Possibility of quenching the integer-quantum-Hall behavior with increasing lattice asymmetry

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Conventional theories of integer-quantum-Hall behavior require the presence of generalized Landau levels separated by gaps. By numerically studying several models of electron systems in a magnetic field, we find that an asymmetry in the hopping strength parameters can lead to a closing of the gaps. The systems studied are the tight-binding case, as well as generalizations including next-nearest-neighbor interactions. The gap closing begins at the center of the band and leads to a curve in Fermi-energy and asymmetry-parameter space that we interpret as a phase boundary. Too much asymmetry leads to a phase transition from quantum Hall to nonquantum Hall behavior.

INTRODUCTION

The explanation of the integer-quantum-Hall effect by Laughlin,[1] Thouless *et al.*,[2] and Středa[3] is remarkably general. It seems that integer-quantum-Hall (QH) behavior is an almost commonplace property, implied by gauge invariance and the existence of an elegant topological invariant.[1,2] The circumstances where QH behavior would *not* be observed are therefore interesting, so that much study has been invested recently on negative conditions such as mixing of Landau levels[4] and tunneling effects.[5]

Equally interesting are the effects of asymmetry of the periodic two-dimensional (2D) lattice. The subject of electrons in a periodic system and magnetic field is far from trivial, and the applied magnetic field introduces incommensurability between lattice and magnetic translation periodicity to the 2D electron systems.[6] The localization and delocalization properties of electron states in a magnetic field and its relation to disorder in the material are still being debated.[7,8]

In this paper we study the effects of lattice asymmetry using model 2D systems of several types. One familiar system is the tight-binding Hamiltonian

$$H=-\sum_{\langle i,j\rangle} t_{ij}\hat{c}_i^{\dagger}\hat{c}_j e^{i\vartheta_{ij}} , \tag{1}$$

where $\hat{c}_i$ is the usual Fermion operator at site i, and the magnetic phase factor $\vartheta_{ij}=(2\pi e/hc)\int_i^j \mathbf{A}\cdot\mathbf{dx}$ is defined on the link from i to j. The coupling constants t_{ij} can introduce lattice asymmetry. For example, for a square lattice with nearest-neighbor couplings, we take the ratio of hopping constants in the y direction (t_y) to the x direction (t_x) to be given by $\lambda=t_y/t_x$. We call λ the "asymmetry parameter," as this ratio plays an important role. For $\lambda\simeq 1$, the case of small asymmetry, the spectrum exhibits generalized Landau levels and gaps necessary for QH behavior. This is the usual case. Surprisingly, for $\lambda\gg 1$, the case of large asymmetry, a region of the spectrum shows unexpected closing of the gaps, implying loss of mobility gaps (in the presence of disorder) and loss of QH behavior. It is as if large λ channeled some of the electrons into quasi-one-dimensional (1D) flow, quenching the 2D phase coherence needed for QH behavior, at least for some of the states. The phenomenon of quenching by asymmetry seems to occur rather generally, as we conclude by investigating other types of periodic potentials. It does *not* require t_x or t_y to be "large" because the critical Fermi energy (defined later) ε_{FC} depends only on the ratio λ.

The critical value of λ at which QH behavior is lost depends on the Fermi energy ε_F of the system. Phenomenologically, we argue that too much asymmetry leads to a phase transition of the magnetoconductance of some states from QH to non-QH (NQH) behavior. Then, in Fermi energy and asymmetry parameter space, a critical line separates QH and NQH regions. Searching for the predicted phase boundary experimentally can be a test of existing theories of the QH effect.

METHODOLOGY

In the first-quantized formalism the Hamiltonian H is given by replacing the operators $\hat{c}_i$ and $\hat{c}_i^{\dagger}$ by the wave function c_i and its complex conjugate c_i^*. The energy eigenstates of (1) are given by Harper's equation for the nearest-neighbor coupling case. The equivalence is well known, but we will review it here for completeness and to specify our notation.

Considering $H(c_i,c_i^*)$ as a functional of c_i and c_i^*, we first generate the equation of motion

$$\frac{\delta H}{\delta c_i^*}=i\hbar\dot{c}_i ,$$

obtaining

$$\sum_{j\ (\neq i)} t_{ij}c_j e^{i\vartheta_{ij}}=Ec_i \tag{2}$$

for the energy eigenstates. We let the index $i=m,n$, where m and n are discrete site coordinates in the x and y directions, respectively. Choosing Landau gauge equivalent to $\mathbf{A}=B(0,x,0)$, the phase factors ϑ_{ij} are zero for links along the x direction and $2\pi m\phi/\phi_0$ in the y direction. Here $\phi_0=hc/e$ is the elementary quantum of magnetic flux, and the flux through a unit cell is ϕ. Our nearest-neighbor hopping parameters are

$$t_{mn;m'n'}=t_x\delta_{nn'}\delta_{m,m'\pm1}+t_y\delta_{mm'}\delta_{n,n'\pm1} .$$

