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Calculation of the mobility of two-dimensional excitons in a GaAs/$Al_xGa_{1-x}As$ quantum well

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We have calculated the mobility of two-dimensionally free excitons in a quantum well by considering various scattering processes. The relaxation times are derived by using a variational wave function for the 1*s* exciton in the lowest subband in an infinitely deep well. The calculated mobilities for the GaAs well at first increase with temperature due to the predominance of the interface-roughness scattering, then attain a peak, and finally decrease with temperature as the deformation-potential scattering dominates. The piezoelectric scattering has less importance. The calculated values agree with the experimental data for 8- and 15-nm-wide wells up to about 150 K; however, the disagreement between the values for a 4-nm-wide well indicates that the effect of a finite barrier should be considered. The polar-optic-phonon scattering has been included to explain the rapid decrease of the experimental data above 150 K; however, the single-subband approximation gives too high a value for the mobility, while the values for bulk excitons are quite low compared to the experimental data.

I. INTRODUCTION

The properties of excitons confined in a semiconductor quantum-well (QW) structure have received attention from many workers over the past decades in view of novel physical phenomena[1] and potential device applications.[2,3] In a QW, the excitons are free to move along the QW layer plane, but the motion is restricted along the normal direction. As a result, a two-dimensional (2D) exciton gas is formed and the energy levels of the excitons are modified. The free motion of the center of mass of 2D excitons in the layer plane is affected by the collision with lattice imperfections like phonons, impurities, and interface roughness. The scattering processes govern the shape of some experimentally observed properties of the excitons, a typical example being the absorption or photoluminescence linewidth. The role of different scattering mechanisms in limiting the linewidths has been investigated by several workers.[4–10]

Recently Hillmer *et al.*[11] reported a time-of-flight measurement of diffusivity and the mobility of 2D excitons in a GaAs/$Al_xGa_{1-x}As$ QW. They also presented the values of the mobility for different temperatures and well widths, calculated by two different methods. In their ambipolar scattering model, the electron and the hole are treated as separate particles affected by different scattering mechanisms and the values of the mobility of the individual species are combined to give the ambipolar mobility. On the other hand, in the excitonic scattering model, the particle is considered as a unit having a proper excitonic mass and being scattered by an effective deformation potential. The piezoelectric scattering was not considered, since it was found to be less important in 2D electronic transport. The polar-optic (PO) scattering limited mobility was estimated by properly scaling the mobility for the bulk electrons. The authors compared their theoretical and experimental values and concluded that the excitonic scattering model works better.

The present work reports a theoretical calculation of the exciton mobility in the relaxation-time approximation for QW's including several scattering processes, and assuming Mathiessen's rule. Our work is an extension of the available theories[12–15] of the mobility for bulk excitons to the 2D excitonic systems. In the theory the scattering matrix elements have been calculated and the statistical distribution of the excitons among various states is taken into account. The scattering of 2D excitons has been considered previously to calculate the energy relaxation time of the excitons[4] and photoluminescence linewidths.[5–10] Similar calculations for the mobility of 2D excitons have not been reported.

In Sec. II, the method of calculating the matrix element for different scattering mechanisms using a generalized wave function is described, and an approximate form of the exciton envelope function is selected. Here we restrict ourselves to QW's having an infinite barrier height, and consider only the 1*s* excitonic state formed by an electron and a heavy hole belonging to the first subband of the QW. The results for the mobility in a GaAs/$Al_xGa_{1-x}As$ QW, limited by different scattering processes, are presented in Sec. III, and the nature of its variation with the well width and temperature is discussed there. A comparison between the theoretical values and the experimental data is made in Sec. IV. The conclusions drawn from the present study are given in Sec. V.

