Magnetization instabilities at tilted magnetic fields in the quantum Hall regime

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In high-mobility samples, we show that a magnetization instability occurs at filling factor $\nu=2$ in the quantum-Hall-effect regime under the application of a parallel magnetic field. The instability is caused by feedback from the screened exchange energy of the electrons. We perform calculations on $Ga_xIn_{1-x}As$-InP heterostructures and show that a surprising jump in the magnetization occurs at experimentally accessible magnetic fields and sample mobilities. This phenomenon is experimentally observable and possible experiments are proposed.

We demonstrate that a two-dimensional electron gas (2DEG) in the quantum-Hall-effect (QHE) regime at $\nu=2$, where ν is the ratio of density to half the total degeneracy of a Landau level (LL), undergoes a magnetization instability when a parallel magnetic field is applied. For high-mobility samples, when a constant perpendicular component of the field holds the system at $v=2$ and the parallel component of the field is varied, the system jumps from one polarized state to another at a critical value of the parallel field.

A system at $\nu=2$, in the absence of broadening, jumps from an unpolarized state to a completely polarized state at a critical parallel magnetic field when the energies of spin up lower LL and spin down upper LL coincide.[1] However, when the effects of LL broadening and screening are included, it is not obvious whether the system changes its polarization gradually from an unpolarized state to a totally polarized state or there is a jump in magnetization at some partially polarized state. In this paper, for a system with small LL broadening, we show that there is a dramatic jump in the magnetization as the system changes its polarization due to a varying parallel magnetic field. To bring out the basic physics, we begin by considering the simple Hartree-Fock (HF) case of constant broadening and no screening. The spin magnetization is defined as

$$M=\frac{\mu_B}{2\pi l^2}(n_0^{\downarrow}+n_1^{\downarrow}-n_0^{\uparrow}) , \tag{1}$$

where μ_B is the Bohr magneton and l is the ground-state cyclotron radius and is equal to $(\hbar c/eH_{\perp})^{1/2}$, $H_{\perp}$ being the perpendicular component of the applied magnetic field. Furthermore, in the above equation n_N^{σ} is the filling fraction of the spin σ Nth LL and is given by

$$n_N^{\sigma}=2\pi l^2\int_{-\infty}^{E_F}dE\, D_N^{\sigma}\left[\frac{E-\varepsilon_N^{\sigma}}{\Gamma}\right] , \tag{2}$$

where $D_N^{\sigma}[(E-\varepsilon_N^{\sigma})/\Gamma]$ is the density of states (DOS), ε_N^{σ} is the energy of an electron at the maximum of the DOS, E_F is the Fermi energy, and Γ is the broadening of the LL. For a symmetric DOS, the energy of the electron at the maximum of the DOS has no contribution from the self-energy due to impurity scattering and is expressed in the HF approximation as follows:[2]

$$\varepsilon_N^{\sigma}=(N+\tfrac{1}{2})\hbar\omega_C+\frac{\sigma}{2}g\mu_B H-\sum_{\mathbf{q},M}V_q J_{NM}^2(q)n_M^{\sigma} , \tag{3}$$

where $\hbar\omega_C(=\hbar eH_{\perp}/mc)$ is the noninteracting cyclotron energy, $g\mu_B H$ is the bare Zeeman energy, V_q is the Fourier transform of the bare Coulomb potential, and $J_{NM}(q)$ is proportional to the probability amplitude of scattering an electron between LL's N and M. Assuming that the spin down lowest LL is completely occupied, we obtain from Eqs. (1)–(3) and the condition that $\nu=2$ (i.e., $H_{\perp}$ is constant), the following expression for the change in spin magnetization as the parallel magnetic field $H_{\parallel}$ is varied:

$$\frac{dM}{dH_{\parallel}}=\mu_B D_0^{\uparrow}\frac{d(\varepsilon_0^{\uparrow}-\varepsilon_1^{\downarrow})}{dH_{\parallel}}=\frac{D_0^{\uparrow}\mu_B^2 g\, dH/dH_{\parallel}}{1-\sum_{\mathbf{q}}V_q(J_{00}^2+J_{11}^2)D_0^{\uparrow}\pi l^2} , \tag{4}$$

where $D_0^{\uparrow}(=D_1^{\downarrow})$ is the DOS at E_F. In the above equation, the second term in the denominator is due to a feedback effect caused by the exchange enhancement of the energy separation $\varepsilon_0^{\uparrow}-\varepsilon_1^{\downarrow}$ [see Eq. (3)].

