Single-electron tunneling in systems of small junctions coupled to an electromagnetic environment

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(Received 6 June 1991; revised manuscript received 6 August 1991)

A simple approach is proposed to describe the influence of the electromagnetic environment on the sequential single-electron tunneling in systems of ultrasmall tunnel junctions. As an application we consider a system of two junctions in series coupled to an Ohmic environment. An increase of the enivironment resistance (i) widens the Coulomb-blockade region and (ii) suppresses the peaks that the conductance shows as a function of the gate voltage. The environment is responsible for an unusual temperature dependence of these peaks, which may explain recent experiments with GaAs lateral microstructures.

Tunnel junctions with small capacitance C show single-electron effects. The tunneling of one electron can appreciably alter the conditions for the tunneling of other electrons. This gives rise to correlations between single-electron tunneling (SET) processes, which lead to various experimentally observable phenomena.[1,2] They show up most pronounced at low temperatures $k_B T \ll e^2/2C$ in junctions with small tunnel conductance $G^{(T)}$. Single-electron effects were observed, for instance, in systems composed of two junctions in series,[3] where they lead to the so-called Coulomb staircase in the I-V characteristic[1,4] and to a periodic dependence of the conductance on the gate voltage coupled to the central electrode.[3] A similar dependence, predicted for the electron transport through a quantum dot,[5] was recently observed in GaAs-$Al_xGa_{1-x}As$ microstructures.[6] More complex and, from a practical point of view, more interesting systems also have been studied. One example is the turnstile device,[7] where the transfer of single electrons is controlled by an external ac signal, and which can serve as a current standard.

Much of the theory of single-electron effects in composite systems of junctions was based on a perturbative ("orthodox") treatment,[1,4,5] or generalizations of it[8,9] which take into account energy quantization effects. In this case it is assumed that, after the electron tunneling, the charge distribution in the leads attached to the junction and in other elements of the circuit relaxes instantly. Charging effects then show up only in systems of tunnel junctions. On the other hand, they also show up in *single* junctions, provided that the charge relaxes in the electromagnetic environment.[10,11] This relaxation depends on the low-frequency impedance $Z(\omega)$ of the environment. If $Z(\omega)$ for $\omega \ll eV/\hbar$ is small, $\mathrm{Re}Z(\omega) \ll h/e^2$, the relaxation is fast, $C\,\mathrm{Re}Z(\omega) \ll \hbar C/e^2$, and single-electron effects are weak. In the opposite limit the relaxation is slow, and the tunneling electron feels the (back) influence of the electromagnetic field which was created by the tunneling current, and which depends on the relaxation processes in the environment.[10] The interaction of the electron with this field leads to the Coulomb blockade of tunneling at low voltages $V < e/2C$.

The charge relaxation in the environment also influences the characteristics of *systems* of tunnel junctions. Since they are of a high practical interest, we generalize here the nonperturbative quantum description of the environment, which was given for a single junction in Refs. 10 and 11, to the case of multijunction systems. We consider a circuit composed of normal junctions and arbitrary linear elements [impedances $Z(\omega)$ and voltage sources] forming the electromagnetic environment.[11] In the absence of tunneling the state of the system is completely determined by the charges $n_i e$ (n_i is an integer) on the internal electrodes ($i=1,\ldots,N$) of the system and the applied voltages. These charges can change only due to single-electron tunneling. The tunneling current can be determined in second-order perturbation theory in the tunneling Hamiltonian using, for example, the approach of Devoret *et al.*[11] The expression for the tunneling current $I_{i,j}$ between two neighboring electrodes i and j contains two terms corresponding to the rates of forward ($\Gamma_{i,j}$) and backward tunneling ($\Gamma_{j,i}$). It can be rewritten in the form

$$I_{i,j} = e(\Gamma_{i,j} - \Gamma_{j,i}) , \tag{1}$$

$$\Gamma_{i,j} = \int_{-\infty}^{\infty} dt\, \gamma_{i,j}(t) \times \exp\left[i\frac{eV_{i,j}}{\hbar} t + \langle [\phi_{i,j}(t) - \phi_{i,j}(0)]\phi_{i,j}(0) \rangle \right] , \tag{2}$$