II. THEORY

A. Outline of the scattering theory

We assume that the excitons are quantized in a QW of width L along the z direction and the energy (E)–wavevector (K) relation for the excitons is given by

$$E(\mathbf{K})=E_{ex}+\hbar^2K^2/2M\ , \tag{1}$$

where E_{ex} is the energy of the excitonic state measured from a suitable reference, $M=m_e+m_h$, $m_{e(h)}$ being the mass of the electron (heavy hole), and $\mathbf{K}$ is the conjugate to the center-of-mass coordinate $\mathbf{R}$ defined as

$$\mathbf{R}=(m_e\mathbf{r}_e+m_h\mathbf{r}_h)/M \ . \tag{2}$$

The general theory of excitonic scattering for 2D systems is developed in the literature. Following the work of Takagahara,[4] we may write the wave function for the quasi-2D exciton in the nth state as

$$|n,\mathbf{K}\rangle=(v/L_0)\sum_{r_e,r_h} e^{i\mathbf{K}\cdot\mathbf{R}} f_n(\mathbf{r}_e-\mathbf{r}_h,z_e,z_h)a^\dagger_{cr}a_{vr}|0\rangle \ , \tag{3}$$

where v is the volume of the unit cell, $a^\dagger_{ir}(a_{ir})$ is the ith band creation (annihilation) operator for electrons at position $\mathbf{r}$ in the Wannier representation, $|0\rangle$ is the crystal ground state, and f_n is the envelope function for an exciton in the nth state.

Now writing the operators in the Bloch representation and converting summation into integration, one may write

$$|n,\mathbf{K}\rangle=\sum_{\mathbf{k},\mathbf{k}'} F_n(\mathbf{k},\mathbf{k},\mathbf{K})\delta_{\mathbf{k}-\mathbf{k}',\mathbf{K}}a^\dagger_{c\mathbf{k}}a_{v\mathbf{k}}|0\rangle \ , \tag{4}$$

where $\mathbf{k}$ ($\mathbf{k}'$) is the wave vector for the electron (hole) and

$$F_n(\mathbf{k},\mathbf{k}',\mathbf{K})=S^{-1}\int d^2r\int dz_e\int dz_h f_n(\mathbf{r},z_e,z_h) \times\exp[i(\alpha_e\mathbf{K}-\mathbf{k})\cdot\mathbf{r} -ik_zz_e+ik'_zz_h] \ . \tag{5}$$

In Eq. (5), $\alpha_{e(h)}=m_{e(h)}/M$ and S is the normalization area.

B. Matrix element for phonon scattering

The interaction between electrons (holes) and the three-dimensional phonons belonging to the jth mode may be expressed as[4]

$$H^j_{e\text{-ph}}=\sum_{\mathbf{k},\mathbf{q}}E^j_c(\mathbf{q})a^\dagger_{c,\mathbf{k}+\mathbf{q}}a_{c\mathbf{k}} +E^j_v(\mathbf{q})a^\dagger_{v,\mathbf{k}+\mathbf{q}}a_{v\mathbf{k}}(b_{\mathbf{q}}+b^\dagger_{-\mathbf{q}}) \ , \tag{6}$$

where $E^j_{c(v)}(\mathbf{q})$ is a factor that measures the strength of interaction between electron (hole) and the jth mode phonon of 3D wave vector $\mathbf{q}$, and $b^\dagger_{\mathbf{q}}(b_{\mathbf{q}})$ is the creation (annihilation) operator for the phonons.

The matrix element for the transition of an excitonic state $|n,\mathbf{K}\rangle$ to another state $|n,\mathbf{K}'\rangle$ due to an interaction with the jth mode phonon is[4]

$$|M^\pm_j(\mathbf{K},\mathbf{K}')|^2=\sum_{q_z}\{E^j_c(\mathbf{K}'-\mathbf{K},q_z)H_n[-\alpha_h(\mathbf{K}-\mathbf{K}'),q_z] -E^j_v(\mathbf{K}'-\mathbf{K},q_z)H_n[\alpha_e(\mathbf{K}-\mathbf{K}'),q_z]\}^2 \times(N_{\mathbf{q}}+\tfrac{1}{2}\pm\tfrac{1}{2}) \ , \tag{7}$$

where

$$H_n(\alpha\mathbf{q},q_z)=\int d^2r\int dz_e\int dz_h|f_n(\mathbf{r},z_e,z_h)|^2 \times\exp(iq_zz_e+i\alpha\mathbf{q}\cdot\mathbf{r}) \ . \tag{8}$$