Inserting these into the sum in (2) and making the ansatz $c_{mn}=c_m\exp(-i\nu n)$, we obtain

$$t_x(c_{m+1}+c_{m-1})+t_y(c_m e^{-i\nu+i2\pi m\phi/\phi_0} + c_m e^{i\nu-i2\pi m\phi/\phi_0})=Ec_m$$

or, setting $t_y/t_x=\lambda$,

$$c_{m+1}+c_{m-1}+2\lambda\cos\left[2\pi m\frac{p}{q}-\nu\right]c_m=\varepsilon c_m , \tag{3}$$

with $\varepsilon=E/t_x$, and setting the reduced flux $\phi/\phi_0=p/q$, a ratio of integers p and q. Equation (3) is known as Harper's equation. We will be concerned with the dependence of the eigenvalues on the parameter λ.

The case $\lambda=1$ of no asymmetry is called the self-dual point and is rather special. For $\lambda=1$, Hafstadter[9] solved (3) for the union of all eigenvalues ε associated with bound wave functions in an infinite system. The results are well known: a spectacular, butterflylike pattern of eigenvalues versus magnetic flux showing interesting fractal structure. This case is assumed in most discussions which consider the effects of periodic lattice potentials. The behavior of the system for $\lambda\neq1$ is not obvious.

DUALITY AND THE CASE OF LARGE ASYMMETRY

For $\lambda>1$ it is known that the solutions to (3) are exponentially localized in the j variable.[10,11] This result can be made intuitively plausible for large λ at least. Equation (3) has the form of a discretized 1D Schrödinger equation with a potential V_m given by

$$V_m=2\lambda\cos\left[2\pi m\frac{p}{q}-\nu\right]+2 .$$

The periodic wells in this analogy are deep for $\lambda\gg1$, and the conflict between the lattice and p/q periodicities should lead to localization, as claimed. Now, by the duality symmetry of Harper's equation[11] electrons are delocalized in the y direction if localized in the x direction (and vice versa). For $\lambda\gtrsim1$, electrons should move in the delocalized y direction much more easily than in the localized x direction. For $\lambda\gg1$ we argue that the system becomes quasi-one-dimensional.

For very small λ, which is again a case of high asymmetry, we have the same conclusion. For example, setting lattice lengths $a=b$ for simplicity, switching $\lambda\rightarrow1/\lambda$ is equivalent to changing x and y. The spectrum of the square-lattice system is invariant under a rotation by 90°, so Harper's equation must have a symmetry under this transformation.[6,12] Indeed, the symmetry is none other than duality symmetry, and is important in establishing the exponential localization. From the duality symmetry, then, we anticipate quasi-1D behavior for either large or small λ.

There is another, complementary way of obtaining Harper's equation, used by Thouless *et al.*,[2] which we will also review. We consider the 2D continuum single-particle Hamiltonian

$$H=\frac{1}{2m}(\mathbf{p}-e\,\mathbf{A})^2+V(\mathbf{x})=H_0+V(\mathbf{x}) , \tag{4}$$

with

$$V(\mathbf{x})=2V_1\cos(G_1x)+2V_2\cos(G_2y) ,$$

where $G_1=2\pi/a$, $G_2=2\pi/b$, and a, b are lattice parameters. Solving $H_0\psi_n^0=E_n^0\psi_n^0$ in Landau gauge analytically gives Landau levels with energies $E_n^0=(n+1/2)\hbar\omega_c$, where $\omega_c=eB/m$ is the cyclotron frequency, and eigenfunctions

$$\psi_{nk}^0(x',y)=e^{iky}\phi_n(x') ,$$

where

$$\phi_n(x')=N_n e^{-(1/2)\alpha^2x'^2}H_n(\alpha x') ,$$
$$x'=x-\hbar k/eB . \tag{5}$$

Here $\alpha=\sqrt{m\omega_c/\hbar}$, $N_n=(\alpha/\sqrt{\pi}2^n n!)^{1/2}$, and H_n is the nth Hermite polynomial. Each Landau level is highly degenerate because E_n^0 is independent of k. The lattice potential $V(\mathbf{x})$ will break the symmetry associated with k degeneracy and cause mixing of different k's. Assuming $V(x,y)$ is weak enough so that broadened Landau levels do not overlap with each other, we expand the single-electron wave function $\psi(x,y)$ in terms of the unperturbed states within one Landau level as follows:

$$\psi_{nk}(x,y)=\sum_j f_j\exp[i(k+jG_2)y]\phi_n(x'-jG_2) . \tag{6}$$

The above ansatz implements Bloch translational symmetry $\psi_{nk}(y+b)=e^{ikb}\psi_{nk}(y)$. After some algebra, it can be shown that

$$V_2'(f_{j+1}-f_{j-1})+2V_1'\cos\left[2\pi j\frac{\phi_0}{\phi}+\nu\right]f_j=\tilde{E}f_j , \tag{7}$$

