Theory of the small photocarrier grating under the application of an electric field

I. Balberg
The Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel
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Presently, a highly reliable method for the determination of the minority-carrier diffusion length in noncrystalline materials is the photocarrier grating (PCG) technique. The theory and the application of this technique have been essentially limited to the low-electric-field regime, i.e., where the carriers drift lengths are negligible in comparison with the carriers diffusion lengths. Recently, closed-form solutions that connect the microscopic mobility-lifetime ($\mu\tau$) products with the experimentally derived parameter β have been given for the above regime, thus allowing for the determination of the conditions under which the interpretation of the experimental results in terms of the minority carrier $\mu\tau$ is justified. In contrast, for the high-electric-field regime only numerical solutions are available at present. These solutions are not very useful for the unique determination of $\mu\tau$ products, and they do not provide physical insight into the transport and kinetic processes taking place in the PCG. In this paper we present analytic solutions for the PCG conductance in the high-field regime by using a diffusion-drift "perturbation" approach. The results reveal the physical processes in the grating and yield a general explicit relation between the measured quantities and the microscopic transport and kinetic parameters. The important prediction of the present theory is that the PCG technique in the high-field regime can yield direct and unique values for the *majority*-carrier $\mu\tau$ product. The advantage of the PCG, in comparison with the conventional photoconductivity measurement, which is used for the same purpose, is that it is independent of experimental parameters, which are hard to determine. In view of the application of the PCG method to materials where trapping plays a dominant role, our analysis considers the PCG when trapping takes place, yielding closed-form solutions that include the trapping parameters.

I. INTRODUCTION

In view of the importance of the determination of the characteristic parameters of photoconductors for the evaluation of their transport and kinetic mechanisms, the experimental determination of these parameters has become an area of extensive research.[1,2] The relevance of these properties to semiconductor devices in general,[3] and to solar cells[4] in particular, has made the accurate measurement of the parameters an important stage in the assessment of the suitability of the corresponding semiconductor materials for various applications. The most informative single parameters that characterize the photoelectronic quality of a material are the minority- and majority-carrier mobility-lifetime $\mu\tau$ products, which combine the transport and kinetic aspects of the microscopic processes. The common way to derive the value of the majority carrier[5] $\mu\tau$ is from the measurement of the photoconductivity[6,7] (though only within a factor as will be discussed below), while the experimental determination of the minority-carrier $\mu\tau$ is usually more complicated.[1,2,8] Most of the methods suggested and used for the determination of the latter quantity are suitable only for crystalline semiconductors where the minority-carrier $\mu\tau$, or the corresponding room-temperature diffusion length (which is proportional to $\sqrt{\mu\tau}$), is large[1,2,9] (>10 μm). The very few methods that were applicable to crystalline materials and seemed initially suitable for small $\mu\tau$ materials, such as amorphous and some polycrystalline semiconductors, were found to be inappropriate for the latter systems. For example, the photoelectromagnetic method[1,2], which is known to be inaccurate even for crystals,[8] was found to be "impractical"[10] for amorphous materials, and the surface-photovoltage (SPV) method, which was found to be successful for crystalline semiconductors,[9] was shown to yield dubious results for amorphous semiconductors.[11]

Following the need to determine the minority-carrier diffusion length[12,13] in materials with small $\mu\tau$ values, a technique, which seems to be suitable for amorphous[14,15] and polycrystalline[16] materials has been developed. This is the photocarrier grating method, which was suggested four years ago by Ritter, Zeldov, and Weiser.[14] More recently, these authors have presented a small-signal analysis[17] of the method, which connects the quantity determined experimentally (the normalized grating amplitude, see below) with the microscopic transport and kinetic parameters. In their analysis, which will be referred to herein as the RZW analysis, they have considered the linearized equations that are obtained under small-signal steady-state conditions, and they have presented numerical solutions of these equations.

Very recently[18] we have derived closed-form solutions for these equations in the limit of zero applied electric field, i.e., when the drift lengths of both carriers are much smaller than the carrier diffusion lengths. This, in contrast with the numerical solutions, has enabled the derivation of the conditions under which the interpretation of experimental results in terms of the ambipolar diffusion length L is justified.

It turns out, when one considers the analytic solutions in the other extreme, that the corresponding numerical results of RZW are even less informative. Such numerical results not only obscure the physical process but they do not reveal the possibility of deriving accurate as well as unique values for the microscopic parameters (i.e., the two $\mu\tau$ products) from the experimental data. Moreover, previous experimental works[19,20] in which the electric-field dependence was considered, were limited to the low- (but finite) field regime and yielded ill-defined $\mu\tau$ products. The use of the numerical results,[17] on the other hand, required a number of adjustable parameters and this does not enable a unique derivation of $\mu\tau$ products. In addition, the fact that trapping effects have not been considered explicitly in the RZW analysis makes its use in studying materials with strong trapping effects difficult. For example, one realizes[18] that while in the zero-field-ambipolar limit, the ratios between the carrier concentration amplitudes are determined by trapping effects; in the high-field the role of these effects becomes more subtle.

In this paper we reconsider the pioneering analysis of RZW noting that the many groups[19–21] that already utilize the photocarrier grating (PCG) technique did not make any use of the high-electric-field regime for the determination of the microscopic parameters. We also note that the explicit relation between the measurable quantity (β, see below) and the microscopic amplitudes have not been given previously. Our present comprehensive extensions of the RZW analysis include a qualitative physical picture and the basic phenomenological relation of the PCG as well as a presentation of analytic solutions for the most general case and in forms by which experimental results can be interpreted uniquely. Our extension also includes the effect of trapping in every stage of the analysis. This is, of course, necessary since the materials for which the PCG technique is relevant have appreciable trapping effects. As we go along, numerical criteria that are pertinent to the application of the present method to hydrogenated amorphous silicon, *a*-Si:H, will be given. We have chosen *a*-Si:H as our reference materials in view of the suitability of the PCG method for the study of its samples[14] and devices[15] (proper optical gap and thin-film form). Indeed most of the previous PCG studies[19–21] were carried out on this material.

In Sec. II of this paper we give an illustrated description of a carrier grating in a photoconductor under spatially modulated illumination, and we define the PCG measurable parameters. The connections between the qualitative features, which are discussed in that section, and the quantitative analytic solutions, to be given in subsequent sections, are pointed out. In Sec. III we present the most general form of the differential equations that govern the transport process in the presence of a small spatially modulated carrier generation, as well as the most general expression for the recombination rate, which determines the measured quantities. This was done in some detail in view of the confusion one encounters in the case of two-carrier kinetics and transport.[6,22] Finally, the general algebraic equations that result from the differential equations are presented. While the analytic solutions of these equations in the *general* small-signal case, i.e., the case of an "intermediate" applied electric field, can be derived from the equations given in Sec. III, we have chosen to omit this solution in the present paper. The reason is that the analytic solution in this case has a very cumbersome[23] expression, and thus it is quite useless for obtaining unique microscopic parameters from the corresponding measurements. In contrast, we show in Sec. IV that in the regime of a high applied dc electric field (but weak illumination) the analytic results can be made simple enough to enable the derivation of unique microscopic material parameters by a simple comparison with experimental results. This is done by the application of a "perturbation" approach to the diffusion versus drift contributions. The results that follow from such an approach and the experimental evaluation that they enable will be discussed in Sec. V. Finally in the Appendix we argue that under the conditions of the PCG the possible solutions of the coupled homogeneous continuity equations are the same as those of the inhomogeneous equations given in Sec. III. Hence the solutions presented in this paper are the general solutions of the problem.

II. THE PHYSICAL PRINCIPLES OF THE PHOTOCARRIER GRATING

For the general understanding of the PCG method and for the clarification of the analysis to be presented in the following sections we start with a phenomenological derivation and a qualitative description that illustrate the physical principles involved in the relation between the microscopic transport properties and the macroscopic quantities determined by the experiments. Such a phenomenological derivation was given previously[14] only for the $E_0=0$ case while the corresponding qualitative physical description for $E_0>0$ has not been given previously at all.

Let us consider a film or a slab of a semiconductor that has a cross section of unity and a uniform conductivity σ_0, and on which a square-wave carrier-concentration modulation is imposed. Such a modulation, which has an amplitude $\Delta\sigma_0$, is illustrated in Fig. 1. The modulation is assumed small ($\Delta\sigma_0 \ll \sigma_0$) and of period Λ. Hence the length of the sample between the two electrodes is $N\Lambda$, where N is an integer. For simplicity we assume that the modulation is only in the carrier concentration, such as the one that may be achieved by injection or illumination. Using a notation, the reason for which will become apparent later (see also Ref. 14), we may define and "average" conductivity of the structure

$$\sigma_{\perp}=\tfrac{1}{2}[(\sigma_0+\Delta\sigma_0)+(\sigma_0-\Delta\sigma_0)] ,$$

which is just σ_0. On the other hand, the measured conductance (say, under constant voltage conditions) will be determined by the series connection of N resistors, each of which has the value

$$R_{\parallel}=(\Lambda/2)[(\sigma_0+\Delta\sigma_0)^{-1}+(\sigma_0-\Delta\sigma_0)^{-1}] . \qquad (2.1)$$

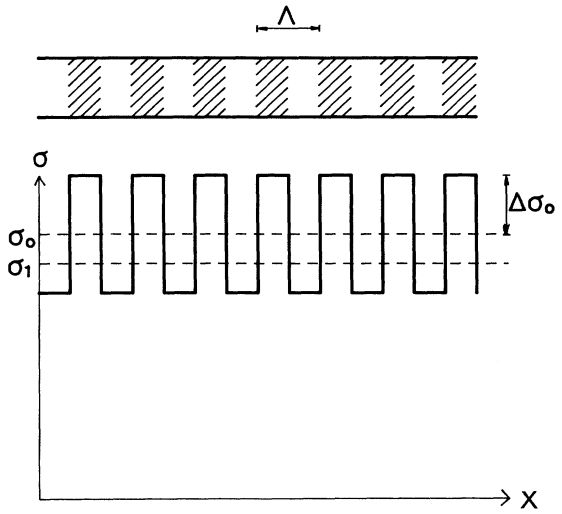

FIG. 1. An illustration of the square-wave conductivity grating in a semiconductor. The hatched regions represent the half-periods where the conductivity is higher than the average conductivity. The exhibited dependence of the conductivity on the distance connecting the two end electrodes is characterized by the period Λ, the uniform conductivity σ_0, and the modulation amplitude $\Delta\sigma_0$.

Since the measured conductivity, i.e., the conductance of the structure per unit length, is $(NR_{\parallel})^{-1}$ (where $N=1/\Lambda$), the measured conductivity of the sample will be

$$\sigma_{\parallel}=\sigma_{\perp}[1-(\Delta\sigma_0/\sigma_{\perp})^2] . \tag{2.2}$$

Hence, the most conspicuous effect to be appreciated for this configuration is that a first-order periodic perturbation induces a second-order decrement in the measured value of the sample conductivity.

Turning from the above general case of a conductivity-modulated system to the case of a photocarrier grating, we consider a photoconductor under steady-state conditions. Let us assume that there is a uniform carrier generation rate G_1 which yields the photoconductivity

$$\sigma_1=B_0G_1^{\gamma} , \tag{2.3}$$

where B_0 is a constant and γ is the well-known photoconductivity exponent.[5] If we apply an additional uniform illumination that causes a carrier-generation rate ΔG_2, we have to replace Eq. (2.3) with (see Fig. 1)

$$\sigma_0=\sigma_{\perp}=B_0(G_1+\Delta G_2)^{\gamma} . \tag{2.4}$$

Now assume that on top of the $G_1+\Delta G_2$ uniform generation rate we add a square-wave-modulated illumination $\Delta G(x)$, with period Λ and amplitude ΔG_a, such that $\Delta G_a \ll G_1$. Under these conditions one expects to obtain the local modulated photoconductivity

$$\sigma(x)=B_0[G_1+\Delta G_2+\Delta G(x)]^{\gamma}$$
$$\approx B_0(G_1+\Delta G_2)^{\gamma}[1+\gamma\Delta G(x)/(G_1+\Delta G_2)] . \tag{2.5}$$

However, this expression is correct only if the carriers reside where they are generated. Since carriers diffuse, from regions of higher concentration to regions of lower concentration, the amplitude of the modulated photoconductivity grating in the *steady state* will be always lower than expected from Eq. (2.5). Hence, in the case where the carrier diffusion lengths are finite but small compared to Λ (so that the "square-wave" shape is not significantly distorted[23]), we should rewrite Eq. (2.5) as

$$\sigma(x)\approx B_0(G_1+\Delta G_2)^{\gamma}\{1+[\gamma C(x)\Delta G(x)]/(G_1+\Delta G_2)\} , \tag{2.6}$$

where $0<C(x)\leq 1$.