In the above, $N_{\mathbf{q}}$ is the equilibrium number of phonons obeying Bose statistics, and + (−) corresponds to the emission (absorption) process. The transition probability for the interaction between an exciton and a jth-mode phonon is then expressed as

$$W^\pm(\mathbf{K},\mathbf{K}') =(2\pi/\hbar)|M^\pm_j(\mathbf{K},\mathbf{K}')|^2\delta(E(\mathbf{K}')-E(\mathbf{K})\pm\hbar\omega_{\mathbf{q}}) \ . \tag{9}$$

C. Excitonic wave function

In the present work, we shall consider only the $1s$ state of the excitons formed by the electrons and the heavy holes belonging to the first subband of the QW. The following simple variational function will be chosen for the envelope function:[6,7]

$$f_{1s}(\mathbf{r},z_e,z_h)=A\exp(-\beta r/2) \times\cos(\pi z_e/L)\cos(\pi z_h/L) \ , \tag{10}$$

where A is the normalization constant. Equation (10) applies when the barrier height in the QW is infinite and the relative motion of the electrons and holes along the z direction is ignored. β is the usual variational parameter used to minimize the exciton binding energy.

D. Relaxation time and mobility

Using the above envelope function, the transition probability from a state $|1s,\mathbf{K}\rangle$ to another state $|1s,\mathbf{K}'\rangle$ may be calculated using Eq. (9). The relaxation time for excitonic transport may then be related to the matrix elements in the same way as in the theory of the electrons.[16,17] The relaxation time is given by

$$\frac{1}{\tau_j(K)}=\frac{2\pi}{\hbar}\sum_{+,-}\sum_{\mathbf{K}'}|M^\pm_j(\mathbf{K},\mathbf{K}')|^2(1-\cos\theta) \times\delta(E(\mathbf{K}')-E(\mathbf{K})\pm\hbar\omega_{\mathbf{q}}) \ , \tag{11}$$

where θ is the angle between $\mathbf{K}$ and $\mathbf{K}'$. The mobility is then given by

$$\mu_j=(e/M)\langle\tau_j\rangle \ , \tag{12}$$

where $\langle\ \rangle$ denotes the average over the distribution function defined in the same way as in the case of electrons.[16,17] Since the exciton density is quite small in the actual situation, a Boltzmann distribution may be assumed, and the average relaxation time for the 2D exciton may be written as

$$\langle\tau_j(E)\rangle=\frac{\int E\tau_j(E)e^{-E/k_BT}dE}{\int Ee^{-E/k_BT}dE} \ , \tag{13}$$

k_B being the Boltzmann constant. The expressions for the momentum relaxation time for various scattering processes are given below.

E. Deformation-potential acoustic-phonon scattering

In this case one may write[16,17]

$$E^{\mathrm{DP}}_{c(v)}(\mathbf{q})=(\hbar|q|/2\rho uV)^{1/2}D_{c(v)} , \tag{14}$$

where $D_{c(v)}$ is the deformation potential (DP) for the conduction (valence) band, $\hbar$ is Planck's constant divided by 2π, ρ is the mass density, V is the crystal volume, and u is the velocity of acoustic waves in the material. The interaction potential is assumed to be unscreened.

We introduce the usual equipartition approximation for the phonon number $N_{\mathbf{q}}$ and consider the scattering to be elastic, so that the absorption and the emission processes are identical. Converting the summation over q_z and $\mathbf{K}'$ into integrals and performing the integrations we get

$$[\tau^{\mathrm{DP}}(E)]^{-1}=\frac{3\beta^6 k_B TM}{4\pi\hbar^3\rho u^2 L}\int_0^{2\pi}d\theta(1-\cos\theta) \times(D_c\gamma_c-D_v\gamma_v)^2 , \tag{15}$$

where

$$\gamma_{c(v)}=(\beta^2+\alpha^2_{h(e)}q^2)^{-3/2},\ q^2=4K^2\sin^2(\theta/2) .$$

