with typical electron densities $n = 2\times10^{11}$ cm^{-2} and mobilities $\mu = 2\times10^5$ cm^2/V s. The exact parameters for different samples are given in the figure captions. Samples with square or rectangular shapes of side lengths $L = 1$–6 mm have been studied in a nonresonant radio frequency measurement cell, which is commonly used for the investigations of EMP.[1–3] The metal cylinder cell, which acts as a grounded screen, has a diameter of 25 mm and a height of 15 mm. The sample is placed with the backside mounted on an isolated floating metal plate in the center of the cell. The sample and this plate are positioned between two electrodes which are acting as antenna probes. The height of the electrodes is 1 mm and its length is usually one-half of the adjacent sample side length. The distance between the electrode and the sample edge is usually 0.5 mm and thus couples capacitively to the 2DES. One of the electrodes is connected to the generator, the other one to the receiver. Both electrodes are matched with 50 Ω to the grounded cell. The generator thus generates an ac electric field in the plane of the 2DES. We call the corresponding potential of the ac electric field in the 2DES plane, which would occur without 2D electrons, the external potential V_{ext}. The signal on the receiver is proportional to the potential difference between the region near to the sample edge and the screen. We have measured the normalized amplitude of this signal, $U_B = U(f=\text{const},B)/U(f=\text{const},B=0)$, at different generator frequencies f in sweeps of the magnetic field B. For a constant amplitude of the generator the measured amplitude of the probe electrode corresponds to the changing of the potential drop in the 2DES if we sweep B. Thus the measured signal characterizes the process of the dynamic electron-density redistribution.

Original experimental spectra are shown in Fig. 1. From many of such curves we have reconstructed, as shown in Fig. 2(a), the f dependences $U_f = U(f,B=\text{const})$ at different magnetic fields B and the related filling factor ν. The inset of Fig. 2(a) shows a wide frequency region and the prominent feature at $f > 150$ MHz is the well-known EMP resonance. The interesting observation is that at lower frequencies the normalized amplitude U_f becomes smaller than 1. This is a very unusual behavior, which has so far not been noticed in previous investigations.[1–5] We have investigated this "$U < 1$" effect in great detail and found the following unexpected behavior. The effect is, as shown in Fig. 1(a), filling factor dependent and more pronounced at the lowest filling factor ν. The most interesting observation shown in Fig. 2(a) is that this response occurs abruptly above a well-defined onset frequency f_0. With increasing f the strength of this response increases until the process of the EMP excitation is superimposed onto this process. Figure 2(b) shows that this onset frequency is by itself filling factor dependent, moreover, comparing the f_0 dependence with the longitudinal conductivity we find a direct proportionality between

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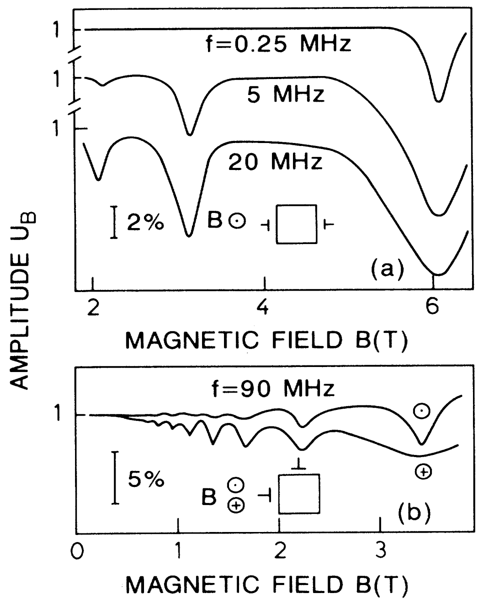

FIG. 1. Dependences of the normalized amplitude U_B for various frequencies at $T=4.2$ K. (a) Symmetrical geometry. The sample size is 6×6 mm^2, mobility $\mu=2\times10^5$ cm^2/Vs, density $n=2.9\times10^{11}$ cm^{-2}. (b) Asymmetrical geometry and opposite directions of B. ($L=3.5$ mm, $\mu=2\times10^5$ cm^2/Vs, $n=3.2\times10^{11}$ cm^{-2}.)