Following Eqs. (2.2) and (2.6), we may conclude that the measured photoconductivity of the sample in the presence of the grating will be

$$\sigma_{\parallel}=\sigma_{\perp}\{1-[C'\gamma\Delta G_a/(G_1+\Delta G_2)]^2\} , \tag{2.7}$$

where $C'\Delta G_a$ is some average of $C(x)\Delta G(x)$.

When both the diffusion length of the electrons L_e, and the diffusion length of the holes L_h are much larger than Λ, the carrier lifetime is long enough for both carriers to cross the regions of low generation and one obtains a uniform carrier distribution. Correspondingly, the values of the steady-state carrier concentrations will be the same as those obtained by an "average" generation rate, $G_1+\Delta G_2$, throughout the photoconductor, and thus $C'=0$. Hence C' will have values between 1 (zero diffusion lengths) and 0 (infinite diffusion lengths). One may then define a measurable quantity β such that

$$\beta=(\sigma_{\parallel}-\sigma_1)/(\sigma_{\perp}-\sigma_1) , \tag{2.8}$$

which for $G_1 \gg \Delta G_2$ will be given [using Eqs. (2.3)–(2.6)] to first order by

$$\beta=1-\gamma(C'\Delta G_a)^2/(G_1\Delta G_2) . \tag{2.9}$$

From the above physical picture and from dimensional considerations one may guess that since C' is physically determined by the relation between the quantities L_e, L_h, and Λ (see above), it will be a function of L_e/Λ and Λ_h/Λ. Hence the aim of the theory is to provide a relation between the measurable quantity β (i.e., the quantities $\sigma_{\parallel}$, $\sigma_{\perp}$, and σ_1 or their differences) and the microscopic quantities L_e and L_h. One notes that β of Eq. (2.9) may be negative even if $\Delta G(x)\ll G_1$. For example, in the square-wave-modulation case, if $\Delta G_a=(2G_1\Delta G_2)^{1/2}$, one may have as low a value of β as -1. On the other hand, from the above discussion it is obvious that with any periodic grating the highest value of β cannot exceed $+1$. In the case where ΔG_a is a function of ΔG_2 (as in

the above example), the definition of β [Eq. (2.8)] can be generalized to make it more pertinent to the (most convenient though unnecessary[15]) experimental procedure[14] where ΔG_2 is also a function of the time t. Under these conditions we may define $(d\sigma/dG)/(dG/dt)$ as the variation of the conductivity due to the time variation of the excess carrier-generation illumination ΔG_2. Correspondingly, the generalized definition of β will be

$$\beta=(d\sigma/dG)_{\parallel}/(d\sigma/dG)_{\perp}=(dI_{\parallel}/dG)_{\parallel}/(dI_{\perp}/dG)_{\perp} , \tag{2.10}$$

where the subscripts refer to the configurations that yield $\sigma_{\parallel}$ and $\sigma_{\perp}$. We have here that β is expressed by the experimental ratio of the photocurrent variations, which are obtained, due to the time modulation of ΔG_2, under the grating-on conditions $I_{\parallel}$, and under the grating-off conditions $I_{\perp}$.

While the square-wave carrier-generation case is helpful for illustration, it does not represent a very useful grating in practice since it is hard to impose and accurately analyze[23] such a grating. On the other hand, it is very easy to impose a sinusoidal grating by using the interference of two coherent and monocohromatic light beams.[14,15] The analysis in this case is in principle similar to the one used above, except that now $R_{\parallel}$ is expressed by the integral[14]

$$R_{\parallel}=\int_0^{\Lambda}[\sigma(x)]^{-1}dx \tag{2.11}$$

rather than by the sum given in Eq. (2.1).

The configuration of two interfering light beams, which is designed for creating a carrier-generation grating, is illustrated in Fig. 2. A light beam of a plane wave $y_1=A_1\exp(i\mathbf{K}_1\cdot\mathbf{r})$, with a wave vector $\mathbf{K}_1=(k_x,k_y)$, interferes at the point $\mathbf{r}=(x,y)$ of the photoconductor with another light beam of a plane wave $y_2=A_2\exp(i\mathbf{K}_2\cdot\mathbf{r})$, with a wave vector $\mathbf{K}_2=(-k_x,k_y)$. The corresponding light intensity of the first beam is $F_1=A_1^2$, while that of the other beam is $F_2=A_2^2$. If the wavelength of the light is λ and the angles of incidence are $\delta/2$, we have $k_x=(2\pi/\lambda)\sin(\delta/2)$. Hence, the light intensity distribution along the x direction of the photoconductor's surface (xz plane where the x axis is defined as the direction perpendicular to the planar electrical electrodes which are presumed to be attached to the sample) is obtained by a simple superposition of y_1 and y_2. This superposition yields

$$F(x)=F_1+F_2+2\sqrt{F_1F_2}\cos(kx)\equiv F_0+\Delta F(x) , \tag{2.12}$$

where

$$k=2(2\pi/\lambda)\sin(\delta/2) . \tag{2.13}$$

Hence the spatial grating period associated with this wave vector is

$$\Lambda\equiv(2\pi/k)=\lambda/[2\sin(\delta/2)] . \tag{2.14}$$

Due to the experimentally imperfect light polarization, one should take into account that the modulated carrier generation is somewhat weaker than the last term of Eq. (2.12), and thus an experimental quality factor γ_0 (which we found to be very close to unity[15]) was introduced.[14] Using this factor and making the assumptions (which will be unimportant throughout this paper) of no reflection, a quantum efficiency of 1, and a uniform illumination absorption[1] we obtain the carrier-generation function

$$G(x)=\alpha[F_0+2\gamma_0\sqrt{F_1F_2}\cos(kx)]\equiv G_{\perp}+A_g\cos(kx) , \tag{2.15}$$

where α is the light-absorption coefficient. Following these assumptions we have here $\Delta G(x)=A_g\cos(kx)$. Hence the local photoconductivity should be given by

$$\sigma(x)=\sigma_{\perp}[1+A\cos(kx+\Psi)] , \tag{2.16}$$

where Ψ is a possible phase shift due to a "broken" symmetry introduced by an applied electric field E_0. Using Eq. (2.5) we see that in the limit of zero mobility or zero diffusion length the carriers reside where generated and $A=\gamma A_g/G_{\perp}$. However, because of the redistribution of the carriers as explained above [with regard to Eq. (2.6)] the excess photoconductivity will be of a smaller amplitude than this A. In the particular case of the sinusoidal grating, A_g is simply given by Eq. (2.15) and thus

$$A=A_g\gamma\gamma_{\text{eff}}/G_{\perp}=2\gamma_0\gamma\gamma_{\text{eff}}\sqrt{F_1F_2}/F_0 , \tag{2.17}$$

where γ_{eff} plays here the same role as the factor C' in the square-wave grating. For reasons that will become apparent below, the term γ_{eff} is called[17] the "normalized grating amplitude." This term is introduced here to reflect the microscopic redistribution of the carriers due to diffusion and drift in the sinusoidal grating. For example, in the $E_0\to 0$ case for very large L_e/Λ and L_h/Λ

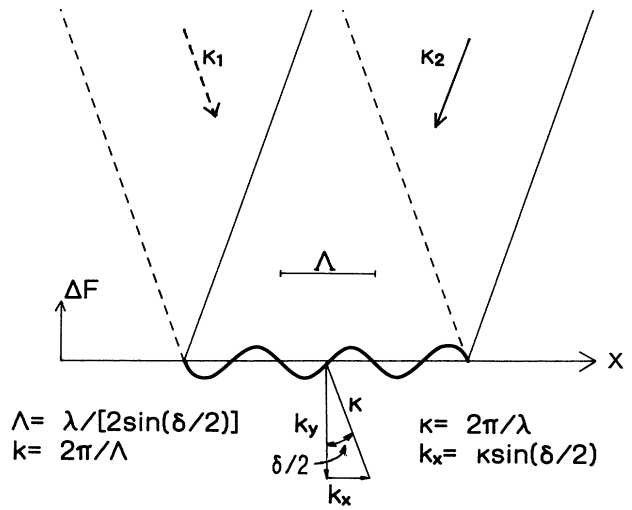

FIG. 2. A cross section (xy plane) of the modulated excess light intensity ΔF in a photoconductor, along the spacing between the two end electrodes attached to it. This excess light intensity can be obtained by the interference of two coherent monochromatic light beams (propagating in the xy plane) as shown in the figure.

values, $\gamma_{\text{eff}}=0$, while for very small L_e/Λ and L_h/Λ values, $\gamma_{\text{eff}}=1$. The important point to note is (again) that the steady-state amplitude of the carrier grating is always smaller than the amplitude of the carrier-generation grating (see below).

Substituting $\sigma(x)$ of Eq. (2.16) in Eq. (2.11) yields the sample resistance given now by $R_{\parallel}=[\Lambda\sigma_{\perp}(1-A^2)^{1/2}]^{-1}$, and thus the measured photoconductivity in the presence of a grating of $N(=1/s)$ periods will be[14]

$$\sigma_{\parallel}=\sigma_{\perp}(1-A^2)^{1/2}\approx\sigma_{\perp}(1-A^2/2)\ , \tag{2.18}$$

where the last approximation follows the fact that $F_2 \ll F_1$. Hence, the measurable grating parameter β is given by[14]

$$\beta=[\sigma_{\perp}(1-A^2/2)-\sigma_1]/(\sigma_{\perp}-\sigma_1)\ , \tag{2.19}$$

where the microscopic information is embedded in the normalized grating amplitude γ_{eff}.