F. Piezoelectric phonon scattering

The strength of interaction for the piezoelectric scattering neglecting screening may be written as[16–18]

$$E^{\mathrm{PZ}}_{c(v)}(\mathbf{q})=(e/\epsilon)(\hbar/2\rho\omega V)^{1/2}h^{c(v)} , \tag{16}$$

where $h^{c(v)}$ is the piezoelectric constant for the conduction (valence) band, e is the electronic charge, ϵ is the permittivity of the material, and ω is the frequency of the acoustic phonon. Following the same procedure as above, one may write for the relaxation time

$$[\tau^{\mathrm{PZ}}(E)]^{-1}=\frac{1}{\hbar(2\pi)^2}(e/\epsilon)^2(k_B T/\rho u^2)(4M\beta^6/L^2\hbar^2) \times\int_0^{2\pi}d\theta(1-\cos\theta)I(q,\eta)(h^c\gamma_c-h^v\gamma_v)^2 , \tag{17}$$

where $\eta=2\pi/L$ and the factor $I(q,\eta)$ is given by[6,7,19]

$$I(x,y)=[\pi/2x^2(x^2+y^2)] \times\left[\frac{3x^2+2y^2}{y}\pi-\frac{y^4(1-e^{-2\pi x/y})}{x(x^2+y^2)}\right] . \tag{18}$$

G. Polar-optic-phonon scattering

The scattering by the PO phonon is neither elastic nor random, and it has been found in the case of electron transport that a relaxation time cannot be defined.[16,17,20] This is also true for the excitonic transport. Nevertheless, in order to obtain an order-of-magnitude value of the mobility, we shall assume, following some work on electronic transport,[21] that a relaxation time exists. Both the initial and final states in the scattering will be assumed to belong to the first excitonic state, and $1s\rightarrow 2s$ and $1s\rightarrow 2p$ transitions, transitions to higher excitonic subbands and the total dissociation of an exciton into an electron and a hole, will not be considered in the present work. The strength of interaction between 2D electrons (holes) and the bulk polar-optic phonons without considering screening is given by[16,17,19–21]

$$E^{\mathrm{PO}}_{c(v)}=(e/q)[(\hbar\omega_0/2V)(1/\epsilon_\infty-1/\epsilon_s)]^{1/2} , \tag{19}$$

where ω_0 is the frequency of the longitudinal-optic phonon and ϵ_∞ (ϵ_s) is the optical (static) permittivity of the material. Using Eq. (19) the following expression for the relaxation time is obtained:

$$[\tau^{\mathrm{PO}}(E)]^{-1}=\sum_{+,-}(e^2\omega_0\beta^6M/\pi^2L^2\hbar^2\epsilon')(N_{\mathbf{q}}+\tfrac{1}{2}\pm\tfrac{1}{2}) \times\int_0^{2\pi}d\theta(1-\cos\theta)I(q,\eta)(\gamma_c-\gamma_v)^2 , \tag{20}$$

where $1/\epsilon'=1/\epsilon_\infty-1/\epsilon_s$.

In some studies of the 2D electron transport the polar-optic-phonon scattering around room temperature has been described in terms of the interaction between phonons and the bulk (3D) electrons. The relaxation time when the excitons are bulklike is given by

$$[\tau^{\mathrm{PO}}(E)]^{-1}_{\mathrm{3D}}=\sum_{+,-}(2\pi e^2\beta^8\omega_0M/\hbar^2\epsilon')(N_{\mathbf{q}}+\tfrac{1}{2}\pm\tfrac{1}{2}) \times\int_0^{\pi}d\theta\sin\theta(1-\cos\theta)(K'^2/q^2) \times[(\beta^2+\alpha_h^2q^2)^{-2} -(\beta^2+\alpha_e^2q^2)^{-2}]^2 , \tag{21}$$

where $\mathbf{q}$ is the 3D phonon wave vector and β is the variational parameter employed in the envelope function for the 3D excitons.