where

$$V_1'=V_1\exp[-\tfrac{1}{4}2\pi(\phi_0/Ba^2)] ,$$
$$V_2'=V_2\exp[-\tfrac{1}{4}2\pi(\phi_0/Bb^2)] ,$$
$$\nu=G_1x_0=G_1(\hbar k/eB) ,$$

and

$$\tilde{E}=2(E-\tfrac{1}{2}\hbar\omega_c) .$$

Again we recover Harper's equation (3), except that in this case $\lambda=V_1'/V_2'$ and $\varepsilon=\tilde{E}/V_2'=(\tilde{E}/V_2)\exp[\frac{1}{4}2\pi(\phi_0/Bb^2)]$. Semiclassically it is easier for the electrons to move along the direction of the most shallow

potential. For a potential that is deep in the x direction and shallow in the y direction, we have $\lambda \gg 1$. Similarly, in the tight-binding picture we have $\lambda = t_y/t_x \gg 1$ for the same effect. But the remarkable thing about the continuum picture is that the overall scale of the potential V is exponentially suppressed and becomes irrelevant in (7); only the ratio λ is important. This explains the remark made earlier that the size of the hopping constants did not need to be large, in any physical sense, for us to discuss large asymmetry.

QUANTIZATION AND FINE STRUCTURE

We adopt the tight-binding picture, in which the magnetic field splits a particular band into generalized Landau levels. The Kubo formula for the off-diagonal magnetoconductance tensor element σ_{xy} from a single level is

$$\sigma_{xy} = \frac{e^2}{h} \frac{1}{2\pi i} \int \int d\mathbf{s}_{\mathbf{k}} \cdot \nabla_{\mathbf{k}} \times \langle \psi(\mathbf{k}) | \nabla_{\mathbf{k}} | \psi(\mathbf{k}) \rangle \ , \tag{8}$$

where $|\psi(\mathbf{k})\rangle$ is the eigenstate, $ds_{\mathbf{k}}$ is the area element in $\mathbf{k}$ space, and integration is over the reduced Brillouin zone. Applying Stoke's theorem (8) becomes

$$\sigma_{xy} = \frac{e^2}{h} \frac{1}{2\pi i} \oint d\mathbf{k} \cdot \langle \psi(\mathbf{k}) | \nabla_{\mathbf{k}} | \psi(\mathbf{k}) \rangle \ . \tag{9}$$

The value of the Hall conductivity is thus related to a global quantity, the winding number (or Chern number) of the phase of the wave function. The winding number is necessarily an integer when the mapping from the 2D wave-number space $\mathbf{k}$ to the phase is everywhere continuous.

Much discussion has focused on the topological necessity for nontrivial integer quantization of the conductance[2] when the $\mathbf{k}$ space is periodic in the x and y directions, i.e., equivalent to a torus. However, topology alone does not take into account the peculiar sensitivity of the size of the magnetic Brillouin zone to the magnetic flux ϕ/ϕ_0. For $\phi/\phi_0 = p/q$ rational, the magnetic Brillouin zone is q times smaller than its naive periodic size of $2\pi/a$, where a is the lattice spacing. A slight change in the magnetic field can create an enormous change in the size of the zone boundary in the topological discussion. Related to this, the number of subbands and clusters inside a given band is extremely sensitive: there are p subbands inside each generalized Landau level. For a physical field, p and q are not very well defined since nearly equal rational numbers can have drastically different p's and q's. Thus it is not obvious how the Hall quantization value is stable under tiny changes in the B field.

The issue has been discussed in the literature,[13] and some numerical calculations,[9,14] support the fact that the cluster gaps are stable under small variations of magnetic field. MacDonald has discussed the recursive fractal structure of the symmetric, $\lambda = 1$ tight-binding spectrum which is related to these issues,[15] and applied the same counting technique as that of Thouless *et al.*, but to a finite system which includes the edge states.[16] In the following we show how the details of the fine structure within clusters conspire to make the Hall conductance insensitive to small variations in the rational p/q field values in the tight-binding case.

We consider several nearly equal fields, $p/q \simeq p'/q'$, but q is much different from q'. An example is $p/q = \frac{1}{11}$, $p'/q' = \frac{4}{43}$, $\frac{3}{32}$, and $\frac{2}{21}$, giving rise to 11, 43, 32, and 21 subbands, respectively. The empirically discovered clustering[9] of p or p' subbands leads to spectra which are physically similar, ignoring tiny gaps between subbands in each cluster.

The appearance of the spectra is far from the whole story. Laughlin's argument[1] shows that each subband, no matter how infinitesimally split, carries a whole unit of Hall current. Since tiny changes in the B field induce possibly huge changes in the number of subbands, there is potentially a problem for the stability of the Hall current.