We will consider the situation where the system is unpolarized when $H_{\parallel}=0$. In Eq. (4) the denominator ≈ 1 for the two extreme cases when $H_{\parallel}=0$ and $H_{\parallel}\to\infty$ (i.e., the completely polarized situation) because in both these cases the DOS $D_0^{\uparrow}\approx 0$. However, for the case when spin up lower LL and spin down upper LL overlap perfectly, i.e., $\varepsilon_0^{\uparrow}=\varepsilon_1^{\downarrow}$, the Fermi energy lies at the center of these LL's and $D_0^{\uparrow}$ is at its maximum value. Then, $D_0^{\uparrow}2\pi l^2\sim 1/\Gamma$, and for sufficiently high-mobility samples the denominator in Eq. (4) will have zeros leading to a spin magnetization instability. For a strictly 2DEG, in the HF approximation, the instability condition for a Gaussian DOS (Ref. 3) is $e^2/\varepsilon l\Gamma\geq 16/7$. It is of interest to note that when the spin magnetization undergoes an instability, so do the orbital and total magnetization.

Next, we will study other filling factors for possible magnetization instabilities. For the case where the Fermi energy lies in the spin up Nth LL, the spin magnetization is given by

$$M=\frac{\mu_B}{2\pi l^2}(n_N^{\downarrow}+n_{N+1}^{\downarrow}-n_N^{\uparrow}) . \tag{5}$$

Again assuming $n_N^{\downarrow}=1$, we obtain

$$\frac{dM}{dH_{\parallel}}=\frac{\tilde{D}\mu_B^2 g\, dH/dH_{\parallel}}{1-\sum_{\mathbf{q}} V_q(J_{NN}^2+J_{N+1N+1}^2)\tilde{D}\pi l^2} . \tag{6}$$

In Eq. (6), the maximum value of the term $\tilde{D}=2D_N^{\uparrow}D_{N+1}^{\downarrow}/(D_N^{\uparrow}+D_{N+1}^{\downarrow})$ is obtained for $\nu=2N$ when energy separation $\varepsilon_N^{\uparrow}-\varepsilon_{N+1}^{\downarrow}=0$. Furthermore, for even integer filling factors, the magnitude of the second term in the denominator decreases as ν increases. This is because the exchange energy of an electron in the Nth LL is inversely proportional to the cyclotron radius in that level. Hence, for $n_N^{\downarrow}=1$, the instability condition is best met at $\nu=2$.

We will now proceed to give a more realistic consideration of the magnetization instabilities at $\nu=2$ by taking screening effects into account. For the sake of simplicity we assume that the screening is static. Then, the energy separation $\varepsilon_1^{\downarrow}-\varepsilon_0^{\uparrow}$ is given as follows:

$$\varepsilon_1^{\downarrow}-\varepsilon_0^{\uparrow}=\hbar\omega_C-g\mu_B H+\sum_{\mathbf{q}}E_X(q) , \tag{7}$$

where

$$E_X(q)=\frac{V_q}{\epsilon(q,0)}(J_{00}^2 n_0^{\uparrow}-J_{11}^2 n_1^{\uparrow}-J_{10}^2 n_0^{\downarrow}) . \tag{8}$$