H. Interface-roughness scattering

We shall assume, in conformity with the study of the electronic transport,[22–24] that the scattering potential is due to the local fluctuations in the well width, such that

$$\delta E(\mathbf{r})=\frac{\partial E_0}{\partial L}\Delta(\mathbf{r})=-(\pi^2\hbar^2/m_{e(h)}L^3)\Delta(\mathbf{r}) , \tag{22}$$

where E_0 is the energy of the lowest subband for infinite barrier height. The fluctuation $\Delta(\mathbf{r})$ in the well width is characterized by a height Δ and a lateral correlation length λ and is assumed to have Gaussian autocorrelation, i.e.,

$$\langle\Delta(\mathbf{r})\cdot\Delta(\mathbf{r}')\rangle=\Delta^2\exp(-|\mathbf{r}-\mathbf{r}'|^2/\lambda^2) . \tag{23}$$

The relaxation time for the interface roughness (IFR) scattering becomes

$$[\tau^{\mathrm{IFR}}(E)]^{-1}=\frac{\pi^4\hbar\lambda^2\Delta^2M\beta^6}{2L^6} \times\int_0^{2\pi}d\theta(1-\cos\theta)e^{-\lambda^2q^2/4} \times(m_e^{-1}\gamma_c-m_h^{-1}\gamma_v)^2 . \tag{24}$$

III. CALCULATED RESULTS

We have calculated the mobility of excitons in $GaAs/Al_xGa_{1-x}As$ QW's by using the following values of parameters: $m_e=0.067m_0$, $m_h=0.45m_0$, $D_c=12.0$ eV, $D_v=6.1$ eV, $|h^c-h^v|=0.160$ C/m^2, $\epsilon_\infty=10.9\epsilon_0$, $\epsilon_s=12.8\epsilon_0$, $\rho=5.346\times10^3$ kg/m^3, $u=5.20\times10^3$ m/sec, and $\hbar\omega_0=36.2$ meV. The calculated values of the mobility as a function of temperature are plotted in Figs. 1–3, respectively, for 4-, 8-, and 15-nm-wide wells. The curves labeled μ_{expt} in these figures represent the experimental data obtained by Hillmer *et al.*[11]

The values of the mobility limited by the deformation-potential acoustic-phonon scattering (μ_{DP}) increase linearly with increasing well width as Eqs. (12) and (15) suggest. The trend agrees with the theoretical results obtained by Hillmer *et al.* for both the ambipolar and the excitonic scattering models. In these models a T^{-1} dependence of the mobility μ_{DP} was predicted for changes in temperature. In the present work, in which the statistical distribution of the excitons is taken into account, the mobility decreases at first at T^{-1}, but more slowly afterward. A straight line showing the T^{-1} variation is included in each figure to point out the difference.

It should be noted that the values of μ_{DP} are sensitive to the choice of the values of D_c and D_h. There is still some controversy about the proper value of D_c in the literature.[25] We have used the currently accepted value (the average of 11 and 13 eV) for D_c. The value of D_h is more uncertain.

The piezoelectric-scattering-limited mobility (μ_{PZ}), calculated in the present work, is found to increase with well width, but the increase is sublinear with an increase of L. The mobility values show a very weak dependence on temperature. The values of μ_{PZ} are higher than the values of μ_{DP} and at a lower range of temperature μ_{DP} and μ_{PZ} are somewhat closer. At higher temperatures, however, the deviation between them increases to make the piezoelectric scattering less significant. It should again be noted that the value of h^v is not known exactly. We have used the value of $|h^c-h^v|$, quoted in Ref. 6, for our calculation.