Thouless *et al.* resolved this paradox for the perturbative case (4) and argued that the same is true for the tight-binding case due to the duality symmetry. We show here explicitly the stability of the cluster currents in the tight-binding case. Let t_r be the integer Hall conductivity, i.e., $\sigma_{xy} = (e^2/h)t_r$. Recall the Diophantine relation

$$r = qs_r + pt_r \tag{10}$$

for the rth level, where $1 \le r \le q$, t_r and s_r are integers, and $|t_r| \le q/2$.[2] The current in the rth band is $I_r = t_r - t_{r-1}$. By direct calculation one can then verify that the total current in one cluster is insensitive to the detailed subarrangements of currents in the subbands within clusters, though the currents in the subbands themselves show rather drastic variations. Some examples for $p/q = \frac{1}{11}$, $\frac{4}{43}$, $\frac{3}{32}$, and $\frac{2}{21}$ are shown in Fig. 1. Reflecting on this, one finds that the cancellation across sums over r inside clusters,

$$\sum_{r=r_0}^{r_0+p} I_r = \sum_{r=r_0}^{r_0+p} (t_r - t_{r-1}) \ ,$$

has indeed been concocted to guarantee a stable cluster current. For our purposes, the conclusion is that the subband currents are practically irrelevant, and so is the fine

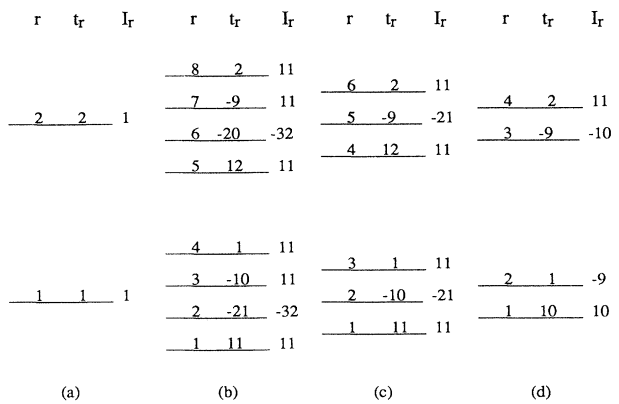

FIG. 1. Hall conductances t_r and currents I_r for the two lowest Landau levels calculated by Diophantine equation (10) for adjacent rational fields p/q. (a) $p/q = \frac{1}{11}$, (b) $p/q = \frac{4}{43}$, (c) $p/q = \frac{3}{32}$, and (d) $p/q = \frac{2}{21}$.

structure, however beautiful. Similar conclusions supporting this point have been given in Refs. 17–20. The cluster currents are the real physical measurable currents. Our calculations will concentrate on dependence of large gaps between clusters which can have measurable effects. In particular, we study how large enough λ can close large cluster gaps.

GAP CLOSING

It is commonly accepted that the existence of band or mobility gaps is a necessary condition for quantization of the Hall conductance. We assume this to be true, associating the mobility gaps due to the presences of impurities or disorder with the cluster band gaps we calculate. Thus, if the cluster gaps close, there are reasons to believe that the quantization of the Hall conductance will fail. At least one study, done by Aoki,[19] provides a specific example where this is true (although the context was a case in which large disorder caused the gaps to close). Topologically, two independent tori on which the wave function is specified become joined at a gap closing region. Unless there are special boundary conditions, the nontrivial integer winding number can disappear at such a point, i.e., deformed continuously to zero.

This point can be illustrated simply, but a complete discussion would rapidly become very involved. Consider the magnetic Brillouin zone of two bands which touch as two tori, joined at a "neck." The winding number can be physically visualized as due to a topological "flux" running inside the holes of the tori. Once tori are joined, certain loops around one torus can be deformed around the other. Now, the flux inside one torus can cancel that of the other, implying a change in the QH quantum number. It is tempting to pursue the mathematics of the mapping onto two- and multihole tori but we are not sure it is relevant. Indeed there are, for a two-state system, many new degrees of freedom, equivalent to a U(2) system rather than $U(1)\times U(1)$. Unless there is a miraculous match between the space involved, one expects little of topologically importance. This is our expectation after not only one, but two, three, or more gaps between clusters close: the space becomes so large that topological barriers become rare, if not impossible, and irrelevant. For practical purposes, we believe QH behavior will cease after many states mix. We emphasize that this is a practical statement: one cannot forbid something interesting, but in an experiment one should not expect it.