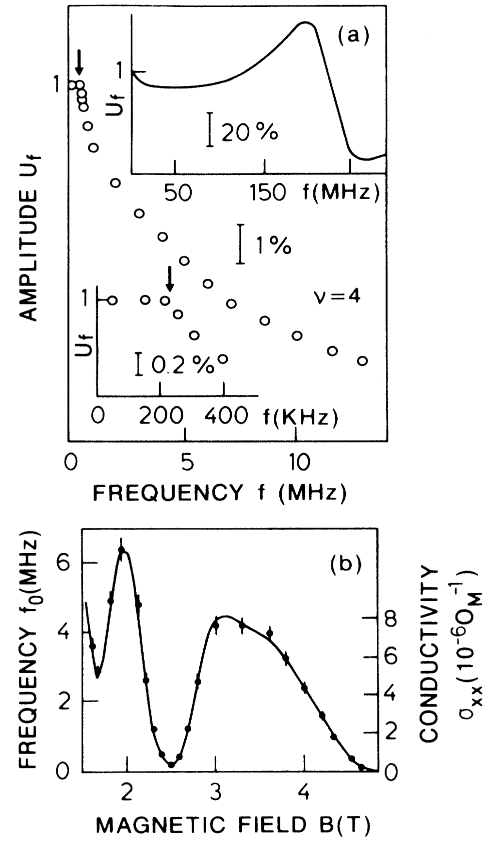

FIG. 2. (a) Dependences U_f for the filling factor $\nu=4$ in different frequencies f regions at $T=4.2$ K. Arrows indicate the positions of f_0. The inset shows a wide f region with the EMP resonance as a pronounced feature at $f_p>150$ MHz. (b) Dependences f_0 (solid circles) and σ_{xx} (line) vs B. The sample parameters are $L=5$ mm, $\mu=1.5\times10^5$ cm^2/Vs, $n=2.4\times10^{11}$ cm^{-2}.

the values $f_0(B)$ and $\sigma_{xx}(B)$. (The latter was measured at the same frequencies on a Corbino disk sample with contacts. We note that in this experiment we have not observed an essential f dependence on σ_{xx} up to 10 MHz.) Variation of the temperature leads to a direct influence on the f_0 dependence exactly as expected from the corresponding variation of σ_{xx} with temperature. We note further that this novel response does not depend qualitatively on the sample form. Quantitatively, however, we could make the additional observation that, for a given sample quality and temperature, i.e., the same longitudinal conductivity, the frequency f_0 depends in a characteristic way on the side lengths L of the sample. From a fit to experimental values of f_0 for a series of samples from the same wafer but with different sizes, we find in Fig. 3 that, for $L>2$ mm, f_0 is inversely proportional to the square of the side length, L^{-2}. [On rectangular samples with side lengths a and b and large asymmetry ($a\leq 3b$) the frequency f_0 depends basically on the side length in the direction between the antenna electrodes.] As demonstrated in Fig. 1(b), we additionally find that the amplitudes U_B depend on the direction of B if the exciting and measuring electrodes are arranged asymmetrically. From this observation we can exclude that it arises from the properties of the measuring cell.

We explain these results by the formation of a dynamical edge potential which has a very special behavior in a 2DES. Its spatial distribution is characterized by a length l_E which depends on f and σ_{xx}. In the following, we first give a qualitative explanation and then present the results of a calculation[6] which confirm our qualitative explanation and allow us to make a quantitative comparison.

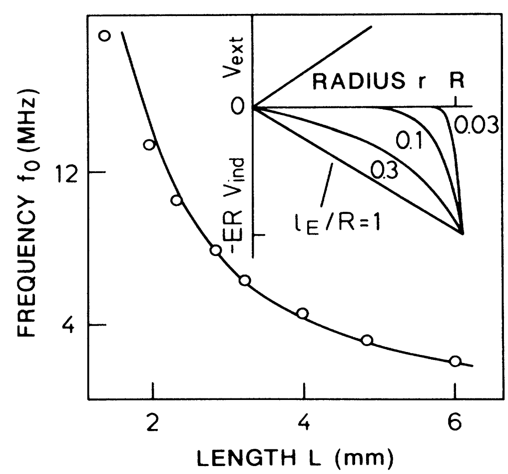

FIG. 3. Measured (open circles) dependences f_0 vs L for the square sample at $T=4.2$ K, $\mu=1.5\times10^5$ cm^2/Vs, $n=2.4\times10^{11}$ cm^{-2}. The solid line gives a fit of a L^{-2} dependence to the experimental values of f_0. The inset represents the calculated spatial dependences $V_{\rm ind}(\phi=0)$ for different ratios l_E/R in the case of a strong B ($\sigma_{xy}\gg\sigma_{xx}$), and the spatial dependence $V_{\rm ext}$.

At small f, $f < f_0$, an in-plane external electric field causes electrons to redistribute over the entire area of the sample and screen this field in the 2DES. The induced amplitude $V_{\rm ind}$ is determined by the charge-density amplitude and thus has the same spatial dependence as that of the amplitude $V_{\rm ext}$ taken with an opposite sign. This situation is kept until f becomes comparable with a certain characteristic frequency which depends on σ_{xx} and L. Then there occurs a qualitative change in the process of the charge relaxation which is a special property of 2D systems. In a 3DES it is known that the charge relaxation time, sometimes called "Maxwell relaxation time," is independent on the size of the region involved and is characterized by a time $\tau = 4\pi\epsilon\epsilon_0/\sigma$, which depends only on the conductivity σ and the dielectric constant $\epsilon\epsilon_0$.[7] In a 2DES, however, a perturbation charge density spreads out on a 2D layer and the time, at which it is redistributed completely, depends on the size of the 2DES. This has been shown theoretically[8,9] for $B=0$. In the case of a 2D disk with radius R it is[8,9] $\tau = 2\epsilon\epsilon_0 R/\sigma_0$, where σ_0 is the 2D conductivity at $B=0$. In a magnetic field the situation is more complex because the conductivity is characterized by two different components, σ_{xx} and σ_{xy}. Before our work it was not clear which or what combination of these quantities determines relaxation processes in a magnetic field. Our results show that (i) the relaxation in a 2DES is indeed size dependent and (ii) that in a magnetic field it only depends on σ_{xx} and not on σ_{xy}.

