RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 44, NUMBER 23 15 DECEMBER 1991-I

# Quantum noises in mesoscopic conductors and fundamental limits of quantum interference devices

Akira Shimizu and Hiroyuki Sakaki*

*Quantum Wave Project, Exploratory Research for Advanced Technology, Research Development Corporation of Japan, Keyaki-house 302, 4-3-24 Komaba, Meguro-ku, Tokyo 153, Japan*

(Received 21 March 1991; revised manuscript received 28 June 1991)

We investigate quantum fluctuations of an electron-interference current in a *perfectly fabricated, single-mode* conductor. We first point out that the magnitude of the Johnson-Nyquist noise at zero temperature, when expressed as the fluctuation in the electron number, depends on the transmission coefficient only. We then clarify the physical origin of the "excess noise." Discussions are also given for a more general case when an interference current is modulated by an external variable $I$, which may be either a classical or quantum-mechanical one. It is found that a possible fluctuation of $I$ and the excess noise generate a new coupled noise. These results are used to discuss fundamental limits of quantum interference devices.

An electric current in a mesoscopic conductor fluctuates randomly for various reasons. For example, the universal conductance fluctuation is caused by some imperfections in the conductor.[1] It does not vanish even after a *long-time average,* so the long-time average is usually considered. A chaotic fluctuation was also found in some conductors, which is due to the multimode nature of the conductors.[2] However, these fluctuations are absent in a *perfectly fabricated, single-mode* conductor, where other fluctuations of more fundamental origins would be found: Measured values would fluctuate from measurement to measurement if a measurement is performed in a finite detection time.[3–6] The purpose of the present work is to study these fundamental fluctuations. We first point out that the magnitude of the Johnson-Nyquist noise,[4,5] at zero temperature, when expressed as the fluctuation in the electron number, depends on the transmission coefficient only. We then clarify the physical origins of the "excess noise," or "quantum shot noise" (QSN).[3,4] Discussions are also given for a more general case when an interference current is modulated by an external variable $I$. The $I$ may be either a classical variable such as a gate voltage,[7] or a quantum-mechanical one such as a photon number.[6] It is found that when $I$ has a finite fluctuation, such as a quantum fluctuation of the photon number,[6] the fluctuation and the QSN generate a new coupled noise. These results are finally used to discuss fundamental limits of quantum interference devices.

*a. Model structure.* Most discussions on electronic conduction in a mesoscopic conductor assumed that large reservoirs are connected to ends of the conductor.[1,4–9] We also employ this assumption, and take one of the reservoirs as a source reservoir $S$, from which electrons are emitted into the conductor. In general, a part of the emitted electrons are reflected by the conductor back to $S$, and only the *difference* between the emitted and reflected currents can be detected in usual experiments. In order to discuss separately a noise arising from random emissions of electrons and other noises, we consider a structure in which the reflected electrons go to another reservoir, a drain $D_-$. The structure is a Mach-Zehnder interferometer shown in Fig. 1, where the "reflected" (or "backward-scattered") electrons will be detected as a current $J_-$ and the remaining "forward-scattered" electrons as another current $J_+$ at a third reservoir, a drain $D_+$. The emission current $J$ is thus the sum $J=J_+ + J_-$.[10] We further assume that an external variable $I$ acting on the gate region can modulate $J_\pm$. This allows us to discuss not only various types of noises, but also fundamental limits of quantum interference devices.

The operation principle of our structure is as follows. An electron emitted from $S$ is split into two portions by the beam splitter (BS).[7] In the gate region an external perturbation $I$ causes a phase difference $\Delta\theta_I$ between the two portions. When $I$ is a classical variable, we assume

$$\Delta\theta_I = \xi I , \tag{1}$$

where $\xi$ is a coupling constant. The above linear relationship approximately holds in many cases, including the cases when $I$ is a magnetic field,[1] a gate voltage,[7] or a light intensity.[8] For a quantum $I$, we expand the density operator $\rho_I$ of a quantum state of the input in terms of eigenfunctions of $I$:

$$\rho_I = \sum_{n,m} \rho_I^{nm} |I_n\rangle\langle I_m| , \tag{2}$$

where $I|I_n\rangle = I_n|I_n\rangle$. It is sufficient for the following dis-

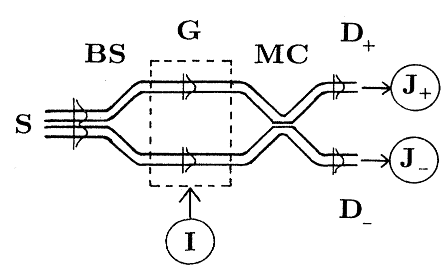


FIG. 1. Schematic structures of the model device used in the analysis. The pair of quantum wires compose an electron interferometer, which consists of the source ($S$), beam splitter (BS), gate ($G$), mode converter (MC), and drain ($D_+, D_-$) regions. The electron subband function in each region is also schematically plotted. In the gate region, an external input $I$ causes a phase difference between the split electron waves. The currents $J_\pm$ detected by the ammeters vary with the phase difference.

 

cussions to know $\Delta\theta_I$ for the eigenstates, $\rho_I = |I_n\rangle\langle I_n|$, for which we assume

$$\Delta\theta_I = \xi I_n \,. \tag{3}$$

This is a quantum analog of the classical relationship (1), and approximately holds also in many cases, including the case when $I$ is a photon number and $|I_n\rangle$ is a number state.[6] Having the phase difference $\Delta\theta_I$, the two portions of the electron wave are then superposed at the mode converter (MC), which consists of crossed quantum wires coupled via a thin barrier layer of 50% transmittance.[6] Then, the overall transmittance from $S$ to $D_+$ and $D_-$ are given by $T_+ = \cos^2(\Delta\theta/2)$ and $T_- = \sin^2(\Delta\theta/2)$, respectively, where the overall phase difference $\Delta\theta = \Delta\theta_I + \Delta\theta_0$. Here $\Delta\theta_0$ is an additional phase determined by detailed structures of MC.[6] Note that $T_+ + T_- = 1$, which means, as mentioned before, *no reflections back to the source region occur* in our structure, which makes the following discussion transparent.[10]

*b. The Johnson-Nyquist noise.* We formulate a measurement of $J_\pm$ as a *counting process* of the number $N_\pm$ of electrons detected during a detection period $\tau$, which is assumed to be longer than the transit time of an electron across the conductor. Here,

$$N_\pm(t) = \frac{1}{e}\int w_\tau(t'-t) J_\pm(t')dt' \,, \tag{4}$$

where $w_\tau(t)$ is a weight function of width $\tau$, the detailed form of which is irrelevant to the following discussions. We first estimate the Johnson-Nyquist noise (JNN). Although Refs. 4 and 5 reported a formula that vanishes at zero temperature, a more rigorous treatment[11] based on the fluctuation-dissipation theorem predicted a nonzero JNN, from which we here calculate the fluctuation of $N_\pm$. When the source-drain voltage $V_{SD} = 0$, and if $\tau$ is longer than any of the scattering times and the transit time of an electron, we find

$$\delta N^2_{\pm\,\mathrm{JNN}} \simeq A T_\pm \,, \tag{5}$$

for the JNN at zero temperature, where $A$ is a constant of order unity, which is determined by a detailed form of $w_\tau$. Interestingly, $\delta N_{\pm\,\mathrm{JNN}}$ *depends only on the transmittance.* It will be possible to confirm this prediction by a measurement of the JNN at very low temperatures. In the following we will discuss other noises, and will find that they are much larger than the JNN as long as $\{\langle N_\pm\rangle\} \gg 1$ [Eq. (8)]. We will therefore ignore the JNN hereafter.