We consider systems where only short-range scatterers are important (e.g., $Ga_xIn_{1-x}As$-InP heterostructures) and perform our calculations assuming a Gaussian DOS. We find that the DOS obtained using the self-consistent Born approximation (SCBA) is not appropriate owing to the sharp cutoffs of the semielliptic DOS predicted by this approximation.[2] The dielectric function is treated within the random-phase approximation (RPA) by ignoring in the polarizability the vertex corrections due to impurities. The inter-LL screening is treated exactly within this approximation. However, the intra-LL screening is taken to be proportional to the DOS so as to satisfy a Ward identity. The resulting intra-LL screening is of the Thomas-Fermi type and is thus an overestimation of the actual screening. Then, we obtain

$$\epsilon(q,0)=1+V_q\sum_{\sigma}\pi_{\mathrm{RPA}}^{\sigma}(q,0) , \tag{9}$$

where

$$\pi_{\mathrm{RPA}}^{\sigma}(q,0)=\sum_N D_N^{\sigma}+\frac{1}{2\pi l^2}\sum_{N,M}{}' J_{NM}^2\frac{n_N^{\sigma}-n_M^{\sigma}}{\hbar\omega_C(M-N)} , \tag{10}$$

where the prime denotes omission of the case $N=M$. The last term in the above equation corresponds to inter-LL screening.

To understand our results, we present the following analytic expression for $dM/dH_{\parallel}$ when inter-LL screening is ignored:

$$\frac{dM}{dH_{\parallel}}=D_0^{\uparrow}\mu_B^2 g\frac{dH}{dH_{\parallel}}\left[1-\sum_{\mathbf{q}}\frac{V_q}{\epsilon(q,0)}D_0^{\uparrow}\times\left[\pi l^2(J_{00}^2+J_{11}^2)+E_X(q)\frac{\varepsilon_1^{\downarrow}-\varepsilon_0^{\uparrow}}{2\Gamma^2}\right]\right]^{-1} . \tag{11}$$

The value of $dM/dH_{\parallel}$ in the above equation could diverge, i.e., an instability can occur because the second term in the denominator resulting from the exchange contribution is negative. Furthermore, the last term in the denominator, obtained from $d\epsilon(q,0)/dH_{\parallel}$, is also negative for values of $n_1^{\downarrow}$ that are small or >0.5. For SCBA, the derivative of the dielectric function with respect to $H_{\parallel}$ diverges when the Fermi energy is at the edge of the DOS. A Gaussian DOS does not show this unphysical effect and is thus a better choice.

The change in spin magnetization as the parallel magnetic field is varied, i.e., $dM/dH_{\parallel}$, is similar to the static long-wavelength limit of the spin susceptibility χ_S of a 2DEG.[4] If one were to ignore screening then only exchange energy contributes to the enhancement of χ_s over its value for noninteracting electrons. However, upon including static screening effects, $dM/dH_{\parallel}$ is enchanced not only due to screened exchange as in the treatment of χ_s, but also due to change in screening, an effect not present in χ_s.

To illustrate the magnetization instability effect we choose $Ga_xIn_{1-x}As$-InP heterostructure for which the ratio of the Zeeman energy ($\approx 4.06\mu_B H$) (Ref. 5) to the bare cyclotron energy ($\approx\hbar eH/0.047m_e c$) (Ref. 6) is ≈ 0.1. For this system we expect the instability to set in at experimentally accessible values of the magnetic fields. However, for GaAs heterostructures this ratio is much smaller and thus they are not experimentally suitable systems to observe the effect in question. In $Ga_xIn_{1-x}As$-InP heterostructures the dominant scattering mechanism is alloy scattering and this leads to low mobilities. We performed our calculations for two different densities (0.7×10^{11} and 1.4×10^{11} cm $^{-2}$) and at two different experimentally achievable values of the mobility μ (70 000 and 100 000 cm^2/V s). The broadening Γ is taken to be $(e\hbar^2\omega_c/2\pi 0.047 m_e\mu)^{1/2}$ which is half the broadening obtained within SCBA.[2] For the sake of simplicity, the extension of the wave function in the direction perpendicular to the 2DEG is assumed to be of the Fang-Howard[7] type, even though the electron can penetrate the InP layer. The average extent of the wave function was taken to be 100 Å and the dielectric constants of both $Ga_xIn_{1-x}As$ and InP layers were taken to be 13.8. The magnetization results of our HF calculations are displayed in Fig. 1(a) and those for the screening given by Eqs. (9) and (10) are shown in Figs. 2(a) and 3(a). Although for the higher-density sample the instability occurs at a higher field, the features are qualitatively similar to the lower-density case. These calculations were performed by using the fact that the magnetic field is a single-valued function of the magnetization and also by using the symmetry that $E_F=(\varepsilon_0^\uparrow+\varepsilon_1^\downarrow)/2$. We will now explain these figures qualitatively. Owing to the nonlinear nature of Eq. (7) with respect to the energy separation $\varepsilon_1^\downarrow-\varepsilon_0^\uparrow$, more than one solution is possible for the energy separation or the magnetization. Qualitatively, one can argue that small (large) energy separation implies small (large) values of $n_0^\uparrow-n_1^\downarrow$ and large (small) screening when screening is considered. From Eq. (7), we see that both small and large energy separations (or spin magnetizations) are consistent with it. In all the figures multiple solutions exist. Among the possible magnetization solutions at a given magnetic field, only the one that corresponds to the lowest energy is realized physically. We compute the total energy E of the system using the following expression:

$$E=\sum_{N,\sigma}\int_{-\infty}^{E_F}dE(E-\tfrac{1}{2}\Sigma_{N,\mathrm{el-el}}^{\sigma})D_N^{\sigma}\left[\frac{E-\varepsilon_N^{\sigma}}{\Gamma}\right], \quad (12)$$

where $\Sigma_{N,\mathrm{el-el}}^{\sigma}$ is the electronic self-energy due to screened exchange. We plot the total energy curves in Fig. 1(b) for the HF case and in Figs. 2(b) and 3(b) for the screened HF case. In all the figures a jump in the magnetization is indicated by dotted lines. The figures, in

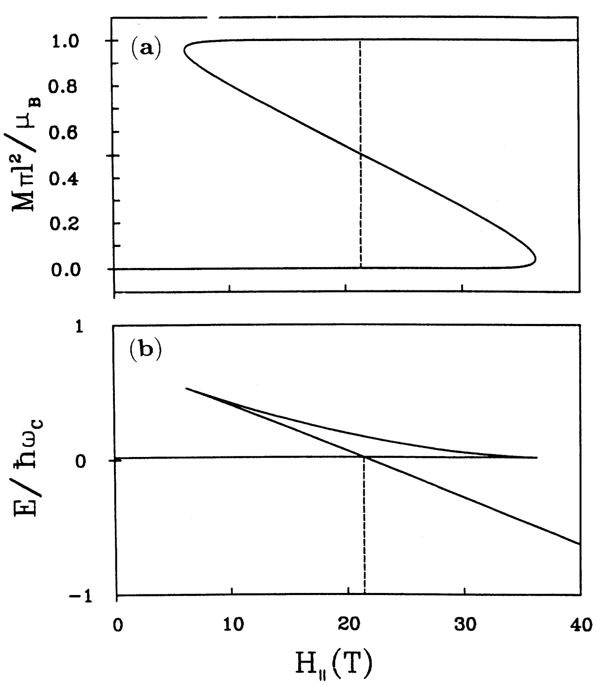

FIG. 1. (a) Hartree-Fock result of the dimensionless spin magnetization $M\pi l^2/\mu_B$ vs parallel magnetic field $H_{\parallel}$ for $Ga_xIn_{1-x}As$-InP heterostructure for the following parameter values: density, 1.4×10^{11} cm^{-2}; mobility, 100 000 cm^2/V s; bare g factor, 4.06; band mass, $0.047m_e$; average extent of wave function, 100 Å; and dielectric constants of both layers, 13.8. (b) HF value of the dimensionless total energy $E/\hbar\omega_c$ [see Eq. (12)] vs parallel magnetic field $H_{\parallel}$ for the same parameter values. The dotted lines indicate jump in magnetization.