QUENCHING

The 1D localization is something of an indication that QH behavior may disappear for large λ. Thus we numerically study the gaps, and the entire spectrum, for Harper's equation with both infinite and strip-geometry finite systems. For an infinite, symmetric system we show the splitting of a single band into subbands for several p/q in Fig. 2. Consistent with symmetry arguments[9] for integer $p>1$ there are p subbands which tend to cluster together and hence create bigger gaps between clusters. The number of subbands in the central cluster is given by

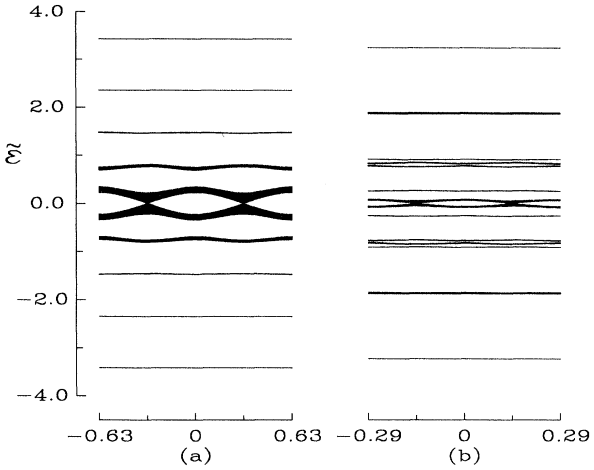

FIG. 2. Reduced energy $\tilde{\varepsilon}$ vs wave number ν showing the split of a single band into subbands for the infinite system with no anisotropy ($\lambda=1$) for two rational fields (a) $p/q=\frac{1}{10}$ and (b) $p/q=\frac{3}{22}$. Twice the size of the reduced Brillouin zone $-2\pi/q\le\nu\le 2\pi/q$, is shown.

the remainder of $q/2p$; for example, if $q=11$ and $p=3$, then there are five subbands in the central cluster. Numerical studies on the symmetric $\lambda=1$ system confirm this behavior.[9,14,17–19] For comparison, a strip-geometry system has been solved with an independent numerical method and it serves to check our calculation. The spectrum of the strip-geometry system (Fig. 3) indeed shows very similar structure except there are extra states intruding into the gap region. The gaps in the finite system are thus "quasigaps." These extra states are called edge states due to the fact that electrons in these states are localized at the edges of the system.[8,16,18,21]

For $\lambda\neq 1$, we found, by Hofstadter's method,[9] that the

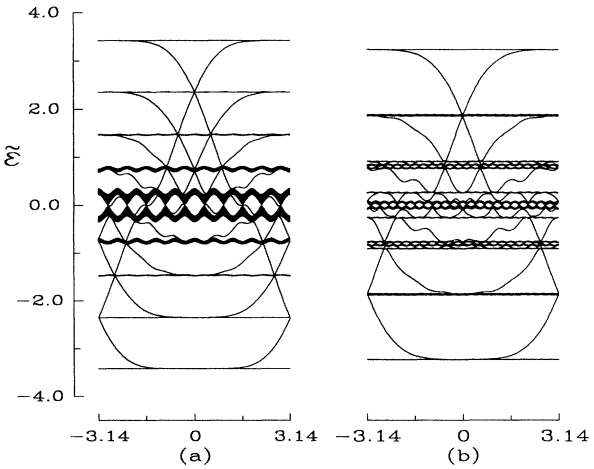

FIG. 3. Reduced energy $\tilde{\varepsilon}$ vs wave number ν showing the split of a single band into subbands for the finite system with no anisotropy ($\lambda=1$) for two rational fields (a) $p/q=\frac{1}{10}$ and (b) $p/q=\frac{3}{22}$. Notice edge states appear in the gap region.

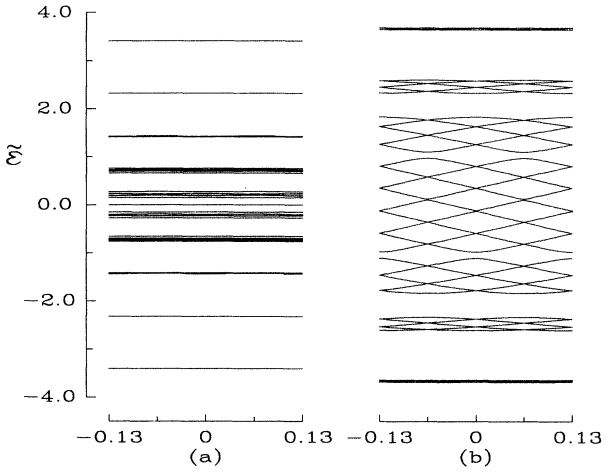

FIG. 4. Reduced energy $\tilde{\varepsilon}$ vs wave number ν for two different asymmetry parameter λ's for the infinite system with $p/q=\frac{5}{49}$. $\lambda=1.0$ and 2.0 for (a) and (b), respectively. Twice the size of the reduced Brillouin zone, $-2\pi/q \le \nu \le 2\pi/q$, is shown.

upper and lower bound on the eigenvalue ε obeys $|\varepsilon| \le 2(\lambda+c)$, where $c<1$ is magnetic-field dependent. Thus, for easy comparison with the $\lambda=1$ case, we plot the normalized eigenvalue $\tilde{\varepsilon}=2\varepsilon/(\lambda+1)$ which is bound within the closed interval $[-4,4]$. Figures 4 and 5 show the subband structure for different λ's for infinite and finite systems, respectively. It is evident that lattice asymmetry reduces the band-gap width for both systems. A gap at a particular energy will eventually vanish as the lattice asymmetry reaches a critical value. We note that the gaps between subbands in one cluster require only a small amount of asymmetry to close, while the gaps between clusters are relatively difficult to close. The closing of gaps always starts from the band center.