For a quantitative comparison we have calculated[6] the response for a disk of electrons with radius R in the x-y plane placed between two metal plates at $z=d$ and $z=-d$ in a dielectric medium of relative permittivity ϵ. This is the so-called local capacitance approximation.[10–12] We find that the induced potential $V_{\rm ind}$ depends in a polar system of coordinate (r,ϕ) in the following way on an external ac electric field applied with a constant amplitude E in the x-direction ($\phi=0$).

$$V_{\rm ind}(r,\phi) = -\frac{ERJ_1[(1+i)r/l_E]}{\sigma_{xy}^2 I^2+\sigma_{xx}^2 Y^2}[(\sigma_{xy}^2 I+\sigma_{xx}^2 Y)\cos\phi - \sigma_{xx}\sigma_{xy}(Y-I)\sin\phi] \quad (1)$$

where J_1 is the Bessel's function of the first order, $I=J_1(Z)$, $Y=Z\partial J_1(Z)/\partial Z$, $Z=(1+i)R/l_E$, and

$$l_E^2 = d\sigma_{xx}/2\pi\epsilon_0\epsilon f\,, \quad (2)$$

where ϵ_0 is the permittivity of vacuum.

The most important result is that the spatial distribution is characterized by a length l_E which depends on σ_{xx}, f, and the dielectric surrounding. Microscopically there is within the length l_E a dynamical accumulation of charge near the edge of the 2DES which compensates the current flowing in the direction perpendicular to the edge. The spatial dependence $V_{\rm ind}(r,\phi=0)$ for different ratios l_E/R is plotted in the inset of Fig. 3 for a case of a strong B ($\sigma_{xy} \gg \sigma_{xx}$). At high f the induced potential is localized within a stripe near the edge. With decreasing f this stripe extends until l_E becomes equal to $\sim R$; it occurs at $f \simeq f_0$. For $f < f_0$ the $V_{\rm ind}$ distribution is practically flat and is the same as that of $V_{\rm ext}$ taken with the opposite sign. Using (2) and $l_E(f_0)=L/2$, we expect an L^{-2} dependence for f_0. This is indeed observed in Fig. 3. Moreover, using an estimated value of $\epsilon \simeq 4$ we find an excellent fit to the experimental results in Fig. 3 by inserting $d=0.6$ mm which is the sample thickness and a reasonable assumption to model the distance of the metal screen within the local capacitance approximation. As seen from Fig. 3 the L^{-2} dependence in Eq. (2) describes the experiment quite well. The discrepancy at small L occurs because the inequality $d \ll l_E$, which is required for the local capacitance approximation, is not exactly valid here. This case occurs also for the experimental situation of a large spacing between the sample and the electrodes or the screen surfaces. Under these conditions we indeed observed experimentally a weaker variation of the signal near f_0. This can be explained if one assumes that the $V_{\rm ind}$ distribution in the case $d > l_E$ is described by a more slowly varying function (nonexponential) compared to the case $d < l_E$. This assumption seems to be confirmed by theory[10] which shows that for the case $d > l_E$ the distribution of the potential in the EMP wave near the edge of a 2D half-plane is algebraic. As another remark on Eq. (1) we note that $V_{\rm ind}$ depends on the direction of B in the asymmetrical case ($\sin\phi \neq 0$) as was observed in the experiment, Fig. 1(b).

The important feature of the response is that, although both dissipative and Hall current flow in the sample, l_E is determined only by σ_{xx}. This allows us to realize a rather simple method of a contactless determination of very small σ_{xx} values in the quantum Hall regime which is not easy in samples with contacts. Small σ_{xx} corresponds to small f_0 which can be measured reliably. We propose the following procedure. First, one determines the ratio f_0/σ_{xx}, i.e., the effective ϵ and d, in a sample with known values of σ_{xx} and for the same geometry as that of the sample under test. Then, after measuring f_0 in the test sample, the values of σ_{xx} are determined via these effective ϵ and d. We have used this procedure to measure a sample with a mobility of $\mu \gtrsim 10^6$ cm^2/V s and $L=2$ mm and find, e.g., at $T=1.5$ K and $\nu=2$, a frequency $f_0=30$ Hz which corresponds to a value of $\sigma_{xx} \simeq 2 \times 10^{-11}\ \Omega^{-1}$. This was within the 30% experimental accuracy identical to the value determined from a convential measurement with contacts.

As a final remark we would like to note that in high mobility samples with very small values of σ_{xx} one can easily achieve conditions where, at frequencies smaller than f_p, Eq. (2) gives l_E values comparable to the magnetic length. Of course, in this case a microscopic model, beyond our macroscopic calculation [Eq. (1)], is required for an accurate description, however, under these conditions our excitation strongly resembles the edge current model that is used for the interpretation of the quantum Hall effect.

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