*c. Role of the Pauli principle.* We consider the usual case, in which (i) the electron system is a normal Fermi liquid, and (ii) a current is carried by low-energy, single-particle excitations (SPEs) rather than collective excitations. The mean free path of these SPEs is assumed to be longer than the distance between BS and MC. According to the Fermi-liquid theory, we can focus on these SPEs only, and the other electrons forming the Fermi sea can be considered as a background. The SPEs are normally induced by a voltage $V_{SD}$ applied between $S$ and $D_\pm$. At zero temperature, the energies of the SPEs have a distribution of width $\simeq eV_{SD}$.[9] Hence, we must keep $V_{SD}$ small enough to get good coherence. In an ideal case of very small $V_{SD}$, which is actually the case in usual experiments, the average distance $l_{sp}$ between the SPEs will exceed the source-drain distance $l_{SD}$.[12] That is, the SPEs pass through the conductor *one by one.* Obviously, the Pauli principle *among the SPEs* becomes irrelevant in this low-density limit. We thus expect a noise formula which takes the *same* form for both fermions and bosons in this limit. On the other hand, the Pauli principle *among all electrons* does play a role; it makes the SPEs (quasi)monochromatic. Once a monochromatic, low-density particle flow is thus obtained, however, the noise depends on neither the particle statistics nor the detailed features of the host system. That is, in contrast to a claim of Ref. 4, the noise in a mesoscopic conductor is *not* characteristic of the mesoscopic system, as explicitly shown below.

*d. The case of no fluctuation in I.* When a fluctuation of $I$ is absent, noise formulas for an interferometer were previously given for both bosons[13] and fermions.[14] However, the previous derivations used the commutation or anticommutation relations of particle operators, so that the irrelevance of the particle statistics might not be seen easily. We here reproduce the formula in a different manner, which allows us to see the underlying physics more clearly.

Let $N \equiv N_+ + N_-$, which is the number of emitted SPEs during $\tau$. Since SPEs pass through the conductor one by one in the limit of low current density, we can safely assume that there are *no correlations* among the $N$ SPEs in each choosing either drain for the destination. That is, the probability $T_\pm$ for each SPE to go to drain $D_\pm$ is *not* affected by other SPEs. For a given $N$, this leads to the simple binomial form for the probability $P_N(N_+)$ of detecting $N_+$ SPEs in $D_+$ (and, simultaneously, $N_-$ SPEs in $D_-$):

$$P_N(N_+) = \begin{pmatrix} N \\ N_+ \end{pmatrix} T_+^{N_+} T_-^{N_-} \,. \tag{6}$$

We denote the average over this distribution by the brackets $\langle \cdots \rangle$. When $N$ has a finite statistical distribution, we must also take the averages over it, which is denoted by the curly brackets $\{\cdots\}$. We then find

$$\{\langle N_\pm\rangle\} = \{N\} T_\pm = \{\langle J_\pm\rangle\}\tau/e = e\tau T_\pm V_{SD}/\pi\hbar \,, \tag{7}$$

where the second and the third equalities, respectively, come from Eq. (4) and the Landauer formula[9] (which is valid for the average value). We also find the fluctuation

$$\{\langle \delta N_\pm^2\rangle\} = \{\delta N^2\} T_\pm^2 + \{N\} T_+ T_- \,. \tag{8}$$

Equations (7) and (8) agree with the previous result for bosons[13] and for fermions,[14] apart from the last equality of Eq. (7) (which is specific to mesoscopic conductors) and the first term in the right-hand side (rhs) of Eq. (8) which was absent in Ref. 14 because it considered the case of a given $N$. In the rhs of Eq. (8), the first term is the scaled emission noise,[13] whereas the second is the "granularity noise,"[13] which comes from the fact that although the square of the wave function, $T_\pm$, can take an arbitrary value between 0 and 1 each SPE count is limited to either 0 or 1. This granularity noise can also be understood as a manifestation of the number-phase uncertainty principle.[6,13,14]