The value of the energy at which the gaps close is independent of the magnetic field. This is true for all fields studied (Fig. 6); for small p/q, the physical case, the critical Fermi energy (defined later) is quite sharp. We have not found an analytic explanation for this phenomenon.

A global survey of the spectrum is shown in Fig. 7. One can compare Fig. 7(a), reproducing Hofstadter's calculation[9] at $\lambda=1$; with Fig. 7(b), the same system at $\lambda=1.5$. Already at $\lambda=1.5$ the spectrum has rearranged drastically, while retaining some of the fractal butterfly pattern.

The gap and nongap regions have a boundary as shown in Fig. 8. Suppressing an additive constant energy scale, we define the energy which separates two regions as the critical Fermi energy ε_{FC}. The system in the nongap region should behave rather like a metal and a simple Drude model gives Hall conductance $\sigma_H=nec/B$, a linear function of inverse magnetic field. As λ is varied with the Fermi energy of the system in the vicinity of the critical Fermi energy, there should be a phase transition in the magnetoconductance of the system. Similarly, shifting the Fermi level by changing carrier density at fixed λ can also accomplish the transition of the system from QH to NQH regions. For small p/q the gap and nongap regions have a well-defined boundary and therefore a sharp transition is expected. In ε_F and λ space, the solid line in Fig. 8 shows a phase boundary line separating QH and NQH regions for an infinite system of the nearest-neighbor-coupling case. Notice that the tiny gap separating the QH and NQH regions has a finite width, the error bars in Fig. 8 reflecting this uncertainty in the definition of the critical Fermi energy. In Fig. 8, the phase boundary for $\lambda<1$ is obtained by the duality symmetry. Our numerical results show that ε_{FC} is rather in-

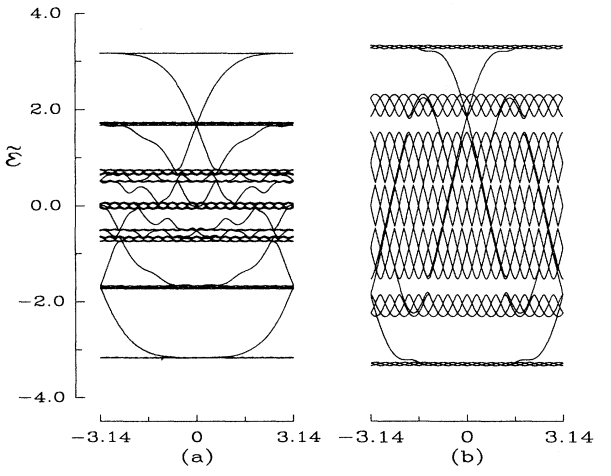

FIG. 5. Reduced energy $\tilde{\varepsilon}$ vs wave number ν for two different asymmetry parameters λ's for the finite system with $p/q=\frac{3}{20}$. $\lambda=1.0$ and 3.0 for (a) and (b), respectively. Starting from the center of the spectrum, gap closing also occurs, similar to that of an infinite system.

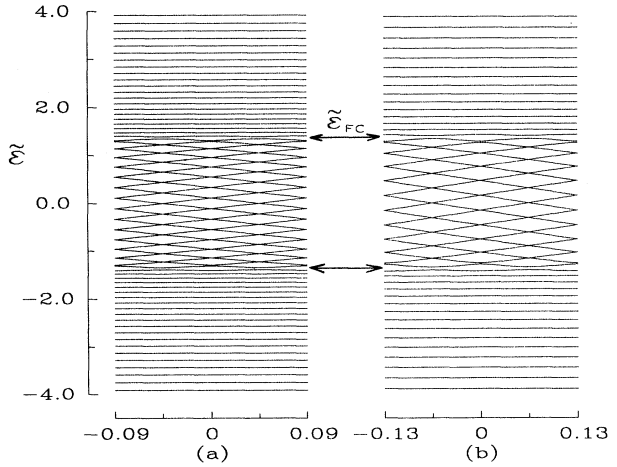

FIG. 6. Reduced energy $\tilde{\varepsilon}$ vs wave number ν for two rational fields at fixed asymmetry parameter λ of 2.0, showing the insensitivity of $\tilde{\varepsilon}_{FC}$ vs magnetic fields. Twice the size of the reduced Brillouin zone, $-2\pi/q \le \nu \le 2\pi/q$, is shown. (a) $p/q=\frac{1}{70}$, (b) $p/q=\frac{1}{50}$.