where

$$\gamma_{i,j}(t) = \frac{G_{i,j}^{(T)}}{2\pi\hbar e^2} \int_{-\infty}^{\infty} dE_i \int_{-\infty}^{\infty} dE_j f(E_i)[1-f(E_j)] \times \exp\left[\frac{i}{\hbar}(E_i - E_j)t \right] , \tag{3a}$$

$$\gamma_{i,j}(\omega) = G_{i,j}^{(T)} \frac{\hbar}{2\pi e^2} \frac{\omega}{\exp(\hbar\omega/T) - 1} . \tag{3b}$$

Here $V_{i,j} \equiv V_{i,j}(n_1, \ldots, n_N)$ is the difference of the electric potentials of the two electrodes before the tunneling. It depends on the charge state $(n_1, \ldots, n_N)$ of the system. The tunneling conductance of the junction involved is $G_{i,j}^{(T)}$, and $f(E)$ is the Fermi function. The average $\langle \cdots \rangle$ over the equilibrium voltage fluctuations of the environment is determined by the impedance $Z_{i,j}(\omega)$ of the circuit between the electrodes i and j,

$$\langle \phi_{i,j}(t)\phi_{i,j}(0)\rangle = \frac{e^2}{\hbar}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{\mathrm{Re}Z_{i,j}(\omega)}{\omega} \times \left[\coth\left[\frac{\hbar\omega}{2T}\right]\cos(\omega t) - i\sin(\omega t)\right] . \quad (4)$$

There is a complete analogy to the problem of a single junction. The only extension is that the impedance seen by one junction $Z_{i,j}$ is modified by the capacitances of the other junctions. We remark that the expression (2) of the tunneling rates as an integral over time[12] has a technical advantage over the approach used in Ref. 11, where an integration over the energy was used, since the analytic expression for the function $\gamma_{i,j}(t)$ is known

$$\gamma_{i,j}(t) = \frac{\hbar G_{i,j}^{(T)}}{2\pi e^2}\left[\pi i \frac{d}{dt}\delta(t) - \left[\frac{\pi T/\hbar}{\sinh(\pi T t/\hbar)}\right]^2\right] . \quad (5)$$

This implies that for the calculation of the tunneling rates only two integrations are needed, rather than three as in Ref. 11.

We consider the limit, where the tunneling conductances of the junctions are small in comparison to the admittances of the environment,

$$G_{i,j}^{(T)} \ll \mathrm{Re}Z_{i,j}^{-1}(\omega) \quad \text{for } \hbar\omega < eV_{i,j} . \quad (6)$$

In addition we ignore, as is done in nearly all circumstances, higher-order tunneling processes in the junctions. These conditions are usually satisfied in the experiments (see, e.g., Ref. 13). In the limit (6) the time between subsequent tunneling events is much larger than the relaxation time of the environment. It implies that there exist no *quantum* correlations between subsequent tunneling events, and that the tunneling is sequential. The tunneling process thus can be described in terms of rates Γ_{ij} of transitions between the charge states of the system.

Next we introduce the probability distribution function $\rho(n_1, \ldots, n_N)$ to find the system in the state $(n_1, \ldots, n_N)$. This function satisfies the balance equation

$$\frac{\partial}{\partial t}\rho(n_1, \ldots, n_N) = \sum_{i\neq j}[\Gamma_{i,j}(n, \ldots, n_i+1, n_j-1, \ldots, n_N)\rho(n_1, \ldots, n_i+1, n_j-1, \ldots, n_N) - \Gamma_{i,j}(n_1, \ldots, n_N)\rho(n_1, \ldots, n_N)] . \quad (7)$$

The rates $\Gamma_{i,j}$ depend on the state of the system $(n_1, \ldots, n_N)$ via the voltages $V_{i,j}$. The solution of Eq. (7) enables us to determine the tunneling currents through the junctions

$$I_{i,j} = \sum_{n_1, \ldots, n_N}\rho(n_1, \ldots, n_N)[\Gamma_{i,j}(n_1, \ldots, n_N) - \Gamma_{j,i}(n_1, \ldots, n_N)] . \quad (8)$$

Equations (7) and (8), combined with the transition rates $\Gamma_{i,j}$ [Eq. (2)], describe the dynamics of single-electron tunneling in an arbitrary system of junctions and arbitrary linear environment—provided that the tunneling is weak as given by (6).