Let us look now more closely at the processes that determine γ_{eff} in an *intrinsic* photoconductor. For clarity we start with the case when no electric field is applied. Under steady-state conditions and a uniform illumination of intensity F_1+F_2 there will be a uniform concentration of electrons n_0 and holes p_0 such that $n_0=p_0=G_{\perp}\tau_0$, where τ_0 is the common recombination time[6] under this illumination level (see Sec. III B). For simplicity, we ignore here trapping processes (which will be addressed in Secs. III B and IV B) and use the notation $n_0=n_{\perp}$ $[\propto(F_1+F_2)^{\gamma}]$ for the concentration of the two types of carriers when the two light beams, impinging on the sample, do not interfere (e.g., by using two coherent beams that have perpendicular polarizations[14–16]). When a small-amplitude light grating is imposed (the beam polarizations are made parallel) the carrier-generation function is as given by Eq. (2.15) and as illustrated in Fig. 3. In this simple case where a dc field is not applied, there is no preferred direction in space, and thus there is a symmetry of the carrier distributions around the carrier-generation grating which is imposed on the uniform background of n_0 and p_0. Under small-signal conditions the recombination time is essentially the same in the somewhat stronger and somewhat weaker illuminated regions. Hence, the steady-state carrier concentrations are such that the excess carriers that have diffused from the regions of the excess illumination are present in the regions of deficient illumination. This suggests a sinusoidal dependence (see the Appendix) of the carrier concentrations such that the grating amplitudes of the two carrier concentrations can be expressed by the single values A_e and A_h. For the sake of consistency we define here the amplitudes A_h and A_e in accordance with Ref. 17 (P_1 and N_1 there). These are the carrier-concentration amplitudes that would have resulted if the photoconductor would have been subjected only to an illumination of amplitude $2\gamma(F_1F_2)^{1/2}$ [but with $\tau_0=\tau_0(p_0)$]. Hence, the actual amplitudes of the excess carriers in the PCG configuration are γA_e and γA_h. The advantage of this definition is that it yields a normalized γ_{eff} as described above, and that it has a simple physical meaning (see below). The disadvantage of this definition is that A_e and A_h are not the actual carrier-concentration amplitudes under the PCG conditions (except when $\gamma=1$). This point has not been clarified in Ref. 17 and indeed Eqs. (35) and (36) in that paper are not consistent in the $\gamma\neq1$ case. Having these definitions we can express the photoconductivity of the PCG by[14]

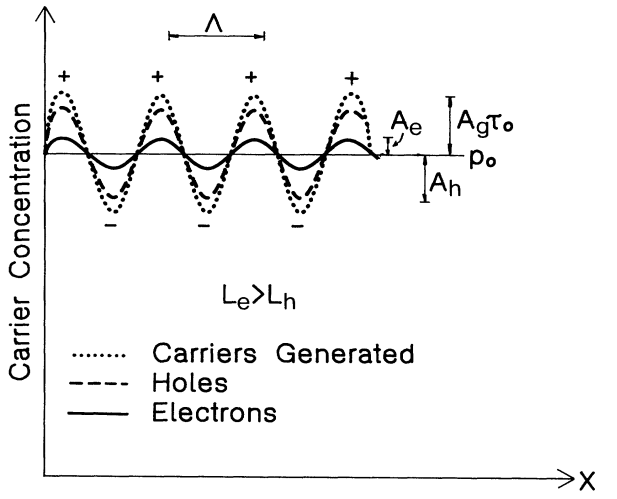

FIG. 3. An illustration of the distribution of the holes and electrons under steady-state conditions in the PCG configuration. The amplitudes of the corresponding carrier gratings are smaller than the amplitude of the carrier-generation grating common to the two carriers. Also indicated in the figure are the regions of excess positive- or negative-carrier space charge. One notes that in the asymptotic ambipolar case $A_h=A_e$, and space-charge neutrality prevails everywhere.

$$\begin{aligned}\sigma(x)&=\sigma_{\perp}+q\gamma(\mu_e A_e+\mu_h A_h)\cos(kx)\\&\equiv\sigma_{\perp}[1+A\cos(kx)]\ ,\end{aligned} \tag{2.20}$$

where q is the (positive) electronic charge and μ_e and μ_h are the carrier's (so-called band or microscopic) mobilities. One notes of course that while both A_e and A_h are proportional to $(F_1F_2)^{1/2}$, they are not equal to each other in the general case, due to the difference in the diffusion lengths of the two charge carriers. It is also clear that the larger the diffusion length the smaller the value of these amplitudes. In the case illustrated in Fig. 3, we assume that $L_e>L_h$ (or that $\mu_e>\mu_h$; see Sec. III) and thus the electrons are distributed more evenly (between the region of excess illumination and the region of deficient illumination) than the holes. On the other hand, we note that while charge neutrality over a single period is maintained, within the period the region of excess illumination is positively charged while the region of deficient illumination is negatively charged. This is since in the first half of the grating period there are more excess holes than excess electrons (A_h is assumed larger than A_e) while in the second half there is a larger deficiency of holes than of electrons. Hence, in the gen-

eral case, this induced sinusoidal space charge opposes the diffusion and determines the final resultant steady-state carrier concentrations. Since the desired physical information is enclosed in A_h and A_e, we must find the relationship between these quantities and the measurable quantity γ_{eff}. This can be done by comparing Eq. (2.20) with the phenomenological value of A, as given by Eq. (2.17). From this comparison we obtain

$$\gamma_{\text{eff}}=q(\mu_e A_e+\mu_h A_h)/[2\gamma_0\sqrt{F_1F_2}(\sigma_\perp/F_0)] . \quad (2.21)$$

Under the intrinsic conditions[6,18] there is the single recombination time τ_0 that determines the recombination process (see Sec. III B) and one can use the simplest relation between the photoconductivity and recombination time[5,6] to write

$$\sigma_\perp/G_\perp=q(\mu_h+\mu_e)\tau_0 . \quad (2.22)$$

Substitution of Eq. (2.22) into (2.21) then yields

$$\gamma_{\text{eff}}=q\gamma(\mu_e A_e+\mu_h A_h) \times[q\gamma(\mu_e+\mu_h)\tau_0(2\gamma_0\alpha\sqrt{F_1F_2})]^{-1} . \quad (2.23)$$

The denominator in Eq. (2.23) is the photoconductivity that would have resulted had there been an additional, small, *uniform* generation rate of an amplitude $2\gamma_0\alpha(F_1F_2)^{1/2}$. We see then that γ_{eff} has a simple physical meaning: it is the ratio between the amplitude of the additional photoconductivity in the presence of the grating and the additional photoconductivity that would have resulted under uniform generation of the same amplitude. Hence the name "normalized grating amplitude."

We may summarize now the relation between the measurable parameter β [or A, see Eqs. (2.18)–(2.20)] and the microscopic carrier concentrations and mobilities by

$$[(1-\beta)/(2\gamma\gamma_0^2)]^{1/2}\approx\gamma_{\text{eff}} =(\mu_e A_e+\mu_h A_h)/[(\mu_e+\mu_h)\tau_0 A_g] . \quad (2.24)$$

It is obvious then that the task of the theory is to determine A_e and A_h, and to express these amplitudes in terms of $\mu\tau$-like quantities, thus making it possible to determine the latter quantities from the measurable β. We note in passing, one of the *important advantages* of the photocarrier grating method: Since $A_e \propto A_g$ and $A_h \propto A_g$, we have A (and thus γ_{eff}) determined from ratios of measurable quantities [Eq. (2.8)] rather than from their absolute values, which are usually much harder to determine accurately in the experiments (see Sec. V).

From the above physical picture it is clear that for $L_e=L_h$, one obtains $A_e=A_h$, and no space charge will be accumulated in either half of a grating period. In the case shown in Fig. 3 ($L_e>L_h$) we have assumed that the electrons' diffusion is essentially independent of the holes' diffusion, so that most of the excess generated holes recombine before reaching, say, the center of the region of lower generation rate, while most of the excess generated electrons reach this center before they recombine. If, however, one considers the electrostatic attraction between the two types of carriers, one must conclude that, if the distance that the carriers diffuse during the dielectric relaxation time τ_d is short compared to Λ, the system is essentially "locally" neutral. Hence, on the relevant length scale of the problem, Λ, we have charge neutrality and thus $A_h=A_e$. If this is not the case the diffusion "overcomes" the electrostatic attraction and ("local") space charge is formed. As we have shown in Ref. 18, one can obtain an exact analytic criterion for the fulfillment of the *asymptotic* $A_e=A_h$ case, i.e., the ambipolar case. Under these conditions the diffusion length is the same for the two carriers, and it is known as the ambipolar diffusion length L.

The question of interest in the present work is what happens when a dc electric field is applied to the system illustrated in Fig. 3. The most obvious effect to be expected is that of charge separation. Under these conditions the peak of the hole concentration will be shifted in the direction of the field and the peak of the electron concentration will be shifted against this direction. Hence, as illustrated in Fig. 4, the hole grating and the electron grating will be shifted by the corresponding phases ϕ and ν with respect to the carrier-generation grating. As was mentioned above, under small-signal conditions, the separation of charges is such that the amplitude of the excess carriers in the region of stronger illumination will be equal to the amplitude of deficient carriers in the region of weaker illumination. We expect[23] then that the carriers concentrations will be described by

$$p=p_0+A_h\cos(kx+\phi)$$

and

$$n=n_0+A_e\cos(kx+\nu) . \quad (2.25)$$

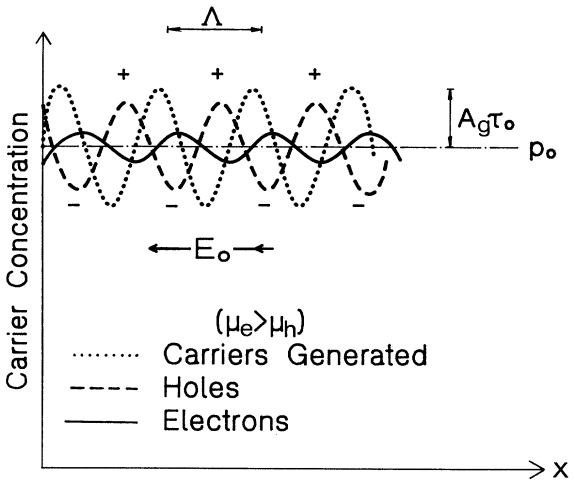

FIG. 4. An illustration of the distribution of holes and electrons under steady-state conditions of the PCG configuration when an external electric field E_0 is applied. Also indicated in the figure are regions of excess positive- or negative-carrier space charge.

Again, there will be a competition between the dielectric relaxation process that tends to pull the two types of carriers together, and the electric-field and the diffusion processes that tend to pull them apart. It is further clear that the carriers with the larger mobility will have the larger phase and the smaller amplitude. In the extreme case of a very high electric field, i.e., when the drift process dominates the diffusion process and the dielectric relaxation, the phases of the carrier concentrations will approach $\pm\pi/2$ with respect to the generation grating. From continuity considerations one would expect that their amplitudes will be related by

$$A_e/A_h=\mu_h/\mu_e \ . \tag{2.26}$$

Using Eqs. (2.25) we can generalize the zero-field case given by Eq. (2.24) to the case that includes an applied dc electric field. Under these general small-signal conditions the local conductivity (in the intrinsic $n_0=p_0$ or $n_\perp=p_\perp$ case) will be given by

$$\sigma(x)=q(\mu_e+\mu_h)n_0 + q\gamma[\mu_e A_e\cos(kx+\nu)+\mu_h A_h\cos(kx+\phi)] \ . \tag{2.27}$$

Hence by writing Eq. (2.27) in terms of

$$\sigma(x)=\sigma_\perp[1+A\cos(kx+\Psi)] \ , \tag{2.28}$$

we obtain

$$\sigma_\perp A/q=\gamma[(\mu_e A_e\cos\nu+\mu_h A_h\cos\phi)^2 + (\mu_e A_e\sin\nu+\mu_h A_h\sin\phi)^2]^{1/2} \ , \tag{2.29}$$

and the "effective phase" Ψ of the PCG conductivity can be defined by

$$\tan\Psi=(\mu_e A_e\sin\nu+\mu_h A_h\sin\phi)/(\mu_e A_e\cos\nu+\mu_h A_h\cos\phi) \ . \tag{2.30}$$

It is apparent already, from the values of ν and ϕ and Eqs. (2.26) and (2.29), that under very high electric fields, which separate the charges completely, we have $A=0$. The physical consequence is that although the gratings of the two carriers are different and separated, their contribution to the conductance is equal and opposite. Hence $\gamma_{\rm eff}=0$ as if none of the carrier gratings exist. In the general case (i.e., for any finite value of the electric field) applying the integration [Eq. (2.11)] to $\sigma(x)$ [as given by Eq. (2.28)] yields the general expression for $\sigma_\parallel$. One notes, however, that in this general case the integration limits are $\Psi/k\le x\le(2\pi+\Psi)/k$ rather than $0\le x\le 2\pi/k$ in the zero-field case. Hence $\sigma_\parallel$ is given as in Eq. (2.18), but the value of A is determined by Eq. (2.29) instead of Eq. (2.20). Correspondingly, one finds that

$$\gamma_{\rm eff}=[(\mu_e A_e\cos\nu+\mu_h A_h\cos\phi)^2 + (\mu_e A_e\sin\nu+\mu_h A_h\sin\phi)^2]^{1/2} \times[(\mu_e+\mu_h)\tau_0 A_g]^{-1} \ . \tag{2.31}$$

Finally, for completeness, let us use Eq. (2.19) in order to derive the general $\beta(\gamma_{\rm eff})$ relationship. Since $A^2\ll 1$ for the small-signal conditions ($F_1F_2\ll F_0^2$), one can use Eqs. (2.17) and (2.19), and the approximation $\sigma_\perp=\sigma(F_1)(1+\gamma F_2/F_1)$, to find that the more accurate expression for β will be

$$\beta=1-2\gamma_0^2\gamma\gamma_{\rm eff}^2[(1+\gamma F_2/F_1)]/[(1+2F_2/F_1)] \ . \tag{2.32}$$

In the $F_2\ll F_1$ limit this reduces of course to the original $\beta(\gamma_{\rm eff})$ relationship proposed by RZW (Ref. 14) [Eq. (2.24)], i.e., to

$$\beta=1-2\gamma_0^2\gamma\gamma_{\rm eff}^2 \ . \tag{2.33}$$

We see now that the measurable quantity β is simply related to the normalized grating amplitude $\gamma_{\rm eff}$ which is given by the concentration amplitudes and concentration phases of the holes and the electrons. As will be shown below, one can determine these amplitudes and phases even for the most general case, i.e., the case that includes the presence of an intermediate electric field.