Considering the values given in Figs. 1–3, one finds that the mobilities limited by polar-optic-phonon scattering ($\mu_{\text{PO-2D}}$) decrease slightly with increasing well width. The values for each well, however, decrease strongly with

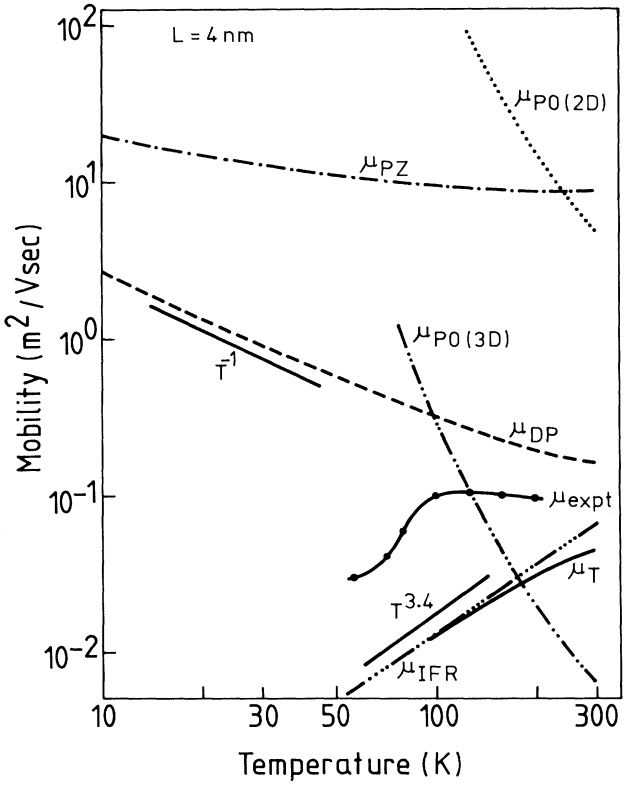

FIG. 1. Mobility of two-dimensional excitons as a function of temperature for a 4-nm-wide QW for scattering due to the deformation-potential (DP) acoustic phonons, the piezoelectric (PZ) phonons, the polar-optic (PO) phonons, and the interface roughness (IFR). μ_{expt} denotes the experimental values and $\mu_T^{-1}=\mu_{\text{DP}}^{-1}+\mu_{\text{IFR}}^{-1}$. The values of μ_{PO} are obtained for two-dimensional (2D) and bulk (3D) excitons.

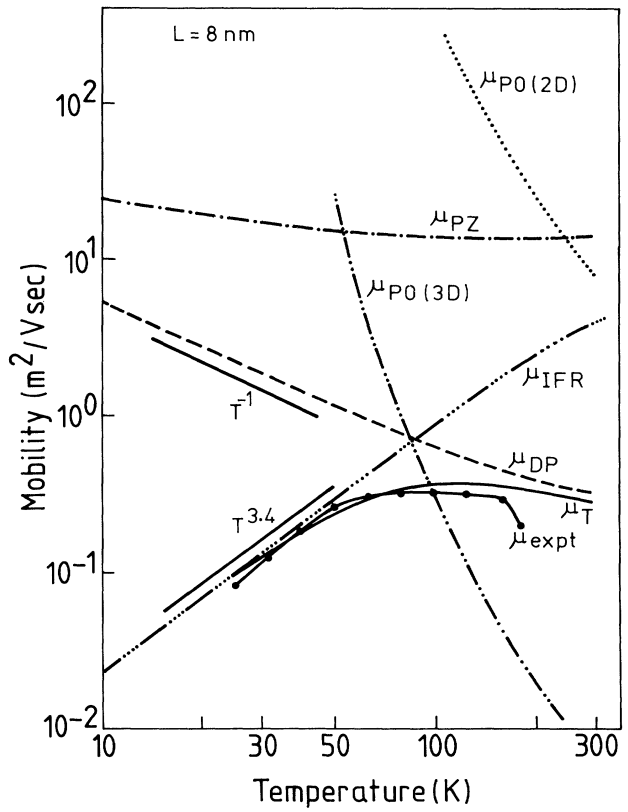

FIG. 2. Mobility of two-dimensional excitons as a function of temperature for an 8-nm-wide QW for scattering due to the DP acoustic phonons, the PZ phonons, the PO phonons, and the IFR. μ_{expt} denotes the experimental values and $\mu_T^{-1}=\mu_{\text{DP}}^{-1}+\mu_{\text{IFR}}^{-1}$. The values of μ_{PO} are obtained for two-dimensional and bulk (3D) excitons.