References 3 and 4 evaluated an excess noise, or quantum shot noise, of a current in mesoscopic conductors. We can easily see using Eq. (7) that this QSN is *equivalent* to the above granularity noise. Hence, the QSN is *not* a characteristic noise of mesoscopic conductors, but is the usual granularity noise. As for the emission noise, the above references (implicitly) assumed its absence by assuming that an ideal electron source of no randomness is connected to the conductor. That is, their formula corresponds to the special case of Eq. (8) when $\{\delta N^2\}=0$. In real systems, however, the statistics of $N$ is determined by the nature of the *whole circuit*, and the neglect of the emission noise is *not* justified. For example,[15] when a low-noise battery and a large resistance $R$ ($\gg \pi\hbar/e^2$) of temperature $T$ are connected to the conductor, the emission noise is super-Poissonian ($\{\delta N^2\} > \{N\}$) when $RJ < 2k_BT/e$, and sub-Poissonian ($\{\delta N^2\} < \{N\}$) when $RJ > 2k_BT/e$. When $RJ = 2k_BT/e$, it becomes Poissonian ($\{\delta N^2\}=\{N\}$), and Eq. (8) coincides with the classical shot noise (CSN), $\{\langle \delta N_\pm^2 \rangle\}_{cl}=\{N\}T_\pm$. Although the QSN is always smaller than the CSN, the *total* noise can be either smaller (when $\{\delta N^2\} < \{N\}$) or larger (when $\{\delta N^2\} > \{N\}$) than the CSN, depending on the nature of the whole circuit.

Note that the sub-Poissonian emission noise means that the emitted SPEs are "antibunched," i.e., SPEs are equally spaced.[13,16,17] If the density of SPEs were high enough such that $l_{sp}$ is less than a coherence length (in the sense of Refs. 13 and 16) $l_c$, this antibunching would be automatically induced by the Pauli principle among the SPEs.[16] However, since $l_{sp} \gg l_{SD}$ in usual mesoscopic conductors under a high-coherence condition (small $V_{SD}$) as mentioned before, $l_{sp} \gg l_c$ because $l_c$ cannot exceed $l_{SD}$. Hence, the antibunching can only be caused by other means like a large resistance in series or a feedback. Note also that the antibunching does not conflict with our starting point of uncorrelated $T_\pm$ because $T_\pm$ is independent of the spacing between SPEs when their density is low. If higher density and good coherence could be simultaneously obtained, it would become possible to reduce the noise less than Eq. (8) by introducing some many-body correlation among the SPEs, as pointed out by Yurke.[14]

*e. The case of a finite fluctuation in I.* The above consideration indicates that the emission noise and other noises which are truly specific to the conductor must be considered *separately*. In order to extract the latter noises only, we introduce a "normalized output variable" $R$ defined by[6,14]

$$R \equiv \frac{N_+ - N_-}{N_+ + N_-}, \tag{9}$$

which takes a value $-1 \le R \le 1$. Since both the numerator and denominator are proportional to $N$, $R$ is insensitive to $N$, and the emission noise becomes irrelevant as long as $\{\delta N^2\} < (\{N\})^2$. This can be confirmed using the relation

$$\{N^p\} \simeq \{N\}^p[1+p(p-1)\{\delta N^2\}/2\{N\}^2] \simeq \{N\}^p, \tag{10}$$

where $p$ is a real number. On the other hand, we now take a *fluctuation* of $I$ into account, whose origin may be a quantum fluctuation,[6] a thermal noise, an input error, and so on. The statistical distributions of $I$ is given, say, by $\rho_I^{nn}$ in the case of quantal $I$.[6] Denoting the average over the distribution of $I$ by the overbar, we find

$$\langle R \rangle = \sin\Delta\theta_I, \quad \overline{\{\langle R \rangle\}} = \overline{\langle R \rangle} = \overline{\sin\Delta\theta_I}, \tag{11}$$

$$\overline{\{\langle \delta R^2 \rangle\}} = (1-\overline{\langle R \rangle^2})/\{N\} + \overline{(\langle R \rangle - \overline{\langle R \rangle})^2}, \tag{12}$$

where $\Delta\theta_0 = -\pi/2$ has been assumed for a later convenience. These are *quite general* formulas in the sense that they are valid (i) for any origin of the fluctuation of $I$, (ii) for both classical and quantal $I$, and (iii) for any statistics of $N$ [as long as $\{\delta N^2\} < (\{N\})^2$]. The second term in the rhs of Eq. (12) arises from the fluctuation of $I$ only. On the other hand, the QSN and the fluctuation of $I$ are *coupled* in the first term. This can be seen in the later discussion on a digital logic element, where this coupled noise will play a striking role.