III. THE LINEARIZED PHOTOCARRIER GRATING EQUATIONS

The basic nonlinear differential equations that govern the steady-state charge flow in intrinsic photoconductors have been known for many years.[1] However, the linearized equations in the simple ambipolar case[24] and in the more general case[17,18] have been considered only recently in connection with the measurements of the two-carrier transport in *a*-Si:H. In this section, we discuss the linearization procedure, and point out the limitations involved in its application to the PCG configuration. In order to extend the free-carrier-only approach of RZW to the more general case of trapping-controlled recombination and transport, which is relevant to *a*-Si:H and other noncrystalline systems, we consider the form of the recombination term. Finally, we present the general algebraic equations that are derived from the linearized differential equations in this case. Substituting the solutions of these equations for A_h and A_e in Eq. (2.31) and using Eq. (2.33) yields the desirable relationship between the experimental PCG conductivity ratio β and the material's $\mu\tau$ parameters.

A. The linearization of the small-signal equations

The carrier concentrations under carrier generation G and carrier recombination U, in the steady state, are determined[1] by the continuity equation for holes,

$$G-U-\mu_h E(\partial p/\partial x)-\mu_h p(\partial E/\partial x)+D_h(\partial^2 p/\partial x^2)=0, \tag{3.1}$$

the continuity equation for electrons,

$$G-U+\mu_e E(\partial n/\partial x)+\mu_e n(\partial E/\partial x)+D_e(\partial^2 n/\partial x^2)=0, \tag{3.2}$$

and the Poisson space-charge equation,

$$\partial\Delta E/\partial x=(q/\epsilon)(p-n) \ , \tag{3.3}$$

where ϵ is the dielectric constant. In this set of equations one assumes a one-dimensional geometry, i.e., that a constant dc electric field E_0 is applied in the x direction and that the carrier-generation rate G may vary only along this axis. The total photoexcited carriers concentrations are p and n (see below), and the resulting electric field E is composed of E_0 and a component $\Delta E(x)$ which may result from the deviations from neutrality in the material. For simplicity we assume that the diffusion terms result from a Boltzmann distribution such that[1]

$$D_h = k_B T \mu_h / q \tag{3.4}$$

and

$$D_e = k_B T \mu_e / q \ , \tag{3.5}$$

where $k_B T$ is the thermal energy. In the more general case (which is relevant to amorphous materials where there is a continuous-state distribution in the pseudogap) one has to use the Fermi-Dirac distribution function, and correct the values of the diffusion coefficients in Eqs. (3.4) and (3.5) by a factor.[17] This correction amounts to different effective mobilities and does not change the results given in the present paper. For the present work and for comparison with experiments it is sufficient to consider D_h and D_e as phenomenological diffusion coefficients which depend linearly on the temperature and the mobility.

The nonlinear coupled equations (3.1)–(3.3) cannot be solved analytically and are usually solved numerically for particular systems and under some simplifying assumptions.[1,12] The situation can be improved considerably when a small signal (or a small perturbation) is considered since mathematically a linearization of the continuity equations is possible. In the general case of a carrier generation $G = G_0 + \Delta G(x)$, where G_0 corresponds to a uniform generation and $\Delta G(x)$ to a perturbation $[\Delta G(x) \ll G_0]$, one expects solutions of the form

$$\begin{aligned} U &= U_0 + \Delta U(x) \ , \\ E &= E_0 + \Delta E(x) \ , \\ p &= p_0 + \Delta p(x) \ , \end{aligned} \tag{3.6}$$

and

$$n = n_0 + \Delta n(x) \ .$$

Here U_0, E_0, p_0, and n_0 are the quantities that prevail when $G = G_0$ and $E = E_0$. Under those conditions $G_0 = U_0$, and thus substituting Eqs. (3.6) in Eqs. (3.1)–(3.3) yields the small-signal equations

$$\Delta G - \Delta U - \mu_h (E_0 + \Delta E)(\partial \Delta p / \partial x) - \mu_h (p_0 + \Delta p)(\partial \Delta E / \partial x) + D_h (\partial^2 \Delta p / \partial x^2) = 0 \ , \tag{3.7}$$

$$\Delta G - \Delta U + \mu_e (E_0 + \Delta E)(\partial \Delta n / \partial x) + \mu_e (n_0 + \Delta n)(\partial \Delta E / \partial x) + D_e (\partial^2 \Delta n / \partial x^2) = 0 \ , \tag{3.8}$$

and

$$\partial \Delta E / \partial x = (q / \epsilon)(p_0 - n_0 + \Delta p - \Delta n) \ . \tag{3.9}$$

In an "intrinsic" photoconductor[1,6] for which solutions are derived in this work, we note that $p_0 = n_0$.

As was pointed out above, the motivation in turning to the small-signal ($\Delta G \ll G_0$) case was the prospect of linearization of Eqs. (3.7)–(3.9). RZW have neglected terms such as $\mu_h \Delta E(\partial \Delta p / \partial x)$ and $\mu_h \Delta p(\partial \Delta E / \partial x)$ by simply labeling them as "second-order" terms. Mathematically this is justified, since one can choose as a small ΔG as one wishes. In practice however, if one is interested in utilizing the results of the theory for the analysis of experimental data, pertinent to real systems and real conditions, one has to ensure that these "second-order" terms are indeed smaller than the other, "first-order" terms. In particular one notes that even if Δp and Δn are small, their derivatives may be non-negligible, and thus in the most general case one cannot ignore these terms. The conclusion is then that one has to find the experimental conditions under which it is justified to neglect these "second-order" terms before comparing one's experimental results with the results of the linearized PCG analysis. Since we are concerned in this paper mainly with the sinusoidal carrier-generation system, which was found to be experimentally convenient[14] [Eq. (2.15) and Fig. 2], i.e., for

$$G = G_0 + \Delta G(x) \equiv G_\perp + A_g \cos(kx) \ , \tag{3.10}$$

where $A_g = 2\gamma_0 \alpha (F_1 F_2)^{1/2}$, we examine the linearization of the continuity equations in the presence of such a generation.

As is argued in the Appendix, in the very-small-signal limit (i.e., when the linearization is justified) the general solutions of the second-order differential equations (3.7) and (3.8) with $\Delta G(x)$ as given by Eq. (3.10) will be of the form [see Eqs. (2.20) and (2.25)]

$$\Delta p(x) = A_h \cos(kx + \phi) \tag{3.11}$$

and

$$\Delta n(x) = A_e \cos(kx + \nu) \ . \tag{3.12}$$

By substituting these solutions in Eqs. (3.7)–(3.9) one can compare the first-order and second-order terms and find[18] the specific conditions under which the linearization is justified. The important practical conclusion of such analysis[18] is that in order to obtain true material parameters, comparisons of the experimental data with results of the linearized theories[17,18] should be made under as low light intensities, of the time-modulated beam F_2, as possible. However, in contrast to the $E_0 \to 0$ case discussed previously[18] it is desirable for the determination of unique microscopic parameters in the $E_0 \to \infty$ case that the F_1 intensity also be as low as possible (see Sec. IV).

In the rest of this paper we consider then the linearized continuity equations

$$\Delta G - \Delta U - \mu_h E_0 \left[\frac{\partial \Delta p}{\partial x} \right] - \mu_h p_0 \left[\frac{\partial \Delta E}{\partial x} \right] + D_h \left[\frac{\partial^2 \Delta p}{\partial x^2} \right] = 0 \tag{3.13}$$

and

$$\Delta G-\Delta U+\mu_e E_0\left[\frac{\partial \Delta n}{\partial x}\right]+\mu_e n_0\left[\frac{\partial \Delta E}{\partial x}\right] +D_e(\partial^2\Delta n/\partial x^2)=0 , \quad (3.14)$$

which for an intrinsic photoconductor ($n_0=p_0$) are coupled via

$$\partial \Delta E/\partial x=(q/\epsilon)(\Delta p-\Delta n) . \quad (3.15)$$

In order to proceed with a more specific form of these equations we must give a specific form to the excess recombination term ΔU. This is done below.

B. The recombination term

In the preceding subsection we have not considered the kinetics of the recombination process, and have denoted the recombination term symbolically by $U=U_0+\Delta U$. Since, by definition, the additional recombination rate ΔU under steady-state conditions has to be the same for electrons and holes, and since in general there are quite a few recombination mechanisms that may be involved, the best expression of ΔU seems to be the phenomenological expression which has been suggested by RZW.[17] They have written the small-signal (additional) recombination rate as

$$\Delta U=(\partial U/\partial n)_p\Delta n+(\partial U/\partial p)_n\Delta p\equiv(\Delta n/t_n+\Delta p/t_p) , \quad (3.16)$$

where t_n and t_p are phenomenological recombination times which are defined for an "intrinsic photoconductor" (i.e., under strong enough background illumination[5] such that $p=n$) via Eq. (3.16). Indeed we shall use these t_n and t_p throughout this paper. However, in order to relate these to well-defined kinetics parameters such as the deep trapping times[6] we will give everywhere the results obtained using a Shockley-Read-like recombination expression which has been shown[6] to describe a much more general recombination process. The Shockley-Read recombination rate in an insulator is given by[1,6]

$$U=pn/(n\tau_p+p\tau_n) , \quad (3.17)$$

where p and n are the concentrations of all photoexcited free holes and free electrons, and τ_p and τ_n are the deep trapping times of holes and electrons, respectively. Hence by applying Eq. (3.16) we find that in this case

$$t_n=\tau_n[1+n\tau_p/(p\tau_n)]^2 \quad (3.18)$$

and

$$t_p=\tau_p[1+p\tau_n/(n\tau_p)]^2 . \quad (3.19)$$

Since under steady-state conditions, in an "intrinsic" uniformly illuminated photoconductor $n=p$, we have

$$U=n/(\tau_n+\tau_p)=p/(\tau_n+\tau_p) . \quad (3.20)$$

We may then call $\tau_c=\tau_n+\tau_p$ the "common" recombination time for both carriers. That the same common recombination time also prevails in the special case of a small perturbation (of uniform or nonuniform illumination) under ambipolar transport conditions (i.e., $\Delta n=\Delta p$) is seen as follows. Equation (3.16) implies that

$$\Delta U=\Delta n(1/t_n+1/t_p) , \quad (3.21)$$

and thus substituting Eqs. (3.18) and (3.19) into Eq. (3.21) yields

$$\Delta U=\Delta n/(\tau_n+\tau_p)=\Delta p/(\tau_n+\tau_p) . \quad (3.22)$$

The result (3.22) is indeed the result that has been obtained (however, differently) in the literature[24] for the special case of ambipolar transport and Shockley-Read recombination. We have redeveloped it here in order to demonstrate that the general approach given by Eq. (3.16) also yields this expected result (and that the factor of $\frac{1}{2}$ in the definition of t_n and t_p as given in Ref. 17 is incorrect). The important point to realize is that in the general case of nonambipolar transport there is no "common" recombination time, and one has to use explicitly the two different recombination times t_n and t_p.