We now apply the formalism to the single-electron transistor[1] composed of two tunnel junctions in series as shown in Fig. 1. The state of the system is characterized by the charge ne of the common electrode of the junctions. In the absence of fluctuations the voltages V_1 $(=V_{0,1})$ and V_2 $(=V_{1,2})$ across the left and right junctions for the given state n are

$$C_\Sigma V_{1\,(2)} = \left[C_{2\,(1)} + \frac{C_g}{2}\right]V_{(\mp)}C_g V_g - ne , \quad (9)$$

where $C_\Sigma \equiv C_1 + C_2 + C_g$. The electromagnetic environment consists of resistors in the source-drain (R) and gate (R_g) parts of the circuit. They cause fluctuations of the transport voltage (V) and gate voltage (V_g). Under the conditions

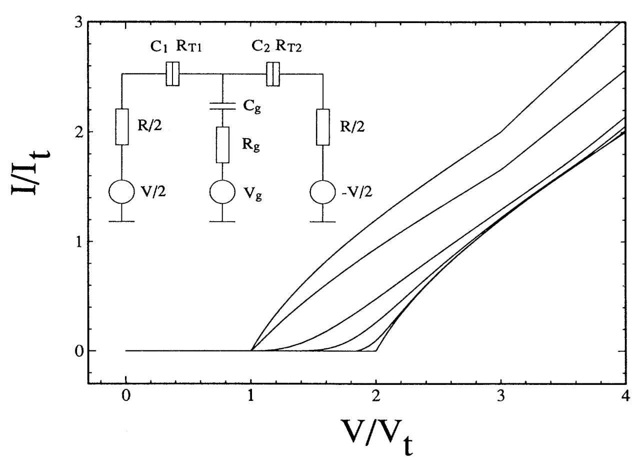

FIG. 1. Influence of the transport voltage fluctuations on the I-V characteristics of the SET transistor for $Q_g=0$. The inset shows the equivalent circuit of the system. The curves from top to bottom correspond to the different values of the environment resistance $h/e^2R = \infty, 5, 0.5, 0.05, 0.005, 0$. The other parameters $R_{T1}=R_{T2}, C_1=C_2, V_t \equiv e/(C_1+C_2), I_t \equiv V_t/(R_{T1}+R_{T2})$.

$$C_g \ll \min(C_1,C_2), \quad R_g/R \ll [\min(C_1,C_2)/C_g]^2 , \quad (10)$$

the fluctuations of the transport voltage dominate over those of the gate voltage in the whole frequency range; and the impedances Z_1 ($=Z_{0,1}$) and Z_2 ($=Z_{1,2}$) seen by the junctions can be presented in the form[14]

$$Z_{1\,(2)}(\omega)=\frac{1}{i\omega C_\Sigma+\varepsilon}+\frac{1}{i\omega C'_{1\,(2)}+1/R'_{1\,(2)}} , \quad (11)$$

where $C'_1=C_\Sigma C_1/C_2$, $C'_2=C_\Sigma C_2/C_1$, $R'_1=R(C_2/C_\Sigma)^2$, $R'_2=R(C_1/C_\Sigma)^2$, and $\varepsilon\to+0$. The first term in the expressions (11) leads to the orthodox Coulomb effects due to the discreteness of the charge on the central electrode. The second term describes the charge-relaxation process due to the dissipative electromagnetic environment. In the limit considered, where we ignore fluctuations of the gate voltage, it has the same functional form as for a single junction. This, of course, is just a manifestation of the fact that the relaxation of charge after the tunneling is determined by the effective circuit formed by the resistance of the environment and the capacitances of the junctions.