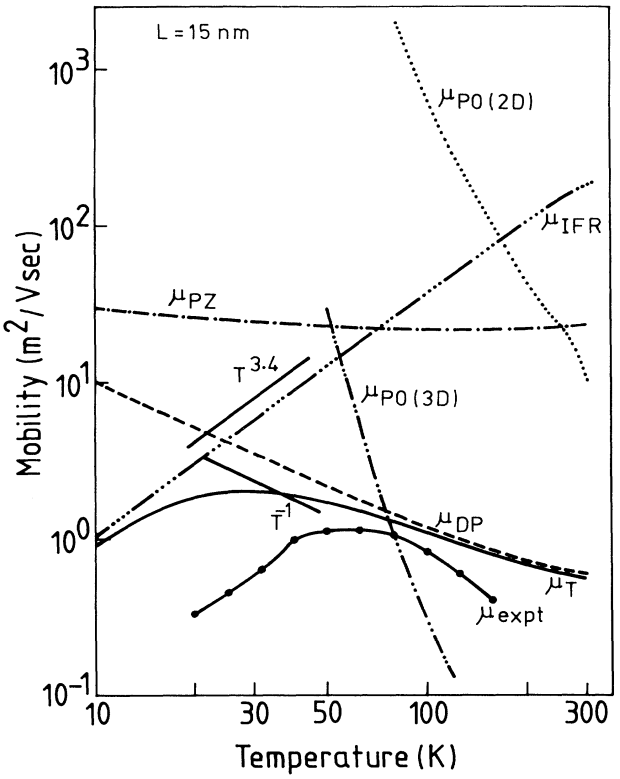

FIG. 3. Mobility of two-dimensional excitons as a function of temperature for a 15-nm-wide QW for scattering due to the DP acoustic phonons, the PZ phonons, the PO phonons, and the IFR. μ_{expt} denotes the experimental values and $\mu_T^{-1}=\mu_{DP}^{-1}+\mu_{IFR}^{-1}$. The values of μ_{PO} are obtained for two-dimensional and bulk (3D) excitons.

increasing temperature. It is also noticed that the values are higher than the values of μ_{DP}, even at room temperature. In the present calculation, only the 1s excitonic state has been considered. In actual practice many subbands and other hydrogenic states are occupied and the actual mobility calculation should take all these into consideration.

In the study of 2D electronic transport, it has been argued[26] that around room temperature many subbands are occupied and the transport in a heterojunction should be bulklike. Several authors have used the polar-optic-phonon scattering-limited mobility of bulk (3D) electrons to explain the mobility data for electrons in heterojunctions at room temperature. In the present work we have calculated similar mobility values assuming the excitons to be bulklike, and the values are shown in the figures by curves labeled $\mu_{PO\text{-}3D}$. Again only the 1s state has been considered. The values are found to be almost three orders of magnitude lower than the values represented by $\mu_{PO\text{-}2D}$.

The values of the interface-roughness scattering-limited mobility (μ_{IFB}) have been obtained by assuming the mean height of the fluctuations to be about half a monolayer (2.83 Å) and the correlation length to be 100 Å. These values are not much different from the results quoted by different authors[23,24] from the electronic mobility studies. One may notice from Eqs. (12) and (24) that μ_{IFR} varies as Δ^{-2} and L^{+6}. The effect of the change in λ on the values of μ_{IFR} is not easy to guess, since the integral in Eq. (24) involves λ. We have studied the variation numerically and found that μ_{IFR} increases almost exactly as λ^1 for lower values of λ, but the increase is less rapid for higher values of λ. As may be seen from the figures, μ_{IFR} increases roughly as $T^{3.4}$ with temperature.