In most photoconductors[5,6] shallow trapping also takes place, and thus its effect on the recombination rate has to be considered. This is usually done[5,24] by defining the ratio θ_n between the concentration of the free photoelectrons and the sum of the concentrations of both the free and the trapped photoelectrons. Using the same definition for holes yields then the condition for charge neutrality[6,24] $n/\theta_n=p/\theta_p$. Hence Eq. (3.17) takes the form

$$U=p/(\tau_p+\theta_p\tau_n/\theta_n)=n/(\tau_n+\theta_n\tau_p/\theta_p) . \quad (3.23)$$

We can now apply the above concepts to obtain an expression for the measurable steady-state photoconductivity[6]

$$\sigma=q(\mu_e n+\mu_h p) . \quad (3.24)$$

Since under uniform illumination the steady state is determined by the condition $G=U$ one can use this condition and Eqs. (3.23) and (3.24) to write[6,22]

$$\sigma=qG(\mu_e\theta_n+\mu_h\theta_p)(\tau_n/\theta_n+\tau_p/\theta_p) \equiv qG\mu_\theta\tau_\theta\equiv qG(\mu_e+\mu_h)\tau_c , \quad (3.25)$$

and thus define a common recombination time $\tau_c=(\mu_\theta\tau_\theta)/(\mu_e+\mu_h)$. One notes, however, that strictly speaking the recombination times for the two carriers are different [Eq. (3.23)]. The results for the small-signal perturbation, (3.18) and (3.19), can be written in the present case of shallow trapping as

$$t_n=\tau_n[1+\theta_n\tau_p/(\theta_p\tau_n)]^2 \quad (3.26)$$

and

$$t_p=\tau_p[1+\theta_p\tau_n/(\theta_n\tau_p)]^2 . \quad (3.27)$$

For the special case of a small signal of additional illumination, under which the conservation of charge neutrality of the additional (free and trapped) carriers ΔN_0 is

maintained, we have

$$\Delta N_0=\Delta n/\theta_n=\Delta p/\theta_p \ . \tag{3.28}$$

Using Eqs. (3.16), (3.26), and (3.27), we determine that the excess recombination rate ΔU can be expressed again by the recombination time of the background carriers, i.e., by

$$\Delta U=\Delta p/(\tau_p+\theta_p\tau_n/\theta_n)$$
$$=\Delta n/(\tau_n+\theta_n\tau_p/\theta_p)\equiv\Delta N_0/\tau_\theta \ . \tag{3.29}$$

We point out again that the result (3.29), which we derived using the definition (3.16), is the same as the result derived without this definition previously.[24] As in the case for U we do not have here the same recombination time for the two types of excess mobile carriers. We note, however, that this is the situation that is definitely relevant[6,24] to a-Si:H under the application of a PCG.

Following the above discussion (in which it was assumed that only free carriers recombine) we can conclude that the continuity equations (3.1) and (3.2) (which describe the conduction processes in the "extended states") stay much the same under trapping. The only term that changes with trapping is the space-charge term, which is affected by both the stationary and the mobile charges. Hence Eq. (3.3) is to be changed to

$$\partial\Delta E/\partial x=(q/\epsilon)(p/\theta_p-n/\theta_n) \ . \tag{3.30}$$

In the case of charge neutrality, this term vanishes since one has the condition

$$n/\theta_n=p/\theta_p \ . \tag{3.31}$$

In the "intrinsic" case considered throughout this work we only assume that

$$n_0/\theta_n=p_0/\theta_p \ . \tag{3.32}$$

Hence the fulfillment of (3.31) [or (3.28)] represents an ambipolar case in the sense that charge neutrality is maintained but not that $\Delta n=\Delta p$, as in the case with no trapping. In the more general nonambipolar case to be discussed in Sec. IV, relation (3.32) is fulfilled while relation (3.31) is not. Hence the Poisson equation for this condition will be given by replacing p by Δp and n by Δn in Eq. (3.30).

C. The continuity equations under a small photocarrier grating

Following our interest in the carrier distributions under the sinusoidal carrier generation (3.10) and having the general excess recombination rate [Eq. (3.16)] and the Poisson relation [Eq. (3.15)] we can now write the corresponding continuity equations (3.13) and (3.14) explicitly. These equations are[17,18]

$$-D_h(\partial^2\Delta p/\partial x^2)+\mu_h E_0(\partial\Delta p/\partial x)+\Delta p(1/t_p+q\mu_h p_0/\epsilon)+\Delta n(1/t_n-q\mu_h p_0/\epsilon)=A_g\cos(kx) \tag{3.33}$$

for holes, and

$$-D_e(\partial^2\Delta n/\partial x^2)-\mu_e E_0(\partial\Delta n/\partial x)+\Delta p(1/t_p-q\mu_e n_0/\epsilon)+\Delta n(1/t_n+q\mu_e n_0/\epsilon)=A_g\cos(kx) \tag{3.34}$$

for electrons. As we see, these are essentially two coupled, linear, second-order equations. They become considerably simple in the $E_0=0$ ambipolar ($\Delta n=\Delta p$) limit.[17,18] As pointed out already, under an applied electric field we expect[23] sinusoidal solutions for these equations such that

$$\Delta p=A_h\cos(kx+\phi) \tag{3.35}$$

and

$$\Delta n=A_e\cos(kx+\nu) \ . \tag{3.36}$$

In principle, all one has to do, in order to get the most general result, is to substitute Eqs. (3.35) and (3.36) in Eqs. (3.33) and (3.34), and solve for A_e, ν, A_h, and ϕ. Once these quantities are known, one can substitute the results in Eq. (2.31) in order to make a comparison with the experimentally determined γ_{eff}.

Carrying out the first step suggested above we have substituted the steady-state solutions, (3.35) and (3.36), into the coupled differential equations, (3.33) and (3.34), obtaining the following four coupled algebraic equations with the four unknowns, A_h, ϕ, A_e, and ν. These equations are

$$Ag-(A_h\cos\phi)(1/t_p+\mu_h p_0 q/\epsilon+k^2D_h)$$
$$-(A_e\cos\nu)(1/t_n-\mu_h p_0 q/\epsilon)$$
$$+(A_h\sin\phi)\mu_h E_0 k=0 \tag{3.37}$$

and

$$(A_h\cos\phi)\mu_h E_0 k+(A_h\sin\phi)(1/t_p+\mu_h p_0 q/\epsilon+k^2D_h)$$
$$+(A_e\sin\nu)(1/t_n-\mu_h p_0 q/\epsilon)=0 \tag{3.38}$$

for the holes, and

$$A_g-(A_e\cos\nu)(1/t_n+\mu_e n_0 q/\epsilon+k^2D_e)$$
$$-(A_h\cos\phi)(1/t_p-\mu_e n_0 q/\epsilon)$$
$$-(A_e\sin\nu)\mu_e E_0 k=0 \tag{3.39}$$

and

$$(A_e\cos\nu)\mu_e E_0 k+(A_e\sin\nu)(1/t_n+\mu_e n_0 q/\epsilon+k^2 D_e)$$
$$+(A_h\sin\phi)(1/t_p-\mu_e n_0 q/\epsilon)=0 \quad (3.40)$$

for the electrons. These algebraic equations are the most general equations that one obtains for the linearized differential equations which correspond to the conditions of a small sinusoidal carrier generation superimposed on a larger uniform carrier background. While in principle one can simply proceed by solving the algebraic equations, in practice, the general solutions of Eqs. (3.37)–(3.40) have very cumbersome expressions.[23] Hence one should search for some more specific conditions that can yield simple and unique relations between the experimentally determined γ_{eff} and the $\mu\tau$ products so that one will be able to derive unique values for these products. One set of conditions (the ambipolar case) was considered previously[18] and another set will be derived in the following section.

IV. ANALYTIC SOLUTIONS OF THE CONDUCTANCE IN THE PHOTOCARRIER GRATING UNDER AN APPLIED ELECTRIC FIELD

In the $E_0\to 0$ limit one assumes[17,18] that the electric-field terms of Eqs. (3.1) and (3.2) [or Eqs. (3.7) and (3.8)],

$$\mu_h E_0(\partial\Delta p/\partial x)\ , \quad (4.1)$$

are negligible compared with the other terms in those equations. This situation corresponds to the case where competition between the diffusion processes and the space-charge effects is determining the steady-state photocarrier distributions in the PCG. Since we found[18] that the experimental conditions required in the $E_0\to 0$ limit are not too stringent, the second question that arises is, what information can one extract from the electric-field dependence of the measured quantities β, or γ_{eff} (apart from gaining physical understanding of the PCG in the presence of an electric field; see Sec. II). Trying to answer this question one realizes that the analytic solutions of the general case, which includes the electric-field terms, are essentially available, since one can simply solve the algebraic equations (3.37)–(3.40). However, as is apparent from these equations and the solutions[18] of the much simpler $E_0=0$ case, the expressions of these solutions in the $E_0\neq 0$ case are very cumbersome. Since the purpose of applying the PCG technique is to enable unique derivation of parameters such as the $\mu\tau$ products, one has to find conditions that will yield simple analytic results for γ_{eff}. The most natural approach to obtain a simple expression seems to be that of considering another extreme case, e.g., the $E_0\to 0$ limit. The latter case is obtained when the electric-field term (4.1) dominates the diffusion and space-charge terms [in the continuity equations (3.1) and (3.2) or (3.37)–(3.40)] so that the latter terms can be neglected. As we show below, while this limit is very easy to solve and the analytic results obtained are indeed simple and interesting, they are useless in the sense that no information can be obtained by their comparison with experimental results. The solution in this case shows that while the carrier gratings are not blurred, the contributions of the two types of carriers to the conductance cancel each other. This $E_0\to\infty$ result on the one hand, and the cumbersome solution expected for the intermediate E_0 conditions on the other hand, indicate that for the purpose of getting a reasonable simple expression for γ_{eff} one has to consider more specific cases. Of these cases the more useful ones are of course those that are generally achievable experimentally and that are consistent with the linearization described in Sec. III A (rather than those that may take advantage of particular material parameters). Correspondingly we discuss here the case of low illumination level and high electric field. As we show below the corresponding results enable the most accurate available determination of the $\mu\tau$ products of the *two types* of carriers under steady-state conditions.

A. Definition of the high-field regime

In order to quantify our definitions of the various field regimes, let us evaluate the magnitude of the various terms in the algebraic continuity equations (3.37)–(3.40). This is done by using the same reasoning as that proposed in Ref. 18 and mentioned in Sec. III A.

The diffusion term under the small signal PCG is given (e.g., for holes) by $k^2 D_h\Delta p$, while the electric-field term [Eq. (4.1)] is given by $\mu_h k E_0\Delta p$. Hence the criterion for the applicability of the $E_0=0$ approximation is

$$E_0 \ll (kD_h/\mu_h)\ . \quad (4.2)$$

Assuming the Boltzmann approximation for the diffusion term [Eq. (3.4)], we obtain a field that is smaller than the "diffusion field," i.e., a field that obeys the relation

$$E_0 \ll k(k_B T/q) \quad (4.3)$$

is to be considered a small field. In practice, for the common experimental conditions (in mks units), i.e., for $k=2\pi/10^{-6}$ and room temperature [where $(k_B T/q)=\frac{1}{40}$], this means that the $E_0=0$ approximation is valid for

$$E_0 \ll 1600\ \text{V/cm}\ . \quad (4.4)$$

This criterion for the low-electric-field regime is independent of the material measured and depends only on the temperature and the experimentally controllable parameter k. We further note that this implies very convenient working conditions since for typical fields of 100 V/cm one can easily carry out the measurements on various amorphous[14,15] and polycrystalline[16] materials.