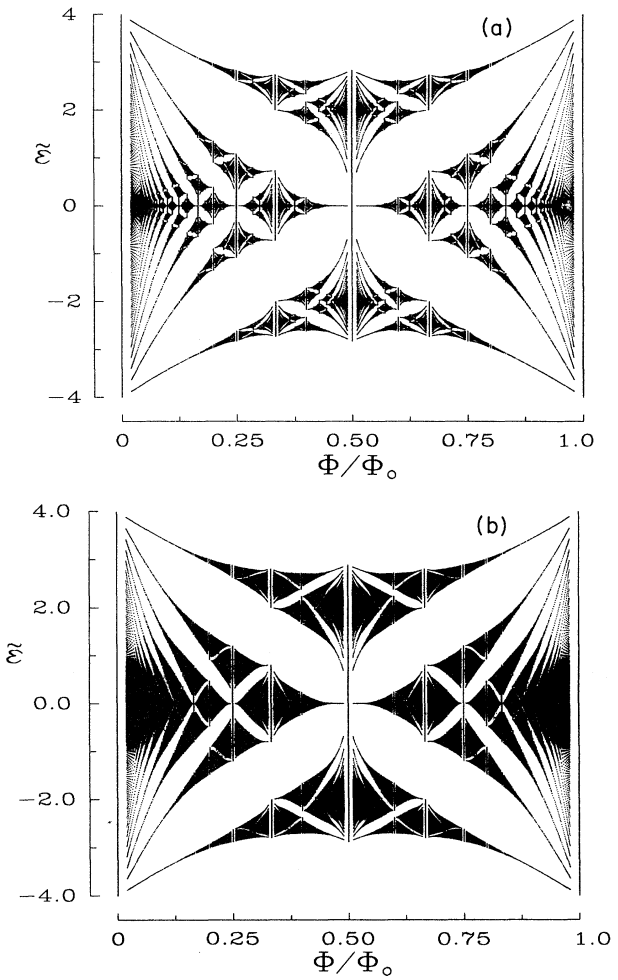

FIG. 7. Complete spectrum of $\bar{\varepsilon}$ vs ϕ/ϕ_0, the reduced flux, for the infinite system. Asymmetry parameter $\lambda=1.0$ in (a) and $\lambda=1.5$ in (b).

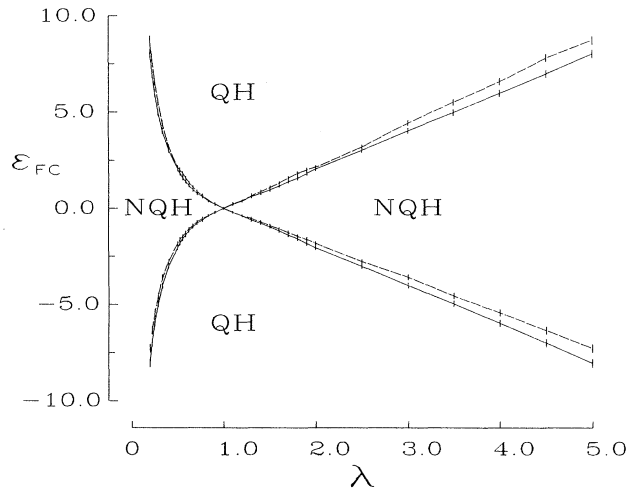

FIG. 8. Regions of quenched and normal quantum Hall behavior. Solid line shows the phase boundary ε_{FC} vs asymmetry parameter λ for the nearest-neighbor hopping Hamiltonian. Dashed line shows the phase boundary calculated for the Hamiltonian including next-to-nearest-neighbor hopping interaction. Error bars display the error in the numerical procedure for calculating the boundary (see text).

sensitive to the value of p/q, i.e., the magnetic field. Thus the predicted critical line should be independent of the magnetic-field strength to a good approximation.

To check that the quenching is a general phenomenon, we considered a family of Hamiltonians given by modifying $t_{mn;m'n'}$ to include next-to-nearest hopping terms, i.e.,

$$t_{mn;m'n'}=t_x\delta_{nn'}\delta_{m,m'\pm1}+t_y\delta_{mm'}\delta_{n,n'\pm1}+t_{2x}\delta_{nn'}\delta_{m,m'\pm2}+t_{2y}\delta_{m,m'}\delta_{n,n'\pm2}\ . \quad (11)$$

Imposing the ansatz $c_{mn}=c_m\exp(-i\nu n)$, after some algebraic manipulations we obtain a generalization of Harper's equation:

$$c_{m+1}+c_{m-1}+\mu(c_{m+2}+c_{m-2})+2\lambda\left\{\cos\left[2\pi m\frac{p}{q}-\nu\right]+\mu\cos\left[2\left[2\pi m\frac{p}{q}-\nu\right]\right]\right\}c_m=\varepsilon c_m\ , \quad (12)$$

where $\mu=t_{2x}/t_x=t_{2y}/t_y$, representing the importance of the next-to-nearest interaction to the nearest. Some study has recently appeared on this kind of system.[20,22] We are interested in the same asymmetry parameter and $\lambda=t_y/t_x$ in (12), the same definition as in (3).