IV. COMPARISON WITH EXPERIMENTAL DATA

We have made an attempt to compare the present theoretical results with the experimental data presented by Hillmer *et al.*[11] for three different values of the well widths. For this purpose, we have calculated the overall mobility of the 2D excitons by employing Mathiessen's rule. The contribution by μ_{PO} is not considered in the calculation and the overall mobility (μ_T) is presented as a solid curve in each figure. The use of Mathiessen's rule is questionable when all the scattering mechanisms are equally important.[27] In the present case, however, we find that the overall mobility is almost coincident with μ_{IFR} at lower temperatures and with μ_{DP} at higher temperatures, signifying that only one type of scattering is dominant in these two temperature ranges. A more exact calculation of the mobility considering all the scattering processes should not, therefore, be much different from the results obtained by Mathiessen's rule, so that we can employ the present μ_T values for comparison with the experimental data.

The above-mentioned comparison is meaningful only at higher values of the well width, since the theory is based on the assumption of an infinite barrier height and complete confinement of the wave function for thick wells. For the results given in Figs. 2 and 3, respectively, for $L=8$ and 15 nm, we find that there is satisfactory agreement between the calculated values and the experimental data over a temperature range from 10 to 150 K. The experimental data at first increase with temperature, indicating the dominance of IFR scattering alone. The peak in the mobility temperature curve arises from the opposite nature of the temperature dependence of the IFR and the DP scatterings. The decrease in the mobility at higher temperatures indicates the dominance of the DP scattering. The agreement between the theoretical and the experimental data could be improved by adjusting the values of the correlation length λ in the IFR scattering, although we made no such attempt, since this is justified only if an exact calculation with a proper wave function is performed.

Experimental values above about 170 K are not reported. The trend of the experimental data indicates a more rapid decrease with temperature than the variation of μ_{DP} exhibits. Considering the conclusions drawn from the study of the electronic mobility,[20,21,25,28] we may state that above this temperature μ_{PO} becomes dominant. The present theoretical calculations are, however, incomplete and do not throw much light on the exact magnitude of

the mobility. The values of $\mu_{\text{PO-2D}}$, considering the 2D nature of the excitons, are quite high. On the other hand, the values of $\mu_{\text{PO-3D}}$, assuming the 3D nature of excitons, are too low. An exact calculation of μ_{PO} is extremely difficult, since a large number of subbands and hydrogenic states are involved.

The calculated results for $L=4$ nm show the dominance of the IFR scattering at low temperatures. However, the value of μ_{IFR} is so low that the overall mobility μ_T shows a monotonic increase with temperature and the peak in the mobility curve does not occur below 300 K. One should note, however, that the assumption of the complete confinement of the wave function in the well and the resultant simple form of the IFR-scattering potential do not hold well for thin wells. The scattering potential is somewhat exaggerated in the infinite barrier model to make the values of μ_{IFR} too low. Hillmer *et al.* observed an $L^{2.5}$ dependence of the mobility, which they have ascribed to the IFR scattering. In the present calculation, the mobility increases as L^6, as described in Sec. III. A more exact calculation of μ_{IFR} for small well widths requires solving the excitonic wave function and subband energies by lengthy numerical techniques.[29]

We have not considered the barrier-alloy-disorder scattering-limited mobility.[30] This mechanism becomes significant the more the wave function penetrates into the barrier, i.e., the thinner the well is. Once again, in order to perform this calculation one needs the knowledge of the wave function for a QW with a finite barrier.

V. CONCLUSIONS

In the present paper, we have derived the expressions for the relaxation times for various scattering processes considering the 1*s* state of the lowest excitonic subband in a QW of infinite barrier. The calculated values of the mobility are then compared with the experimental data for $GaAs/Al_xGa_{1-x}As$ QW's. Satisfactory agreement between experiment and theory is obtained for thick wells. The mobility is found to be limited by the IFR scattering at low temperatures and by the DP scattering up to about 150 K. The mobility at higher temperatures is expected to be limited by the polar-optic scattering; however, the present simple model is unable to yield a correct value for this scattering. The present mode is also inapplicable to the IFR scattering for thin wells and the effects of finite barrier height should be considered.

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