The criterion (4.4) can be read in reverse, i.e., that fields of the order of 10^4 V/cm are required for the high-electric-field regime. Of course it is not enough, however, to reverse the criterion (4.3) and one has to consider also the space-charge term $\mu_h p_0 q(\Delta p-\Delta n)/\epsilon$ [see Eqs. (3.13)–(3.15)]. This term is bounded by $\mu_h p_0 q(|\Delta p|+|\Delta n|)/\epsilon$. Correspondingly (assuming $\mu_e>\mu_h$ and thus $|\Delta p|>|\Delta n|$), we see that the criterion for the high-field regime [when the sinusoidal solutions given by Eqs. (3.11) and (3.12) are assumed; see the Appendix] consists of the

reversal of Eq. (4.4) as well as the requirement [considering Eq. (4.1) and Eqs. (3.13)–(3.15)] that

$$2qp_0/(k\epsilon) \ll E_0 \ . \tag{4.5}$$

Hence in a material with no trapping (see Sec. IV C) the criterion for the high-field regime is that the value of the field will satisfy both the reversal of (4.4) and the fulfillment of (4.5). To get a feeling for the magnitude of the required fields we must use the material parameters $p_0(=G_0\tau_0)$ and ϵ. As an example, let us consider a typical "intrinsic" photoconductor (a-Si:H) under AM1 illumination.[17] In such a material [$G_0 \approx 2\times 10^{21}$ cm^{-3} s^{-1}, $\tau_0 \approx 2\times 10^{-6}$ s, and $\epsilon = 10^{-10}$ (mks units)], we have $p_0 = 4\times 10^{15}$ cm^{-3}, and thus for our typical grating ($k = 2\pi/10^{-6}$) we must require that

$$2\times 10^4 \text{ V/cm} \ll E_0 \ . \tag{4.6}$$

As we will see, however, in Sec. IV C the conditions become more stringent when trapping is present. Since criterion (4.6) yields a much higher value than criterion (4.4), we conclude that for a photoconductor of $p_0 \simeq 4.10^{15}$ cm^{-3} fields higher than, say, 10^5 V/cm belong to the very-high-field regime. For the same photoconductor under a 0.1 AM1 illumination we see that the very-high-electric-field regime will be for fields higher than 10^4 V/cm. This value satisfies both (4.5) and the reversal of (4.4). Because of the latter reversal the lower bound of E_0 *cannot* be further reduced by lowering the high intensity. For the very high fields we can neglect the diffusion and space-charge effects and solve the continuity equations (3.1) and (3.2) by excluding the corresponding terms. As we show in Sec. IV B this is easily done and one obtains sinusoidal solutions such as those given by Eq. (2.25).

We should also check whether fields of the order of 10^4 V/cm are also high enough to be consistent with the linearization of the continuity equations, i.e., high enough to ensure that $E_0 \gg \Delta E$ [see Eqs. (3.7) and (3.8)]. For this purpose we have to estimate the value of ΔE. Considering the sinusoidal solutions[18] and Eqs. (3.7) and (3.8) we may conclude that the linearization is justified if

$$\Delta E \leq (2q/k\epsilon)A_p \ll E_0 \ . \tag{4.7}$$

Since in the PCG configuration $A_p \ll p_0$, the fulfillment of (4.5) ensures the fulfillment of (4.7). In fact, this restriction is less stringent than the restriction on the illumination intensities required for the linearization under the $E_0 = 0$ conditions.[18] We expect then that applied electric fields larger than at least 10^4 V/cm (depending on the illumination intensity, see above) will ensure both the linearization and the domination of the electric-field term. We further note [see Eqs. (4.4) and (4.5)] that if we also want the diffusion term to be significantly larger than this space-charge term (see below) we have to substitute 1600 V/cm for the value E_0 in Eqs. (4.5) and (4.7). Hence, the regime for which the field term dominates the diffusion term corresponds to electric fields of the order of 10^4 V/cm, and if under these conditions the diffusion term has to dominate the space-charge term the light intensity should not exceed about 0.1 AM1 (for more details see Sec. IV C). As we show below, the latter case is the one which is of the most practical value.

B. The photocarrier grating in the very-high-field regime

As was pointed out in Secs. I and III, there is a limited practical use for the full analytic solutions that can be obtained from the algebraic equations (3.37)–(3.40). Considering the relatively simple results obtained for the $E_0 = 0$ case,[18] one would hope that simple enough results would be obtained for the other extreme, i.e., the $E_0 \to \infty$ limit.

The meaning of the $E_0 \to \infty$ limit is that the drift terms in Eqs. (3.13) and (3.14) are larger than the space-charge and diffusion terms in these equations. We note that the structure of the algebraic solutions (3.37)–(3.40) does not change by just neglecting one of these terms, and thus the complexity of the solution is not reduced by this type of approximation. Hence, the only way to simplify the structure of the algebraic equations (which appears to be the only way to modify the form of the solutions; see, however, Sec. IV C) is by neglecting both terms.

Applying this approximation reduces equations (3.13) and (3.14) to

$$\Delta G - \Delta U - \mu_h E_0(\partial \Delta p/\partial x) = 0 \tag{4.8}$$

and

$$\Delta G - \Delta U + \mu_e E_0(\partial \Delta n/\partial x) = 0 \ . \tag{4.9}$$

We see immediately from Eqs. (4.8) and (4.9) that

$$\mu_h(\partial \Delta p/\partial x) = -\mu_e(\partial \Delta n/\partial x) \ , \tag{4.10}$$

and since the symmetry and the linearization of the problem imply that there are no dc terms in Δp and Δn, one would expect that

$$\mu_h \Delta p(x) = -\mu_e \Delta n(x) \ . \tag{4.11}$$

As will be shown below, this expectation is obeyed and with ΔG given by (3.10) the corresponding solutions are

$$\Delta p = A_h \sin(kx) = A_h \cos(kx - \pi/2) \tag{4.12}$$

for holes and

$$\Delta n = -(A_h\mu_h/\mu_e)\sin(kx) = (A_h\mu_h/\mu_e)\cos(kx + \pi/2) \tag{4.13}$$

for electrons.

Physically, Eq. (4.11) shows that the contribution of the holes to the conductivity, at some point x, is exactly canceled out by that of the electrons and Eqs. (4.8) and (4.9) show that the amplitudes decrease with increasing E_0. We further note that while the carrier grating amplitudes are different for electrons and holes, the fact that the net contribution of both carriers is zero, makes it, from the experimental point of view, indistinguishable from the case where both amplitudes are zero (i.e., when there is no grating at all). The above result is interesting for the understanding of the PCG since it shows that the

effect of the application of a very high electric field is separating the two types of carriers as well as blurring the carriers gratings. The role of these two effects could not be deduced from the numerical results of RZW. On the other hand this result is of no practical use since it shows that the conductivity is the same as the one obtained under complete blurring. Correspondingly the measurement of the conductivity under the $E_0 \rightarrow \infty$ conditions does not enable the derivation of any physical parameter.

Following the above conclusion it may appear at first that one has to use the full cumbersome solutions of Eqs. (3.37)–(3.40) in order to get a nonzero γ_{eff}, even in the high-electric-field limit. As will be shown in Sec. IV C, there is a solution to this problem and with the experimentally accessible conditions of weak enough illumination and high, but not too high, electric fields, useful comparison with the experimental quantity γ_{eff} can be made. For exploring this possibility let us reconsider the very high-electric-field case [Eqs. (4.8) and (4.9)] rigorously using the explicit recombination term [Eq. (3.16)], i.e.,

$$\Delta U = \Delta p / t_p + \Delta n / t_n \ . \tag{4.14}$$

This is done in order to establish the conclusion (4.11) and to provide the results needed for the procedure to be suggested in Sec. IV C.

Using a sinusoidal solutions [Eqs. (3.11) and (3.12)] the differential equations (4.8) and (4.9) become the algebraic equations

$$Ag - (A_h \cos\phi)/t_p - (A_e \cos\nu)/t_n + \mu_h E_0 k A_h \sin\phi = 0 \ , \tag{4.15}$$

$$\mu_h E_0 k A_h \cos\phi + (A_h \sin\phi)/t_p + (A_e \sin\nu)/t_n = 0 \ , \tag{4.16}$$

$$Ag - (A_h \cos\phi)/t_p - (A_e \cos\nu)/t_n - \mu_e E_0 k A_e \sin\nu = 0 \ , \tag{4.17}$$

and

$$-\mu_e E_0 k A_e \cos\nu + (A_h \sin\phi)/t_p + (A_e \sin\nu)/t_n = 0 \ . \tag{4.18}$$

Solving the above equations we obtain

$$A_h \cos\phi = A_g t_p t_n \mu_e (\mu_e t_n - \mu_h t_p)/B \tag{4.19}$$

and

$$A_h \sin\phi = -A_g k E_0 \mu_e^2 \mu_h t_n^2 t_p^2 / B \ , \tag{4.20}$$

where

$$B = (\mu_e t_n - \mu_h t_p)^2 + (k E_0 \mu_e \mu_h t_p t_n)^2 \ . \tag{4.21}$$

Similarly we obtain

$$A_e \cos\nu = A_g t_n t_p \mu_h (\mu_h t_p - \mu_e t_n)/B \tag{4.22}$$

and

$$A_e \sin\nu = A_g k E_0 \mu_e \mu_h^2 t_n^2 t_p^2 / B \ . \tag{4.23}$$

Using these results one sees immediately that $\mu_e A_e = \mu_h A_h$ and that the only possible solution for $\sin\phi = -\sin\nu$ and $\cos\phi = -\cos\nu$ is $-\phi = \nu = \pi/2$, hence the expected solution given by (4.12) and (4.13). Having all the quantities that appear in γ_{eff} [Eq. (2.31)] we see again that the solutions (4.20)–(4.23) yield the value $\gamma_{\text{eff}} = 0$. We can, however, utilize these results in our search for γ_{eff} in the case where we keep terms of the order of $k_B T k/(qE_0)$ but neglect the space-charge terms. As we show below, this (high field, low illumination level) approach is found to be useful for the derivation of a simple expression for γ_{eff} in terms of the photoelectronic parameters.

C. The grating photoconductance under weak illumination and strong electric field

Let us consider now the expression obtained for the photoconductance of the grating under the experimentally achievable conditions of very low light intensity and high electric field, i.e., when $kE_0 > k_B T k^2/q \gg q p_0/\epsilon$. This means that the space-charge term is negligible compared with the diffusion term while the diffusion term is smaller than, but not negligible in comparison with, the drift term. Here, we show that this expression is amenable to comparison with the experimental quantity γ_{eff}, and is thus useful for a unique derivation of the photoelectronic parameters of a photoconductor.

Since very much of the interest in the present method is associated with the experimental work on amorphous materials, where trapping takes place, let us reconsider first the relationship between the diffusion terms and the space-charge terms under the presence of trapping and high applied field. We saw already that the only terms in the continuity equations [(3.13) and (3.14)] that change in the presence of trapping are the space-charge terms. These terms, under trapping, are given by [see Eqs. (3.15) and (3.30)]

$$(\mu_h q p_0/\epsilon)(\Delta p/\theta_p - \Delta n/\theta_n) \tag{4.24}$$

and

$$(\mu_e q n_0/\epsilon)(\Delta p/\theta_p - \Delta n/\theta_n) \ . \tag{4.25}$$

In the "intrinsic" case discussed in this paper these terms may be written as

$$(\mu_h q N_0 \theta_p/\epsilon)(\Delta p/\theta_p - \Delta n/\theta_n) \tag{4.26}$$

and

$$(\mu_e q N_0 \theta_n/\epsilon)(\Delta p/\theta_p - \Delta n/\theta_n) \ , \tag{4.27}$$

where N_0 is the sum of free and trapped carrier concentrations of each of the carriers (i.e., $N_0 = n_0/\theta_n = p_0/\theta_p$).