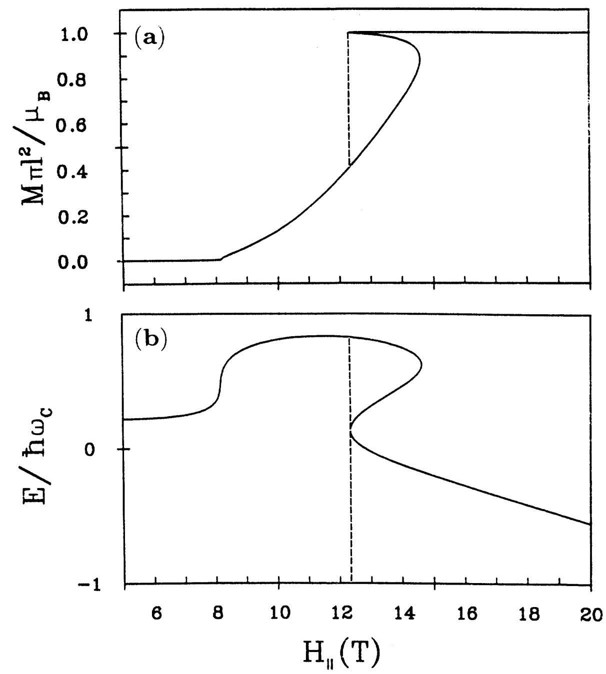

FIG. 2. (a) Plot of the dimensionless spin magnetization $M\pi l^2/\mu_B$ vs parallel magnetic field $H_{\parallel}$ for $Ga_xIn_{1-x}As$-InP heterostructure for the screening given in Eqs. (9) and (10). The values of the density and mobility are, respectively, 0.7×10^{11} cm^{-2} and 70 000 cm^2/V s. The values of the other parameters are the same as in Fig. 1. (b) Plot of the dimensionless total energy $E/\hbar\omega_c$ [see Eq. (12)] vs parallel magnetic field $H_{\parallel}$ for the screening given in Eqs. (9) and (10). The dotted lines indicate jump in magnetization.

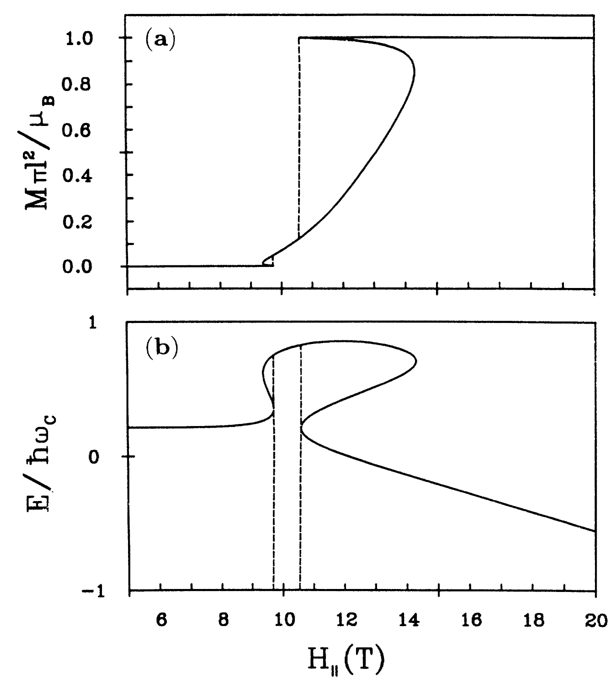

FIG. 3. (a) and (b): same as in Figs. 2(a) and 2(b) but for mobility of 100 000 cm^2/V s.

agreement with previous arguments, show an increase in the size of the jump in magnetization as the mobility increases.