We can now evaluate the *I-V* characteristics of the single-electron transistor. For simplicity we consider the symmetric case ($C_1=C_2=C$) at zero temperature. At low voltages

$$|V|<V_t^{(0)}\equiv\frac{e}{2C}-\frac{Q_g}{C}, \quad Q_g\equiv\min_n|C_gV_g-ne| \quad (12)$$

the tunneling current vanishes, independent of the source-drain resistance R. The complete blockade of tunneling arises since the Coulomb energy of the state after the electron tunneling to or from the central electrode and *after* the charge relaxation is higher than the energy of the initial state. At higher voltages

$$V_t^{(0)}<V<V_t^{(1)}, \quad V_t^{(1)}\equiv\frac{e}{C}-\frac{Q_g}{C} \quad (13)$$

such a state has energy lower than the initial one. However, the state arising just after the tunneling but *before* the relaxation is higher in energy than the initial state. Therefore the *I-V* characteristics in the region (13) strongly depends on the rate of charge relaxation and hence on R. With increasing R the Coulomb blockade region gradually expands from $|V|<V_t^{(0)}$ for small $R\ll h/e^2$, to $|V|<V_t^{(1)}$ for large $R\gg h/e^2$, as shown in Fig. 1. In the second limit the Coulomb blockade occurs for any value of V_g. The buildup of the blockade is accompanied by the total shift of the *I-V* curve towards higher voltages. If the temperature is increased to values of order $T\sim e^2/2C$, the Coulomb blockade and the Coulomb staircase in the *I-V* curve are washed out.

Another interesting property of the system is the dependence of the conductance $G(V_g)$ (for low transport voltage $V\to0$) on the gate voltage. It is periodic in the gate voltage.[3,5,6] In Fig. 2 we show results for various values of the resistance of the environment and different temperatures. For a symmetric system ($C_1=C_2=C$) at low temperatures the conductance near the maximum is

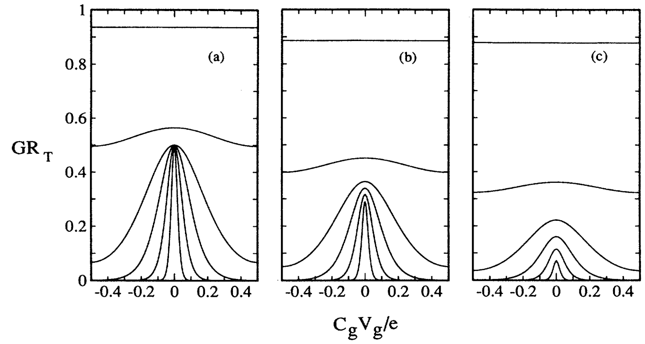

FIG. 2. Temperature dependence of the conductance G as a function of the gate voltage V_g for different values of the environment resistance (a) $h/e^2R=\infty$, (b) 5, and (c) 1. The parameters are $R_T=R_{T1}+R_{T2}$, $C_1=C_2$. For the curves from top to bottom we have $T/(e^2/2C)=2.5, 0.25, 0.1, 0.05, 0.025, 0.01$.

$$G(V_g,T)=\frac{g(T)}{2(R_{T1}+R_{T2})}\frac{E(V_g)/T}{\sinh[E(V_g)/T]} \quad \text{for } E(V_g)<T\ll\frac{e^2}{2C} . \quad (14)$$

Here R_{T1} and R_{T2} are the tunneling resistances of the junctions, and $E(V_g)=(0.5-Q_g/e)e^2/2C$, where Q_g is defined in Eq. (12). The function $g(T)$ is the normalized conductance of a (hypothetical) single junction with capacitance $C'=2C$ in the presence of an Ohmic environment with the resistance $R'=R/4$ [see (11)]:

$$g(T)=\frac{\pi^{1/2}}{2}\frac{\Gamma(1+R'e^2/h)}{\Gamma(1.5+R'e^2/h)}(\pi R'C'T)^{2R'e^2/h} \quad \text{for } T\ll\frac{\hbar}{R'C'} , \quad (15a)$$

$$g(T)=\left[\frac{8C'T}{\pi e^2}\right]^{1/2}\exp\left[-\frac{e^2}{8C'T}\right] \quad \text{for } \frac{\hbar}{R'C'}\ll T\ll\frac{e^2}{2C'} . \quad (15b)$$