When we approach the problem from the very-high-field end we may approximate $|\Delta n/\Delta p|$ by μ_e/μ_h (as shown in Sec. IV B), and thus we may use the estimate

$$|\Delta p/\theta_p - \Delta n/\theta_n| \approx |\mu_e \theta_n/(\mu_h \theta_p) - 1| \Delta p/\theta_p \ . \tag{4.28}$$

For carrying out our development in $k_B T k/(qE_0)$ (see above) we have to find the conditions under which the space-charge terms (4.26) and (4.27) are indeed negligible compared to the diffusion terms $k^2 D_h \Delta p$ and $k^2 D_e \Delta n$.

Using the Einstein relations [Eqs. (3.4) and (3.5)] these conditions become

$$(N_0\theta_p q/\epsilon)|(\Delta p/\theta_p-\Delta n/\theta_n)| \ll (k^2 k_B T\Delta p/q) \quad (4.29)$$

and

$$(N_0\theta_n q/\epsilon)|(\Delta p/\theta_p-\Delta n/\theta_n)| \ll (k^2 k_B T\Delta n/q) . \quad (4.30)$$

Applying now the very-high-field relation [Eq. (4.28)] these two conditions are reduced to

$$(qN_0/\epsilon)|(\mu_e\theta_n)/(\mu_h\theta_p)-1| \ll k^2 k_B T/q . \quad (4.31)$$

In practice (e.g., in intrinsic a-Si:H where $\mu_e\theta_n \gg \mu_h\theta_p$, see below) this condition finally can be simplified to

$$(qN_0/\epsilon)(\mu_e\theta_n)/(\mu_h\theta_p) \ll k^2 k_B T/q . \quad (4.32)$$

To get a feeling for this constraint we recall that (in mks units) $k^2 k_B T/q \approx 10^{12}$. The left-hand side of Eq. (4.32) has the value $1.6\times10^{-9}\ N_0[\mu_e\theta_n/(\mu_h\theta_p)]$ and thus for a drift mobility ratio of 10, N_0 must be smaller than 6.7×10^{13} cm^{-3}. This means that for τ_θ [Eq. (3.25)], which was measured[17] in a-Si:H to be $\approx 2\times10^{-6}$ s, we determine that the highest allowed carrier-generation rate consistent with our condition is less than $G_0=N_0/\tau_\theta \approx 3\times10^{19}$ photons/cm^3 s (i.e., about 0.015 AM1 or 1.5 mW/cm^2). The larger the mobility ratio the lower the allowed light intensity if the present "diffusion-field" approximation is to be used. Fortunately in a-Si:H the ratio of the drift mobilities decreases with decreasing light intensity,[22,25] so that the conditions considered here can be approached experimentally.[26] One may further improve the experimental conditions by working[23] with as a small grating period [smaller than the common[14-16] smallest grating of $(\Lambda=6328/2)$ Å] as possible. This will allow an improvement of the method to be used by enabling larger and thus more convenient light intensities.

To obtain the desired solution that has a simple enough expression let us reconsider Eqs. (3.37)–(3.40) when the space-charge terms (which contain p_0 or n_0) are neglected. Under this approximation the equations are reduced to

$$A_g-[(A_h\cos\phi)/t_p](1-k^2 D_h t_p) + (A_h\sin\phi)\mu_h E_0 k-(A_e\cos\nu)/t_n=0 , \quad (4.33)$$

$$(A_h\cos\phi)\mu_h E_0 k+[(A_h\sin\phi)/t_p] \times(1+k^2 D_h t_p)+(A_e\sin\nu)/t_n=0 , \quad (4.34)$$

$$A_g-(A_h\cos\phi)/t_p-[(A_e\cos\nu)/t_n] \times(1-k^2 D_e t_n)-(A_e\sin\nu)\mu_e E_0 k=0 , \quad (4.35)$$

and

$$-(A_e\cos\nu)\mu_e E_0 k+[(A_e\sin\nu)/t_n] \times(1+k^2 D_e t_n)+(A_h\cos\phi)/t_p=0 . \quad (4.36)$$

Subtraction of Eq. (4.36) from Eq. (4.34), and the use of D_h and D_e [according to (3.4) and (3.5)] yields

$$\mu_h A_h\cos\phi+\mu_e A_e\cos\nu =-[k_B Tk/(qE_0)](\mu_h A_h\sin\phi-\mu_e A_e\sin\nu) . \quad (4.37)$$

Similarly one obtains, by subtracting Eq. (4.35) from Eq. (4.33),

$$\mu_h A_h\sin\phi+\mu_e A_e\sin\nu =[k_B Tk/(qE_0)](\mu_h A_h\cos\phi-\mu_e A_e\cos\nu) . \quad (4.38)$$

The terms on the right-hand sides of Eqs. (4.37) and (4.38) are then the "corrections" of order $(k_B Tk/qE_0)$ to the (zero value, in the very high-field regime; see Sec. IV B) quantities in the left-hand sides of these equations. Using the very-high-field solutions [i.e., the zeroth-order solutions in $(k_B Tk/qE_0)$] (4.19)–(4.23), we obtain

$$\mu_h A_h\cos\phi+\mu_e A_e\cos\nu=2A_g(k_B Tk^2/q)(\mu_e^2\mu_h^2 t_n^2 t_p^2/B) \quad (4.39)$$

and

$$\mu_h A_h\sin\phi+\mu_e A_e\sin\nu =2A_g(k_B Tk/qE_0)\mu_e\mu_h t_n t_p(\mu_e t_n-\mu_n t_p)/B . \quad (4.40)$$

Finally using Eq. (2.31) we find for $\gamma_{\rm eff}$ the expression

$$\gamma_{\rm eff}(\mu_e+\mu_h)A_g\tau_0=[2A_g k_B Tk\mu_e\mu_h t_n t_p/(qE_0B)] \times[(kE_0\mu_e\mu_h t_n t_p)^2 +(\mu_e t_n-\mu_h t_p)^2]^{1/2} , \quad (4.41)$$

where τ_0 is the common recombination time of the uniformly illuminated "intrinsic" material (in which $n_0=p_0$); for example, in the Shockley-Read-like case $\tau_0=\tau_n+\tau_p$ (see Sec. III B). Hence the most concise form of Eq. (4.41) is

$$\gamma_{\rm eff}=2k_B Tk\mu^* t_n t_p/(qE_0\tau_0\sqrt{B}) , \quad (4.42)$$

where $\mu^*=\mu_e\mu_h/(\mu_e+\mu_h)$ is the "reduced mobility." We obtain a closed-form result for the case of very low light intensities and high electric fields. Using this result under more specific conditions yields a further simplification of the expression for $\gamma_{\rm eff}$. For example, from Eqs. (4.21), (4.41), and (4.42) one finds that for

$$\mu_e t_n-\mu_h t_p \gg kE_0\mu_e\mu_h t_n t_p \quad (4.43)$$

[see Eq. (4.21)] $\gamma_{\rm eff}$ can be approximated by

$$\gamma_{\rm eff}=2k_B Tk\mu^* t_n t_p/[qE_0\tau_0(\mu_e t_n-\mu_h t_p)] . \quad (4.44)$$

On the other hand, for higher fields, i.e., when

$$kE_0\mu_e\mu_h t_p t_n \gg (\mu_e t_n-\mu_h t_p) , \quad (4.45)$$

or rather (for the common case of a-Si:H, i.e., where $\mu_e t_n \gg \mu_h t_p$) if

$$k\mu_h t_p E_0 \gg 1 , \quad (4.46)$$

we obtain

$$\gamma_{\text{eff}}=2k_BT/[qE_0^2\tau_0(\mu_e+\mu_h)] . \quad (4.47)$$

We see then that γ_{eff} has a reciprocal parabolic approach to zero with increasing electric field. The results (4.44) and (4.47) explain the inflection point in the γ_{eff} (E_0) dependence which was found in the numerical solutions[17] when E_0 increases from the very-low-field regime to the very-high-field regime. In a forthcoming paper[26] we will report the first experimental results for E_0 beyond this inflection point, i.e., for the region considered here. For the Shockley-Read recombination the last two expressions for γ_{eff} [Eqs. (4.44) and (4.47)] are reduced to

$$\gamma_{\text{eff}}=kL^2/[E_0(\mu_e\tau_n-\mu_h\tau_p)] \quad (4.48)$$

and

$$\gamma_{\text{eff}}=2k_BT/[qE_0^2(\tau_n+\tau_p)(\mu_e+\mu_h)] , \quad (4.49)$$

where L is the ambipolar diffusion length,[18] i.e., $L=[2(k_BT\mu^*/q)(\tau_n+\tau_p)]^{1/2}$. The importance of these results is that by using γ_{eff} vs $1/E_0$ or $1/E_0^2$ plots one can obtain $\mu\tau$ information that cannot be obtained otherwise. In particular Eq. (4.49) yields accurately the most important quantity associated with the phenomenon of photoconductivity, i.e., $(\mu_e+\mu_h)\tau_0$. We should emphasize that this quantity cannot be determined accurately even from the most accurate photoconductivity measurements because of the always existing uncertainty in the generation rate of the carriers[26] (see Sec. V). It is also important to note that since $q(\mu_e+\mu_h)\tau_0G_0$ just represents the photoconductivity when no grating is present the result (4.47) is entirely independent of the recombination mechanism [Eq. (3.16)] and can be written, in the most general case, in terms of $q(\mu_e\tau_n^R+\mu_h\tau_p^R)$ where τ_n^R and τ_p^R are the corresponding carrier-recombination times. Hence, the γ_{eff} measurement under the conditions discussed yields always the sum of the $\mu\tau$ products of the two carriers.

Turning to the effect of trapping on the behavior of γ_{eff}, we realize that trapping enters the continuity equations in our diffusion-field "perturbation" approach only through the values of t_p and t_n. Correspondingly we have to introduce the proper values of t_p, t_n, and τ_0 in Eqs. (4.33)–(4.36).

To carry out this substitution explicitly we need to assume a recombination mechanism. (See Sec. III B.) If we apply our standard Shockley-Read-like expressions for τ_0, t_p, and t_n [Eqs. (3.25)–(3.27)], we obtain

$$\tau_0=\mu_\theta\tau_\theta/(\mu_e+\mu_h) \quad (4.50)$$

and Eq. (4.44) becomes

$$\gamma_{\text{eff}}=[2k_BTk/(qE_0)](\tau_n\theta_p+\tau_p\theta_n)^2 \times(\mu_e\mu_h)/\{\tau_n\tau_p[(\mu_e\theta_n)^2-(\mu_h\theta_p)^2]\} . \quad (4.51)$$

This expression is useful for obtaining $\mu\tau$ information when the drift mobility of one carrier exceeds that of the other since, as can be clearly seen from Eq. (4.44), it yields essentially the ratio between the minority- and majority-carrier $\mu\tau$ values.[26] On the other hand for higher electric fields [which correspond to the condition given by Eq. (4.46)] we have the extremely simple relationship

$$\gamma_{\text{eff}}=2k_BT/(qE_0^2\mu_\theta\tau_\theta) . \quad (4.52)$$

We found then that for this higher end of the high-electric-field regime we get the very desired $\mu_\theta\tau_\theta$ product. Combining either of the above results with the ambipolar diffusion length yields then accurate values for the $\mu\tau$ products of both carriers. In the present "intrinsic" case, but when $\mu_e\theta_n \gg \mu_h\theta_p$, this result provides an accurate determination of the majority-carrier $\mu\tau$ product. Note that in *a*-Si:H under those conditions one actually measures[22] $\mu_\theta\tau_\theta \simeq (\mu_e\tau_p)\theta_n/\theta_p$, in contrast with the quantity measured in time-of-flight experiments which is $\mu_e\tau_n$.