This magnetization instability can be observed by performing transport measurements. If one were to measure the diagonal resistivity ρ_{xx} as a function of parallel magnetic field, in the absence of a magnetization instability, one would expect the value of ρ_{xx} to vary smoothly from zero value at $H_{\parallel}=0$ (unpolarized situation) to a maximum value for the symmetric situation ($\varepsilon_1^{\downarrow}=\varepsilon_0^{\uparrow}$) and then back to zero value at large values of $H_{\parallel}$ corresponding to the completely polarized situation. However, if an instability sets in there would be a sudden change in ρ_{xx} for cases similar to the ones shown in Figs. 2 and 3. For situations corresponding to a jump from an unpolarized state to a totally polarized state, there would not be any change at all in ρ_{xx}. In this context, we would like to point out that the measurements of ρ_{xx} under tilted fields, carried out by Nicholas *et al.*,[8] were done on low-mobility samples at $\nu>2$. We think that due to these reasons no instability was observed by the authors. One could also measure the nuclear spin relaxation rate as a function of parallel magnetic field.[9] Since the relaxation rate is approximately proportional to the product, at the Fermi energy, of the DOS of opposite spin LL's, a sudden jump in the relaxation rate would be a signal for magnetization instability. Alternately, one could also measure de Haas–van Alphen effect and thus obtain the magnetization directly.[10] Lastly, one could perform cyclotron resonance experiments[11] to detect this magnetization jump. However, there could be complications due to coupling between different subbands.[12]

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[1]G. F. Giuliani and J. J. Quinn, Phys. Rev. B **31**, 6228 (1985); Solid State Commun. **54**, 1013 (1985); Surf. Sci. **170**, 316 (1986).

[2]T. Ando and Y. Uemura, J. Phys. Soc. Jpn. **37**, 1044 (1974).

[3]R. R. Gerhardts, Z. Phys. B **21**, 275 (1975); **21**, 285 (1975); Surf. Sci. **58**, 227 (1976).

[4]S. Yarlagadda and R. R. Gerhardts (unpublished); S. Yarlagadda and G. F. Giuliani, Phys. Rev. B **40**, 5432 (1989); Surf. Sci. **229**, 410 (1990).

[5]M. Dobers, J. P. Vieren, Y. Guldner, P. Bove, F. Omnes, and M. Razeghi, Phys. Rev. B **40**, 8075 (1989).

[6]Y. Guldner, J. P. Vieren, P. Voisin, M. Voos, M. Razeghi, and M. A. Poisson, Appl. Phys. Lett. **40**, 877 (1982).

[7]F. F. Fang and W. E Howard, Phys. Rev. Lett. **16**, 797 (1966).

[8]R. J. Nicholas, M. A. Brummel, J. C. Portal, M. Razeghi, and M. A. Poisson, in *Conference on Application of High Magnetic Fields in Semiconductor Physics, Grenoble, 1982,* edited by G. Landwehr (Springer, Berlin, 1983), p. 98.

[9]A. Berg, M. Dobers, R. R. Gerhardts, and K. v. Klitzing, Phys. Rev. Lett. **64**, 2563, (1990); S. Yarlagadda and R. R. Gerhardts, in *Proceedings of the 20th International Conference on the Physics of Semiconductors, Thessaloniki, 1990,* edited by E. M. Anastassakis and J. D. Joannopoulos (World Scientific, Singapore, 1990), p. 993; D. Antoniou and A. H. MacDonald, Phys. Rev. B **43**, 11 686 (1991); S. V. Iordanskii, S. V. Meshkov, and I. D. Vagner, *ibid.* (to be published).

[10]F. F. Fang and P. J. Stiles, Phys. Rev B **28**, 6992 (1983); Surf. Sci. **142**, 290 (1984); T. Haavasoja, H. L. Stormer, D. J. Bishop, V. Narayanamurti, A. C. Gossard, and W. Weigmann, *ibid.* **142**, 294 (1984); J. P. Eisenstein, H. L. Stormer, V. Narayanamurti, A. Y. Cho, A. C. Gossard, and C. W. Tu, Phys. Rev. Lett. **55**, 875 (1985).

[11]J.-W. Wu, P. Hawrylak, and J. J. Quinn, Phys. Rev. B **31**, 6592 (1985).

[12]T. Ando, Phys. Rev. B **19**, 2106 (1979); J. J. Koning, R. J. Haug, H. Sigg, K. v. Klitzing, and G. Weimann, *ibid.* **42**, 2951 (1990).