*f. Fundamental limits of quantum interference devices.* We now use the above results to discuss the *fundamental limits* of our model device. Let us first consider the case when the device is used as a detector of $I$.[6,8] One may define the "readout variable" $I_r$ by

$$\sin\xi I_r \equiv R. \tag{13}$$

We then find that $\overline{\{\langle \sin\xi I_r \rangle\}} = \overline{\sin\xi I}$, which reduces for small $I$ to $\overline{\{\langle I_r \rangle\}} \simeq \overline{I}$. That is, the average value of $I_r$ indeed gives the average value of $I$, as is desired. On the other hand, Eq. (12) yields

$$\overline{\{\langle (\sin\xi I_r - \overline{\{\langle \sin\xi I_r \rangle\}})^2 \rangle\}} \simeq \overline{\cos^2\xi I}/\{N\} + \overline{(\sin\xi I - \overline{\sin\xi I})^2}, \tag{14}$$

which reduces for small $I$ to

$$\overline{\{\langle \delta I_r^2 \rangle\}} \simeq 1/\xi^2\{N\} + \overline{\delta I^2}. \tag{15}$$

In the rhs, the second term just reflects the fluctuation of $I$, which is a "desired term" because an accurate detector must reflect any fluctuations of an input faithfully. On the other hand, the first term represents the *fundamental lower limit of the measurement error.* Although this was previously discussed for a specific example,[6] we have found the general formula here.

We next consider an analogous case when the device is used as a *linear transformer* of $I$ into $R$ or $J_\pm$. For $R$ to be linear in $I$, we must restrict the range of $I$ to

$$|I| \le I_{\max} \ll \pi/2\xi \equiv I_0, \tag{16}$$

which yields $\overline{\{\langle R \rangle\}} \simeq \xi\overline{I}$. Let us evaluate the *upper limit* $\mathrm{SNR}_0$ of the signal-to-noise ratio at $I = I_{\max}$. When the fluctuation of $I$ is absent, Eqs. (10) and (12) yield

$$\mathrm{SNR}_0 \simeq \pi I_{\max}\sqrt{\{N\}}/2I_0, \tag{17}$$

which increases as $\sqrt{\{N\}}$. For example, if we require $\mathrm{SNR}_0 \ge 10$ when $I_{\max} = I_0/10$, then $\{N\}$ must be larger than 4000. In order to get such a large $\{N\}$, it is effective to connect many devices in parallel and use them as a single device. Let $N_{\mathrm{dev}}$ be the number of the connected devices, then $\{N\} = e\tau N_{\mathrm{dev}} V_{SD}/\pi\hbar$. If we assume, say, $V_{SD} = 0.1$ mV, the requirement $\{N\} \ge 4000$ for the above example reduces to $\tau N_{\mathrm{dev}} \ge 80$ ns. This implies, for example, $N_{\rm dev} \geq 8000$ for $\tau = 10$ ps, or, $\tau \geq 80$ ns for $N_{\rm dev} = 1$.

Let us finally consider the case when the device is used as a *digital logic element*. One may switch $I$ between $-I_0$ and $I_0$, for which $\langle R \rangle = -1$ and $+1$, respectively. We are interested in the *lower limit* $\mathrm{ERR}_0$ of the error rate, which may be defined as the rate at which a *negative* $R$ is obtained when $I = I_0 \pm \delta I/2$, where $\delta I(\ll I_0)$ is an input error. If the QSN were absent, $R = \cos(\pi\delta I/4I_0) > 0$ for $I = I_0 \pm \delta I/2$, so that negative $R$ would never be obtained, i.e., $\mathrm{ERR}_0 = 0$. On the other hand, when $\delta I = 0$, the QSN, $(1 - \langle R \rangle^2)/\{N\}$, vanishes at $I = \pm I_0$, and $\mathrm{ERR}_0 = 0$ again. The error rate becomes finite only when *both* the QSN and the input error are present, i.e., nonvanishing $\mathrm{ERR}_0$ is a result of a coupling of both noise sources. We find

$$\mathrm{ERR}_0 = \left\{ \sum_{N_+ = 0}^{N/2} [P_N(N_+)]_{I = I_0 \pm \delta I/2} \right\} . \tag{18}$$