The spectrum of the new system was recalculated and gaps were studied as a function of λ. The phase boundary between QH and NQH behavior was redetermined for $\mu=\frac{1}{5}$ and is shown as the dashed line in Fig. 8. In accordance with our expectations, the phase boundary moves but does not disappear with a reasonably small next-nearest-neighbor interactions. It is rather striking that both curves (and all intermediate curves) show that ε_{FC} is roughly linear in λ for $\lambda\gtrsim1$.

From this study we conclude that QH quenching may be a generic effect of asymmetry. The localization, delocalization, and quasi-1D behavior seem to be generally true and are certainly not pathologies of Harper's equation. Of course, the exact location of the phase boundary does depend on the details of the potential. But, for the examples studied, any natural definition of the asymmetry leads to a universal linear $\varepsilon_{FC}\sim\lambda$ phase boundary for large λ.

PHENOMENOLOGY AND CONCLUSIONS

It would be interesting to test the quenching effect of asymmetry experimentally. We do not pretend to have modeled real materials with the oversimplified models used so far, so our results are hardly quantitative. Nevertheless, we believe the results could be made quantitative without changing the main conclusions.

The most obvious way to test the quenching behavior

would be to use a metal-oxide-semiconductor (MOS) structure in which the semiconductor material has some kind of lattice asymmetry. The MOS system is a three-layer sandwich structure consisting of a gate, the top layer usually made of heavily doped polysilicon or aluminum, a substrate of p- or n-type semiconductor material, and an insulating layer usually made of oxide of semiconductor materials, such as SiO_2, which separates the gate and substrate. The great advantage of the MOS system is its ability to control the density of the electron gas in the 2D inversion layer at the SiO_2-Si interface and hence the Fermi energy of the system by varying the gate-substrate voltage. Experimentally, one would wish to move the Fermi energy to the vicinity of ε_{FC} so that the subsequent variation of the magnetic field is sufficient to move the Fermi energy in and out of the QH region. Therefore one expects a fairly spectacular signal showing the turning on and off of the quantization of the Hall conductance as the Fermi energy of the system is moved back and forth across the critical Fermi energy. However, we note that only a limited number of Landau levels are available for study in the current technology of MOS devices. One may have to custom make a device with high carrier density so that the Fermi energy can be pushed into the region close to the critical Fermi energy ε_{FC}.

One can consider varying λ with hydrostatic pressure. Consider the Hall conductance $\sigma_H(B)$ with λ fixed. For $\lambda=1$, the symmetric case, the width of the plateaus in σ_H is determined by the mobility gap width, assuming a fixed distribution of density of states for localized impurity states in the gap. Recall that $\varepsilon_{FC}(\lambda)$ is essentially independent of B but the gaps becomes narrower for $\lambda > 1$.

Assuming squeezing of a sample will not change the impurity states in any significant way, then the plateau widths in the σ_H versus $1/B$ plot became narrower as λ is increased. If λ can be varied by 10%, there should be a measurable, if not very dramatic, effect.

A second method is to vary λ with B fixed. It is advantageous to adjust B first so that σ_H is close to the edge of a step. Then, squeezing the sample might move the mobility edge and produce an integer jump in the conductance. Enough squeezing, in principle, could push the sample into the NQH region and hence no further integer jumps would be observed by more squeezing. A change in the crystal structure from very high pressure is not necessarily disastrous: it would just correspond to an increase in λ. A transition from QH to NQH behavior here would be a spectacular signal. We note that there are experimental measurements performed under high hydrostatic pressure.[23–25] It is claimed by Gregoris *et al.*[23] that the lattice parameter changes by about 0.1% for 1 kbar pressure, but the systems tested are not conventional in the sense that they do not show QH behavior without applied pressure. Therefore there is a need to test the response of more conventional systems, such as a MOS structure, to an applied hydrostatic pressure, and data may show surprises.

Yet another way to test the ideas is to search for a "scaling law" under which different materials should show regularity. For a given material, the maximum Fermi energy at which QH behavior quenches might be determinable by careful doping. The asymmetry parameter of the material and ε_{FC} fix a point in the $\varepsilon_{FC} \sim \lambda$ plane for each material. Materials with different lattice symmetry have different effective λ's and would exhibit a regularity by lying along a universal, approximately straight line. The problem, of course, is that λ has to be calculated appropriately for the materials, so the assignment of points on the scaling curve would be somewhat model dependent.

Our suggestions are somewhat qualitative since much more study is needed. The basic point that asymmetry effectively decreases the 2D system to something acting like one living in less than two dimensions is certainly fascinating. Quantum Hall quenching will definitely attract increasing attention if it can teach us more about the profoundly interesting behavior of electrons in a magnetic field.

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