V. SUMMARY AND DISCUSSION

In view of the failure of known experimental methods to determine the $\mu\tau$ product of the minority carrier in amorphous and polycrystalline (thin-film) materials and the general inaccuracy in determining the $\mu\tau$ product of the majority carrier under steady-state conditions, it appears that the photocarrier grating (PCG) method is a unique tool for both purposes. From the basic physics point of view this tool can yield information regarding the state distribution in the lower half of the band gap (e.g., by temperature[20] and light intensity[25] variations) as well as information regarding surface transport and kinetic processes[14] (e.g., by film thickness[27] and light wavelength[28] variations). The other advantage of the PCG method is its potential of becoming a good predictor for device optimization. From this point of view one would like to find a characteristic property of the semiconducting material that may predict the quality of the device made of it.[29] In spite of the recent progress in the development of thin-film devices in general, and those made of *a*-Si:H in particular,[30] this challenge has not been met in a satisfactory manner. Some of the properties (such as photoconductivity), while being very sensitive to the "quality" of the material, do not yield an indication as to the performance of the device (e.g., solar cell) made of this material. The other group of properties, in particular those based on sensing the density of states [such as the photothermal deflection spectroscopy (PDS) method] while being excellent coarse predictors, are insensitive to material variations in the range over which solar-cell performance changes drastically.[23] Hence, there is a need for an experimental property that will be both a good predictor and a sensitive one. For thin-film devices these two requirements seem to be fulfilled by the PCG technique. In particular, since solar-cell performance is very sensitive to the minority-carrier diffusion length[13,30] or the two-carrier drift lengths[12,30] and since experimentally the PCG method[14,15] enables the determination of the corresponding $\mu\tau$'s with a high accuracy (e.g., 5%, for L) this technique appears to be the *best* method, known at present, for the fulfillment of the above two requirements. Preliminary experimental results indicate indeed that very fine changes in the material (e.g., minute doping[25] or gentle light soaking[26]) significantly affect both the solar-

cell performance[23] and the $\mu\tau$'s. We note that the methods previously used for this purpose[31] such as SPV (even if their interpretation is accepted; see, however, Sec. I and Ref. 11) have yielded a much lower accuracy in the determination of L. Hence, even if the results of these methods have some predictive value they are definitely, unlike the PCG method, impractical for the fine tuning of materials used in devices.

Following the above considerations and the wide use of the PCG technique[19–21] one has to understand the experimental results that are obtained by it in order to make it useful for reliable derivation of the photoelectronic parameters. Since the experimental application of the method was shown to be relatively simple,[14,15] there is only a need to ensure a correct interpretation of the measurements. In particular, it is important to evaluate the experimental situations under which the experimental results can be associated uniquely with the $\mu\tau$ products of both carriers. The detailed description of the physical processes in the PCG configuration and the analysis presented in this paper are a response to this need.

Using the coupled continuity equations of an intrinsic photoconductor, we have found that for a PCG of a small enough amplitude (which is superimposed on a uniform distribution of photogenerated carriers) a general analytic solution of the conductance can be derived. This system provides one of the very few cases where analytic solutions of the coupled electron and hole continuity equations can be found.

A result of significant importance, made possible by the $E_0 \to 0$ analytic solution,[18] is our definition of the "ambipolarity" coefficient f, in the case where the applied electric field is much smaller than the "diffusion field" (1600 V/cm) at room temperature. This f is given by[18]

$$f=[1/(\tau_d\theta_p)+k^2D_h]/[1/(\tau_d\theta_n)+k^2D_e] . \quad (5.1)$$

The knowledge of this coefficient is essential for the determination of the correct ambipolar diffusion length:

$$L_\theta=(2k_BT/q)(\mu_e\theta_n\mu_h\theta_p\tau_\theta)/\mu_\theta , \quad (5.2)$$

from the experimentally measurable parameter γ_{eff} [or β, see Eq. (2.24)]. If f is not known or is not equal to its ambipolar value, $f=\theta_n/\theta_p$, the apparent ambipolar diffusion length, which is determined from the measurement of β by using Eqs. (2.24), may only yield an upper bound for the true ambipolar diffusion length. We note that this diffusion length is the quantity that is associated with the minority-carrier transport in devices, i.e., in uniformly illuminated photoconductors. The analytic expression given by (5.1) further tells us how to approach the ambipolar limit in order to determine the desired diffusion length L_θ. It is seen that the stronger the applied uniform illumination F_0 and the larger the grating period Λ, the closer will be the experimentally derived length L_{eff}, which may be defined from the measured γ_{eff} by

$$\gamma_{\text{eff}}=1/(1+k^2L_{\text{eff}}^2) , \quad (5.3)$$

to the true L_θ. As we pointed out previously,[18,26] for *a*-Si:H under AM1- like illumination $L_{\text{eff}} \approx L_\theta$. This is due to trapping effects which yield for this material $\theta_n, \theta_p < 10^{-1}$.

Turning to the most general case, which includes the drift terms (that result from the application of the field E_0), we found that the analytic solutions are complicated expressions of the basic transport parameters (i.e., mobilities), the basic kinetic parameters (i.e., recombination times), and the uniform carrier concentration n_0. Hence, while the academic interest in the general behavior of the conductance of the PCG configuration is satisfied, comparison with the experiment does not enable the derivation of the $\mu\tau$ parameters (or a useful combination of them). Following the success of the $E_0 \to 0$ case discussed above,[15–21] it was natural to expect that the other extreme $E_0 \to \infty$ (i.e., neglecting the diffusion and space-charge terms) would yield a simple expression for the conductance of the PCG. Indeed, this expectation is fulfilled but the result is that the contributions of the two carriers to the conductance cancel each other. Hence, conductance measurements under these conditions cannot distinguish between this $E_0 \to \infty$ limit, and the complete blurring of the carrier gratings. Correspondingly, this limit is useless for the derivation of the above parameters from experimental data.

A useful result was achieved in this work by considering the case where the space-charge term is negligible, compared with the diffusion term while the latter is small, but not negligible, compared with the drift term. This situation can be achieved experimentally since it can come about as a result of using low illumination levels and high electric fields. It turns out that for *a*-Si:H, light intensities of 1.5 mW/cm^2 ($\approx$0.015 AM1) and electric fields of the order of 10^4 V/cm will yield these conditions. We found that under such conditions and with increasing electric fields the measurable parameter γ_{eff} is given by

$$\gamma_{\text{eff}}=[(2k_BT/q)/(\mu_\theta\tau_\theta)](1/E_0^2) . \quad (5.4)$$

Hence, the slope of a γ_{eff} vs $1/E_0^2$ plot will yield the sum of the mobility-lifetime products. For the rather general Shockley-Read-type recombination (when trapping effects are considered) this sum is given by the value of $\mu_\theta\tau_\theta$. Combining the experimental result under these conditions with the result for L_θ, which is achieved under ambipolar carrier distribution [Eq. (5.2)] enables a unique determination of the $\mu\tau$ products of *both* carriers. While one may argue that the $\mu_\theta\tau_\theta$ product can be derived directly from the simple measurement of the photoconductivity, which is given by [Eq. (3.37)]

$$\sigma=qG\mu_\theta\tau_\theta , \quad (5.5)$$

one knows that the accuracy derived from the corresponding measurement is much smaller. The reason is that the uncertainties[26] in G (due to light scattering, light reflection, light intensity distribution and back reflection, and unknown quantum efficiency) and in σ (sample dimensions, contact effects) can accumulate to an error of a factor of about 2. A factor of 2 in the value of $\mu_\theta\tau_\theta$ may be important, both from the basic physics (e.g., ratio be-

tween time-of-flight and steady-state $\mu\tau$'s[22]) and the device physics (e.g., such a factor is expected to cause a significant variation[12] between fill factors, 0.66 instead of 0.56, in the operation of *a*-Si:H solar cells) points of view. We should point out that our proposed determination of the $\mu_\theta\tau_\theta$ product under specific field and light intensity conditions does not limit the experimental derivation of the products to these conditions. This is since one can use a single γ_{eff} vs E_0 measurement [see Eq. (5.4)] in order to determine a reference $\mu_\theta\tau_\theta$ point, and then obtain the various dependences of $\mu_\theta\tau_\theta$ by the measurements of the corresponding dependences of σ.

In conclusion, the present analysis makes the photocarrier grating method a practical and reliable method for determining the minority- and majority-carrier $\mu\tau$ products. This is true in particular for materials for which the determination of these products is difficult because of their relatively small $\mu\tau$ values.

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APPENDIX

In all previous calculations of the photoconductance in the PCG configuration,[17,18] as well as in the present paper, the inhomogeneous transport equations have been solved for Δp and Δn. The question that arises is whether those solutions are the most general ones, i.e., whether the solutions of the corresponding homogeneous equations are the same as those of the inhomogeneous equations. Apart from the interest in the mathematical completeness of the problem this question has an interesting nontrivial[23] physical meaning, i.e., do the carriers form sinusoidal gratings that follow the exciting sinusoidal illumination or is the grating periodic but nonsinusoidal? In principle, while the periodicity is inherent to the problem, the carrier diffusion and drift may distort the sinusoidal shape (by creating, say, a squarelike or, in the case of an applied field, a sawtoothlike grating). This is not a trivial problem since, for example, it is obvious that for a square-wave-imposed illumination the carrier gratings under finite diffusion will not be square waves. Hence the form of the homogeneous solutions can shed light on the physical processes in the PCG. Here we argue that even in the most general case discussed in this paper (i.e., of the intermediate fields) the carrier gratings due to a sinusoidal generation are always sinusoidal. Consequently with the boundary conditions of the problem the solutions of the homogeneous equations are the same as those of the inhomogeneous equations. A rigorous proof for this argument, which is based on the two-carrier picture, will be given elsewhere.[23] For brevity we just outline here the one-carrier solution of this problem.

It is well known[1] that the solutions of the one-carrier homogeneous equation are of the form

$$\Delta p \propto \Delta G \exp[-(x-x_0)/L_d] , \quad x > x_0 , \tag{A1}$$

where L_d is a typical decay distance of the carrier concentration in the direction of the applied field and x_0 is the position of the carrier generation. Similarly,

$$\Delta p \propto \Delta G \exp[-(x_0-x)/L_u] , \quad x < x_0 , \tag{A2}$$

where L_u is a typical decay distance in the opposite direction. One notes, of course, that in the two-carrier problem the corresponding L_d and L_u are different from those of the one-carrier problem given in the classical textbooks[1,2] and more exponential terms are to be included.[23] If we assume that the carriers are generated at a narrow slit of width dx_0 and the absorbed illumination flux is ϕ_0, then $\Delta G \propto \phi_0 dx_0$. In our case $\phi_0 \propto \cos(kx_0)$ and thus the combined contribution of all narrow slits to the carriers concentration at x will be

$$\Delta p \propto \Delta G \exp\left[\frac{-x}{L_d}\right] \int_{-\infty}^{x} \exp\left[\frac{x_0}{L_d}\right] \cos(kx_0) dx_0 + \exp\left[\frac{-x}{L_u}\right] \int_{x}^{\infty} \exp\left[\frac{-x_0}{L_u}\right] \cos(kx_0) dx_0 . \tag{A3}$$

Hence,

$$\Delta p \propto [k^2+(1/L_d)^2]^{-1}\{[\cos(kx)]/L_d + k\sin(kx)\} + [k^2+(1/L_u)^2]^{-1}\{[\cos(kx)]/L_u - k\sin(kx)\} . \tag{A4}$$

In the general two-carrier case more terms of the same type are to be included[23] but this does not change the end result, i.e., that the general solution of the homogeneous equations can be written in the form

$$\Delta p \propto \cos(kx+\Psi) . \tag{A5}$$

If no electric field is introduced, $L_d = L_u$ and $\Delta p \propto \cos(kx)$ as suggested previously.[17,18] Hence, the solutions of the homogeneous equations have the sinusoidal form. One can further show[23] that the PCG boundary conditions provide exactly the same solutions for the homogeneous equations as those of the inhomogeneous equations. This leads to the conclusion that the solutions [Eqs. (3.11) and (3.12)] suggested in this paper [i.e., solutions of Eqs. (3.37)–(3.40)] are indeed the general solutions of the PCG configuration.

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