To see the $\{N\}$ dependence, it is convenient to approximate the rhs by

$$\sqrt{2/\pi\{N\}}(\pi\delta I/4I_0)^{\{N\}} , \tag{19}$$

where use has been made of Eqs. (VI 3.4) and (II 9.1) of Ref. 18. For example, if we require $\mathrm{ERR}_0 = 10^{-9}$ when $\delta I = I_0/10$, $\{N\}$ should be larger than 8. When $V_{\rm SD} = 0.1$ mV, it reduces $\tau N_{\rm dev} \geq 160$ ps. This implies, for example, $N_{\rm dev} \geq 16$ for $\tau = 10$ ps, or, $\tau \geq 160$ ps for $N_{\rm dev} = 1$.

Although the above fundamental limits have been derived for the specific model device of Fig. 1, we consider that most quantum devices based on an electron interference are subject to these limits, because the limits have very fundamental origins such as the granularity noise.

The authors acknowledge valuable discussions with M. Ueda (NTT) and H. Akiyama (JRDC).

*Also with Institute for Industrial Science (IIS), and Research Center for Advanced Science and Technology (RCAST), the University of Tokyo, Tokyo, Japan.

[1]For brief but insightful reviews, B. L. Al'tshuler and P. A. Lee, Phys. Today **41** (12), 36 (1988); R. A. Webb and S. Washburn, *ibid.* **41** (12), 46 (1988).

[2]C. W. J. Beenakker and H. van Houten, Phys. Rev. Lett. **63**, 1857 (1989), and references therein.

[3]G. B. Lesovik, Pis'ma Zh. Eksp. Teor. Fiz. **49**, 513 (1989) [JETP Lett. **49**, 592 (1989)].

[4]M. Büttiker, Phys. Rev. Lett. **65**, 2901 (1990).

[5]R. Landauer, Physica D **38**, 226 (1989).

[6]A. Shimizu, Phys. Rev. A **43**, 3819 (1991).

[7]S. Datta, M. R. Melloch, S. Bandyopadhyay, and M. S. Lundstrom, Appl. Phys. Lett., **48**, 487 (1986); M. Okuda, K. Fujii, and A. Shimizu, *ibid.* **57**, 2231 (1990).

[8]M. Yamanishi, Superlattices Microstruct. B **6**, 403 (1989); A. Shimizu, K. Fujii, M. Okuda, and M. Yamanishi, Phys. Rev. B **42**, 9248 (1990).

[9]R. Landauer, IBM J. Res. Dev. **32**, 306 (1988).

[10]To relate the present results with properties of a usual two-terminal conductor, in which a part of electrons are reflected back to the source region, $J_+$ or $N_+$ (*neither J nor N*) of our structure corresponds to the *total* current through the two-terminal conductor.

[11]W. Jones and N. H. March, *Theoretical Solid State Physics* (Dover, New York, 1985), Sect. 6.11.3.

[12]For example, $l_{sp} \sim 50$ $\mu$m for typical values of $J \sim 1$ nA and $v_F \sim 3 \times 10^7$ cm/s, whereas $l_{\rm SD} \sim 1$ $\mu$m typically.

[13]R. Loudon, *The Quantum Theory of Light*, 2nd ed. (Oxford Univ. Press, Oxford, 1983), Sect. 6.9.

[14]B. Yurke, Phys. Rev. Lett. **56**, 1515 (1986).

[15]P. R. Tapster, J. G. Rarity, and S. Satchell, Europhys. Lett. **4**, 293 (1987).

[16]M. P. Silverman, Physica B **151**, 291 (1988).

[17]For the antibunching of bosons and the resulting reduction of quantum noise, see papers in J. Opt. Soc. Am. B **4** (1987).

[18]W. Feller, *An Introduction to Probability Theory and Its Applications*, 3rd ed. (Wiley, New York, 1968), Vol. 1.