From (15) we easily find the height $G_{\max}(T)$ of the conductance peaks. In the limit $R'\to0$ the results (14) and (15a) reduce to that of Ref. 5. If the resistance is low enough, $R'\ll h/e^2$, the conductance $G_{\max}(T)$ has a weak power-law dependence on the temperature (15a). In the opposite limit $R'\gg h/e^2$ the conductance drastically diminishes with the decrease of temperature (15b) for $T<e^2/2C'$. In this limit the Coulomb blockade occurs irrespective of the value of the gate voltage.

We consider now the fluctuations of the gate voltage. They dominate over the fluctuations of the transport voltage in the whole frequency range if

$$C_g\gg\max(C_1^2/C_2, C_2^2/C_1), \qquad R_g/R\gg[\max(C_1,C_2)/C_g]^2 . \quad (16)$$

In this case the impedances seen by both the junctions are the same. Moreover, they are still described by the expression (11), with the parameters

$$C' = C_\Sigma (C_1 + C_2)/C_g, \quad R' = R(C_g/C_\Sigma)^2 . \tag{17}$$

Thus the results obtained so far can be easily generalized to the present case. When neither (10) nor (16) are satisfied both the transport and gate voltages fluctuations have to be taken into account, and (11) is replaced by the true impedances, as seen by the junctions.

So far we have not considered the higher-order processes of electron tunneling through both the junctions (cotunneling),[15,16] which give rise to the electron transport in the Coulomb-blockade region and yield a finite conductance of the double junction at zero temperature.[16] As far as cotunneling is concerned, our system is qualitatively equivalent to a single junction with a resistor in series. Using the result of Ref. 15 for this system we conjecture that the rate of cotunneling decreases rapidly with increase of the resistance R. A further study of the influence of environment on electron cotunneling is still needed.

To conclude we discuss the possibilities of observing the described effects in an experiment. If conventional (metallic) tunnel junctions are used, the main problem is to place a high Ohmic resistor with low stray capacitance in the vicinity of the junction (see Chap. 4.1 of Ref. 1). The best result, to our knowledge, has been achieved by Kuzmin and Haviland,[13] who fabricated thin-film resistive leads with the resistance of 95 kΩ and a stray capacitance of 6×10^{-17} F/μm. For these parameters the effects of the electromagnetic environment can be detected reliably.

Single-electron tunneling effects also arise in semiconductor nanostructures containing quantum dots.[6,17,18] If the energy-level spacing $\Delta\varepsilon$ in a quantum dot and the traversal time τ_{tr} are small enough [$\Delta\varepsilon \ll \max(eV,T)$ and $\tau_{tr}^{-1} \gg \max(eV,T)$], then one can neglect the effects of discreteness of levels[8,19] and of finite traversal time.[20] Under these conditions our approach describes the electron tunneling also in semiconductor systems. Our estimates show that the effective impedance[20] of a narrow one-dimensional electron gas channel can be large enough [$\mathrm{Re}Z(\omega) \sim h/e^2$] to give rise a pronounced effect of the environment. Such an effect could be observed in the temperature dependence of the conductance peaks in experiments similar to those of Refs. 6 and 18. In the experiment[18] the spacing between the energy levels is large and our approach is not sufficient. However, a combination of the mechanism proposed here with that considered in Ref. 19 can explain the observed increase of the heights of some conductance peaks with temperature. A further analysis of electron tunneling in semiconductor nanostructures taking into account the discreteness of energy levels,[19] finite traversal time,[20] and coupling to an electromagnetic environment is needed.

After completion of the present article we learned that Grabert *et al.*[21] have derived a similar extension to the case of networks of tunnel junctions. However, they have not pursued the analysis as far as we did, for instance, to the result shown in Fig. 2.

This work was supported by the Dutch science foundation FOM. The authors wish to thank D. V. Averin, K. K. Likharev, Yu. V. Nazarov, and V. A. Volkov for stimulating discussions and V. Bubanja for the help in the numerical calculations.

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